DOE/ER/14355--T2

**DEPARTMENT OF ENERGY
GRANT DE-FG03-93ER14355**

**LINEAR KINETIC THEORY AND PARTICLE TRANSPORT IN
STOCHASTIC MIXTURES**

**Third Year and Final Report
(Performance Period: 6/15/93 - 12/14/96)**

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Submitted by

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May 1997

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SUMMARY OF COMPLETED WORK

The goal in this research was to continue the development of a comprehensive theory of linear transport/kinetic theory in a stochastic mixture of solids and immiscible fluids. Such a theory should predict the ensemble average and higher moments, such as the variance, of the particle or energy density described by the underlying transport/kinetic equation. The statistics studied correspond to N -state discrete random variables for the interaction coefficients and sources, with N denoting the number of components in the mixture. The mixing statistics considered were Markovian as well as more general statistics.

In the absence of time dependence and scattering, the theory is well developed and described exactly by the master (Liouville) equation for Markovian mixing, and by renewal equations for non-Markovian mixing. The intent of this research was to generalize these treatments to include both time dependence and scattering. A further goal of this research was to develop approximate, but simpler, models from any comprehensive theory. In particular, a specific goal was to formulate a renormalized transport/kinetic theory of the usual nonstochastic form, but with effective interaction coefficients and sources to account for the stochastic nature of the problem.

In the three and one-half year period of research summarized in this final report, we have made substantial progress in the development of a comprehensive theory of kinetic processes in stochastic mixtures. This progress is summarized in 16 archival journal articles, 7 published proceedings papers, and 2 comprehensive review articles. In addition, 17 oral presentations were made describing these research results.

1. PROJECT TECHNICAL DESCRIPTION AND RESULTS - CITATIONS

This research was concerned with linear kinetic/transport theory in stochastic mixtures. It was performed under a U. S. Department of Energy grant, DE-FG03-93ER14355 (Basic Energy Sciences, Division of Engineering and Geosciences), covering the period June 15, 1993 through December 14, 1996. Many new and significant research results were obtained, and we list here the archival publications, conference proceedings, review papers, and oral presentations that resulted from this grant. For the written contributions, we have included the abstracts, where available, taken from the original papers. If an abstract is not available, we have included a short summary.

Archival Publications

1. Bingjing Su and G.C. Pomraning, "Benchmark Results for Particle Transport in Binary Non-Markovian Mixtures," *J. Quant. Spectrosc. Radiat. Transfer* 50, 211 (1993).

Abstract - Numerical benchmark results are reported for particle transport in a randomly mixed binary medium, with the stochastic mixing described by a variety of stationary and homogeneous renewal statistics. Such statistics are quantified by chord length distributions for the two immiscible components of the mixture. A simple Monte Carlo procedure is used to generate physical realizations of the statistics, and the transport equation is solved numerically for each realization. The ensemble-averaged solutions and certain variance results are obtained by appropriately averaging over a large number of such calculations. The geometry considered is the rod model of transport, in which particles are constrained to move along a single axis.

2. F. Malvagi and G.C. Pomraning, "Stochastic Atmosphere Radiative Transfer," *Atmos. Oceanic Opt.* 6, 610 (1993).

Abstract - A radiation treatment of the broken cloud problem is described, based upon a class of stochastic models of the equation of radiative transfer, which considers the clouds and clear sky as a two component random mixture. These models, recently introduced in the kinetic theory literature, allow for both Markovian and non-Markovian statistics as well as spatial variations of the cloudiness. Numerical results are given which compare different models of stochastic radiative transfer, and which point out the importance of treating the broken cloud problem as a stochastic process. It is also shown that an integral Markovian model proposed within the atmospheric radiation community by Titov is equivalent to one of our differential models.

3. Bingjing Su and G.C. Pomraning, "A Variational Approach to Stochastic Transport," *J. Quant. Spectrosc. Radiat. Transfer* **51**, 467 (1994).

Abstract - We consider the problem of radiation transport through a stochastic background material described by arbitrary statistics. A variational principle is developed to estimate the ensemble average of a general linear functional of the solution. Numerical results based upon this principle are compared with exact benchmark results using a variety of trial functions. The test problem employed is a source-free rod, in a monochromatic, time independent setting. The variational formalism is used to estimate reflection, transmission, absorption, and the intensity at the rod center for a binary background mixture of immiscible fluids. The numerical calculations employ various renewal statistics, including Markovian, to describe the mixing of the two fluid components.

