Perturbation of the Periodic Dispersion Under Beam Crossing Optics in LHC

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Perturbation of the Periodic Dispersion Under Beam Crossing Optics in LHC

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Abstract
Beam crossing and separation schemes in the LHC interaction regions impose non-zero closed orbit in the low-β triplets. The related perturbative dispersion is derived; propagation, multi-crossing interference, perturbative effects around the ring are investigated and quantified. Horizontal and vertical compensation schemes are presented.

I. Introduction
Crossing angle and orbit off-centering schemes at the interaction points (IP) in the LHC ring are foreseen [1][2], for the purpose of early separation of the beams so as to reduce harmful effects related to beam-beam interactions in that region where they share a common vacuum pipe. Such closed orbit (c.o.) geometry imposes horizontal and vertical off-centering in the low-β triplets, which has sensible effect on dispersion in collision optics when betatron functions reach very large values. This report provides an understanding and study of the building-up and effects of the anomalous dispersion in the LHC ring (Version 4.2), and investigates compensation schemes.

II. Anomalous dispersion

A. Equation of the anomalous dispersion
The perturbative dispersion $d_y(s)$ due to $y_{co}(s)$ c.o. in the low-β triplets is the closed solution of [3]

$$\frac{d^2 d_y}{ds^2} + K(s) d_y = -\Delta R(s) / B_0 + K(s) y_{co}$$

(1)

with $y \equiv x$ or $z$, $B_0 = \text{particle rigidity}$, $K(s) = \text{quadrupole strength}$, and the field term $\Delta R(s)/B_0$ is introduced by the c.o. dipole. Eq. (1) can be solved in the elementary kick approximation $K(s) y_{co}(s) = \int K(s) y_{co}(s) \delta(s - s_q) ds_q$, which yields the periodic solution (Fig. 1)

$$d_y(s)+y_{co}(s) = \frac{\sqrt{\beta}}{2 \sin \tau} \sum (KL) q y_{co}(s) \sqrt{\beta(s_q) \cos \varphi [s-\delta(s)-\delta(s_q)]}$$

(2)

where $\phi(s) = 1/\nu \int ds / \beta = \text{normalized betatron phase}$, $\phi(s_q) = \text{phase at the kick}$, $\beta = \text{betatron function}$, $\nu = \text{machine tune}$. The closed orbit $y_{co}(s)$ at the kick can be expressed in terms of its transport from the IP (optical functions $\beta^*$, $\phi^*$ while $\alpha^* = 0$ is assumed). This yields

$$d_y(s) = -y_{co}(s) + \{ \frac{\sqrt{\beta(s)} / \beta^*}{2 \sin \tau}$$

$$+ y^* \sum (KL) q \beta(s_q) \cos \varphi \{ \phi(s) - \phi(s_q) \} \cos \varphi[s-\delta(s)-\delta(s_q)]$$

$$+ y^* \beta^* \sum (KL) q \beta(s_q) \sin \varphi \{ \phi(s) - \phi(s_q) \} \cos \varphi[s-\delta(s)-\delta(s_q)] \}$$

(3)

B. Upper limits of the perturbation
Beyond the low-β triplets associated with the non-zero c.o. Eq. (2) can be written under the form $d_y(s)/\sqrt{\beta(s)} = -y_{co}(s)/\sqrt{\beta(s)} + \tilde{D}_y \cos \varphi \{ \phi(s) + \Omega(s) \}$, with

$$\tilde{D}_y = (\sum (KL) q y_{co}(s) \sqrt{\beta(s) \cos \varphi \{ \phi(s) + \epsilon(s_q) \}})^2$$

$$+ (\sum (KL) q y_{co}(s) \sqrt{\beta(s) \sin \varphi \{ \phi(s) + \epsilon(s_q) \}})^2 \}^{1/2} (2 \sin \tau)$$

(4)

(4)

with $\epsilon = \pm 1$ for $\phi(s) > \phi(s_q)$, $\pm \varphi$. Numerical calculation of the sums from first order optics yields [3][6]

$$\tilde{D}_y (x^* = 0) \approx 170, \tilde{D}_y (x^* = 0) \approx 158, \tilde{D}_y (x^* = 0) \approx 2 \approx 2$$

(5)

Since $\beta_0$ and $\beta_2$ have similar shapes Eq. (5) tells that the perturbation due to $10^{-4} \text{rad c.o.}$ angle ("$d_x^*$" plot in Fig. 1) is about ten times that due to $10^{-3} \text{m c.o.}$ off-centering ("$d_x^*$" plot in Fig. 1). Extrema of $d_y(s) = \tilde{D}_y \sqrt{\beta(s)}$ can be derived, this is studied in more details in Section III.

