Field Investigation of Keyblock Stability

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(Doctor of Philosophy)

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Field Investigation of Keyblock Stability

By

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Discontinuities in a rock mass can intersect an excavation surface to form discrete blocks (keyblocks) which can be unstable. This engineering problem is divided into two parts: block identification, and evaluation of block stability. Keyblocks can be identified from discontinuity and excavation geometry using a whole stereographic projection. Once a block is identified, the forces affecting it can be calculated to assess its stability. The normal and shear stresses on each block face before displacement are calculated using elastic theory and are modified in a nonlinear way by discontinuity deformations as the keyblock displaces. The stresses are summed into resultant forces to evaluate block stability. Since the resultant forces change with displacement, successive increments of block movement are examined to see whether the block ultimately becomes stable or fails.

One stable keyblock and thirteen fallen keyblocks were observed in field investigations at the Nevada Test Site. Nine blocks were measured in detail sufficient to allow back-analysis of their stability. Measurements included block geometry, and discontinuity roughness and compressive strength. Back-analysis correctly predicted stability or failure in all but two cases. These two exceptions
involved situations that violated the stress assumptions of the stability calculations. Keyblock faces correlated well with known joint set orientations. The effect of tunnel orientation on keyblock frequency was apparent. Back-analysis of physical models successfully predicted block pullout force for two-dimensional models of unit thickness.

Two-dimensional (2D) and three-dimensional (3D) analytic models for the stability of simple pyramidal keyblocks were examined. Calculated stability is greater for 3D analyses than for 2D analyses. Calculated keyblock stability increases with larger in situ stress magnitudes, larger lateral stress ratios, and larger shear strengths. Discontinuity stiffness controls block displacement more strongly than it does stability itself. Large keyblocks are less stable than small ones, and stability increases as blocks become more slender. Rock mass temperature decreases reduce the confining stress magnitudes and can lead to failure. The pattern of stresses affecting each block face explains conceptually the occurrence of pyramidal keyblocks that are truncated near their apex.
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<tr>
<td>a, b</td>
<td>coefficients used in hyperbolic stiffness equation</td>
</tr>
<tr>
<td>a₀</td>
<td>average no-load aperture of discontinuity</td>
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<tr>
<td>D</td>
<td>displacement</td>
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<tr>
<td>E</td>
<td>Young's modulus</td>
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<tr>
<td>Eₘ, Eₘₛ</td>
<td>rock mass deformation modulus</td>
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<tr>
<td>F/W</td>
<td>ratio of support force to keyblock weight</td>
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<tr>
<td>G</td>
<td>shear modulus</td>
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<tr>
<td>i</td>
<td>dilatancy angle of discontinuity</td>
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<tr>
<td>JCS</td>
<td>compressive strength of joint surface</td>
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<tr>
<td>JRC</td>
<td>coefficient of roughness of joint surface</td>
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<tr>
<td>kₙ</td>
<td>normal stiffness</td>
</tr>
<tr>
<td>kₙᵢ</td>
<td>initial normal stiffness</td>
</tr>
<tr>
<td>kₛ</td>
<td>shear stiffness</td>
</tr>
<tr>
<td>L</td>
<td>length of block face in direction of sliding</td>
</tr>
<tr>
<td>P</td>
<td>pressure in tunnel</td>
</tr>
<tr>
<td>r, R</td>
<td>radius</td>
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<tr>
<td>S</td>
<td>spacing between discontinuities of a given set</td>
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<tr>
<td>Sᵥ</td>
<td>horizontal stress</td>
</tr>
<tr>
<td>Sᵥ</td>
<td>vertical stress</td>
</tr>
<tr>
<td>ΔT</td>
<td>change in temperature</td>
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<tr>
<td>ΔU</td>
<td>shear displacement</td>
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<tr>
<td>ΔV</td>
<td>normal displacement</td>
</tr>
<tr>
<td>Vₘ</td>
<td>maximum closure of discontinuity</td>
</tr>
<tr>
<td>W</td>
<td>weight of keyblock</td>
</tr>
<tr>
<td>X, Y, Z</td>
<td>Cartesian coordinate system</td>
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NOMENCLATURE (continued)

\[ \begin{align*}
\alpha & \quad \text{coefficient of linear thermal expansion} \\
\beta & \quad \text{angle between block face and displacement vector} \\
T & \quad \text{weight per unit volume} \\
\nu & \quad \text{Poisson's ratio} \\
\sigma & \quad \text{normal stress} \\
\sigma_1, \sigma_2, \sigma_3 & \quad \text{principal stresses} \\
\sigma_r, \sigma_\theta & \quad \text{radial and tangential normal stresses} \\
\tau & \quad \text{shear stress} \\
\tau_a & \quad \text{available shear strength} \\
\phi & \quad \text{friction angle of discontinuity} \\
\theta & \quad \text{tangential coordinate}
\end{align*} \]
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Sola Deo Gloria
Definition of the Block Stability Problem

Rock masses in which excavations are constructed almost always contain discontinuities such as joints, shears, and faults. Often, these discontinuities will intersect to create blocks of rock in the perimeter of an excavation. These blocks can have shapes so as to be able to displace into the excavation, unobstructed by adjacent rock. A simple form of such a block is shown in two dimensions in Figure 1.1. Blocks of more complex shape may be encountered, but are usually formed by combinations of geometrically simpler blocks (Goodman and Shi, 1985). The behavior and stability of these relatively simple blocks is therefore a problem of fundamental interest in the design and use of underground excavations.

Many block stability problems have been described in the rock mechanics literature in the past several years, and the stabilizing influence of confining stresses in situ has been recognized qualitatively as well (Brekke, 1968). The blocks themselves have been called wedges, keystones, or keyblocks by various investigators. Keyblocks are encountered on a range of scales, varying in size from small pieces of rock found ubiquitously along a tunnel rib to large monoliths that dominate an entire excavation surface. Cording et al (1971) described examples of the latter: a 2000 cubic yard (1529 m³) block in an excavation at the Nevada Test Site, and a block measuring over 50 feet (15 m) from base to apex at Morrow Point power
Figure 1.1 Two-dimensional schematic diagram of a keyblock in the roof of an underground excavation.
Figure 1.2 Sketch of large block encountered in Morrow Point powerhouse cavern (after Cording et al, 1971).
station (Figure 1.2). Cording and Mahar (1974) summarized the forms of blocks of lesser scale found in New York City and Washington, D.C. subway excavations (Figure 1.3).

The behavior of keyblocks can seriously affect the construction and use of an underground opening. While failure of a very large keyblock could entirely halt the use of an excavation, even small keyblock failures can be a safety hazard and can cause construction delays, rockfall cleanup expenses, and unexpected ground support costs. In extreme cases, keyblock displacement could lead to ravelling of the overlying ground; Ward (1978) described how a progressive failure can occur in regularly jointed rocks (Figure 1.4). Even if a fall did not occur, partial displacement of a keyblock adjacent to a tunnel would cause dilation and opening of the discontinuities bounding the keyblock. This would create a zone of increased permeability and decreased rock mass stiffness next to the excavation. In addition to other areas of mining and civil works, the problem is of interest in nuclear waste disposal, where excavation performance over several decades is critical for waste repository operation. In particular, waste package retrieval from emplacement holes (which may not be lined) must be possible for a specified length of time.

Analysis of the stability and behavior of a keyblock adjacent to an underground opening is a two-step problem. The first step is identification of the keyblock, while the second is a quantitative analysis of its response to the stresses and loads acting upon it.
Figure 1.3 Typical blocks encountered in Washington, D.C. subway tunnels (after Cording and Mahar, 1974).
Figure 1.4 Fallen block sequence in model of Kielder Tunnel (after Ward, 1978).
The first step is based upon an understanding of the discontinuities in the geologic setting and the orientation and size of the planned excavation. The second step requires measured or assumed values for discontinuity parameters and for the stress and load conditions around the excavation. Both steps need to be carried forward as far as possible prior to construction of the opening so as to allow the analysis to be of benefit in excavation design. Preliminary analysis of keyblocks can help evaluate how well discontinuity orientations and other parameters must be known for design purposes.

Conventional design approaches for excavation stability analysis usually include assumptions regarding the rock mass and its properties. Figure 1.5 illustrates common ways of treating the rock mass in calculations of excavation stability or deformation. Figure 1.5a shows a tunnel sited in a continuous rock mass, i.e., rock containing no discontinuities. This allows continuum methods of elasticity to be used in either closed form expressions or finite element approaches to estimate stresses and deformations around the excavation. Since rock is usually discontinuous, this approach is a simplification that often underestimates excavation deformations and does not allow all possible failure modes to be addressed.

Figure 1.5b shows a tunnel in rock containing a single ubiquitous set of discontinuities; this concept can be used with closed form solutions (Daemen, 1983) or with finite element schemes to locate and determine the size of zones around the excavation in which slip is expected to occur as construction or post-construction loading
proceeds. Scoping calculations can be readily made with this type of model; the results are useful to get an overall impression of the limits of opening behavior.

Figure 1.5c illustrates a tunnel in a rock mass containing several discrete discontinuities. Like the ubiquitous discontinuities, this can be examined with a finite element method of analysis to obtain excavation behavior estimates for specific situations (e.g., Heuze et al, 1983). However, a considerable amount of information on the discontinuity and excavation geometries is needed, and each situation to be analyzed requires setting up a new finite element mesh for the numerical code that is used for the solution.

Figure 1.5d shows a tunnel in rock containing discontinuities that intersect to form keyblocks. This work is an approach for solving this class of problems which is intended to complement rather than replace the three preceding methods of analysis. Each of the three preceding methods can be readily used for a two-dimensional analysis or, with some difficulty, a three-dimensional analysis. However, the keyblock approach allows keyblocks in three dimensions to be identified using procedures developed by Goodman and Shi (1981), and allows the stability and displacement behavior of keyblocks to be estimated using methods developed here. The calculations involved are simple enough to allow use of the method for general scoping calculations as well as for analysis of specific problems.
Figure 1.5a Tunnel in continuous rock mass.

Figure 1.5b Tunnel in rock mass containing a single ubiquitous discontinuity set.
Figure 1.5c  Tunnel in rock mass with several discrete discontinuities.

Figure 1.5d  Tunnel in rock mass with discontinuities intersecting to form keyblocks.
Scope of Work

The scope of this research can be summarized in four parts: development of methods with which to analyze keyblock behavior, field observations of keyblocks, back-analysis of the stability of observed and modeled keyblocks, and interpretation and extrapolation of the analytic results.

Development of methods to analyze keyblock behavior requires that the keyblocks be identified from geologic data. Methods with which keyblocks can be identified for analysis are reviewed and compared briefly in Chapter 2. The most versatile of the available methods, the Keyblock Method by Goodman and Shi (1981) is then described in somewhat greater detail. Complete details about the Keyblock Method and its basis have been published by Goodman and Shi (1985). The second step, analysis of keyblock behavior, requires that techniques be devised to evaluate and relate keyblock behavior to factors that include block and excavation geometry, pre-existing and excavation-induced stresses, thermal stresses, and discontinuity strengths and stiffnesses. This is the thrust of the first part of the work. Fluid pressures within the discontinuities and dynamic loads, however, were beyond the scope of this stage of the work. These new techniques for analyzing keyblock behavior are developed in Chapter 3.