4. Bingjing Su and G.C. Pomraning, "Limiting Correlation Length Solutions in Stochastic Radiative Transfer," *J. Quant. Spectrosc. Radiat. Transfer* **51**, 893 (1994).

Abstract - The problem of particle transport through a Markovian stochastic mixture of two immiscible materials is considered. In the special case of rod geometry, both small and large correlation length limiting solutions are explicitly constructed. The analysis is general in the sense that independent fluctuations are allowed in both the single scatter albedo and the particle mean free path. These results are used to provide two new closures for a low order ensemble averaging of the stochastic equation of transfer. Numerical comparisons with Monte Carlo benchmark results are given to assess the accuracy of the proposed closures.

5. Bingjing Su and G.C. Pomraning, "A Stochastic Description of a Broken Cloud Field," *J. Atmos. Sci.* **51**, 1969 (1994).

Abstract - The chord length distributions within a cloud and between clouds are considered. Such information is needed as input to certain statistical models of cloud-radiation interaction. Modeling the clouds as azimuthally symmetric ellipsoids, the authors find that the chord length distribution through a cloud of fixed size is proportional to the chord length. The proportionality constant depends upon the semiaxes of the ellipse as well as the angle of incidence of the radiation. This linear behavior is easily convolved over an arbitrary size distribution of the clouds to obtain the chord length distribution through a statistical mixture of different cloud sizes. The chord length distribution between clouds is also considered for an atmospheric layer of finite thickness. In this case, both analytic and numerical methods are needed to obtain results. In the limit of an infinite thickness atmospheric layer described by homogeneous statistics and fixed cloud chord lengths, our considerations reduce to a Markovian (exponentially distributed chord lengths) model for the intercloud spacing.

6. Anil K. Prinja and G.C. Pomraning, "On the Propagation of a Charged Particle Beam in a Random Medium. I: Gaussian Statistics," *Trans. Th. Statis. Phys.* **24**, 535 (1995).

Abstract - A model is presented for the transport of energetic charged particles in a medium whose density is a continuous random function of position. Using the straight-ahead continuous slowing down approximation and assuming Gaussian statistics for the density fluctuations, exact solutions for the ensemble-averaged flux and dose are obtained. It is demonstrated that the ensemble-averaged flux satisfies an exact closed equation of the Fokker-Planck type in energy. The effect of fluctuations is to introduce straggling in space and energy, resulting in the dose profile extending well beyond the ion range for a corresponding deterministic medium. Very reasonable results are obtained for a fluctuation amplitude on the order of the mean density, in spite of the negative densities admitted by the Gaussian process. However, for very large fluctuations, the Gaussian model leads to a considerable overestimate of the dose near the surface.

7. G.C. Pomraning and Anil K. Prinja, "On the Propagation of a Charged Particle Beam in a Random Medium. II: Discrete Binary Statistics," *Trans. Th. Statis. Phys.* 24, 565 (1995).

Abstract - We consider the linear transport of energetic charged particles through a background stochastic mixture consisting of two immiscible fluids or solids. The transport model used is the continuous slowing down description in the straight-ahead approximation. Under the assumption of homogeneous Markovian mixing statistics and separable (in space and energy) stopping powers with a common energy dependence, the problem of finding the ensemble-average intensity and dose is reduced to simple quadrature. The use of the Liouville master equation offers an alternate approach to this problem, and leads to exact differential equations whose solution gives the ensemble-averaged intensity and dose. This master equation approach applies to inhomogeneous Markovian statistics as well as non-separable stopping powers. Both treatments can be extended, in an approximate way, to non-Markovian statistics. Typical numerical results are given, contrasting this stochastic treatment with the standard treatment which ignores the stochastic nature of the problem.

8. Bingjing Su and G.C. Pomraning, " P_1 , P_2 , and Asymptotic Approximations for Stochastic Transport," *Nucl. Sci. Eng.* 120, 75 (1995).

