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FRAGMENT PRODUCTION RATES**

Author(s):

Gregory H. Canavan, DDP

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COMPARISON OF CALCULATIONS OF FRAGMENT PRODUCTION RATES

Gregory H. Canavan

Differences between NASA and DoD estimates of fragment production rates in space debris collisions are shown to be due primarily to different choices of the exponent in the debris distribution. The sensitivity to this parameter over the range of values consistent with experimental data is discussed.

This note compares NASA and DoD estimates of fragment production rates in space debris collisions. The calculations are performed for a consistent set of debris distribution functions, which are implicit in the set of integrated collision frequencies provided by NASA Johnson Space Center (JSC) in Attachment A¹ for comparison with calculations in the report by the USAF Scientific Advisory Board (SAB).² Attachment A gives a useful table of collision frequencies, defines NASA's equation for fragment production, and provides references that are used as the basis for the comparisons. This information makes it possible to understand and achieve agreement with JSC's average masses for all collisions and to discuss the discrepancies in the fragment production rate estimates based on them. The agreement on the average mass involved in all collisions is on the order of 1%; the agreement on average masses in catastrophic collisions is on the order of 6%, which is within the uncertainties in averaging procedures, once it is recognized that only about 52% of the collisions in the JSC table are catastrophic.

Empirical fragment distribution kernels are reviewed and proper methods for averaging them over debris and collision frequency distributions are derived, leading to a prediction of 102 fragments per collision for the highly cascaded distributions presented. While the substitution of average masses into fragmentation kernels for the estimation of fragment production rates is technically incorrect, the agreement with the corrected JSC fragment production rates is within a factor of two and readily traceable to the different parameters used in the two calculations. For JSC's collision frequencies, the JSC production rate overestimates that from the SAB study by about 90% due to the choice of the parameter used in the fragmentation kernel. The SAB results are also compared to the predictions of the FASTT model developed by DNA to model the DoD impact experiments, which predicts a lower number of fragments per collision than that calculated by the SAB for the average parameters from the DNA test series.

This comparison was stimulated by the apparent contradiction between JSC and SAB estimates of the average number of fragments per collision. The differences between the two are shown to result primarily from improper normalization, confusion between catastrophic and total collisions, and the use of a parameter in describing the fragment distribution that is not supported by the empirical DoD data base on hypervelocity collisions. The sensitivity to this parameter due

to the range of values consistent with experimental data is discussed. The remaining discrepancy is within the errors expected for the spuriously cascaded distributions used for comparison.

JSC debris collision rate estimates are given in Table 1 of Appendix A. These estimates of cumulative collision rates are derived from 10 Monte-Carlo simulations of debris growth over the next century from current conditions for the same launch rate synthesized by repeating that of 1985-1995 in each decade for every simulation. The growth of cumulative collisions in each case is shown in Fig. 1 of Attachment A, which indicates variances on the order of 10-20%, as is appropriate for the small number of cases. These numbers are sufficiently well behaved to make the average quantities computed below meaningful, if a bit noisy.

Table 1 gives the collision frequency between objects of various sizes, averaged over bins with ratios of upper to lower masses of about a factor of 2.8. Summing the entries gives an average of 58.4 collisions in 100 years or 0.58 collisions/yr. That is a factor of $\approx 0.58/0.05 = 11.6$ -fold greater than the present rate, but JSC indicates³ that the increase due to cascading can be normalized to first order by dividing each element in the matrix by a factor of 11.6. Table I is used as is below and the results are so normalized where necessary.

Figure 1 shows the average number of collisions of all types per century from Table 1 of Attachment A as a function of the diameters of the projectile and target fragments, where the projectile is defined as the colliding partner with the smaller diameter D_p and mass M_p and the target as the partner with the larger diameter D_t and mass M_t , which prevents double counting of collisions. This average collision frequency is denoted by $F(D_p, D_t)$. The fragment sizes defined in Table 1 of Attachment A range from a bin from 0.1 to 0.16 m, which contains the smallest fragments, to a bin from 6.3 to 10 m, which contains the largest. The coordinates are oriented so that the origin (0.16, 0.16) is at the lower left and the largest fragments (10,10) are at the right.

The value at the origin of about 0.2 collisions/century for fragments from 0.1 to 0.16 m corresponds to the top entry of Table 1 of Attachment A. There is a detectable number of collisions of these fragments despite their small size because of their large number.⁴ For $D_p = 0.16$ m, as D_t increases there are a few local maxima, a sharp increase at $D_t = 1.6$, and a pronounced peak at $D_t = 6.3$ m, after which the number falls, indicating that the most numerous collisions are between the numerous small particles and the few large ones.