C. Comparison with the effects of D1/D2 dipoles
Dispersive effects due to crossing can be compared to those due to the separator/recombiner dipoles D1/D2, in particular in view of simultaneous compensation by an optical assembly such as proposed in [7]. A single dipole (D1 or D2) with bend $\Theta_D$ excites a dispersion of closed form

$$\frac{D_y(s)}{\sqrt{k(s)}} = \frac{\Theta_D}{2 \sin \tau} \sqrt{\beta(s)} \cos \varphi \{ \phi(s) + \epsilon(s_D) \}$$

(6)

with $\sqrt{\beta(s_D)}$ = mean value of $\sqrt{\beta(s_D)}$ and assuming $\phi(s_D) \approx C^\mu \tau$, over a dipole. The overall perturbation is obtained by superposing the effects of the two pairs D1/D2, which, with $\phi(s_D) \approx \phi(s(s_D))$, leads to

$$\frac{D_D y_{D1/D2}(s)}{\sqrt{\beta(s)}} = \Theta_D D_D y_{D1/D2}(s)$$

$$= \frac{\Theta_D}{2 \sin \tau} \sqrt{\beta(s_D)} \cos \varphi \{ \phi(s) + \epsilon \}$$

(7)

\approx \frac{\Theta_D}{2 \sin \tau} \sqrt{\beta(s_D)} \cos \varphi \{ \phi(s) + \epsilon \}$$

(7)

Given $\sqrt{\beta(s_D)} > \sqrt{\beta(s_D)}$ and $\Theta_D = 2.17 \times 10^{-3} \text{rad}$, it comes out $D_D y_{D1/D2} \approx 400 \times 10^{-4}$, which yields about $0.6 \text{m modulation at } \beta(s) = 180 \text{m}$. This can be readily compared to the analogous coefficients due to $\alpha^* = 10^{-4} \text{rad c.o. angle}$ (Eq. 5), namely

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In other words, the modulation in the arcs due to \( z^* \) is \( \pm 0.35 \times 0.6 \approx \pm 0.2 \text{ m} \) (Fig. 1). It also means that a correction scheme intended to compensate the dispersion due to \( D_1/D_2 \) can take care in addition of \( 10^{-4 \text{ rad}} \) c.o. angle by changing its strength (increase or decrease depending on the crossing sign) by about 35%.

III. Typical effects of crossing angle geometry

We consider the sole crossing scheme \( (y^* = 0, y_{\text{co}}^* \neq 0) \), which has the major perturbative effect as shown above (Eq. 5). Beyond the crossing region Eq. (3) leads to [3]

\[
\begin{align*}
\tilde{d}_y(s < s_{c,y} & , f_1, s) = -\varphi_{s_{c,y}}(s) \\
\pm y^* \beta_{s_{c,y}}/2(2 \sin \varphi_s) & = (1 \pm \delta y^* / \sin \varphi_s) \sum_q (K_L) \hat{\beta} s_{c,y} \\
(1 \pm \delta y^* / \sin \varphi_s) & \geq \varphi_{s_{c,y}}(s), q
\end{align*}
\]

\[d_{x,\text{extr}} \approx 1.6 \text{ m for opposite signs} \quad (c = -1) \quad \text{(Fig. 2)}.