Abstract - The problem of describing particle transport through a Markovian stochastic mixture of two immiscible materials is generally approximated by the so-called Levermore model, consisting of two coupled transport equations. In this paper, the P_2 diffusive equations and the associated boundary conditions for this Levermore model are derived in planar geometry by using a variational principle, and numerical results comparing P_2 , P_1 , and S_{16} (benchmark) calculations are presented. These results demonstrate that the P_2 equations are considerably more accurate than the P_1 equations away from boundary layers. An asymptotic diffusion approximation to this model is also explored with several different boundary conditions, and the overall conclusion is that the asymptotic diffusion treatment is in general inferior to P_2 theory, and its superiority over P_1 theory is not overwhelming and not consistent.

9. G.C. Pomraning, "Flux-Limiting in Three-Dimensional Stochastic Radiative Transfer," *J. Quant. Spectrosc. Radiat. Transfer* 54, 637 (1995).

Abstract - We report the development of a flux-limiting algorithm for radiative transfer in a Markovian mixture consisting of two immiscible materials. The underlying stochastic transport description consists of two coupled equations of radiative transfer for the ensemble-averaged specific intensity of radiation. This transport description, while exact only in the absence of scattering, has been demonstrated to be a robust and accurate model in the presence of scattering. The flux-limited diffusion description developed here is applicable to general three-dimensional geometry, in contrast to an earlier effort where algebraic difficulties prohibited explicit results except in a one-dimensional setting.

10. G.C. Pomraning, "Modification to a Previous Higher Order Model for Particle Transport in Binary Stochastic Media," *J. Quant. Spectrosc. Radiat. Transfer* 54, 779 (1995).

Abstract - The problem of particle transport and radiative transfer through a stochastic mixture of two immiscible materials is considered. When the material mixing process is assumed to obey Markovian statistics, an ensemble average of this stochastic transport equation leads to two coupled transport equations continuing four different ensemble-averaged unknowns. Modification to a previous higher order closure involving additional transport equations is suggested as a new closure. The closure philosophy is to demand that the closed set of equations reproduces certain exact limiting behaviors in rod geometry. To assess the accuracy of the proposed modification, numerical results predicted by this new higher order closure, for both Markovian and the non-Markovian mixing statistics, are given and compared with exact benchmark results as well as with results from the standard closure available in the literature.

11. G.C. Pomraning, "Effective Radiative Transfer Properties for Partially Cloudy Atmospheres," *Atmos. Oceanic Opt.* 9, 7 (1996).

Abstract - In this paper, we suggest a relatively simple radiation treatment of the broken cloud field problem. The underlying equations are based upon a Markovian model, which treats this radiative transfer problem by a stochastic formalism. Specifically, the partially cloudy atmosphere is treated as a two component (clouds and clear sky) mixture, which is described by a set of two coupled deterministic equations for the ensemble-averaged intensity. For Markovian statistics, these equations are exact in a purely absorbing mixture, and are reasonably accurate and very robust in the general case including scattering. This description can also be modified to account for non-Markovian statistics. We show that in two different asymptotic limits this set of two coupled equations can be renormalized to a single radiative transfer equation, involving effective atmospheric properties. These two limits correspond to a nearly transparent atmosphere, and small correlation length mixing statistics. General spatial dependences of cloud and clear sky properties, as well as inhomogeneous and anisotropic statistics, are allowed in the formalism.

12. G.C. Pomraning, "Small Correlation Length Solutions for Planar Symmetry Beam Transport in a Stochastic Medium," *Ann. Nucl. Energy* 23, 843 (1996).

Abstract - This paper considers planar symmetry, steady-state, monoenergetic linear transport of a normally incident particle beam through a purely scattering thin slab, composed of a stochastic mixture of two immiscible materials. The scattering process is assumed sufficiently forward peaked so that the Fokker-Planck description is valid. If the material mixing obeys Markovian statistics, and ignoring particle backscattering, this situation is a joint Markov process. As such, the stochastic Liouville master equation is valid, and we obtain a set of two coupled linear transport equations describing exactly the ensemble-averaged solution for the particle intensity. An asymptotic expansion is used to reduce these two equations, in the small correlation length limit, to a single renormalized equation. This equation is treated by an angular-moments method, exploiting the assumed peakedness in angle of the solution. An analytic solution of these moment equations provides a simple and explicit solution for the spatial distribution of the particle density. The effect of stochasticity is to increase the solution over the corresponding nonstochastic result recently reported in the literature.

