Title:

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The Effect of Interface Damage on the Microbuckling of Unidirectional Fiber-Reinforced Composites

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ABSTRACT

Fiber microbuckling is the primary failure mechanism in unidirectional fiber-reinforced composites under compression. Due to processing or service conditions, damage (e.g., microcracks) exists at fiber/matrix interfaces. The effect of damage on the microbuckling of fibers is investigated in the present study. Based on the micromechanics analysis, the damage at interfaces is modeled as a linear spring against interface sliding, and the spring constant depends on the damage level. It is established that the critical strain for fiber microbuckling is relatively insensitive to the interface damage, but increases rapidly with the fiber volume fraction.

1. INTRODUCTION

Fiber microbuckling has been identified as the mechanism for tension/compression asymmetry of long and aligned fiber-reinforced composites [1]. Due to microbuckling of fibers, the compressive strength is only half of the tensile strength for fibrous composites [1]. The effect of plastic microbuckling and fiber kinking in composites have been investigated [2-6]. It has been observed repeatedly in experiments [7,8] that the compressive failure of fibers originates from a traction-free surface and subsequently propagates into the interior of the specimen, triggering a global failure of the test specimen. A simple model [9] has been developed to show that the initial microbuckling of fibers indeed starts from a traction-free surface. The model has been extended to incorporate the effect of elastic anisotropy of composites [10]. It is established that the fiber buckling strain accounting for the anisotropy of composites is more than twice that for isotropic solids. However, these studies are limited to ideal fiber-reinforced composites without any interface damage.

Fig. 1 A fiber beam bonded to a composite foundation with damage at the interface

Interface damage exists due to processing or service conditions of composites. The effect of damage at fiber/matrix interfaces on the microbuckling of unidirectional fiber-reinforced composites is investigated in the present study. Based on a micromechanics model, the damage at interfaces is represented by a linear spring against interface sliding, and the spring constant depends on the damage level. The critical strain for fiber microbuckling is an important parameter for fibrous composites and is investigated in detail.

2. CONSTITUTIVE RELATIONS AND DAMAGE AT FIBER/MATRIX INTERFACES

As shown in Fig. 1, the matrix material is reinforced by fibers aligned in the x_1 direction. The composite is subjected to remote uniform compressive strain ϵ_0 along the fiber direction (x_1). The microbuckling of fibers starts from a traction-free surface [7-10]. The fiber at the surface is modeled as an elastic beam bonded to an elastic foundation of the composite material (Fig. 1). The constitutive relations of the fiber, matrix, and composite, and the modeling of damage at fiber/matrix interfaces are discussed below.

2.1 Constitutive Relations

The fiber material is assumed isotropic and has the constitutive relations

$$\epsilon_{11} = (\sigma_{11} - \nu_f \sigma_{22})/E_f, \quad \epsilon_{22} = (-\nu_f \sigma_{11} + \sigma_{22})/E_f, \quad \epsilon_{12} = \sigma_{12}/(2\mu_f), \quad (1)$$

where E_f , ν_f , and μ_f are the Young's modulus, Poisson's ratio, and shear modulus of the fiber material, respectively, and $\mu_f = E_f/[2(1+\nu_f)]$. The corresponding relation for the matrix material is obtained by replacing E_f , ν_f , and μ_f with the matrix's properties E_m , ν_m , and μ_m , respectively. The unidirectional fiber-reinforced composite exhibits an orthotropic stress-strain relation

$$\epsilon_{11} = b_{11}\sigma_{11} + b_{12}\sigma_{22}, \quad \epsilon_{22} = b_{12}\sigma_{11} + b_{22}\sigma_{22}, \quad \epsilon_{12} = b_{66}\sigma_{12}/2, \quad (2)$$

where the elastic constants b_{ij} of the composite are related to the fiber and matrix properties by [10]

$$b_{11} = [cE_f + (1-c)E_m]^{-1}, \quad b_{12} = -b_{11}[c\nu_f + (1-c)\nu_m], \\ b_{22} = b_{11}[1 - c(1-c)(2 - 2\nu_f\nu_m - E_m/E_f' - E_f/E_m')], \quad b_{66} = c\mu_f^{-1} + (1-c)\mu_m^{-1}. \quad (3)$$

Here c is the fiber volume fraction, and $E_f' = E_f/(1 - \nu_f^2)$, $E_m' = E_m/(1 - \nu_m^2)$. An orthotropic solid under plane-stress condition gives three nondimensional material parameters [10,11],

$$\lambda = \frac{b_{11}}{b_{22}}, \quad \rho = \frac{2b_{12} + b_{66}}{2\sqrt{b_{11}b_{22}}}, \quad \kappa = \frac{3\sqrt{b_{11}b_{22}} + b_{12}}{\sqrt{b_{11}b_{22}} - b_{12}}. \quad (4)$$

For an isotropic solid, λ and ρ become 1, and κ becomes $(3-\nu)/(1+\nu)$, where ν is the Poisson's ratio.