The second part of the work involved field observations of keyblocks. Fallen and stable keyblocks were observed in tunnels in
granite and in tuff at the Nevada Test Site. These observations allowed the keyblock analysis techniques to be checked in the third part of the work, where the stability of each keyblock measured in the field was back-analyzed using the methods of Chapter 3. Model studies published by Crawford and Bray (1983) were also back-analyzed. Comparison of the observations and calculations in Chapter 4 gives important insights about keyblock behavior and the sensitivities of the models used to describe it.

The fourth part of the work involves interpretation of the analyses and extrapolation for other field conditions. Limitations of the new methods of stability analysis and the effects of keyblock displacement, discontinuity strength, discontinuity stiffness, in situ stress magnitude, lateral stress ratio, thermal stresses, and keyblock geometry are each described in Chapter 5. Chapter 6 then elaborates on the engineering implications of the results. Two-dimensional and three-dimensional models for keyblock behavior were coded in BASIC; computer code listings are provided in the Appendices.
CHAPTER 2 IDENTIFICATION OF KEYBLOCKS

Approaches for Identifying Critical Blocks

Solution of the problem of potentially unstable rock blocks (keyblocks) in the perimeter of an underground opening can be divided into two steps: identification of the keyblocks, and analysis of their stability. Methods for identification and analysis of the failure modes, surface areas, and volumes of blocks defined by discontinuities have been presented by Goodman (1976). Subsequently, procedures have been published with which sets of rock mass discontinuities of known orientation can be examined to identify combinations forming keyblocks in underground openings. In order of publication these methods are the Ubiquitous Joint Method (Cartney, 1977), the Inclined Hemisphere Projection Method (Priest, 1980), the Keyblock Method (Goodman and Shi, 1981), and a method that uses the Keyblock Method in conjunction with principles from graph theory (Chan and Goodman, 1983). Other methods have been developed for defining kinematic modes of block failure (e.g., Lucas, 1980, and Warburton, 1981), but these other approaches do not address the necessary first step of block identification.

The three-dimensional procedures listed above all use either graphical techniques or vector analysis to examine the ways in which discontinuity planes may intersect to form keyblocks in excavation surfaces. The methods also include an assessment of possible translational keyblock failure modes (falling or sliding); rotational
failures such as toppling are not considered. Some of the procedures require more geologic data than others, and the procedures are not equally versatile in handling a range of excavation surface orientations. Since keyblock behavior should be considered as part of the excavation design for underground openings in jointed rock, it is advantageous to be able to commence the analysis with a minimum amount of geologic data. The analysis can then be refined as more information becomes available, and can even help guide the site exploration and data acquisition program.

The first approach published specifically for underground excavations with which all joint sets could be examined simultaneously for block-forming set combinations was the Ubiquitous Joint Method by Cartney (1977). The method, which was used in cavern design for the Dinorwic Power Station (Douglas et al, 1979), involves plotting great circle representations of each discontinuity set on a lower hemisphere stereographic projection. The excavation surface under examination and the discontinuity planes are thus passed through a common point at the center of the sphere, allowing identification of pyramidal wedges (keyblocks) that are kinematically capable of failure. An example plot of a single discontinuity is shown in Figure 2.1b, and a typical stereographic construction from Cartney's 1977 paper is shown in Figure 2.2. "Ubiquitous" refers conceptually to the possibility of encountering joints of a given orientation anywhere in the excavation. This "ubiquitous joint" concept was named by Goodman (1967).
(a) Block diagram showing discontinuity in rock mass.

(b) Lower hemisphere stereographic plot of discontinuity in (a).

(c) Plot of discontinuity (a) on upper hemisphere whole stereographic projection.

Figure 2.1 Planar discontinuity dipping at 30° below horizontal.
Figure 2.2 Typical lower hemisphere stereographic construction for Ubiquitous Joint Method (after Cartney, 1977)
In the Ubiquitous Joint Method, excavation surfaces are all assumed to be either vertical or horizontal. Walls are thus drawn on the stereonet plots as straight lines passing through the center of the plot, while the rim of the stereonet represents a horizontal excavation roof (Figure 2.2). The method is effective for identifying the forms of simple rock wedges that could fall or slide from the walls or roof of an underground opening. These wedges can then be sized from the excavation dimensions or by engineering judgment for subsequent analysis. However, inclined excavation surfaces such as may occur between the roof and springline of a tunnel were not treated by Cartney.

The next development in generalized approaches for identifying keyblocks in tunnels was published by Priest (1980). The Inclined Hemisphere Projection Method can identify pyramidal blocks and their translational modes of failure from given discontinuity and excavation surface orientations. The method uses stereographic projection techniques similar to those of the Ubiquitous Joint Method. However, Priest's approach overcomes Cartney's constraints on excavation surface orientations by rotating the stereographic projection plane to coincide with the excavation surface. Although effective, this requires construction of a new rotated projection for each excavation surface being examined (Figure 2.3).

The most generalized approach available for identifying blocks is the Keyblock Method of Goodman and Shi (1981), which is based on Block Theory (Goodman and Shi, 1985). The method uses stereographic
Figure 2.3 Repeated stereographic constructions needed for the Inclined Hemisphere Projection Method (after Priest, 1980).
construction techniques, and has also been implemented with vector analysis principles for expedient solutions using a computer. The excavation surface limitations of Cartney's method and the repeated graphical constructions of Priest's method are avoided by using what is known as a whole stereographic projection. In the example plot of Figure 2.1c, the region within the reference circle represents a conventional upper hemisphere stereonet plot, while the area outside the reference circle is the remainder of the sphere. By including both upper and lower hemispheres on one construction, the intersection of planar discontinuity sets and excavation surfaces of any orientation can be examined for potential keyblocks and their failure modes. Because of its versatility, the method was used to analyze the field observations presented in Chapter 4 for modes of failure. Some of the procedures of the method are summarized in the next part of this chapter.

Chan and Goodman (1983) developed an application of the Keyblock Method that incorporates variations in discontinuity spacings and sizes. This new approach, like the methods described above, requires that discontinuity set orientations be defined. In addition, though, distributions of discontinuity spacings and extents are needed. These statistical parameters are used to construct hypothetical discontinuity trace maps that represent the intersections of fractures with an excavation surface. The constructions are scanned using principles from graph theory for trace patterns indicative of keyblocks or of combinations of keyblocks. Once the keyblocks are located, they can be individually analyzed for stability. Acquisition
of the statistical data needed for the method may, however, impede its early use during site exploration and excavation design. The method was not used in this work because all of the keyblocks encountered in the field observations were of simple pyramidal shapes, and because the necessary statistical data were not available.

**Block Theory and Keyblock Identification**

The Keyblock Method allows an investigator to examine the intersections of discontinuity sets and excavation surfaces for plane combinations creating blocks that are kinematically able to fail. The method has been applied to surface excavations (Goodman and Shi, 1981) and to underground excavations (Shi and Goodman, 1981). Graphical methods for block identification are outlined below; ways to assess kinematic modes of failure, computer implementations of the methods, and other applications of Block Theory are covered in a book by Goodman and Shi (1985).

Figure 2.4 indicates the types of blocks that might be expected to occur in a discontinuous rock mass. Of these block types, the removable blocks are of primary interest in excavation design. As explained by Goodman and Shi (1985), the other types of blocks are stable as long as the removable blocks do not fail. Of the three categories of removable blocks, the methods for stability analysis developed in Chapter 3 are particularly relevant for keyblocks and potential keyblocks.
Figure 2.4 Types of blocks in a discontinuous rock mass (after Goodman and Shi, 1985).
Keyblock identification requires that certain assumptions be made about the discontinuities that form removable blocks; they are taken to be planar, extensive, and of known orientation. These assumptions are also required by the methods of Cartney and of Priest which were described previously. Planar discontinuity surfaces are reasonable to assume unless a discontinuity surface has a significant curvature at the problem scale of interest. Even then, an average representation of the surface can be used until specific blocks are singled out for stability analysis. Discontinuities are considered to extend far enough to intersect one another so as to form removable blocks. If this is not true in a given instance, then rock bridges exist between discontinuities that would to some degree stabilize potential keyblocks. This is a separate problem that is not treated here.

Discontinuity orientations must be known for possible blocks to be defined by the stereographic constructions or the numerical procedures of the Keyblock Method. If adequate orientation data is available, an identification analysis can be repeated for widely dispersed (fringe) members of the discontinuity sets, or for unique geologic features that do not belong to a set.

Given the assumptions described above, the Keyblock Method enables an investigator to identify removable blocks from a whole stereographic projection. The data needed for such an analysis consist of the orientations of discontinuities present at the excavation site and orientations of the excavation surfaces to be examined. These planes are plotted on the whole stereographic projection; Figure 2.1 compares the way in which a discontinuity plane
is represented on a conventional lower hemisphere stereonet with its appearance on an upper hemisphere whole projection. Note that the great circle of the discontinuity plane that appears within the reference circle is reversed between the two projections. This is because the conventional plot is a lower hemisphere projection while the whole stereographic projection of Figure 2.1 uses an upper hemisphere. In other words, the projections have opposite focal points, which makes one a mirror image of the other. The other difference between the projections is that the whole projection includes the entire great circle of each plane whereas the conventional projection shows only the portion of the great circle that falls within the reference circle. A single graphical construction can be made for the discontinuities of interest and then examined for any number of excavation surfaces without having to rotate or redraw the discontinuities.

When discontinuity planes are plotted on a stereographic projection, they are shifted through space so as to pass through the center of the three-dimensional sphere that is represented in two dimensions by the projection. Conceptually, the planes divide the sphere into pyramids of rock, where each pyramid apex is at the center of the sphere, and each pyramid base is part of the surface of the sphere. These pyramids are called "joint pyramids" by Goodman and Shi (1985). An excavation surface passing through the center of the sphere divides the sphere into two regions (or half-spaces) that correspond to the excavation opening and to the adjacent rock. Joint pyramids that have their base in the rock mass side of the excavation
surface are tapered such that they get larger with distance into the rock mass, away from their apex. These pyramids represent blocks that cannot move into the excavation. Conversely, joint pyramids that have their base in the free space provided by the excavation must be tapered such that they get smaller with distance into the rock, away from their base. These correspond to blocks that can move into the excavation and are therefore kinematically capable of failure.

It is difficult to represent the three-dimensional situation described above on a two-dimensional diagram. The whole stereographic projection enables the user to make such a representation in a way that allows the removable blocks to be identified. Figure 2.5 shows three example discontinuities that will be intersected by tunnel excavation surfaces. The regions of the projection that are defined by the plotted discontinuities are actually the bases of the pyramids described in the preceding paragraph, projected onto the stereographic plane. Once these regions are established, excavation surfaces of any orientation can be added to the projection to be examined for potentially unstable blocks created by these discontinuities.