As D_p increases, similar scaling on D_t is seen, but the prominence of the peaks at large D_t falls due to the reduction in the number of target particles in the distribution. However, there are secondary peaks at 2.5 and 6.3 m, where the large size of the target particles overcomes their lower numbers to produce peaks about 15% as large as that at (0.16, 0.63).

The bottom curve in Fig. 2 shows collisions summed over targets size, $\sum_{D_t} F(D_p, D_t)$, as a function of projectile size. The value is about 16.8 collisions/century at $D_p = 0.16$ m, due largely to the large peak at $F(0.16, 6.3)$. The sum falls with increasing D_p by about an order of

magnitude by 1 m, where it stabilizes at about 3 through $D_p = 6$ m, after which it falls by another order of magnitude for the largest 10 m bin, consistent with the interpretation of Fig. 1 that collisions primarily involve the large flux of small fragments on the few large targets. The top curve is the cumulative number of collisions, which is obtained by summing the lower curve up to the value of D_p indicated, i.e., $\sum_0^{D_p} \sum_{D_t} F(D_p, D_t)$. At $D_p = 0.16$ m the cumulative is equal to the sum. It increases rapidly with D_p up to about 1 m; more slowly thereafter. It reaches about 50% of the total of 58.4/century by $D_p = 0.25$ m and 80% by 2.5 m, again reflecting the dominance of collisions of small projectiles with large targets in total collisions. Figure 3 shows the sum and cumulative collisions as a function of target size, $\sum_0^{D_t} \sum_{D_p} F(D_p, D_t)$, which increase roughly as $D_t^{3/2}$, which again indicates a strong role for large fragments as targets for the numerous small fragments.

Catastrophic collisions. The dashed lines around the lower left bins in Table 1 of App. A identify collisions that are not catastrophic, i.e., do not satisfy the requirement that the mass of the projectile be within a factor of 1,000 of the mass of the target for complete fragmentation,⁵ determined by JSC from inspection of the DoD-DNA impact experiment data base.⁶ Smaller impactors cause only partial fragmentation of targets.⁷ Excluding these non-catastrophic collisions produces the catastrophic collisions per century $G(D_p, D_t)$ distribution shown in Fig. 4, in which the peaks at small D_p and large D_t seen in Fig. 1 are suppressed by the elimination of the numerous collisions involving projectiles too small to cause catastrophic collisions.

Table 1 of Attachment A indicates that the ≤ 0.16 m objects have a maximum mass of 0.73 kg, so they are too small by an order of magnitude to fragment the largest objects, which have masses up to 8589 kg. Only fragments larger than 0.4 m can fragment them. By eliminating collisions with target to projectile mass ratios over 1,000, JSC reduces the collisions to those in the dashed region. That eliminates 27.8 collisions per century and reduces the number of catastrophic collisions per century to 30.6—a reduction of about a factor of two.

Figure 4 shows that the net result is that the peak seen in Fig. 1 at $D_p = 0.16$ m is eliminated, and the peaks at 0.25 and 0.63 m are attenuated to about the levels of those at 2.5 and 6.3 m, so that the secondary peaks such as the one at (1.6, 1.6) become more visible. Figure 5 shows the sum of G over target size, $\sum_{D_t} G(D_p, D_t)$, which indicates that contributions from all diameter bins are comparable. The cumulative $\sum_0^{D_p} \sum_{D_t} G(D_p, D_t)$ rises rapidly to about 50% of the total of 30.6 by $D_p = 1$ m, increases more slowly thereafter, and reaches about 80% of the total by $D_p = 2.5$ m. Figure 6 shows that the sum and cumulative values as functions of D_t resemble those in Fig. 1, although their slopes fall for larger targets.

The average mass per collision is evaluated by adding D_p and D_t for each element in $F(D_p, D_t)$, forming the product $(D_p + D_t)F(D_p, D_t)$ for each element in the collision matrix, summing over all D_p and D_t , dividing the result by the total of 58.4 collisions per century, and

multiplying by 0.6, which produces the average mass involved per collision. The factor of 0.6 is needed because the JSC bins have a ratio of maximum to minimum mass of ≈ 2.8 , so the geometric average mass of each bin is only $\approx 1/\sqrt{2.8} \approx 0.6$ times the mass of the upper limit used to index the bin. This process is repeated with F replaced by G to determine the average mass per catastrophic collision.