13. G.C. Pomraning, "The Planar Symmetry Beam Problem in Stochastic Media," *J. Quant. Spectrosc. Radiat. Transfer* 55, 771 (1996).

Abstract - This paper considers planar symmetry, steady-state, energy dependent, linear transport of a normally incident radiation beam through a scattering/absorbing thin slab, composed of a stochastic mixture of two immiscible materials. The scattering process is assumed sufficiently peaked in both energy and angle so that the Fokker-Planck description is valid. If the material mixing obeys Markovian statistics, and if particle backscattering can be ignored, this situation is a joint Markov process. As such, the stochastic Liouville master equation is valid, and we obtain a set of two coupled linear transport equations describing exactly the ensemble-averaged solution for the particle intensity. We develop the small correlation length asymptotic limit of this set of two equations, which results in a single renormalized equation. The two-equation set is treated by an angular moments method, exploiting the assumed peakedness of the solution in angle. In a separate development, an explicit result is obtained for the spatial profile of the particle density in the case of monoenergetic, purely scattering transport through a layered statistical mixture.

14. Bingjing Su and G.C. Pomraning, "The Fermi-Eyges Beam Analysis for a Heterogeneous Absorbing Stochastic Medium," *Ann. Nucl. Energy* 23, 1153 (1996).

Abstract - The Fermi-Eyges beam analysis yields an approximate analytic solution for the energy dependent angular flux corresponding to the steady state, finite slab, seachlight problem with a monoenergetic, normally incident, beam. Their analysis, which treats a nearly collimated beam of particles, originally entering a source-free slab through one surface as a perfectly collimated pencil beam, was restricted to a purely scattering, homogeneous medium. The purpose of this paper is to generalize the Fermi-Eyges analysis in two ways. First, in a straightforward generalization, the slab is allowed arbitrary heterogeneous properties, and further the absorption interaction is included. Second, and more significantly, we analyze such beam transport throughout a slab comprised of a stochastic mixture of two immiscible materials. If the material mixing is taken as Markovian, an exact

stochastic model for the ensemble-averaged solution is easily constructed. Explicit scalar flux solutions are obtained in two asymptotic limits, namely: (1) the small correlation length limit; and (2) the limit of a small amount of relatively opaque material admixed with a large amount of relatively transparent material.

15. G.C. Pomraning, "The Variance in Stochastic Transport Problems with Markovian Mixing," *J. Quant. Spectrosc. Radiat. Transfer* 56, 629 (1996).

Abstract - This paper considers steady-state particle transport and radiative transfer through a stochastic mixture consisting of two immiscible materials. The mixing of these two components of the medium is assumed to be described by Markovian statistics. In the absence of scattering, exact equations are developed for the ensemble average of any power of the stochastic intensity field. Particular attention is paid to the first and second powers, which yield algorithms for computing the ensemble-averaged intensity as well as the associated variance. A simplified description is obtained in the asymptotic limit of a small amount of opaque material admixed with a large amount of transparent material. This description is in the form of renormalized transport equations, with effective coefficients, for the ensemble-averaged intensity and variance. An asymptotic equivalence argument is used to ensure robustness of the renormalized variance equation.

16. G.C. Pomraning, "Renewal Analysis for Higher Moments in Stochastic Transport," *J. Quant. Spectrosc. Radiat. Transfer* 57, 295 (1997).

Abstract - This paper considers steady-state particle transport and radiative transfer through a stochastic mixture consisting of two immiscible materials. The statistics of the mixing of the two components of the medium are allowed to be very general; in particular, they are not necessarily Markovian. In the absence of scattering, the theory of alternating renewal processes is used to obtain an exact set of integral equations describing the ensemble average of any integer power of the stochastic intensity field. These equations are solved in an infinite medium with homogeneous statistics, and this solution is used to develop a simpler, but approximate, formalism for treating particle transport in a non-Markovian mixture. This simplified description is in the form of differential, rather than integral, equations, and results from a modification of the differential Markovian equations which arise from the use of the stochastic Liouville master equation.

Proceedings

1. F. Malvagi, R.N. Byrne, G.C. Pomraning, and R.C.J. Somerville, "Stochastic Radiative Transfer Predictions of Functional Cloud Cover," *Proc. Fourth Symp. on Global Change Studies*, American Meteorological Society (Anaheim, California) p. 149 (1993).