Identification of removable blocks is accomplished by looking for regions that are located entirely within the opening indicated by the plotted excavation surface (Figure 2.6). Once the removable blocks associated with an excavation surface have been identified, each discontinuity making up the block can be checked to see if the block is on the upper or lower side of the discontinuity. Procedures published by Goodman (1976) and by Goodman and Shi (1985) can then be
Figure 2.5 Typical plot of discontinuities on a whole stereographic projection.
Figure 2.6 Identification of blocks from whole stereographic projection. Hatched area one indicates a block that is removable into the excavation (a potential key block). Area two indicates a block that is infinite and is not removable.
used to discern potential failure modes, and to calculate the maximum size that the block could have while still being able to fail into the excavation.
CHAPTER 3 KEYBLOCK STABILITY ANALYSIS METHODS

Once a removable block (keyblock or potential keyblock) has been identified, its stability must be evaluated so that the excavation design can include any ground support measures that are needed to maintain the integrity of the opening. In addition to solving specific problems, methods for assessing the stability of a keyblock can be used profitably in a parametric sense to examine a range of conditions which may affect block behavior. An analytical model with which keyblock stability and displacement behavior may be evaluated is developed in this chapter, and can be applied to two-dimensional and three-dimensional cases. Computer models for these two cases are given in the Appendices. The models are used in subsequent chapters to back-analyze field observations and physical model investigations of keyblock failures.

Approaches for Evaluating Keyblock Stability

Keyblock displacement and stability pose a fully three-dimensional problem, just as block identification poses a three-dimensional problem. In general, block behavior is influenced by the shear strength of the discontinuities forming the keyblock, the initial stresses acting on the block faces, the changes in stresses that occur as the block displaces, and the block geometry. Stress changes are a function of discontinuity dilatancy, normal and shear stiffness, and block displacement. Keyblock geometry includes block height and width, block location around the excavation perimeter, displacement
direction, and block size with respect to the size of the opening. Inspection of Table 3.1, which lists previous analytical approaches, reveals that not all of these factors that influence keyblock stability have been addressed by earlier models.

Different approaches have been taken by the models listed in Table 3.1 in the ways in which they evaluate keyblock behavior. Beyond whether a model treats keyblocks in two dimensions or in three dimensions, there is a significant difference in whether or not a model provides for adjustments in stresses as the block moves towards failure. Models that do not include stress changes typically do not include discontinuity stiffnesses or dilatancy, or keyblock displacement. Since some displacement is usually required in order to mobilize shear stresses along a dilatant discontinuity to help resist block failure, these models do not evaluate keyblock behavior under realistic shear stress conditions. Such models are conservative in their approach to ground support needs. Analytic models that do account for stiffness use different methods of solution; some calculate keyblock displacement corresponding to a specified set of stress conditions, while others calculate the stresses and resultant forces caused by a specified displacement. In either case, the problem is examined parametrically to see if equilibrium will be reached under in situ conditions.
### Table 3.1 Analytic Models for Keyblock Stability

<table>
<thead>
<tr>
<th>Investigator</th>
<th>Block Geometry</th>
<th>Initial Stresses(1)</th>
<th>Discontinuity Stresses</th>
<th>Dilatancy</th>
<th>Stiffness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benson et al, 1971</td>
<td>2D</td>
<td>not used</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Cording et al, 1971</td>
<td>2D</td>
<td>assumed(2)</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Goodman, 1976</td>
<td>3D</td>
<td>not used</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Hoek, 1977</td>
<td>3D</td>
<td>not used</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Croney et al, 1978</td>
<td>3D</td>
<td>assumed(2)</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Bray, 1979(3)</td>
<td>2D</td>
<td>tangent to opening</td>
<td>no</td>
<td>linear</td>
<td></td>
</tr>
<tr>
<td>Coric, 1979</td>
<td>3D</td>
<td>assumed(2)</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Hoek &amp; Brown, 1980</td>
<td>3D</td>
<td>not used</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Warburton, 1981</td>
<td>3D</td>
<td>not used</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Crawford, 1982</td>
<td>2D</td>
<td>tangent to opening</td>
<td>no</td>
<td>linear</td>
<td></td>
</tr>
<tr>
<td>Goodman et al, 1982</td>
<td>3D</td>
<td>tangent to opening</td>
<td>yes</td>
<td>linear</td>
<td></td>
</tr>
<tr>
<td>Zhu &amp; Zhizhong, 1982</td>
<td>2D</td>
<td>assumed(2)</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Crawford &amp; Bray, 1983</td>
<td>2D</td>
<td>tangent to opening</td>
<td>no</td>
<td>linear</td>
<td></td>
</tr>
<tr>
<td>Elsworth, 1983(4)</td>
<td>2D</td>
<td>2D hydrostatic</td>
<td>yes</td>
<td>linear</td>
<td></td>
</tr>
<tr>
<td>Zhifa, 1983</td>
<td>3D</td>
<td>assumed(2)</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Belytschko et al, 1983</td>
<td>2D</td>
<td>assumed(2)</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Boyle, work in progress</td>
<td>3D</td>
<td>tangent to opening</td>
<td>yes</td>
<td>linear</td>
<td></td>
</tr>
<tr>
<td>Yow, this work</td>
<td>3D</td>
<td>3D</td>
<td>yes</td>
<td>linear or</td>
<td>non-linear</td>
</tr>
</tbody>
</table>

**NOTES:**

1. Initial stresses are those affecting block before displacement. All models include friction and block weight.
2. Initial stresses on block faces assumed from separate stress analysis.
Stability Analysis Assumptions

Figure 1.1d illustrated in two dimensions some of the assumptions that are needed for the methods of analysis developed in this chapter. A tunnel or excavation, circular in section, is sited in a rigid rock mass that contains planar discontinuities. The discontinuities themselves are unhealed and are extensive enough that they intersect to form simple, pyramidal keyblocks. Figure 3.1 shows a typical pyramidal block in three dimensions. The circular tunnel section is appropriate for tunnels excavated by boring machines, and is in some cases a reasonable approximation for tunnels of other shapes.

The circular tunnel section allows the use of the Kirsch solution (Goodman, 1980) for calculating initial stresses in two-dimensional analyses. A solution by Amadei (1982) can be used for initial stresses in three-dimensional analyses. The solutions are based on elastic theory; these procedures can be used in numerical models for an expedient estimate of initial stresses around the excavation prior to any displacements along discontinuities. Other investigators have used finite element codes to calculate initial stress values (e.g., Croney et al., 1978), but this makes the analysis more cumbersome to use in evaluating a range of in situ conditions.

Finally, in order to be able to define discontinuity deformation behavior, the rock mass is assumed to deform as a system of springs of varying stiffnesses. The rock that forms the keyblock and its
Figure 3.1 Typical geometry of pyramidal keyblock.
surroundings is taken to be very stiff in comparison to the
discontinuities. All displacement is thus assumed to occur along
discontinuities. A spring system is illustrated in Figure 3.2 for
deformation of a jointed rock in one dimension. Such a conceptual
approach is described by Goodman (1976 and 1980); its application in
keyblock behavior models will be developed below in the discussion of
discontinuity characteristics.

**Keyblock Instability and Displacement**

As sketched in Figure 3.3, keyblocks can fail in a translational
mode either by falling or by sliding. Rotational block failure such
as toppling is beyond the scope of this work. If a keyblock falls,
separation occurs on all of the keyblock faces simultaneously.
Alternatively, sliding can occur on either one or two block faces, and
allows these faces to remain in contact during failure.

There are practical limits on the displacement that can be
expected of a keyblock before either a condition of stability is
reached under prevailing conditions, or the keyblock completely
fails. These limits can be used as a practical check on the
performance of the keyblock behavior model to detect unrealistic
results caused by errors in the data or the computations. The lower
limits of keyblock displacement correspond to the elastic deformation
calculated for a similar excavation in continuous rock.
Two-dimensional plane strain equations for computing radial
defformation of a circular tunnel in an elastic rock without
normal load

\[ \frac{1}{2} S \]

\[ \frac{1}{2} S \]

rock specimen with a discontinuity

spring system

\[ k = \frac{2E}{S} \]

\[ k = k_n \]

\[ k = \frac{2E}{S} \]

Figure 3.2 Representation of rock mass by system of springs of varying stiffness. The discontinuities in the rock mass are assumed to have spacing \( S \) and normal stiffness \( k_n \). The intact rock has an elastic modulus \( E \).
Figure 3.3 Keyblock failure by free fall (all faces separate) and by sliding (one or two faces remain in contact).
discontinuities can be found in texts by Goodman (1980) and by Obert and Duval (1967). The equation given by Goodman is:

\[ u_r = \frac{S_H + S_V}{4G} R + \frac{S_H - S_V}{4G} R \left[ 4(1 - \nu) - 1 \right] \cos \theta \]  \hspace{1cm} (3.1)

where: \( u_r \) = radial displacement of the tunnel wall
\( S_H \) = horizontal stress, unperturbed by the tunnel
\( S_V \) = vertical stress, unperturbed by the tunnel
\( G \) = shear modulus
\( \nu \) = Poisson's ratio
\( R \) = radius of the tunnel
\( \theta \) = tangential coordinate

Conversely, upper limits on keyblock displacement magnitude can be approximated from estimates of the displacement necessary to mobilize the peak shear strengths of the discontinuities that bound the block. This presumes that the keyblock fails once all available peak shear strength has been surpassed. Barton and Choubey (1977) have suggested that peak strength is reached after a shear displacement of about one percent of the discontinuity length as measured in the direction of sliding. Since discontinuity length will depend on keyblock and excavation geometry, an average length based on the height from the excavation to the block apex should suffice for estimating displacement magnitudes.
Discontinuity Shear Strength

Two discontinuity characteristics, strength and stiffness, must be considered in order to properly calculate the stability of a keyblock. The discontinuity shear strengths limit the shear stresses which can act on block surfaces in contact with adjacent rock, and ultimately govern whether or not the keyblock can be stable as situated. The normal and shear stiffnesses in turn control the changes in discontinuity stresses that are induced by keyblock displacement.

The shear strength of an unhealed discontinuity can be represented as a combination of a friction angle ($\phi^0$) for a smooth rock surface and a dilatancy angle ($\iota^0$) for the discontinuity surface roughness. If the shear strength is assumed to vary linearly with the normal stress, it can be calculated as:

$$\tau_a = \sigma_n \tan (\iota + \phi)$$  \hspace{1cm} (3.2)

In this expression $\tau_a$ is the available shear strength and $\sigma_n$ is the normal stress on the discontinuity. Dilatancy also affects changes in normal stress during displacement, as will be explained below.

Barton and Choubey (1977) have given an expression that represents a nonlinear variation of discontinuity shear strength as a function of normal stress, surface strength, and surface roughness:
\[ \tau_a = \sigma_n \tan(JRC \log_{10}(\frac{\text{JCS}}{\sigma_n}) + \phi_r) \]  

(3.3)

JRC is the joint roughness coefficient, which is a measure of surface roughness. JCS is the joint compressive strength, which is a measure of the strength of the asperities that make the discontinuity surface rough. \( \phi_r \) is a residual friction angle that Barton and Choubey relate empirically to \( \phi \). Note that the nonlinear equation can be divided into a dilatancy component and a friction component. The dilatancy component becomes smaller as the normal stress increases, reflecting failures of asperities during shear.

Figure 3.4 plots shear strength as a function of normal stress for both the linear and nonlinear strength equations for typical rock discontinuity properties. Both approaches are available in the keyblock stability models. A choice can be made between the approaches based on the data available for analysis, or based on the magnitude of the expected changes in normal stresses. Normal stresses along the discontinuity planes that form the keyblock will vary from their initial values to zero as discontinuity separation occurs during block failure. The nonlinear strength option can then be selected if indicated by a large magnitude of stress change.