Figure 7 shows the contributions to the average masses per collision summed over D_t as a functions of D_p . The upper curves are the sum and cumulative mass for all collisions. The contributions are a maximum at small D_p , fall until $D_p \approx 1$ m, and then level out. The cumulative value rises rapidly, reaching more than half its asymptotic value of 2293 kg by 0.4 m. The lower curves are the sum and cumulative mass for catastrophic collisions, which are minimized for the small D_p fragments that cannot fragment the large objects. For D_p larger than 1 m, the distribution of contributions to the mass for catastrophic collisions approaches that for all collisions. and then level out at about 1102 kg/bin.

The top curve the cumulative contributions to average mass rises more slowly, reaching half its asymptotic value of 1102 kg by about 0.4 m. Note that this value is the average total mass for the fragments involved in all collisions. Only a fraction $\approx 30.6 / 58.4 \approx 52.4\%$ of the collisions in Table 1 are catastrophic, so the average mass of fragments involved in catastrophic collisions only is $1102 \text{ kg} / 0.524 = 2103 \text{ kg}$. These results are compared in Table I to those from the table of average geometric masses below the collision matrix on Table 1 of Attachment A.

Table I.	Average Mass (kg)		
	<u>all collisions</u>	<u>catastrophic</u>	<u>difference %</u>
JSC	2271	2007	-12
SAB	2293	2103	-9
difference %	1	5.8	

The average masses compare well for both all and catastrophic collisions. The 1% in all collisions is in the noise and the 5.8% discrepancy could be accounted for by differences in averaging. Both calculations agree that the average mass falls by 9-12% for catastrophic collisions as compared to all collisions—largely due to the exclusion of the exclusion of the collisions by the smallest fragments with the large ones. The mass involved in collisions falls by about a factor of two, from 2293 to 1102 kg, as shown above, but only about half of the collisions are catastrophic, so the ratio of the average mass in all to catastrophic collisions appears relatively stable.

The fragment production kernel for hypervelocity impacts has been studied in DoD and NASA experiments,⁸ and the few dozen well diagnosed tests have been summarized and

interpreted.⁹ The results of those experiments can be fit reasonably well by expressions for the cumulative number of fragments with masses greater than m of the form

$$C = A(M_t/m)^B, \quad (1)$$

where M_t is the target mass, approximately the total fragment mass in those experiments, and A and B are parameters that characterize the number of fragments produced. The value of B can be inferred for each experiment from the slope of the plot of $\ln C$ versus $\ln m$; A is determined by the intercept. The value of B so determined is 0.62 ± 0.07 . The SAB report uses this average value of B determines $A = 1/B - 1$ from conservation of mass.¹⁰

The average fragments per collision is determined by averaging Eq. (1) over the collision matrix of Table 1 of Appendix A to determine the average number of fragments produced per collision, which is¹¹

$$\langle N \rangle = \sum' Z(D_p, D_t, M_f) F(D_p, D_t) / \sum F(D_p, D_t), \quad (2)$$

where $Z(D_p, D_t, M_f)$ is the average number of catalog particles of diameter greater than 0.1 m (or mass $M_f \geq 0.26$ kg, according to Table 1 of App. A) produced per collision between objects of mass M_p and M_t . $F(D_p, D_t)$ is the collision rate between them, as represented in this comparison by the elements of the collision frequency matrix in Table 1. The sum is over D_p and D_t , and the prime indicates that the sum is to be taken only over catastrophic collisions, in which F reduces to G , as non-catastrophic collisions do not produce additional particles. Z is in general a function of D_p , D_t , M_f , and the ratio of projectile to target mass for catastrophic interaction. While Z is a function of mass, it is generally not a linear function of mass. Thus, it is not permissible to calculate the average mass per catastrophic collision

$$\langle M \rangle = \sum (M_p + M_t) G(D_p, D_t) / \sum F(D_p, D_t), \quad (3)$$

and use it in Eq. (1) to estimate the average number of fragments per collision, unless Z is linearly proportional to mass, $Z = K(D_p + D_t)$, in which case

$$\langle N \rangle_{\text{linear}} = \sum K(M_p + M_t) G(D_p, D_t) / \sum F(D_p, D_t) = K \langle M \rangle. \quad (4)$$