2.2 Modeling of Damage at Fiber/Matrix Interfaces

The damage at an interface causes the relative sliding between the fiber and the matrix. This has been modeled as the shear resistance against sliding at fiber/matrix interfaces [12,13]. For a perfect interface, the shear resistance is large so that there is no sliding. Once the interface completely delaminates and the fiber and the matrix are separated, the shear resistance against sliding vanishes. Microbuckling of fibers usually occurs before a macroscopic crack is nucleated at the interface. It becomes the driving force for continuous crack propagation after crack nucleation [14]. The normal and shear stresses, $\sigma(x_1)$ and $\tau(x_1)$, are continuous across the interface. The normal displacements (x_2 direction, Fig. 1) are also continuous before the fiber and the matrix are separated. The tangential

displacements, however, are discontinuous due to sliding associated with the damage at the interface. The interfacial shear stress can be related to the relative sliding by a linear spring model,

$$\tau = k(u_1^{matrix} - u_1^{fiber}), \quad (5)$$

where the spring constant k represents the shear resistance against sliding at interfaces. For the limit $k=\infty$, there is no interfacial sliding and the model degenerates to that in [10] for a perfect interface. For the other limit $k=0$, the shear traction vanishes and there is no resistance against sliding. Follow the micromechanics model in [15], the spring constant k is related to the damage at the interface by

$$k = \frac{\pi}{2} \cosh^2(\pi \epsilon) W^{-1} (E_f^{-1} + E_m^{-1})^{-1} \left[\ln \frac{1}{\cos(\pi \omega / 2)} + \int_0^{\pi \omega} \frac{\sinh^2(\epsilon t)}{\sin t} dt \right]^{-1}, \quad (6)$$

where $\epsilon = \ln \left[\left(\kappa_f \mu_f^{-1} + \mu_m^{-1} \right) / \left(\kappa_m \mu_m^{-1} + \mu_f^{-1} \right) \right] / (2\pi)$ is the bimaterial constant, $\kappa_f = (3-\nu_f)/(1+\nu_f)$, $\kappa_m = (3-\nu_m)/(1+\nu_m)$, W is the average spacing between damage (e.g., microcracks) at the interface (Fig. 1), D is the average size of damage, and $\omega = D/W$ is the nondimensional interface damage parameter. The limit $\omega=0$ corresponds to a perfect interface (i.e., no damage) such that there is no relative sliding ($k=\infty$), while $\omega=1$ gives the other limit of vanishing interfacial sliding resistance.

3. MICROBUCKLING OF FIBERS

The fiber on the surface is modeled as an elastic beam, while the composite material underneath is considered as a semi-infinite, homogeneous, orthotropic foundation with the composite's elastic constants b_{ij} (Fig. 1). A uniform compressive strain, ϵ_0 , is imposed in the fiber direction. Before the fiber buckles, the strains are $\epsilon_{11} = -\epsilon_0$, $\epsilon_{12} = 0$, and $\epsilon_{22} = \nu_f \epsilon_0$ for fiber and $\epsilon_{22} = -\epsilon_0 b_{12}/b_{11}$ for composite.

3.1 Deformation Field

Let $u(x_1)$ and $v(x_1)$ be the respective displacements along x_1 and x_2 directions on the central axis of the fiber beam relative to the state just before buckling. The corresponding displacements along x_1 and x_2 directions at the fiber/composite interface are related to u and v by

$$u_1^{fiber} = u(x_1) - \frac{h}{2} \frac{dv(x_1)}{dx_1}, \quad u_2^{fiber} = v(x_1), \quad (7)$$

where h is the thickness of the fiber beam (Fig. 1). The normal and shear stresses at the interface, $\sigma(x_1)$ and $\tau(x_1)$, are governed by the moment and force equilibrium of the beam, i.e.,

$$\frac{E_f h^3}{12} \frac{d^4 v(x_1)}{dx_1^4} + \epsilon_0 E_f h \frac{d^2 v(x_1)}{dx_1^2} - \frac{h}{2} \frac{d\tau(x_1)}{dx_1} - \sigma(x_1) = 0, \quad E_f h \frac{d^2 u(x_1)}{dx_1^2} + \tau(x_1) = 0. \quad (8)$$