Discontinuity Stiffness

The normal stiffness \( k_n \) of a discontinuity can be defined as the change in normal stress that occurs with a given change in
Figure 3.4 Shear strength calculated using linear and nonlinear approaches.
discontinuity thickness (aperture opening or closure). Similarly, shear stiffness $k_s$ is the change of shear stress that results from an incremental amount of shear displacement.

If a load is applied normal to a discontinuity contained in a rock specimen (Figure 3.2) the resulting deformation in one dimension represents the sum of the deformation of the intact rock and the deformation of the discontinuity (Figure 3.5). A jointed rock mass can thus be thought of in a one-dimensional sense as a sequence of springs that act in series. Assuming isotropic, linear elastic rock, the rock is represented by very stiff, linear spring behavior while discontinuities are represented by initially softer, highly nonlinear springs. Correspondingly, the total rock mass deformability and its components are:

$$\frac{1}{E_m} = \frac{1}{E} + \frac{1}{k_n S}$$  \hspace{1cm} (3.4)

In this equation, $E_m$ is the modulus of deformation curve of the rock mass, $E$ is the elastic modulus of the intact rock, $k_n$ is the normal stiffness of a discontinuity, and $S$ is the average discontinuity spacing.

Inspection of the discontinuity deformation plotted in Figure 3.6 reveals that the curve asymptotically approaches a vertical slope as the normal load increases. The vertical line has been designated by Goodman (1976) as the maximum closure of the discontinuity. Although the nonlinear deformation curve can be represented with a hyperbolic
Figure 3.5 Typical form of normal deformations resulting from axial loading of a rock sample containing a discontinuity.
Figure 3.6  Linear and nonlinear representations of discontinuity normal stiffness.
equation, Figure 3.6 shows how the normal stiffness can be approximated with two straight line segments. This useful engineering approximation is available in the keyblock stability models and requires as input a single value for $k_n$. A more correct approach that is described below is also available (as an option) in the keyblock behavior models.

Bandis et al (1983) compared mathematical models that have been suggested for the nonlinear normal stiffness of joints with the results of stiffness tests conducted on discontinuity samples from different rock types. They propose a hyperbolic model for nonlinear behavior in the following form:

$$
\sigma_n = \frac{AV}{a - bAV}
$$

(3.5)

where $\sigma_n$ = normal stress on the discontinuity, and $AV$ = discontinuity closure.

The coefficients $a$ and $b$ can be obtained by rewriting equation 3.5 and considering the limiting extremes of discontinuity closure at very large and very small normal stresses:

$$
\sigma_n = \frac{1}{\frac{a}{AV} - b} = \frac{1/a}{\frac{1}{AV} - \frac{b}{a}}
$$

(3.6)

Large normal stresses imply that the discontinuity closure $AV$ approaches the maximum closure $V_m$. Therefore, as the normal stress
goes to infinity, $a/b = V_m$. Conversely, low normal stresses imply that $\Delta V$ goes to zero. In this case $1/\Delta V$ becomes much larger than $b/a$, and the latter term can be dropped. Thus, $1/a = k_{ni}$, where $k_{ni}$ is the initial normal stiffness.

Once the coefficients $a$ and $b$ in equations 3.5 and 3.6 are defined, equation 3.6 can be changed into an expression for discontinuity closure:

$$\Delta V = \frac{\sigma_n V_m}{k_{ni} V_m + \sigma_n}$$

(3.7)

The slope of the deformation curve (Figure 3.6) represents the normal stiffness of the discontinuity. Since the stiffness varies nonlinearly with normal stress, it can be found from the derivative of equation 3.5 with respect to joint closure:

$$k_n = \frac{k_{ni}}{(1 - \Delta V/V_m)^2}$$

(3.8)

The preceding hyperbolic model for discontinuity normal stiffness (equations 3.5 through 3.8) was developed by Bandis et al (1983). These equations can, however, be used with equation 3.4 to derive an estimate of $k_n$ under in situ stress conditions before the excavation is created. This estimate can then be used with the hyperbolic stiffness equation, enabling nonlinear discontinuity deformation to be included in the keyblock stability model. In the following equations the maximum discontinuity closure $V_m$ is replaced by the
discontinuity aperture under no-load conditions (negligible normal stress), \( a_0 \).

Equation 3.8 can be rewritten to solve for \( k_{ni} \) as follows:

\[
k_{ni} = k_n - \frac{2k_n\Delta V}{a_0} + k_n\frac{\Delta V^2}{a_0^2}
\]  

(3.9)

Equation 3.7 can be rearranged and \( k_{ni} \) can be substituted from 3.9 to obtain an expression relating normal stress, normal stiffness, discontinuity no-load aperture, and discontinuity closure:

\[
\Delta V(a_0k_n - 2\Delta V k_n + \frac{\Delta V^2}{a_0} k_n + \sigma_n) = \sigma_n a_0
\]  

(3.10)

Finally, equation 3.10 can be modified into a third degree polynomial which can be solved for the ratio of discontinuity closure to no-load aperture under ambient in situ conditions:

\[
k_n(\frac{\Delta V}{a_0})^3 - 2k_n(\frac{\Delta V}{a_0})^2 + (\frac{\sigma_n}{a_0} + k_n)(\frac{\Delta V}{a_0}) - \frac{\sigma_n}{a_0} = 0
\]  

(3.11)

The non-trivial solutions of equation 3.11 include two possible values for the discontinuity closure as a fraction of total maximum closure. Inspection of a representative curve (Figure 3.6) for discontinuity normal deformation reveals that aperture closure will typically be greater than 50% for most initial in situ stresses of interest. If the smaller closure value is selected, \( k_{ni} \) must be very close to the in situ \( k_n \), implying that either the in situ stresses are very low.
(low $k_n$) or that the discontinuity is very stiff (high $k_{n1}$). The larger of the two closure values is arbitrarily used in the stability models to represent the condition of the discontinuity before the underground opening is excavated or keyblock displacement occurs.

Once equation 3.11 has been solved, the ratio of closure to aperture can be used in equation 3.9 to calculate $k_{n1}$. The coefficients $a$ and $b$ can then be determined, and equation 3.6 can be used to obtain normal stresses as the discontinuity forming the keyblock face opens or closes.

Figure 3.6 includes a typical nonlinear curve that relates normal load and displacement. This curve was generated with equation 3.6, which completely defines the curve from values of $a$, $b$, and $k_{n1}$. This is implemented as an optional alternative to the linear approximation for normal stiffness in the keyblock behavior models. It should be noted that the discontinuity aperture used as a reference for joint opening or closing during block displacement is calculated from equation 3.7 using the modified in situ stresses around the underground opening, as explained in the final section of this chapter.

The use of equation 3.11 requires values for the normal stress on the discontinuity, discontinuity normal stiffness at the given stress level, and initial (no-load) discontinuity aperture. Principal stresses that affect the discontinuity can be estimated from overburden depth, or may be available from site investigation measurements. The stresses can then be rotated to find normal and
shear stresses on each discontinuity plane using procedures outlined by Goodman (1980). Initial discontinuity apertures can be assumed, or estimated from an empirical equation provided by Bandis et al (1983). However, the normal stiffness of the discontinuity under the ambient stress conditions is difficult to estimate because of its variability.

If we treat the discontinuities that create keyblocks as members of sets, examine only one set at a time, and assume that the discontinuities of a set have an average spacing, we can rearrange equation 3.4 as follows to obtain an estimate of $k_n$:

$$k_n = \frac{E/S}{(E/E_m)^{\alpha} - 1}$$

Such an estimate represents an average $k_n$ for the discontinuity set under the stress conditions that exist before the underground opening is excavated. Values are required for the average discontinuity spacing, the modulus of the intact rock, and the ratio of the rock mass deformability to the intact rock deformability. Discontinuity spacings and the modulus of deformation for intact rock can be measured during site investigations, but now the difficulty lies in estimating the ratio of the deformation moduli. Heuze (1980) has suggested that the ratio of rock mass to intact rock deformation moduli can often be assumed as 0.4. Alternatively, the in situ modulus can be estimated from correlations with rock mass classification schemes. With estimates or with field measurements, the nonlinear normal stiffness of each block face can now be quantified. As an aside, the nonlinear behavior of $k_n$ as a function
of stress implies that the rock mass becomes increasingly stiff with
depth as long as its behavior is elastic. This is because the
discontinuities are more tightly closed with higher confining
stresses. This effect is itself nonlinear, so that the change in rock
mass stiffness should be most noticeable as one excavates downward
from the ground surface, and should become imperceptible at great
depth. Destressed zones around excavations would mask these
observations.

Discontinuity shear stiffness is used in the keyblock behavior
model to modify the shear stresses acting on the keyblock faces that
are created by discontinuities. Before any block displacement occurs,
the stresses on the block faces are determined by the stress field
around the underground opening. When keyblock displacement commences,
the shear stresses increase on each bounding discontinuity in accord
with its shear stiffness until the maximum available shear strength is
reached. However, the normal stress is usually dropping as
displacement occurs, and so the peak available strength may be reached
almost immediately. Either of two approaches can be used to represent
the shear stiffness, corresponding to the linear and nonlinear
definitions of normal stiffness developed above.

For the linear representation, a constant ratio of shear stiffness
to normal stiffness can be used in the model. This approximation for
shear deformation is a constant stiffness model (Figure 3.7) that
assumes elastic behavior as described by Goodman (1976). Goodman et
al. (1982) examined a range of linear stiffness ratios in an analysis
Figure 3.7  Constant stiffness and peak displacement (variable stiffness) representations of shear stiffness.
of a keyblock subjected to a tangential stress field in a planar excavation roof. They found that the stability of the keyblock was less sensitive to the stiffness ratio than to other parameters such as shear strength.

Barton and Choubey (1977) have suggested the following equation for calculation of shear stiffness \( k_s \) from some of the same parameters that are used in the nonlinear representations of discontinuity shear strength and normal stiffness:

\[
k_s = \frac{100}{L} \sigma_n \tan (JRC \log_{10} \left( \frac{JCS}{\sigma_n} \right) + \phi_n)
\]

(3.13)

This equation is based on observations by Barton and Choubey that peak shear strength is reached when shear displacement has reached about one percent of the discontinuity length \( L \) measured in the direction of sliding. Discontinuity length will of course depend on block and excavation geometry. Equation 3.13 is used in the keyblock behavior models in conjunction with the nonlinear normal stiffness and shear strength equations.

Before leaving this discussion of normal and shear stiffness, it should be pointed out that discontinuity surfaces can apparently suffer irreversible changes during repeated cycles of normal loading if the stresses are sufficiently high. This is seen as hysteresis in plots of load versus deformation in the work of Bandis et al (1983). Damage to discontinuity surfaces at high stresses causes some of the joint closure to be unrecoverable as indicated by Figure 3.8. If an
Figure 3.8 Effect of unrecoverable closure on discontinuity normal stiffness.
estimate of damage can be quantified, it could be incorporated in the nonlinear normal stiffness computations by shifting the stiffness curve. This technique is not used in the keyblock behavior models because compressive stresses that are high enough to cause such a significant alteration of discontinuity behavior are also high enough that the assumption regarding the relative deformation of the intact rock and the discontinuities is no longer valid.