For other dependences of Z on M_p and M_t , this approximation does not hold. In particular, for the form of Z inferred from the DoD-DNA impact tests given by Eq. (1)

$$\begin{aligned} \langle N \rangle &= \sum' A[(M_p + M_t)/M_s]^B F(D_p, D_t) / \sum F(D_p, D_t) \\ &\neq A \{ \sum' [(M_p + M_t)/M_s] F(D_p, D_t) / \sum F(D_p, D_t) \}^B. \end{aligned} \quad (5)$$

The errors in such an approximation can be quite large for the DoD-DNA $B \approx 0.62$ for which the scaling of Z is far from linear, which strongly suppresses the effect of large excursions in mass relative to the linear approximation of Eq. (4). In any case, it is not necessary to use the approximation of linearity, as the full collision frequency matrix and kernel are available for this comparison, so the desired averages can easily be evaluated without approximation.

The average number of fragments per catastrophic collision is determined by averaging the fragmentation kernel of Eq. (1) over the catastrophic collision distribution G of

Table 1. The kernel, under the optimistic assumption that all of the mass of both the projectile and target are converted into fragments is given by

$$C = A[(M_t + M_p) / M_f]^B, \quad (6)$$

where M_f is the "mass of the smallest debris considered," which corresponds to 0.26 kg, or a diameter of 0.1 m, the smallest size in the JSC Catalog. Inserting the kernel of Eq. (6) into the full Eq. (2) produces the plot of fragment production as a function of M_p and M_t shown in Fig. 8. This plot for fragment production as a function of projectile and target size resembles the plot of catastrophic collisions in Fig. 4, although the peaks are accentuated by the greater fragment production in collisions between large particles.

Figure 9 shows that the integrated contribution from smaller objects is significant. The contributions to fragment production integrated over target size is roughly the same from bins of all projectile sizes larger than 0.6 m, except that from the largest objects, which is an order of magnitude smaller. The cumulative integral over projectile size reaches about 50% of its full value of 102 fragments by a projectile diameter of about 1.6 m. This value is larger by 25-70% than the value of 60-80 fragments per collision given in the SAB report, but is based on a significantly different, hypothetical cascaded future debris distribution. Figure 10 shows the Fragment production rate as a function of target diameter, which increases rapidly with M_t as a result of collisions of large targets with projectiles large enough to be catastrophic.

The JSC note on "Parts Creation" in Attachment A gives an alternate procedure based on a "standard NASA equation to determine the number of objects greater than a given mass"

$$CN = 0.4478(M_f/M_e)^{-0.7496}, \quad (7)$$

in which M_f is the "mass of the smallest debris considered" 0.26 kg, corresponding to a diameter of 0.1 m, and M_e is the "mass of the ejecta (i.e., total mass for catastrophic collisions)." This expression involves only aggregate quantities, so it can be evaluated directly with the quantities calculated above. Substituting the JSC average masses of Table I into Eq. (1) gives

Table II.	Fragments from average mass of catastrophic collisions			
	JSC all coll	JSC catastr (CN)	SAB catastr	difference
avg. mass	2271	2007	2103	5.8%
fragments	403	368	102	-206%

The average JSC mass for all collisions is that given below Table 1 of Attachment A and in Table I above. The calculated value of 403 fragments for all collisions corresponds to using JSC's average mass of 2271 kg from Table I in Eq. (7). However, Fig. 7 shows that only about half of all collisions are catastrophic, so it is inappropriate to use this full mass in estimating fragment production. Using instead JSC's mass of 2007 kg for catastrophic collisions from

Table I in Eq. (7) gives the value of 368 fragments in the middle column. The third column corresponds to the SAB's calculated average mass of 2103 kg in Table I, which is calculated in an integrated average of Eq. (6) that produces the 102 fragments per collision shown.

The apparent discrepancy in fragments between the JSC and SAB calculations of a factor of $368 / 102 \approx 3.6$ is partly definitional and partly substantive. JSC's average of 368 fragments is only appropriate for the 52.4% of the collisions that are catastrophic, so the average number of fragments per collision of all types is

$$CN = 368 \times 0.524 = 193 \text{ fragment / collision,} \quad (8)$$

which is only a factor of $193 / 102 \approx 1.9$ larger than the value from the SAB calculation. This corrected value of 193 fragment / collision is used for comparisons below. When necessary to avoid confusion, the value of 368 is referred to as the "average number of fragments per *catastrophic* collision." The residual difference of a factor of 1.9 is due primarily to the difference in the choice of exponent B, which is discussed after cumulative fragment production.