The displacements in the orthotropic composite foundation, u_1 and u_2 , are governed by [16-18]

$$b_{22} b_{66} \frac{\partial^2 u_1}{\partial x_1^2} + (b_{11} b_{22} - b_{12}^2) \frac{\partial^2 u_1}{\partial x_2^2} + (b_{11} b_{22} - b_{12}^2 - b_{12} b_{66}) \frac{\partial^2 u_2}{\partial x_1 \partial x_2} = 0, \quad (9a)$$

$$(b_{11} b_{22} - b_{12}^2 - b_{12} b_{66}) \frac{\partial^2 u_1}{\partial x_1 \partial x_2} + (b_{11} b_{22} - b_{12}^2) \frac{\partial^2 u_2}{\partial x_1^2} + b_{11} b_{66} \frac{\partial^2 u_2}{\partial x_2^2} = 0. \quad (9b)$$

The displacements at the interface between the fiber beam and the composite foundation are

$u_{\alpha}^{matrix} = u_{\alpha}(x_1, x_2 = 0)$ ($\alpha=1,2$). In conjunction with Eq. (7), the interface sliding law (5) and the continuity of normal displacements across the interface give

$$u_2(x_1, 0) = v(x_1), \quad \tau(x_1) = k \left[u_1(x_1, 0) - u(x_1) + \frac{h}{2} \frac{dv(x_1)}{dx_1} \right]. \quad (10)$$

Without the loss of generality, the shape of the buckled beam can be described by

$$u(x_1) = a \cos(\omega x_1), \quad v(x_1) = b \sin(\omega x_1), \quad (11)$$

where the constants a , b , and the buckle wavelength L are to be determined, and $\omega=2\pi/L$. The corresponding displacements in the composite foundation have the general form

$$u_1(x_1, x_2) = \phi_1(x_2) \cos(\omega x_1), \quad u_2(x_1, x_2) = \phi_2(x_2) \sin(\omega x_1), \quad (12)$$

where ϕ_1 and ϕ_2 are functions of x_2 to be determined. The governing equation (9) becomes ordinary differential equations of ϕ_1 and ϕ_2 . Their boundary conditions at $x_2=0$ are obtained from Eq. (10). The solution, satisfying that stresses vanish as $x_2 \rightarrow \infty$ (because the remote field is undisturbed), is

$$\phi_1(x_2) = C_1 e^{-\omega \alpha x_2} + D_1 e^{-\omega \beta x_2}, \quad \phi_2(x_2) = \alpha \xi^{-1} C_1 e^{-\omega \alpha x_2} + \beta \xi D_1 e^{-\omega \beta x_2}, \quad (13)$$

where $\alpha = (\sqrt{\rho+1} + \sqrt{\rho-1}) / (4\lambda)^{1/4}$, $\beta = (\sqrt{\rho+1} - \sqrt{\rho-1}) / (4\lambda)^{1/4}$, $\xi = -[(1+\kappa)\alpha^2 + (3-\kappa)\alpha\beta] / [(3-\kappa)\alpha\beta + (1+\kappa)\beta^2]$, and

$$C_1 = \frac{-\beta \xi^2 (a - \omega h b / 2) k h b_{66} + \xi b k h b_{66} + \beta \omega h (\xi - \xi^2) b}{(\alpha - \beta \xi^2) k h b_{66} + \alpha \beta \omega h (1 - \xi^2)}, \quad (14a)$$

$$D_1 = \frac{\alpha (a - \omega h b / 2) k h b_{66} - \xi b k h b_{66} + \alpha \omega h (1 - \xi) b}{(\alpha - \beta \xi^2) k h b_{66} + \alpha \beta \omega h (1 - \xi^2)}. \quad (14b)$$

3.2 Onset of Microbuckling

The displacement field in the composite foundation has been determined in terms of parameters a and b , and the buckle wave length, L , as in Eqs. (12-14). The normal and shear stresses at the fiber/composite interface are then obtained from the constitutive relation (2) as

$$\sigma(x_1) = E_f \omega [-s(a - \omega h b / 2) + (r + t) \sqrt{\lambda} b] \sin(\omega x_1), \quad (15a)$$

$$\tau(x_1) = E_f \omega [r(a - \omega h b / 2) - s b] \cos(\omega x_1), \quad (15b)$$

where r , s and t are functions of the normalized buckle wave length $l = \omega h = 2\pi h / L$, and are given by

$$\begin{bmatrix} r(l) \\ s(l) \\ t(l) \end{bmatrix} = - \frac{1 + \kappa}{E_f b_{11} \Delta(l)} \begin{bmatrix} (1 + \kappa) \sqrt[4]{\lambda} \sqrt{(1 + \rho) / 2} \\ (\kappa - 1) \sqrt{\lambda} \\ l [4 + (1 + \kappa)(\rho - 1)] / (k h b_{66}) \end{bmatrix}. \quad (16)$$