Load and Stress Conditions

Three different types of stresses and forces that influence keyblock stability are included in the keyblock behavior models. These are the stresses in situ as modified by the presence of the underground opening, the changes in the stress field around the opening that result from temperature changes, and the weight of the block itself. The block weight is a matter of rock density and the block volume calculated from the geometry of a given problem, and so will be discussed below as part of the summation of forces acting on the block. Fluid pressures within the discontinuities are not included in this analysis, but can be added using either assumed or calculated pressure distributions for each block face. Ground support forces can also be added to the force summation equations; they are not included here because they do not enter into the more fundamental problem of the behavior of an unsupported keyblock.

Stress in situ around an underground circular opening can be calculated using closed form expressions from elastic theory. In two
dimensions, the stresses at a point can be computed as a function of location using the Kirsch solution:

\[
\sigma_r = \frac{S_Y + S_H}{2} \left(1 - \frac{R^2}{r^2}\right) + \frac{S_H - S_Y}{2} \left(1 - 4 \frac{R^2}{r^2} + 3 \frac{R^4}{r^4}\right) \cos 2\theta + p \frac{R^2}{r^2} \tag{3.14}
\]

\[
\sigma_\theta = \frac{S_Y + S_H}{2} \left(1 + \frac{R^2}{r^2}\right) - \frac{S_H - S_Y}{2} \left(1 + 3 \frac{R^4}{r^4}\right) \cos 2\theta + p \frac{R^2}{r^2} \tag{3.15}
\]

\[
\tau_{r\theta} = \frac{S_Y - S_H}{2} \left(1 + 2 \frac{R^2}{r^2} - 3 \frac{R^4}{r^4}\right) \sin 2\theta \tag{3.16}
\]

where \(r, \theta\) = radial and tangential coordinates

\(R\) = hole radius

\(S_H\) = horizontal stress, unperturbed by tunnel

\(S_Y\) = vertical stress, unperturbed by tunnel

\(p\) = internal pressure

\(\sigma_r, \sigma_\theta, \tau_{r\theta}\) = radial, tangential, and shear stress at point \(r, \theta\)

Daemen (1983) rotated these two-dimensional stresses into the normal and shear stresses affecting a discontinuity that passes through the point (see Figure 3.9 for the problem geometry):
Equations 3.14 through 3.18 are used in the two-dimensional keyblock behavior model to calculate the stresses that initially act on the block at a series of points along each face. The way in which these stresses are adjusted for displacement is discussed in the next section of this chapter. Elsworth (1983) extended two-dimensional keyblock stability methods to include a hydrostatic stress field around a circular underground opening; however, he used a stress solution from elastic theory for thick-walled cylinders rather than the Kirsch solution.

For three-dimensional problems, most previous investigators have assumed that the block under analysis is located in a planar excavation surface, and that the stresses are tangential to the opening (see Table 3.1). A generalized plane strain solution developed by Amadei (1982) is available that allows the three-dimensional stress tensor to be calculated as a function of location around a cylindrical opening in an arbitrary in situ stress field. This solution has been modified for the three-dimensional keyblock behavior model, and is used to compute the stresses acting on a grid of points representing each block face. These initial stresses are then adjusted for the effects of keyblock displacement as described below.
Figure 3.9 Geometry used in Kirsch two-dimensional solution for stresses around a circular hole.
The stresses around an underground opening may change significantly during the operational life of an opening due to variations in rock mass temperature (examples are given in Chapter 6). These stress changes are induced by the thermal expansion or contraction of the rock. This was recognized and examined in the context of its effects on stress measurements by Stephens and Voight (1982), but their emphasis was primarily on stresses in a borehole wall. Zudans et al (1965) have published equations based on theories of elasticity with which components of stress changes caused by thermal loading or unloading within a thick-walled cylinder may be calculated. These equations are for steady state heat transfer, where the temperature is constant at some outer radius b. This represents an extreme case which would bound transient temperature conditions.

\[
\sigma_r = \frac{E \alpha \Delta T}{2(1 - \nu)} \ln \frac{b}{R} \left[ \ln \frac{b}{r} + \frac{R^2}{b^2 - R^2} (1 - \frac{b^2}{r^2}) \ln \frac{b}{R} \right] \tag{3.19}
\]

\[
\sigma_\theta = \frac{E \alpha \Delta T}{2(1 - \nu)} \ln \frac{b}{R} \left[ -1 + \ln \frac{b}{r} + \frac{R^2}{b^2 - R^2} (1 + \frac{b^2}{r^2}) \ln \frac{b}{R} \right] \tag{3.20}
\]

\[
\sigma_Z = \frac{E \alpha \Delta T}{2(1 - \nu)} \ln \frac{b}{R} \left[ -1 + 2 \ln \frac{b}{r} + \frac{2R^2}{b^2 - R^2} \ln \frac{b}{R} \right] \tag{3.21}
\]

where: \( r, \theta \) = radial and tangential coordinates as before
\( Z \) = direction parallel to tunnel axis
\( E \) = modulus of elasticity
\( \alpha \) = coefficient of linear thermal expansion
\[ \Delta T = \text{temperature difference between tunnel wall at radius } R \]
\[ \text{ and rock at radius } b \]
\[ v = \text{Poisson's ratio} \]

As implemented in the two-dimensional and three-dimensional keyblock stability models, the approximation of thermal stresses arbitrarily assumes that the radius \( b \) at which temperature is constant is ten times the excavation radius \( R \). This factor of ten can be readily changed if better temperature distribution data are available.

**Formation of Keyblock Stability Equations**

Evaluation of the behavior and stability of a keyblock requires that the forces affecting the block be summed into a resultant that can be evaluated for magnitude and direction. The factors that enter into such a force summation have been described in preceding sections of this chapter. The forces included are the weight of the keyblock and the normal and shear stresses numerically integrated over each block face that they act upon. These normal and shear stresses are a function of their location on the block surfaces, the initial stresses, keyblock weight, keyblock displacement, and temperature changes. Procedures for computing most of these quantities have been outlined above; they will now be combined to formulate a method for stability analysis. Block weight is brought into the calculation when the stresses are summed into resulting forces; this is not strictly correct. The error that it causes is discussed in the second section of Chapter 5, below.
Only the three-dimensional model will be described since the two-dimensional model represents a simpler application of the same principles. The method calculates a total resultant force in the tunnel coordinate system as a function of keyblock displacement, instead of the opposite approach of finding the displacement that results from a combination of forces. This resultant force can then be examined for a number of specified displacements (1) to see if equilibrium is reached as the block slips, and (2) to see if any excess resisting forces are available to provide a margin of safety against failure for stable blocks.

To begin an analysis of keyblock behavior, the initial (pre-displacement) stresses on the keyblock faces must be calculated. Three sets of coordinate systems are needed as listed in Table 3.2 and shown in Figure 3.10. The unperturbed in situ stresses that exist prior to construction are referenced at first to the global coordinate system, but are rotated into the tunnel coordinate system in which the keyblock resides using Goodman's (1980) procedures. Since the stresses are not uniformly distributed over each block face once the opening is excavated, a grid of points (or nodes) is set up for each discontinuity face of the keyblock (Figure 3.11) in order to allow numerical integration of the stresses into forces. The stresses as modified elastically by the cylindrical excavation are calculated for the grid points using the tunnel coordinate system. Each face has a coordinate system of its own, though, which is based on the normal to the discontinuity that creates the face and the projection of the displacement vector onto the face (Table 3.2). The stress tensor at
Figure 3.10 Coordinate systems used in three-dimensional block analyses.
Figure 3.11 Typical grid of points used to calculate initial stresses for each keyblock face.
Table 3.2 Coordinate Systems Used in Three-Dimensional Keyblock Behavior Analysis

<table>
<thead>
<tr>
<th>System</th>
<th>+X axis</th>
<th>+Y axis</th>
<th>+Z axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global</td>
<td>East</td>
<td>North</td>
<td>Vertical up</td>
</tr>
<tr>
<td></td>
<td>Horizontal</td>
<td>Tunnel Axis</td>
<td>Through Back Perpendicular to X and Y</td>
</tr>
<tr>
<td></td>
<td>Through Springline</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tunnel</td>
<td>Horizontal</td>
<td>Tunnel Axis</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Through Springline</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Vertical up</td>
<td></td>
</tr>
<tr>
<td>Keyblock(1)</td>
<td>Transverse to</td>
<td>Normal to Face</td>
<td>Parallel to Projected Displacement Direction</td>
</tr>
<tr>
<td>Face</td>
<td>Projected Displacement</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Direction in Face</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

NOTE: (1) One face system is used for each keyblock face that is not an excavation surface.
each grid point is rotated from the tunnel coordinate system to the
face coordinate system, and any stress changes caused by thermal
effects are rotated and imposed on these initial stresses at this
time. Once these computations are complete, the initial normal stress
and initial shear stress (for zero block displacement) are available
for each grid point.

If nonlinear discontinuity deformation is to be accounted for,
some intermediate calculations are needed. Once the face grids are
set up, but before the in situ stresses are adjusted for the effects
of the excavation, the normal stresses on the discontinuities are
found and used with equation 3.11 to obtain estimates of aperture
closures. These closures represent average values for each
discontinuity under ambient in situ stresses, and are used in equation
3.9 to compute initial normal stiffnesses. The initial normal
stiffnesses are used to set up a hyperbolic equation for normal
stiffness for each discontinuity. The effects of the underground
opening on the stress field are then calculated as mentioned above;
the stresses usually increase because of the stress concentrations
around the tunnel. These increases in normal stress are used with the
hyperbolic equations to adjust the normal stiffness and aperture value
at each grid point prior to the start of keyblock displacement. Note
that each part of the calculational grid may be at a different point
on the hyperbolic curve for that discontinuity, and that even though
the rock is assumed to be "rigid," the discontinuity is at slightly
varying degrees of closure along its length prior to block
displacement because of the distribution of normal stresses.
The next step in the process is to adjust the initial stresses for an increment of displacement in a specified direction. This is done in a sequence of operations: First, the displacement is resolved into its net effect on each discontinuity grid point in terms of an aperture change (normal) component and a sliding (shear) component. If the angle between the displacement vector and the discontinuity plane is $\theta$, and the face has a dilatancy angle ($i^0$), then the amount of aperture opening $\Delta V$ that occurs with displacement $D$ is:

$$\Delta V = D (\sin \theta - \cos \theta \tan i)$$ \hspace{1cm} (3.22)

In this equation the $\sin \theta$ term represents the movement of the block away from the average discontinuity surface, while the $\cos \theta \tan i$ term is a dilatancy effect that reduces the opening of the surface. Aperture opening is positive and closure is negative in this equation. The dilatancy angle is specified for each face for linear analyses, and is extracted from the shear strength equation (Equation 3.3) for nonlinear analyses.