JSC cumulative fragment production. Figure 2 of Attachment A shows JSC's estimates of cumulative fragment production versus cumulative mass in [catastrophic] collisions. While the calculations that produce the chart are not presented, the implied conclusion that the fragment production is almost linear in the mass involved is worth examining, which can be done with the tools described above. Although Eq. (7) for CN applies to the total mass involved in catastrophic collisions, Fig. 2 of Attachment A is apparently generated by substituting Eq. (7) for Z in the average of Eq. (2). Figure 11 tests that by comparing the result of doing so with the result generated by using Eq. (6) with $B = 0.7496$ to estimate fragment production. The curves are similar and displaced by a small amount, which indicates a small multiple difference.

The ratio of the two curves is almost precisely 1.34. That is almost precisely the ratio of the JSC and SAB A coefficients, which is $0.0.4478 / (1/B - 1) = 0.4478 / (1 / 0.7496 - 1) = 1.343$. Figure 12 shows the A coefficients from the SAB, JSC, and FASTT DNA models. They cross at $B \approx 0.65$, but the JSC A is constant, so it remains above the SAB and FASTT coefficients (i.e., the JSC distribution is not normalized), which is the source of the factor of 1.34. Thus, it is clear how JSC Fig. 2 is generated.

With that understanding, it is possible to compare JSC and SAM versions of JSC Fig. 2 by changing the SAB exponent back to 0.62 and plotting the resulting cumulative fragment productions as a function of cumulative mass involved in producing them, as is done in Fig. 13. The lower curve is the the SAB curve for $B = 0.62$, which is roughly linear although it falls off for large fragments and saturates at about 102 fragments, as discussed in Table II. The top curve is the JSC curve from using Eq. (7) as a kernel in Eq. (2). It asymptotes to about 264 fragments, which is a factor of 2.6 larger than the SAB value for all large fragment sizes. From the above discussion it is clear that this factor is made up of a factor of 1.9 due to the use of $B = 0.75$ rather

than the experimental value of 0.62 and a value of 1.34 due to improper normalization relative to FASTT, i.e., non-conservation of mass. The product of those two factors is $1.9 \times 1.34 \approx 2.6$, which is precisely the factor that separates the two curves.

In the JSC curve an asymptote is shown for the geometric mean number of fragments at 403. As noted in Table II, that corresponds to using the mass for all collisions; however, only half are catastrophic. When the JSC mass of 2007 kg in the second column is used, Eq. (7) produces 368 fragments as shown. As discussed in Eq. (8), when that value is corrected for the fact that only 52.4% of the collisions are catastrophic, the resulting fragment production rate is 193 fragments per collision, which is a factor of 1.9 greater than the SAB value due to the use of the higher $B = 0.75$, as discussed above.

While there is no new information in JSC Fig. 2, the SAB recalculation of it in Fig. 13 does provide the information that for most fragment sizes, the cumulative fragment production is roughly a linear function of cumulative mass. Thus, apart from the large objects, for which fragment production saturates, the derivative of the number of fragments with respect to that of the cumulative mass is about constant, which means that objects of all masses and sizes produce comparable numbers of fragments.

The variation of fragments per collision with B can be assessed by changing the value of B in the kernel of Eq. (16) used in the averaging of Eq. (2). Figure 14 shows that this variation is fairly strong. From a value of about 60 fragments per collision at $B = 0.55$, it increases to a value of about 240 at $B = 0.8$. Its value is about 197 at $B = 0.75$, which is close to the JSC value of 193 in Eq. (8). However, the review of DoD impact experiments in a companion paper shows that a value of $B = 0.75$ is not consistent with the DNA test data, and that the mean value of $B = 0.62$ better characterizes the full data set.¹² The SAB calculations are best represented by the DNA average value of $B = 0.62$, for which the discrepancy between the corrected JSC and SAB estimates is a factor of $193 / 102 = 1.9$, as discussed above.

Figure 11 also shows a curve labeled FASTT, which is the semi-empirical curve developed by DNA to model the DNA experimental results cited by JSC.¹³ It lies slightly below the SAB curve at small B, crosses it at about $B = 0.65$, and rises more sharply for larger values of B. Of interest here is that the curve developed by DNA to model the DoD data gives a slightly lower value of B than the average value of $B = 0.62$ used in the SAB calculations, which represents an independent test of the SAB's reduction of the DNA data and a confirmation of its estimate of the number of fragments produced.