Here $\Delta(l)$ is given by

$$\Delta(l) = 8\kappa + (1 + \kappa)^2 (\rho - 1) + 2l(1 + \kappa) \sqrt{1 + \rho} [4 + (1 + \kappa)(\rho - 1)] / [\sqrt[4]{4\lambda} k h b_{66}]. \quad (17)$$

The substitution of displacements and stresses in Eqs. (11) and (15) into the equilibrium equation (8) of the beam yields the following homogeneous algebraic equations for parameters a and b ,

$$\gamma_{11}(l)a + [\gamma_{12}(l) - \epsilon_0]b = 0, \quad \gamma_{21}(l)a + \gamma_{11}(l)b = 0, \quad (18)$$

where

$$\gamma_{11}(l) = \frac{r(l)}{2} + \frac{s(l)}{l}, \quad \gamma_{21}(l) = 1 - \frac{r(l)}{l}, \quad (19a)$$

$$\gamma_{12}(l) = \frac{l^2}{12} - \frac{r(l)l}{4} - s(l) - \frac{[r(l) + t(l)]\sqrt{\lambda}}{l}. \quad (19b)$$

The determinant of Eq. (18) must vanish in order to have a non-trivial solution for a and b . This gives the compressive strain, ϵ_0 , in terms of normalized characteristic length, $l = \omega h = 2\pi h/L$, as

$$\epsilon_0 = \Gamma(l) \equiv \gamma_{12}(l) - \gamma_{11}^2(l) \gamma_{21}^{-1}(l). \quad (20)$$

It can be verified that the compressive strain ϵ_0 approaches infinity when the normalized length l approaches the limits $l=0$ and $l=\infty$; moreover, ϵ_0 has only one minimum value between these two limits $l=0$ and $l=\infty$. This minimum value gives the smallest possible compressive strain to trigger the microbuckling of the fiber, and is therefore the fiber buckling strain, ϵ_{buckle} , i.e.,

$$\epsilon_{buckle} = \min[\Gamma(l)] \quad \text{for} \quad 0 < l < \infty. \quad (21)$$

Fig. 2 The fiber buckling strain ϵ_{buckle} versus the linear spring constant against interface sliding, khb_{66} ; (h -- fiber thickness, $1/b_{66}$ -- shear modulus of the composite, c -- fiber volume fraction)

The fiber buckling strain ϵ_{buckle} depends on the material properties of the fiber beam, composite foundation, the interface, as well as the fiber thickness. It is an important material property for unidirectional fiber-reinforced composites under compression. For a large range of fiber volume

fraction, $c=0.1, 0.5$ and 0.8 , the buckling strain is shown in Fig. 2 versus the normalized shear resistance against interface sliding, k/b_{66} , where $1/b_{66}$ is the shear modulus of the composite. The fiber to matrix moduli ratio is $E_f/E_m=70$, and Poisson's ratios are $\nu_f=0.30$ and $\nu_m=0.37$, which are typical of graphite fiber and epoxy resin composites. Each curve in Fig. 2 has two asymptotes (marked by the dashed lines), corresponding to a perfect interface ($k=\infty$, no interface sliding) and a completely damaged interface ($k=0$, no resistance against sliding), respectively. The buckling strains for the two asymptotes, as shown in Fig. 2, are not significantly different. At a high fiber volume fraction $c=0.8$, the buckling strain for a completely damaged interface ($k=0$) is only 28% smaller than that for a perfect interface ($k=\infty$). This difference becomes even smaller as the fiber volume fraction decreases. Therefore, *the initiation of fiber microbuckling in a unidirectional fiber-reinforced composite is insensitive to the interface damage*. However, this conclusion only holds for the initiation stage of fiber microbuckling. Once the fiber buckles, the damage at the interface plays an important role in the post-buckling behavior as well as in the propagation of interface delamination [14].

The fiber buckling strain increases rapidly with the fiber volume fraction, as observed in Fig. 2. This is because that the composite foundation becomes stiffer as the fiber volume fraction increases, and provides more lateral support to delay the microbuckling of the fiber.

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