Similarly, shear slip along each average discontinuity surface is given by:

$$\Delta U = D \cos \theta$$ \hspace{1cm} (3.23)

The next operation in the sequence is to adjust the normal stresses at each grid point. For the nonlinear analysis, this is done by entering the hyperbolic stiffness equations with the adjusted aperture values
and calculating new normal stresses for each grid point. For a linear analysis, the initial normal stress ($\sigma_{n1}$) for each point is modified as follows:

$$\sigma_n = \sigma_{n1} - k_n \Delta V$$ (3.24)

With increasing keyblock displacement, the normal stiffness and normal stress follow an unloading path as the block separates from the surrounding rock. The available shear strength also drops because of its relationship to the normal stress, while the shear stresses increase until they reach the level of shear strength that is mobilized. Equation 3.3 or 3.4 is therefore used with the adjusted values for normal stress to determine the available shear strength at each grid point. The shear stress at each point is then set equal to the available shear strength, or the shear stress calculated from shear stiffness and displacement, whichever is less. The shear stress calculated from the initial shear stress $\tau_i$ and the displacement is:

$$\tau = \tau_i + k_s \Delta U$$ (3.25)

During the calculations of block geometry necessary for setting up the grids for each face, the block volume is computed and used to obtain the weight of the keyblock. Face areas are also assigned to each grid point to allow numerical integration of stresses into forces acting normal and parallel to each face. These forces are then rotated into the global coordinate system and summed. Under the
convention of positive being upward in the tunnel Z axis direction, the keyblock weight (W) is a negative force. If the final resultant force (F) is oriented upward, it represents an excess of shear force that must be overcome in order to pull the keyblock out of the surrounding rock mass to the specified level of displacement. If, conversely, F is negative (acting downward) it represents a force which must be resisted by support for the block to be at equilibrium at the given displacement.

The resultant force F can be normalized by dividing by weight W; the expression F/W is a ratio of resultant force to keyblock weight. The sign of the ratio can also be switched so that a positive F/W indicates a support that must be supplied for stability, while a negative F/W implies a negative support, or pullout force. Using this sign convention, the F/W ratio can be related to a factor of safety (FS) by the following equation:

$$FS = 1.0 - \frac{F}{W}$$

(3.26)

It must be remembered that each F/W value for a given keyblock has a specific displacement associated with it. Figure 3.12 plots F/W against displacement for a typical keyblock that might fail by falling. This convention is used because it is conceptually similar to a ground reaction curve.

Inspection of the block reaction curve of Figure 3.12 reveals several interesting points about the displacement behavior of a stable
Figure 3.12 F/W ratio as function of displacement (block reaction curve) for falling keyblock.
keyblock. As the block starts to displace, the block reaction curve (F/W ratio vs displacement) starts out above, rather than at, unity. This reflects an elastic unloading of the discontinuities that occurs during excavation of the opening, and indicates that a force much greater than the block weight would be needed if the block were to be shoved back into a zero displacement condition. Second, F/W drops with increased displacement, and the keyblock reaches equilibrium and presumably stops moving once F/W reaches zero. If additional displacement is specified or allowed, however, the curve continues down until a minimum is reached. After this the peak shear strength of the discontinuities has been exceeded at all points by the mobilized shear stresses, and the curve trends upward until F/W equals unity. At this and larger values of displacement, the block faces have lost all shear strength and are likely to be entirely separated from the surrounding rock, and the entire block weight must be supported.

A plot of F/W versus displacement for a marginally unstable keyblock has a form similar to that of Figure 3.12, except that F/W remains positive for all displacement values. This indicates that the block never reaches a stable condition without support. Completely unstable keyblocks present a curve that never goes below unity, while safe keyblocks cease moving at an F/W ratio of zero or less (factor of safety greater than one). Figure 3.13 shows typical block reaction curves for all three types of blocks for comparison.
Figure 3.13 Block reaction curves for stable, marginal and unstable falling keyblocks.
CHAPTER 4  FIELD OBSERVATIONS OF KEYBLOCKS

The methods developed in Chapter 3 for evaluating the stability and displacement behavior of keyblocks have been implemented in numerical models that are presented in Appendices 1 and 2. Field observations of keyblock disposition (fallen or apparently stable) have been back-analyzed with the three-dimensional model; the field observations and the results of their analyses are described in this chapter. A suite of laboratory studies of blocks performed by Crawford and Bray (1983) with physical models were back-analyzed with the two-dimensional model; these results are also described. These observations and tests were back-analyzed in order to check the validity of the numerical models.

Observations at the Nevada Test Site

Keyblock observations were made in two tunnel complexes at the Nevada Test Site. Figure 4.1 provides a location map for the area. Observations were made in tunnels in a granitic rock mass, the Climax stock, and in welded tuff, under Rainier Mesa. The tunnels in granite were the Spent Fuel Test-Climax (SFT-C) drifts and the Piledriver event access drift (Figure 4.2), while the tunnels in tuff were part of the G-Tunnel complex.

The ribs and crown of over 800 linear feet of tunnel were inspected in the Climax stock. Nine relatively large keyblocks were observed in a one hundred foot long segment of the Piledriver access
Figure 4.1 Location map for the Nevada Test Site (after Langkopf and Eshom, 1982).
Figure 4.2 Layout of drifts at 420 meter level of Climax stock, NTS.
drift, but only four were found in six hundred feet of the Spent Fuel Test-Climax drifts. These four were obscured by steel mesh which hampered measurements as described below; these four blocks were therefore not back-analyzed for stability. Many other keyblocks were seen that were too small to be measured, but the relative frequency of blocks between the two drift settings remained about the same. This variation in keyblock frequency is a direct result of drift orientation; the SFT-C drifts were oriented essentially parallel and perpendicular to the steeply-dipping joint sets (Wilder and Yow, 1984) and thus did not encounter very many pyramid-shaped blocks. In contrast, the Piledriver drift intersected the sets at more oblique angles, and consequently experienced many more failures of pyramidal blocks per hundred feet of drift. This variation in block occurrence is discussed further below.

It should be noted at this point that the Piledriver drifts were excavated prior to the Piledriver weapons effects test, whereas the SFT-C drifts were not. However, both sets of drifts have since been subjected to ground shaking caused by other tests without consequent block falls; keyblocks that were marginally stable were removed during construction or during the Piledriver experiment. The Piledriver drift has apparently been stable for over ten years.

The blocks in the Piledriver drift generally had one of two typical pyramidal shapes based on their location around the drift perimeter. Blocks that occurred at the tunnel springline were of the
form shown in Figure 4.3; these are called shoulder blocks by Ward (1978). Other blocks occurred in the crown (Figure 4.4) or rib of the tunnel. Of these, two of the blocks in the ribs were noticeably truncated (i.e., a stub of the keyblock remained in the apex of the pyramidal cavity).

Over 6000 feet of tunnel in tuff in the NTS G-Tunnel complex were similarly inspected (Figure 4.5). However, most of this footage had been heavily shotcreted or was supported with steel ribs and continuous timber lagging, and only one fallen keyblock was mapped (Figure 4.6) in about 300 feet of open tunnel that was in welded tuff. Block face orientations in G-Tunnel were determined from geometric measurements rather than compass readings because of remnant magnetism within the welded tuff units.

Techniques Used for Field Observations

A total of fourteen relatively large keyblocks, some fallen and some yet in place, were thus observed in tunnels at the Nevada Test Site (Table 4.1). Four of the fallen blocks were in excavation surfaces that were covered with steel chain link mesh anchored by rock bolts. The heavy steel mesh effectively prevented measurement of block dimensions and orientations, and back-analysis of these blocks was thus not possible. Of the remaining ten keyblocks, one was still in place but had apparently displaced slightly. Since the block had not fallen, its dimensions had to be estimated.
Figure 4.3 Typical keyblock at springline (shoulder) of Piledriver drift.
Figure 4.4 Typical keyblocks in crown of Piledriver drift.
Figure 4.5 Location of observations in G-tunnel (map after Langkopf and Eshom, 1982.)
Figure 4.6 Keyblock in rib of G tunnel.
Table 4.1 Summary of Field Observations of Keyblocks

<table>
<thead>
<tr>
<th>Location</th>
<th>Inspected Length of Tunnel</th>
<th>Mapped Keyblocks(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piledriver Drift</td>
<td>100 feet</td>
<td>9</td>
</tr>
<tr>
<td>Spent Fuel Test-Climax</td>
<td>700 feet</td>
<td>4(3)</td>
</tr>
<tr>
<td>G-Tunnel</td>
<td>300 feet(4)</td>
<td>1</td>
</tr>
</tbody>
</table>

NOTES: (1) All tunnels are at the Nevada Test Site (Figure 4.1).
(2) Many keyblocks were seen that were too small to obtain measurements needed for back-analysis of stability.
(3) Complete measurements precluded by steel mesh on rock surface.
(4) Most of the remainder of G-Tunnel was obscured by steel sets, timber lagging, or shotcrete.
The nine keyblocks which had fallen and were accessible were measured in several ways (Table 4.2). The orientations of the block faces (formed by discontinuities) were measured with a Brunton compass. In some cases the location of the block prevented compass measurement of one or more face orientations, so a large adjustable triangle was used to measure angles between adjacent faces. These angular measurements were used to determine missing face orientations using stereonet constructions. A steel tape was used to measure the dimensions of each block face, and each block was photographed from several perspectives. The location of each block was recorded by tunnel survey station, and the height of each block above the tunnel floor was noted. Finally, a carpenter's contour gauge was used to obtain a typical roughness profile of a sample length of each discontinuity in the direction of block displacement, and a Schmidt hammer was used to obtain hammer rebound readings from which the joint compressive strength could be estimated.

Of the various types of measurements made for each block, the most difficult were the face orientation and hammer rebound measurements. This was because the work was often overhead, and because the cavities left by the fallen keyblocks had to be relatively large to allow measurements of face orientation and Schmidt hammer rebound. The latter was especially true because the hammer had to be perpendicular to the face being measured. Many smaller keyblocks necessarily had to be ignored because of their size, and of the accessible keyblocks, complete data sets were obtained for only six. Three other blocks
### Table 4.2 Keyblock Measurements

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Technique</th>
</tr>
</thead>
<tbody>
<tr>
<td>Keyblock face orientations</td>
<td>Brunton compass and adjustable triangle</td>
</tr>
<tr>
<td>Keyblock face dimensions</td>
<td>Steel tape measure</td>
</tr>
<tr>
<td>JRC</td>
<td>Carpenter's profile gauge</td>
</tr>
<tr>
<td>JCS</td>
<td>Estimated from Schmidt hammer readings</td>
</tr>
<tr>
<td>$\phi_r$</td>
<td>Estimated from rock core tilt test results</td>
</tr>
</tbody>
</table>
required shear strength estimates from nearby discontinuities of the
same set, and one block proved to be in too awkward of an overhead
location to allow adequate measurements. Even though some of the
blocks could not be back-analyzed because of a lack of data, keyblock
occurrence in itself was important because of its relationship to the
excavation orientation.

The roughness profiles and Schmidt hammer readings were used to
estimate discontinuity shear behavior with procedures suggested by
Barton and Choubey (1977). The roughness profiles were visually
compared with profiles published by Barton and Choubey to estimate
joint roughness coefficient (JRC). The JRC values were used with
joint compressive strength values derived from the Schmidt hammer
readings in equation 3.3 for shear strength and equation 3.13 for
shear stiffness in the back-analysis work. No-load aperture values
were estimated from the JRC values with a procedure suggested by

Deere and Miller (1966) correlated Schmidt hammer readings with
several different rock properties in an extensive study. However, the
work by Deere and Miller used the Schmidt hammer in a vertical
downward loading direction. All of the readings obtained in the field
work had to be corrected to this common basis because hammer
orientation varied according to which face was being tested. Figure
4.7 was therefore prepared from Barton and Choubey's table of hammer
reading correction values to make these adjustments. In the Nevada
Test Site field work, hammer readings on the keyblock cavity faces and
Figure 4.7 Correction curves for Schmidt hammer orientation.
on freshly sawn rock surfaces were used to estimate the compressive strengths of the discontinuities and the rock matrix, respectively. The joint compressive strength (JCS) suggested by Barton and Choubey (1977) represents the strength of discontinuity surfaces.