Thus, it appears that different choices of the exponent in the fragmentation kernel are the dominant factor in explaining any discrepancies between the JSC and SAB calculations. The source of the SAB value of B is the average of the experimental results in the summary DNA report. The JSC kernel is described as the "standard NASA equation to determine the number of

objects greater than a given mass." It is not otherwise discussed or explained in the documents referenced in Attachment A, and does not agree with the DNA data or with the FASTT model derived to model it.

Summary and conclusions. The JSC Attachment contains much of the information to complete the comparison of SAB and JSC estimates of fragment production. It gives a useful table of collision frequency, NASA's equation for fragment production, and some references, which are used as the basis for the comparisons made above. This information makes it possible to understand and achieve agreement with JSC's average masses for all collisions and to discuss the discrepancies in the fragment production rate estimates based on them. The agreement on the average mass involved in all collisions is on the order of 1%, which is in the noise. The agreement on average masses in catastrophic collisions is on the order of 6%, which is within the uncertainties in averaging procedures, once it is recognized that only about 52% of the collisions in JSC's table are catastrophic and the JSC rates are corrected for that fact.

The empirical fragment distribution kernels are reviewed and the proper methods for averaging them over debris and collision frequency distributions are derived, leading to a prediction of 102 fragments per collision for the highly cascaded distributions presented. While the JSC substitution of average masses into fragmentation kernels for the estimation of fragment production rates is technically incorrect and sometimes misleading, the agreement with JSC on the corrected fragment production rates at that level is within a factor of two and readily traceable to the different parameters used in the two calculations. For the JSC collision frequencies, the JSC production rate overestimates that from the SAB study by about 90% due to the choice of the parameter used in the fragmentation kernel. The SAB results are also compared to the predictions of the FASTT model developed by DNA to model the DoD impact experiments, which predicts a lower number of fragments per collision than that calculated by the SAB for the average parameters from the DNA test series.

This comparison was stimulated by the apparent contradiction between the JSC statement that the average debris collision would "produce about 480 fragments larger than 10 cm"¹⁴ with the result in the SAB report that the average number of fragments per collision is 60-80.¹⁵ For the conditions specified by JSC for this comparison, the number of fragments for JSC's "standard equation" is 368 per catastrophic collision, or 193 per collision, which is only a fraction of $193 / 480 = 40\%$ of the 480 claimed in the literature, but still a factor of $193 / 102 \approx 1.9$ greater than the value calculated by the SAB on the basis of DoD-DNA parameters. The majority of the discrepancy between the two results can be traced to a factor of 1.9 in fragments per catastrophic collision, which is due to JSC's use of a fragmentation formula that is inconsistent with the DoD-DNA test data and a further factor of two due to the confusion between catastrophic and total collisions in JSC's statement.

Overall, a more accurate summary would appear to be that debris collisions produce an average of about 102 fragments, which can be directly related to JSC's estimate through a simple series of corrections: multiply the JSC $\approx 480 \times 403/480$ [normalization] $\times 368/403$ [catastrophic] $\times 0.524$ [catastrophic / total] $\times 1/1.9$ [$B = 0.62$] ≈ 100 , in agreement with the SAB value of 102. That produces a factor of 4.8 reduction in the JSC estimate to a level of about 100 fragments per collision, at which fragmentation would produce little cascading. The remaining discrepancy is within the errors expected for the spuriously cascaded distributions used for comparison.

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Fig. 1. coll per cent 2d

Fig. 1. Collisions per century

Fig. 2. Collisions versus projectile size

Fig. 3. Collisions versus target size

Fig. 4. Catastrophic collisions per century

Fig. 5. Catastrophic collisions versus projectile size

Fig. 6. Catastrophic collisions versus target size

Fig. 7. Average mass

Fig. 8. Fragment production per century per bin

Fig. 9. Fragment production versus projectile size

Fig. 10. Fragment rate versus target size

Fig. 11.Cumulative fragment production vs projectile size

Fig. 12. A coefficients versus B

Fig. 13. Cumulative fragments vs cumulative mass

Fig. 14. Fragment per total collisions versus B exponent

TELECOPY

Send To: ✓ Dr. William Ballhaus
301-897-8816

✓ Mrs. Natalie Crawford
310-451-7038

✓ Lt. Col. James Berke
703-693-8262

Dr. Greg Canavan
505-867-3400 / 3644

Mr. Louis Peach
Mr. George Levin
202-356-2949

Mr. Joseph Loftus
281-483-6636

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From: Nicholas L. Johnson
NASA Johnson Space Center
SN3
Houston, TX 77058