Summary - This paper gives an interpolation formula, between the short and long correlation length limits, in the stochastic treatment of radiative transfer through a broken cloud field.

2. G.C. Pomraning, "Anisotropic Diffusive Limit for Particle Transport in Stochastic Mixtures," *Proc. 18th Rarefied Gas Dynamics Symposium* (Vancouver, Canada), *Rarefied Gas Dynamics: Space Science and Engineering*, Vol. 159, B.D. Shizgal and D.P. Weaver, eds., Progress in Astronautics and Aeronautics, AIAA, Washington, D.C., p. 653 (1994).

Abstract - It has recently been shown that a linear kinetic theory model for particle transport in a binary Markovian stochastic mixture possesses several different diffusive limits. This work was restricted to isotropic mixing statistics and a common scale length for the chord lengths of the two immiscible components of the mixture. Here we allow anisotropic statistics as well as independent length scales for the two components. The formalism used is that of asymptotic expansions, and we find an additional richness of asymptotic diffusive limits by relaxing the prior limitations of the mixing. A short discussion of the derivation of asymptotically consistent initial and boundary conditions of the diffusion equations is also given.

3. R.N. Bryne, D.N. Arion, F. Malvagi, Y. Serra, R.C.J. Somerville, G.C. Pomraning, and B.J. Su, "Stochastic Radiative Transport for Climate Models," *Proc. Third Atmos. Radiat. Measurement (ARM) Science Team Meeting*, U.S. Dept. Energy Report CONF-9303112 (Norman, Oklahoma), p.59 (1994).

Summary - This paper summarizes our current research in four areas, namely : (1) Functional cloud cover; (2) Theory of stochastic transfer; (3) Effect on model calculations; and (4) Tests of stochastic theory.

4. G.C. Pomraning and Bingjing Su, "A Closure for Stochastic Transport Equations," *Proc. International Conf. on Reactor Physics and Reactor Computations*, Ben-Gurion Univ. of the Negev Press (Tel Aviv, Israel), p. 672 (1994).

Abstract - The problem of particle transport through a stochastic mixture of two immiscible materials is considered. The material mixing process is assumed to obey Markovian statistics. An ensemble average of this stochastic transport equation leads to two equations containing four different ensemble-averaged intensities. To close these equations to a set of two equations in two unknowns, certain rod geometry problems are considered. In this geometry, two distinct exact analyses are possible, namely a small correlation length analysis, and a nonstochastic mean number of secondaries per collision analysis. The closure philosophy is to demand that the closed set of two equations reproduces these exact limiting behaviors. Numerical results are given which compare the predictions of this new closure with exact benchmark results as well as with the standard closure available in the literature.

5. G.C. Pomraning and Bingjing Su, "A New Higher Order Closure for Stochastic Transport Closure for Stochastic Transport Equations," *Proc. International Conf. on Mathematics and*

Computations, Reactor Physics, and Environmental Analyses, American Nuclear Society Topical Meeting (Portland, Oregon), p. 537 (1995).

Abstract - The problem of particle transport through a stochastic mixture of two immiscible materials is considered. The material mixing process is assumed to obey Markovian statistics. An ensemble average of this stochastic transport equation leads to two coupled transport equations containing four different ensemble-averaged intensities. A new higher order closure involving additional transport equations is suggested to close these equations. The closure philosophy is to demand that the closed set of equations reproduces certain exact limiting behaviors in rod geometry. Numerical results are given which compare the predictions of this new higher order closure with exact benchmark results as well as with results from the standard closure available in the literature.

5. Bingjing Su and G.C. Pomraning, "P₂ Diffusive Approximation to a Stochastic Transport Description," *Proc. International Conference on Mathematics and Computations, Reactor Physics, and Environmental Analyses*, American Nuclear Society Topical Meeting (Portland, Oregon), p. 564 (1995).

Abstract - The problem of describing particle transport through a Markovian stochastic mixture of two immiscible materials is generally approximated by the so-called Levermore model, consisting of two coupled transport equations. In this paper, the P₂ diffusive equations and the associated boundary conditions for this Levermore model are derived in planar geometry using a variational principle, and numerical results comparing P₂, P₁, and S₁₆ (benchmark) calculations are presented. These results demonstrate that the P₂ equations are considerably more accurate than the P₁ equations away from boundary layers for large stochastic systems. The conditions defining a large stochastic system are discussed.