In some cases the keyblock geometry would not allow the hammer to be used perpendicular to a block cavity face. In these situations JCS values were estimated from adjacent parts of the same discontinuity, or from nearby members of the same discontinuity set. Interestingly, the Schmidt hammer readings on clean, freshly cut rock surfaces indicated a compressive strength for the Climax granite matrix of 33,000 psi (dry); testing of laboratory specimens found the strength to be 29,000 ± 4000 psi (Carlson et al, 1980).

Field Observations of Keyblock Occurrences

Figure 4.8 contours the lower hemisphere poles of discontinuities mapped in the Spent Fuel Test drifts with the poles of keyblock faces superimposed as asterisks. Figure 4.10 presents similar contours of data for the G-Tunnel rock mechanics drift from Langkopf and Eshom (1982). In most cases the block face orientations measured in this work agree with discontinuity orientations from previous mapping. This indicates that block identification techniques listed in Chapter 2 can be used to define the general forms (Figure 4.9, for example) of most blocks that would be encountered by an excavation. However, some mapped blocks were created by fringe members of the joint sets. Block identification should therefore be done with caution during design.
Figure 4.8 Lower hemisphere concentrations of poles of fractures mapped in Spent Fuel Test-Climax drifts. 1% contour interval.
Figure 4.9a Keyblock analysis for north shoulder of Piledriver drift.
Figure 4.9b Keyblock analysis for rib of Piledriver drift.
Figure 4.9c Keyblock analysis for north shoulder of Spent Fuel Test canister drift.
Figure 4.9d Keyblock analysis for rib of Spent Fuel Test canister drift.
Figure 4.10 Lower hemisphere concentrations of poles of fractures mapped in G-tunnel rock mechanics drift (after Langkofl and Eshom, 1982). 1% contour interval.

* = pole of keyblock cavity face
Much of the difference in keyblock frequency between the Climax stock and G-Tunnel is apparently due to the difference in jointing between the two sites. At the Climax stock, Wilder and Yow (1984) identified four pervasive joint sets, four less prominent joint sets, and three shear sets. In contrast, Langkopf and Eshom (1982) identified only steeply dipping sets at Sandia National Laboratories' rock mechanics drift in G-Tunnel, near where the keyblock was mapped. Table 4.3 summarizes the nature of the keyblocks mapped in each setting. It will be noted that three discontinuities intersect to create each observed keyblock. In most cases in the Climax granite, a low angle (shallow dip angle) joint combined with two steeply-dipping features to form and release the block. Inspection of Figure 4.10 reveals that the G-Tunnel tuff has no such set of low angle discontinuities to provide block release, and the one keyblock mapped in G-Tunnel indeed was formed by three steeply-dipping fractures. However, fracturing does change from unit to unit within the tuff, and a considerable amount of scatter can be seen in Figure 4.10. Identification of keyblocks from the tuff discontinuity data may thus not be as straightforward as for the granite, where the jointing is more completely defined.

The contrast in keyblock frequency between the SFT-C drifts and the Piledriver drifts is important because it illustrates the effect of tunnel orientation on keyblock occurrence. As listed in Table 4.1, the SFT-C drift observations included four keyblocks in 700 feet of drift, while the Piledriver drift observations included nine keyblocks in less than 100 feet of drift. Both sets of excavations are located
### Table 4.3 Summary Descriptions of Measured Keyblocks

<table>
<thead>
<tr>
<th>Tunnel</th>
<th>Station/Location</th>
<th>Description of Keyblock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piledriver Drift</td>
<td>2 + 22/shoulder of north rib</td>
<td>formed by one low angle and two near-vertical discontinuities; block fallen</td>
</tr>
<tr>
<td>Piledriver Drift</td>
<td>2 + 45/shoulder of north rib</td>
<td>formed by one low angle and two near-vertical discontinuities; block fallen</td>
</tr>
<tr>
<td>Piledriver Drift</td>
<td>2 + 05/south side of tunnel crown</td>
<td>formed by one low angle and two steeply-dipping discontinuities; block fallen</td>
</tr>
<tr>
<td>Piledriver Drift</td>
<td>2 + 22/in crown</td>
<td>formed by three steeply-dipping discontinuities; block fallen from sagging rock beam</td>
</tr>
<tr>
<td>Piledriver Drift</td>
<td>2 + 22/in crown</td>
<td>formed by three steeply-dipping discontinuities; block still in place</td>
</tr>
<tr>
<td>Piledriver Drift</td>
<td>2 + 55/south rib</td>
<td>formed by one low angle and two near-vertical discontinuities; block fallen but truncated</td>
</tr>
<tr>
<td>Piledriver Drift</td>
<td>2 + 60/south rib at springline</td>
<td>formed by three steeply-dipping discontinuities; located in intensely fractured zone</td>
</tr>
<tr>
<td>Piledriver Drift</td>
<td>2 + 80/south rib</td>
<td>formed by one low angle and two steeply-dipping discontinuities; block fallen but truncated</td>
</tr>
<tr>
<td>G-Tunnel</td>
<td>north rib of extensometer drift</td>
<td>formed by three steeply dipping discontinuities; block fallen(1)</td>
</tr>
</tbody>
</table>

**NOTES:** (1) not back-analyzed because large included angle of apex made failure inevitable.
within the Climax stock, and both encountered essentially the same
geologic structure (Wilder and Yow, 1984). The excavations were in
all cases made with drill and blast techniques. The remaining primary
source of block frequency variation is the orientation of the openings
with respect to the geologic structure.

Figure 4.9a shows a whole stereographic projection of the four
predominant joint sets found by geologic mapping at the 420 meter
level in the Climax stock, along with a fifth less dominant set that
was observed to form one face of some of the keyblocks. The reference
circle for this upper hemisphere projection is shown as a dotted
circle. Superimposed as a dashed line on the plot is a plane
representing an excavation surface dipping at 45° to form the north
shoulder of the drift. The area within the dashed circle represents
the rock mass forming the shoulder of the drift, while the area of the
plot falling outside the dashed circle represents the free space
created by the excavation. Blocks are delineated by regions (joint
pyramids) formed by the intersection of discontinuities (solid lines)
on the projection. The blocks that are shaped such that they can
displace into the excavation are designated by asterisks on the
drawing. Figure 4.9b is a similar projection with the dashed line
indicating a vertical rib of the Piledriver drift. The asterisks in
4.9b indicate potential keyblocks in the north rib of the drift.
Figure 4.9c represents the north shoulder of the SFT-C canister drift
and heater drifts, and Figure 4.9d represents the vertical ribs of
these SFT-C drifts. In 4.9c the asterisks mark potential keyblocks in
the north shoulder of the drifts, and in 4.9d they mark keyblocks potentially removable from the north rib of these drifts.

Inspection of the four whole stereographic projections of Figure 4.9 reveals the cause of the differences in keyblock frequency between the SFT-C and Piledriver drifts. Comparing first the plots that represent the north shoulders of the drifts (Figures 4.9a and 4.9c), it is seen that the Piledriver drift encounters six geometric forms of potential keyblocks, while the SFT-C drifts encounter only five. Since small regions on the stereographic projection represent slender keyblocks, one of the possible keyblock forms in the SFT-C drift shoulders is relatively insignificant in size, leaving four main keyblock forms for this excavation surface. This implies that the north shoulder of the Piledriver drift can encounter about half again as many geometric keyblock forms as the north shoulder of the SFT-C drifts. A similar comparison for keyblocks potentially removable from the north ribs of the two drift settings defines five block types for the Piledriver drift, and five for the SFT-C drifts. Again, however, one of the SFT-C keyblock regions is particularly small, leaving four block types for the SFT-C drift north ribs and five for the Piledriver drift north rib.

A second criteria suggested by Shi and Goodman (1982) can also be used to compare keyblock forms in the two drift settings. If corners of the regions indicating blocks come close to the excavation surface, this implies that corresponding lines of intersection between block faces are nearly parallel to the excavation. Such blocks are
relatively thin and do not impose unusually severe requirements on the excavation ground support system, and may in fact fail immediately upon exposure as part of the excavation overbreak. If the four parts of Figure 4.9 are reconsidered in light of this criteria, the north shoulder of the Piledriver drift has two significant keyblock forms and the north shoulder of the SFT-C drifts also has two significant forms. The Piledriver drift north rib has three significant keyblock forms, while the SFT-C drift north ribs have one significant geometric keyblock form. The Piledriver drifts can thus encounter more geometric forms of keyblocks created by the intersecting discontinuities than the SFT-C drifts can, using either criteria for the assessment.

An important aspect of analyzing potentially unstable keyblocks is the size of the block with respect to the tunnel. Whole stereographic projection techniques such as used in Figure 4.9 to identify keyblocks provide only an indication of the geometric shape of each block, and no quantitative definition of keyblock size. Keyblock size is limited by the dimensions of the excavation into which a block can displace. This concept is illustrated by Figures 4.11, 4.12, and 4.13. Figures 4.11a, 4.12a, and 4.13a show a whole stereographic projection for a Piledriver drift shoulder block, crown block, and rib block, respectively. Figures 4.11b, 4.12b, and 4.13b show the corresponding drift sections, maximum possible keyblock sizes, and actual encountered keyblock sizes for these cases. The fact that no keyblocks were encountered having the maximum possible size may be a
Figure 4.11a  Keyblock in north shoulder of Piledriver drift.
Figure 4.11b Keyblock in north shoulder of Piledriver drift.

Note: View is looking S56W along tunnel axis.

Maximum keyblock possible with joint orientations given

Keyblock actually encountered and back-analyzed

144"
Figure 4.12a  Keyblock in crown of Piledriver drift.
Figure 4.12b  Keyblock in crown of Piledriver drift.
Figure 4.13a Keyblock in south rib of Piledriver drift.
Figure 4.13b Keyblock in south rib of Piledriver drift.
result of a limited length of drift checked for keyblocks, or it may be related to the fact that actual discontinuities are not infinite in extent.