Strong effects may arise from four-IP interference (non-alternating crossing configuration \([8]\)). Consider c.o. angles \( z^* = \epsilon_{\text{IP}} \times 10^{-4} \times \text{rad} \) with signs either identical, \( c_1 = c_2 = c_3 = c_4 = 1 \) or alternate, \( c_1 = c_2 = 1 \) and \( c_3 = c_4 = -1 \). The perturbation at \( D_5 \) low-\( \beta \) triplet reaches

\[d_{x,\text{extr}} = \pm z^* \sqrt{\beta_{\text{max}}^2/2(2 \sin \varphi_s)} \sum_q (K_L) \beta s_{c,y} \frac{\sin \varphi_{s_{c,y}}(s)}{\sin \varphi_s}
\]

\[d_{x,\text{extr}} \approx 0.42 \text{ m if all crossings have identical signs, } d_{x,\text{extr}} \approx 4.1 \text{ m in the second case.}

IV. Correction schemes

A. Self-absorption within regular IR tuning procedures

The simplest way to compensate the anomalous dispersion is by re-tuning the IR. As expected from \( d_y(s) \approx 10^{-4} \times D_y(s) \) under \( 10^{-4} \text{ rad} \) c.o. angle (after Eq. 5), doing so leads to very limited changes in the Q1-Q10 IR quadrupole strengths. As to the optical functions, there is no meaningful difference with the unperturbed ones [3].

B. Quadrupole correction of the horizontal dispersion

**Corrector strength**

Quadrupole correctors take a perturbative dispersion which supersedes with that due to c.o. in the low-\( \beta \) triplets. This translates to additional term \( \sum_q (K_L) \beta s_{c,y} \) in Eq. (1) (index \( Q \) stands for the correctors).

Besides, minimizing the corrector strength imposes on the one hand \( \phi(s_{c,y}) = \phi(s_\text{co}) \pm \pi/\nu [\text{modulo } \pi/\nu] \), on the other hand maximizing \( D_x(s_{c,y}) \sqrt{\beta_x(s_{c,y})} \) (which also minimizes effects on the orthogonal plane). Considering that \( \phi(s_\text{co}) \) and \( D_x(s_{c,y}) \sqrt{\beta_x(s_{c,y})} \) is \( C_{\text{extr}} \) the correction strength writes

\[\sum_q (K_L) \beta s_{c,y} \approx \sum_q (K_L) \beta s_{c,y} \sqrt{\beta_x(s_{c,y})} / D_x(s_{c,y}) (14)
\]

Numerical calculations for odd IR give \( \sum_q (K_L) \beta s_{c,y} \sqrt{\beta_x(s_{c,y})} / D_x(s_{c,y}) \approx -1.12 \times 10^{-4} / 1.50 \times 10^{-2} \) for respectively the left and right low-\( \beta \) triplets. Hence the integrated strengths that independently dose the left and right dispersion bumps \( \sum_q (K_L) \beta s_{c,y} \sqrt{\beta_x(s_{c,y})} / D_x(s_{c,y}) \approx 3.9 \times 10^{-4} / 5.2 \times 10^{-4} \text{ m}^{-1} \).

**Correction with a single quadrupole**

A single quadrupole with strength \( 9 \times 10^{-4} \text{ m}^{-1} \) (after Eq. 5) is sufficient to cure the anomalous dispersion, since the two \( \pi/\nu \) apart low-\( \beta \) triplets sources of the defect excite independent perturbations that add in phase. It may be placed dose to MCBH multipole and would excite a defect in phase opposition thus canceling the anomalous dispersion beyond the local chromatic bump so determined. Fig. (3) shows the resulting second order dispersion at Octant 5, prior to any re-tuning of the IR, to be compared to the uncorrected situation (curve "\( D_x + d_x^* \)" in Fig. 1).

Yet a single quadrupole has sensible effect on the tune and \( \beta \) mismatch, namely, \( \Delta \nu = \beta_x(s_{c,y})(K_L) \nu / 4 \pi \approx 1.3 \times 10^{-2} \), \( \beta_x(s_{c,y}) \approx 178 \text{ m}, \Delta \nu = 0.23 \times 10^{-2} \), \( \beta_x(s_{c,y}) \approx 32 \text{ m} \), and \( \Delta \beta_x/\beta_x < \beta_x(s_{c,y})(K_L) \nu / 2 \sin(2 \pi \nu) \approx 8.5 \% \), \( \Delta \beta_x/\beta_x \approx 1.5 \% \) (with \( \nu = 63.3 \)).
Correlation with two quadrupoles