6. G.C. Pomraning, "Linear Kinetic Theory and Particle Transport in Stochastic Mixtures," *Proc. 13th Symposium on Energy Engineering Sciences*, (Argonne, Illinois), Argonne National Laboratory Report CONF 9505200, p. 96 (1995).

Abstract - We consider the formulation of linear transport and kinetic theory describing energy and particle flow in a random mixture of two or more immiscible materials. Following an introduction, we summarize early and fundamental work in this area, and we conclude with a brief discussion of recent results.

7. G.C. Pomraning, "Radiative Transfer in Discrete Stochastic Mixtures," *Proc. International Radiation Symposium on Current Problems in Atmospheric Radiation* (Fairbanks, Alaska) (in press).

Abstract - We consider the problem of formulating a description of radiative transfer in a stochastic mixture of two immiscible materials. In the no-scattering limit, exact formulations are presented for both Markovian and more general mixing statistics. When scattering is included, several models are discussed which, while not exact, are very robust and reasonably accurate.

Review Articles

1. G.C. Pomraning, "Transport Theory in Discrete Stochastic Mixtures," *Advances in Nuclear Science and Technology* 24, 47 (1996).

Summary - This comprehensive review article discusses all of the research, which began in 1985, in the area of particle transport in discrete stochastic mixtures.

2. G.C. Pomraning, "Radiative Transfer and Transport Phenomena in Stochastic Media," *Int. J. Eng. Sci.* (in press).

Abstract - We consider steady-state radiative transfer through a stochastic mixture of two (or more) immiscible materials. Certain exact results, as well as approximate models, are described for $\langle I \rangle$, the ensemble average of the specific intensity of radiation. In the simplest case of transport in a no-scattering medium, an exact description for $\langle I \rangle$ and higher stochastic moments is easily constructed. If the material mixing is taken as Markovian, the Liouville master equation leads to two (for a binary mixture) coupled deterministic differential equations of transfer describing $\langle I \rangle$, with similar results for higher stochastic moments, such as the variance. For non-Markovian mixing, renewal analysis leads to four coupled deterministic integral equations describing $\langle I \rangle$ and higher moments. In the more general problem involving scattering, no exact results (except in very special cases) are available, but reasonably accurate models can be developed. Certain asymptotic limits of these models lead to a single, renormalized, equation of transfer for $\langle I \rangle$ of the standard form, with effective values of the opacities and Planck function which account for the stochasticity of the problem. Diffusive approximations also result from asymptotic analyses.

Oral Presentations

1. F. Malvagi, R.N. Byrne, and G.C. Pomraning, "Stochastic Radiative Transfer Predictions of Functional Cloud Cover," Fourth Symposium on Global Change Studies (Anaheim, California), 1993.
2. R.N. Byrne and G.C. Pomraning, "Stochastic Radiation Transport for Climate Models," Third Atmospheric Radiation Measurement (ARM) Science Team Meeting (Norman, Oklahoma), 1993.
3. F. Malvagi, R.N. Byrne, G.C. Pomraning, and R.C.J. Somerville, "Functional Cloud Cover," Third Atmospheric Radiation Measurement (ARM) Science Team Meeting (Norman, Oklahoma), 1993.