Phone: 281-483-5313
FAX: 281-483-5276

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National Aeronautics and
Space Administration

Lyndon B. Johnson Space Center
2101 NASA Road 1
Houston, Texas 77058-3696

2 May 1997

Reply to Attn of:

N. L. Johnson, SN3

Dr. William Balhaus
Mrs. Natalie Crawford
Lt. Col. James Berke
Dr. Greg Canavan
USAF Scientific Advisory Board
Washington, DC

Subject: NASA Inputs to Plan for Closure of Orbital Debris Issues between the SAB and NASA

Dear Sirs and Madam:

Re the recent communications among SAB and NASA personnel concerning the subject cited above, attached is the explanation of how NASA determines the average mass involved in a collision and the number of debris which may be produced in such a collision. This summary is submitted to fulfill the NASA data requirement for Item 2, Parts Creation, of the closure plan.

Regarding Item 3, Cascade, NASA wishes to reiterate our recommendation of the acceptance of one of the two definitions submitted to Dr. Balhaus and Lt. Col. Berke on 22 April. We await your response on this issue. To recap, our suggestions were as follows:

"Collisional cascading for a region commences when the first catastrophic collision caused by impact of a previously-generated collision fragment with another resident space object occurs."

or

"Collisional cascading for a region commences when the expected number of succeeding collisions caused by a collision exceeds the value of one."

Concerning Item 4, Traffic Model, since the EVOLVE model is a discrete, event-based Monte Carlo simulation, we must input a specific, itemized launch history for each year. I can script the traffic model, but it must be defined. As I indicated in my correspondence to Dr. Balhaus and Lt. Col. Berke on 22 April, we could take a single year, e.g., 1995, and re-run it continuously for as many years as desired. Alternatively, we could cycle a select number of past years as was done for the exercise in Parts Creation (see attached). These rates could also be reduced by one-half or one-fourth as proposed. These techniques would, of course, lead to some problems since we would not be replicating the normal variation which is seen year-to-year.

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However, the purpose of this exercise was not to examine the details of the output but to compare general characteristics with those of Dr. Canavan's averaging technique. Under the circumstances, I am doubtful that such a comparison would provide meaningful data. NASA remains ready to pursue this issue, and we await the SAB's suggestions.

Sincerely,

A handwritten signature in dark ink, appearing to read "Nicholas L. Johnson". The signature is fluid and cursive, with the first name "Nicholas" and last name "Johnson" clearly distinguishable.

Nicholas L. Johnson
Chief Scientist for Orbital Debris
NASA Johnson Space Center

cc: Mr. Louis Peach, NASA/HQ
Mr. George Levin, NASA/HQ
Mr. Joseph Loftus, Jr., NASA/JSC/SA

Parts Creation

The objectives of this exercise were to determine (1) the "average" amount of mass involved in a natural collision and (2) the number of large debris (diameters greater than 10 cm) created in such a collision. This minimum debris size is important since it is generally sufficient to induce satellite breakups in a subsequent collision.

Methodology. Ten 100-year Monte Carlo runs of the EVOLVE code were executed with a base epoch of January, 1996. A traffic model repeating the historic space launches of 1988-1995 (inclusive) every eight years was employed. Consequently, the large number of LEO communications satellites (along with their rocket bodies and operational debris) scheduled for launch during the next several years were not modeled. Explosions were allowed to occur at historic rates. Only statistics on collisions involving objects both with a diameter of 10 cm or greater (≥ 260 gm) were collected. Each random collision was evaluated to determine if conditions existed for complete (catastrophic) fragmentation. Statistics were compiled for both catastrophic and non-catastrophic collisions.

Results. A total of 554 collisions occurred during the ten Monte Carlo runs, representing an average of 55.4 collisions per 100-year simulation. Figure 1 illustrates the cumulative number of collisions for each of the ten Monte Carlo runs. The non-linear growth is indicative of the growing satellite population greater than 10 cm in diameter. Table 1 summarizes the collision types by size (or mass) categories. For example, on average, two collisions occurred each run between objects in the 0.40-0.63 m and in the 3.98-6.31 m size bins (5.89-16.67 kg and 1071.3-3053.4 kg mass bins, respectively). The geometric and arithmetic means of total mass involved in the collisions are indicated beneath Table 1. The average combined mass of objects involved in all collisions was 2271 kg (geometric) or 2585 kg (arithmetic).