These effects can be taken care of to good level (less than 1% dispersion beating, less than 3% beta-beat and at worst 0.018 tune shift, prior to any re-tuning of the IR) by using two quadrupoles; this could constitute a minimal correction scheme, yet there are several possibilities more or less beneficial w.r.t. residual dispersion, tune shift and beta-beat: the two quadrupoles can be placed one at each end of the IR, or both at the same end, with each one half the strength $[\langle KL \rangle Q/2] \approx 4.5 \times 10^{-4} \text{m}^{-1}$; this has the effect of avoiding tune-shift and beta-beats. They can be placed one at each end of the IR, with strengths $3.9 \times 10^{-4}/5.2 \times 10^{-4} \text{m}^{-1}$ to balance the opposite low-beta triplet; this brings quasi-zero dispersion and derivative at the IP.

Correction with four interleaved quadrupole pairs

Following a correction scheme proposed for SSC [9], the method above has been extended to four pairs of quadrupoles. Such correction scheme is also assimilable within the modular LHC IR tuning scheme [7] and other Q-shift system [10]. As expected from the discussions above, the correction is very efficient in terms of tune-shift, beta and dispersion. More details can be found in [3].

C. Correction of the vertical dispersion

The vertical anomalous dispersion can be compensated by skew quadrupoles (as proposed at SSC [9]) located at arc ends close to MSCBV correctors and maxima of $D_x \sqrt{\beta_z}$ and low $\beta_z$. Their role is to couple the horizontal dispersion into the vertical plane.

**Corrector strength**

The vertical dispersion verifies $d^2 \delta_z/ds^2 + K(s) \delta_z = R(s) D_z$. The closed solution is (after Eq. 2)

$$d_z(s) = \frac{\sqrt{\beta_z(s)}}{2 \sin \pi \varepsilon} \sum (R_L)_{SQ} D_z(s_{SQ}) \sqrt{\beta_z(s_{SQ})} e^{-\epsilon s_{SQ}} \\ \sum (R_L)_{SQ} (KL)_{z \varepsilon} e(s_{SQ}) \sqrt{\beta_z(s_{SQ})} / D_z(s_{SQ}) \sqrt{\beta_z(s_{SQ})}$$

where index SQ denotes the correctors, $R$ = skew quadrupole strength. Taking $\delta_z(s_{SQ}) = \delta_z(s_{SQ}) + \pi/\mu[\pi/\mu]$, while $\phi(s_{SQ})$ and $D_z(s_{SQ}) \sqrt{\beta_z(s_{SQ})} \approx C^{\mu \varepsilon} \varepsilon$ gives the correction strength

$$(RL)_{SQ} / \text{Left} \approx 10.610^{-4}/7.910^{-4} \text{m}^{-1}$$

is necessary for balancing the effects of the left and right low-beta triplets under $\phi = 10^{-4} \text{rad}$ c.o. angle at IP [3]-[6].

**Correction with a single skew quadrupole**

The corrector is placed at an arc end next to a MSCBV multipole with the strength $18.2 \times 10^{-4} \text{m}^{-1}$ (Eq. above). Dispersion does not exceed 0.32 m in the crossing octant (Fig. 4), it is less than 0.05 m everywhere else in the ring (see the uncorrected situation, curve “$d_x$” in Fig. 1).**Interleaved correction scheme**

Residual effects on the first order focusing are weak; however they can be improved by using quadrupole pairs; doing so damps the dispersion to 0.2 m in the crossing low-beta triplet. The philosophy is the same as above, for the horizontal plane; more details can be found in [3].

**Interferences**

If no correction of the vertical dispersion is foreseen, yet some benefit may be drawn from interference, as long as adequate phase relation is fulfilled between IP's of concern. Fig. 5 shows such self-cancellation in the range IR2/IR8 when setting $\phi = 10^{-4} \text{rad}$ c.o. angle at IP2 and IP8 simultaneously. This plot can be readily compared to the situation due to a single crossing (curve “$d_x$” in Fig. 1, and extrema at all IP’s, Eq. (10)).

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**References**

[5] T. Risselada, CERN/SL/AP, provided the MAD files: lhc42.k-collision, lhc42.k-injection, lhc42.sequence.