4. G.C. Pomraning and B. Su., "A Closure for Stochastic Transport Equations," International Conference on Reactor Physics and Reactor Computations (Tel Aviv, Israel), 1994.
5. G.C. Pomraning, "Beam Transport in Deterministic and Stochastic Media," Seminar, University of Wisconsin, 1994.
6. G.C. Pomraning, "Beam Transport in Deterministic and Stochastic Media," Seminar, Rensselaer Polytechnic Institute, 1995.
7. G.C. Pomraning, "Particle Transport in Discrete State Stochastic Mixtures," Seminar, Knolls Atomic Power Laboratory, 1995.
8. G.C. Pomraning, "Radiative Transfer Through a Broken Cloud Field," Seminar, Texas A and M University, 1995.
9. G.C. Pomraning, "Radiative Transfer Through a Broken Cloud Field," Seminar, Georgia Institute of Technology, 1995.
10. G.C. Pomraning and B. Su, "A New Higher Order Closure for Stochastic Transport Equations," American Nuclear Society International Conference on Mathematics and Computations, Reactor Physics, and Environmental Analyses (Portland, Oregon), 1995.
11. B.Su and G.C. Pomraning, " P_2 Diffusive Approximation to a Stochastic Transport Description," American Nuclear Society International Conference on Mathematics and Computations, Reactor Physics, and Environmental Analyses (Portland, Oregon), 1995.
12. G.C. Pomraning, "Small Correlation Length Solutions for Planar Symmetry Beam Transport in a Stochastic Medium," Fourteenth International Conference on Transport Theory (Beijing, China), 1995.
13. G.C. Pomraning, "Beam Transport in Deterministic and Stochastic Media," International Workshop on Electron and Photon Transport Theory Applied to Radiation Dose Calculation (Seattle, Washington), 1995.
14. G.C. Pomraning, "Beam Transport in Deterministic and Stochastic Media," Seminar, University of California, Berkeley, 1995.
15. G.C. Pomraning, "Beam Transport in Deterministic and Stochastic Media," Seminar, Massachusetts Institute of Technology, 1996.
16. G.C. Pomraning, "Radiative Transfer in Discrete Stochastic Mixtures," International Radiation Symposium on Current Problems in Atmospheric Radiation (Fairbanks, Alaska), 1996.

17. G.C. Pomraning, "Radiative Transfer and Transport Phenomena in Stochastic Media," Chandrasekhar Memorial Symposium (Tempe, Arizona), 1996.

ABSTRACT OF DISSERTATION

One Ph.D. dissertation resulted from this grant: *Modeling of Particle Transport Through Binary Stochastic Media* by Bingjing Su, G.C. Pomraning, Chairman.

Abstract - The problem of particle transport or radiative transfer through a stochastic mixture of two immiscible materials is considered. Since the properties of the medium, through which the transport is occurring, are stochastic, the particle or radiation intensity is a random variable. The goal for such a problem is to develop a formalism of computing the ensemble-averaged intensity. However, the complexity of stochastic transport theory will preclude an exact description for the ensemble-averaged intensity in full generality. When the material mixing process is assumed to obey Markovian statistics, an ensemble average of the stochastic transport equation leads to two coupled transport equations containing four different ensemble-averaged unknowns. Thus, one is presented with a closure problem, and from a practical point of view, a usable description will undoubtedly involve an approximate model.

In this dissertation modifications to the existing closure available in the literature, including a classic closure and a higher order closure which involves additional transport-like equations, are suggested as new closures. The closure philosophy is to demand that the closed set of equations reproduces certain exact limiting behaviors in rod geometry. For this purpose, both small and large correlation length limiting ensemble-averaged solutions to the stochastic transport equations, allowing both the single scatter albedo and the mean free path to fluctuate, are explicitly constructed in rod geometry, using asymptotics and the Green's function method. Numerical computations are performed to assess the accuracy of the proposed modifications. Numerical results predicted by these new closures, for both the Markovian and non-Markovian mixing statistics, are given and compared with exact benchmark results as well as with results from the previous closures. It is shown that these new closures in general predict more accurate results than the previous ones. Further simplifying these models that involve coupled transport equations, a P_2 diffusive approximation to the classic model is derived in planar geometry using a variational principle. Numerical results demonstrate that the P_2 diffusive approximation is considerably more accurate than the standard P_1 diffusive approximation away from boundaries.

SCIENTIFIC COLLABORATORS

The published work resulting from this grant involved 7 collaborators. These are:

1. D.N. Arion: Dr. Arion is a staff member at Science Applications International Corporation, La Jolla, California.

2. R.N. Byrne: Dr. Byrne is a staff member at Science Applications International Corporation, La Jolla, California.
3. F. Malvagi: Dr. Malvagi is a staff member at the Nuclear Research Center, Cadarache, France.
4. A.K. Prinja: Professor Prinja is a faculty member at the University of New Mexico, Albuquerque, New Mexico.
5. Y. Serra: Ms. Serra is a graduate student at the Scripps Institute of Oceanography of the University of California, San Diego.
6. R.C.J. Somerville: Professor Somerville is a faculty member at the Scripps Institute of Oceanography of the University of California, San Diego.
7. B. Su: Dr. Su is a postdoctoral fellow at the Los Alamos National Laboratory, New Mexico.