Using the standard NASA equation to determine the number of objects greater than a given mass,

$$CN = 0.4478 (M_r / M_o)^{-0.7486}$$

where, CN = cumulative number of particles with mass M_r or larger
 M_r = mass of the smallest debris considered
 M_o = mass of the ejecta (i.e., total mass for catastrophic collisions),

the number of debris diameter 10 cm or greater is 403 (geometric) or 444 (arithmetic) (Figure 2).

These numbers actually represent the most conservative (least debris generated) NASA assessment. In reality, the mass of each object will be transformed into two separate, distinct debris clouds with greater total debris than assuming a single fragmentation. Hence, if two objects, one 800 kg and one 1400 kg, collided, two debris clouds of 184 debris and 261 debris, respectively, (total = 445) would be created instead of a single debris cloud of 394 debris, as calculated with the equation above with a total mass of 2200 kg. In addition, if a lower mass equivalence of 145 gm for 10 cm diameter debris or if a combination of low intensity explosion/collision fragment generation (part of the mass is converted using low intensity equations and part of the mass is converted using collision equations) techniques are used, as is sometimes seen in the literature, the total number of debris generated is again greater than the values cited above and in Figure 2.

Nicholas L. Johnson
 NASA Johnson Space Center
 1 May 1997

**COLLISIONS BETWEEN OBJECTS GREATER THAN 10 CM DIAMETER
(TEN 100-YEAR MONTE CARLO RUNS)**

Figure 1

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COLLISIONAL INTERACTION COMBINATIONS
(EVOLVE 100 YEAR PROJECTION, BUSINESS AS USUAL)
(10 MONTE CARLO ITERATIONS)

DIAM(M)	TOTAL										
0.10	0.20	0.20									
0.16	0.10	0.00	0.10								
0.25	0.50	0.20	0.30	0.00							
0.40	0.40	0.20	0.00	0.20	0.00						
0.63	1.00	0.30	0.40	0.20	0.10	0.00					
1.00	3.60	0.90	0.60	0.50	0.10	0.40	1.10				
1.58	2.80	0.90	0.60	0.30	0.40	0.20	0.10	0.30			
2.51	9.00	2.80	1.90	0.80	0.80	0.40	1.20	0.80	0.30		
3.98	20.90	8.00	5.50	3.50	2.00	1.50	1.20	2.50	1.60	3.10	
6.31	11.90	3.30	2.20	1.60	0.70	0.70	0.70	0.50	0.90	0.70	0.60
10.00											
TOTAL	58.40	16.80	11.60	7.10	4.10	3.20	4.30	4.10	2.80	3.80	0.60
DIAM(M)	0.10	0.16	0.25	0.40	0.63	1.00	1.58	2.51	3.98	6.31	10.00
MASS(KG)	0.26	0.73	2.08	5.89	16.67	47.20	133.6	378.4	1071.3	3033.4	8588.9

SUM HORIZONTAL: COLLISIONS INVOLVING A GIVEN SIZE AND SMALLER

SUM VERTICAL: COLLISIONS INVOLVING A GIVEN SIZE AND LARGER

TOTAL NUMBERS OF COLLISIONS INSIDE BOXES

DASHED LINE: ALL COLLISIONS ABOVE AND TO THE RIGHT OF LINE ARE CATASTROPHIC

DOTTED LINE: ALL COLLISIONS BELOW AND TO THE LEFT OF LINE ARE NON-CATASTROPHIC

AVERAGE MASS INVOLVED IN COLLISIONS (IN KILOGRAMS):

ALL COLLISIONS INVOLVING OBJECTS LARGER THAN 10 CM:

ONLY CATASTROPHIC COLLISIONS (ABOVE / RIGHT OF DASHED LINE):

MOSTLY CATASTROPHIC COLLISIONS (ABOVE / RIGHT OF DOTTED LINE):

AVERAGE MASS USES GEOMETRIC OR ARITHMETIC MEAN FOR THE SIZE BIN

CATASTROPHIC COLLISION: COMPLETE STRUCTURAL FRAGMENTATION OF BOTH IMPACTING BODIES

GEOMETRIC

ARITHMETIC

2271

2586

2807

2286

2883

2372

Table 1

Figure 2

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