Analyses of Keyblock Stability

Table 4.4 summarizes data (from eight keyblocks in the Piledriver drift) used in the back-analyses done with the three-dimensional block behavior model of Appendix 2, and Table 4.5 summarizes the results. Since all that is known of the behavior of the observed blocks is whether or not they have fallen, and no displacement data is available from development of the failure, the analysis focused on evaluating the block stability at the maximum available resistance to failure. In addition to discontinuity properties, an uncertainty in each calculation was the direction of block displacement. A failure mode analysis was therefore performed for each block using procedures published by Goodman and Shi (1985). The results of the failure mode analysis were used for an initial estimate of displacement direction, because some blocks apparently slid on one plane, some slid on two planes, and some separated on all planes during failure. Once the minimum F/W support ratio was found for the initial displacement direction, the direction was varied in order to locate higher and lower minimum F/W ratios. The block was deemed unstable from the model results if all F/W ratios for the range of displacement directions were greater than zero (factor of safety less than unity). Conversely, the block was indicated to be stable if the F/W ratio went
<table>
<thead>
<tr>
<th>Keyblock</th>
<th>Face</th>
<th>Face Strike</th>
<th>Face Dip</th>
<th>JRC</th>
<th>JCS(2) (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 + 22, north shoulder</td>
<td>1</td>
<td>N65E</td>
<td>85SE</td>
<td>4</td>
<td>8300</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>N45W</td>
<td>85SW</td>
<td>6</td>
<td>4900</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>N40W</td>
<td>20NE</td>
<td>5</td>
<td>5000</td>
</tr>
<tr>
<td>2 + 45, north shoulder</td>
<td>1</td>
<td>N65E</td>
<td>85SE</td>
<td>6</td>
<td>9800</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>N50W</td>
<td>88SW</td>
<td>4</td>
<td>7400</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>N40W</td>
<td>22NE</td>
<td>3</td>
<td>10400</td>
</tr>
<tr>
<td>2 + 05, south side of crown</td>
<td>1</td>
<td>N40W</td>
<td>88SW</td>
<td>2</td>
<td>12100</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>N35E</td>
<td>60NW</td>
<td>3</td>
<td>12100</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>N40W</td>
<td>20NE</td>
<td>3</td>
<td>9600</td>
</tr>
<tr>
<td>2 + 22, crown</td>
<td>1</td>
<td>N15E</td>
<td>60SE</td>
<td>2(1)</td>
<td>5000 (1)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>N52E</td>
<td>82NW</td>
<td>2</td>
<td>5000</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>N42W</td>
<td>76NW</td>
<td>2</td>
<td>5000</td>
</tr>
<tr>
<td>2 + 22, crown</td>
<td>1</td>
<td>N15E</td>
<td>60SE</td>
<td>2(1)</td>
<td>5000 (1)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>N52E</td>
<td>82NW</td>
<td>2</td>
<td>5000</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>N45W</td>
<td>85SW</td>
<td>2</td>
<td>5000</td>
</tr>
<tr>
<td>2 + 55, south rib</td>
<td>1</td>
<td>N63W</td>
<td>85NE</td>
<td>1</td>
<td>7700</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>N20E</td>
<td>85NW</td>
<td>3</td>
<td>10100</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>N50W</td>
<td>26NE</td>
<td>4</td>
<td>17200</td>
</tr>
<tr>
<td>2 + 60, south rib</td>
<td>1</td>
<td>N30E</td>
<td>48NW</td>
<td>3(1)</td>
<td>7500 (1)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>N50W</td>
<td>75NE</td>
<td>1</td>
<td>7500</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>N80E</td>
<td>66S</td>
<td>3</td>
<td>7500</td>
</tr>
<tr>
<td>2 + 80, south rib</td>
<td>1</td>
<td>N80E</td>
<td>80N</td>
<td>4</td>
<td>10500</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>N18E</td>
<td>52NW</td>
<td>4</td>
<td>10500</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>N80E</td>
<td>13N</td>
<td>4</td>
<td>6100</td>
</tr>
</tbody>
</table>

**NOTES:**

1. Assumed from nearby discontinuity segments.
2. $\phi_r$ assumed as 30° from tilt tests on core; in situ stresses after Creveling et al. (1984)
<table>
<thead>
<tr>
<th>Keyblock Status</th>
<th>Keyblock</th>
<th>Failure Mode</th>
<th>Analysis Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>fallen</td>
<td>2 + 22, north shoulder</td>
<td>slide on one plane</td>
<td>unstable</td>
</tr>
<tr>
<td>fallen</td>
<td>2 + 45, north shoulder</td>
<td>slide on one plane</td>
<td>unstable</td>
</tr>
<tr>
<td>fallen</td>
<td>2 + 05, south side of crown</td>
<td>slide on two planes</td>
<td>unstable</td>
</tr>
<tr>
<td>fallen</td>
<td>2 + 22, crown</td>
<td>free fall</td>
<td>stable(1)</td>
</tr>
<tr>
<td>in place</td>
<td>2 + 22, crown</td>
<td>free fall</td>
<td>stable</td>
</tr>
<tr>
<td>fallen</td>
<td>2 + 55, south rib</td>
<td>slide on two planes</td>
<td>unstable</td>
</tr>
<tr>
<td>fallen</td>
<td>2 + 60, south rib</td>
<td>free fall</td>
<td>stable(2)</td>
</tr>
<tr>
<td>fallen</td>
<td>2 + 80, south rib</td>
<td>slide on one plane</td>
<td>unstable</td>
</tr>
</tbody>
</table>

**NOTES:**

(1) This block is at the edge of a sagging slab of rock; the stress assumptions of the analysis may not be valid, or the assumed properties may be wrong.

(2) This block is in a zone of intensely fractured rock; the stress and stiffness assumptions of the analysis may not be valid, or the assumed properties may be wrong.
below zero, since a negative $F/W$ represents a force needed to pull the block out (factor of safety greater than unity).

Table 4.5 lists two cases where the numerical model erroneously predicted a fallen keyblock to be stable. As noted in the table, one block was in the edge of a sagging slab of granite, and the rock may well have experienced a significant drop in confining stress. A similar but larger block at the same tunnel station was stable, but was located somewhat back from the edge of the slab. The model assumptions regarding initial stresses were thus probably incorrect for the block at the slab edge. In the second case, the block was in a very intensely fractured zone, and the stiffness assumptions may have been at fault. Both cases also involved assumed shear strength values, which may have been in error.

In general, Table 4.5 indicates a good correlation between numerical model results and field observations, except for the two special cases that were described above. However, these back-calculations provide only a check on the ability of the keyblock behavior model to evaluate block stability. Physical model studies performed and published by Crawford and Bray (1983) were therefore back-analyzed with the two-dimensional model as a check on its ability to calculate pullout forces necessary to destabilize a block.
Figure 4.14a shows the geometry of the plaster model used by Crawford and Bray (1983). The tests modeled two-dimensional wedges subjected to a tangential stress field in the flat roof of an underground excavation. Plaster wedges were fit into slots or cavities in a slab of plaster that represented the rock over the excavation. Horizontal stresses paralleling the excavation surface were then applied to the edges of the slab, and the block was slowly pulled out of place to simulate failure. A proving ring assembly measured the force required to "fail" the block. Blocks of various apex angles were tested by Crawford and Bray at a range of horizontal stress values.

Figure 4.14b shows the approximation of the test geometry that was used with the two-dimensional numerical model of Appendix 1. The numerical model requires a circular excavation, and so a circular opening of very large radius was used along with far-field stress values necessary to create the required pre-displacement stress regime around the block. The initial horizontal stress distribution in the physical model was uniform; the corresponding distribution of tangential stresses in the numerical model varied by less than 2.5 percent from the uniform distribution. Linear discontinuity behavior was assumed with $\phi = 32^\circ$, and $\gamma = 0^\circ$, based on shear strength properties published by Crawford and Bray. Linear stiffness assumptions affect the magnitude of block displacement at failure much
Plaster block subjected to horizontal stresses

Plaster wedge

100 mm x 50 mm thick (3.94" x 1.97")

Figure 4.14a  Geometry of physical model tested by Crawford and Bray (1983)
Figure 4.14b Geometry used with 2D code to back-analyze model

\[ R = 10,000 \text{ mm} = 393.7'' \approx 394'' \]
more strongly than they do the pullout force; these values were assumed as \( k_n = 10,000 \text{ psi/inch} \) and \( k_s = 100 \text{ psi/inch} \).

Symmetric wedges with included apex angles of 60°, 50°, and 40° were back-analyzed; these correspond to models tested by Crawford and Bray (1983). Typical resulting pullout forces are plotted in Figure 4.15 for numerical model results and physical model tests for the 60° included angle. The slope of the line representing the numerical model results matches well the slope of a line drawn through the physical model results. However, the latter is offset downward, indicating a pullout stress at zero confining pressure of about -0.5 MPa (-72.5 psi). Pullout stress is the pullout force divided by the area of the excavation surface formed by the keyblock; in this case the negative sign indicates a required amount of support pressure. The offset indicates a problem with the physical model since the maximum support force for a completely failed block should be the block weight. This equates to a minimum pullout stress of about -0.006 MPa (-0.1 psi). Crawford and Bray noted that there were problems with the seating of the wedge in the surrounding "rock mass" for each test and this may be the source of the offset. Similar offsets were seen in the test results of the physical models for other apex angles. If the data points and line are shifted upward to correct the offset, the back-analyzed line matches the test data nicely.
Figure 4.15 Back-analysis of pullout force measured in physical model by Crawford and Bray (1983).
CHAPTER 5 PARAMETER EFFECTS ON KEYBLOCK STABILITY

Back-analysis of field observations of fallen and stable keyblocks indicates that the three-dimensional numerical model can be used to estimate keyblock stability. Back-analysis of laboratory tests of block pullout force under various confining pressures indicates that the two-dimensional numerical model can be used to estimate pullout force, which is a different way of expressing block stability. Both numerical models use the same representations of discontinuity strength and deformation, and both apparently can be used to evaluate the stability of keyblocks if appropriate information is available or can be estimated for block geometry, in situ stresses, and discontinuity properties. This chapter describes the results of parametric studies that were used to examine the ways in which stresses, discontinuity stiffness and shear strength, block geometry, and displacement affect stability. Some inherent limitations of the stability analysis methods are discussed before presenting the results of the scoping calculations.

Description of Keyblocks Used for Studies

Figure 5.1 shows the symmetrical two-dimensional and three-dimensional keyblocks used for most of the parametric studies. A horizontal tunnel with a radius of 72 inches (1.8 m) was modeled; the radius from the tunnel centerline to the block apex was 96 inches (2.4 m). Each discontinuity forming a block face dips at 60° below horizontal, and joint apertures for negligible normal stress are 0.04
Figure 5.1a  Typical symmetric keyblock used for two-dimensional studies.
Figure 5.1b Typical symmetric keyblock used for three-dimensional studies.
inches (0.001 m). In situ principal stresses prior to excavation of the tunnel are assumed to act vertically and, in a horizontal plane, at 45° angles from the tunnel axis.

Table 5.1 lists in three categories the parameters that were studied for their effect on keyblock stability. In addition to the parameters listed in Table 5.1, a comparison was made of two-dimensional versus three-dimensional model results and of linear and nonlinear representations of discontinuity behavior. Block stability is a function of all of these parameters, but some parameters influence stability more strongly than do others. In the studies below, only one parameter was varied at a time. The results were plotted on similar scales so that the relative significance of a given parameter in a design problem could be judged from its expected variability and from the gradients of the curves plotted from the parametric studies (Figures 5.2 through 5.15).

Table 5.1 Parameters Examined for Effect on Keyblock Stability

<table>
<thead>
<tr>
<th>Discontinuity Behavior</th>
<th>Stresses</th>
<th>Keyblock Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>shear strength</td>
<td>stress magnitude</td>
<td>block acuteness</td>
</tr>
<tr>
<td>deformation stiffness</td>
<td>lateral stress</td>
<td>block size (tunnel size constant)</td>
</tr>
<tr>
<td></td>
<td>ratio</td>
<td></td>
</tr>
<tr>
<td></td>
<td>thermal loading</td>
<td>block and tunnel size (scale)</td>
</tr>
</tbody>
</table>
All combinations of parameters that were studied used symmetric blocks in the tunnel crown. This was because the stability solution is sensitive to whether or not the selected displacement direction is correct. In a design analysis several different directions of displacement would be examined in a searching sequence that is similar to the way in which a soil mechanics slope stability analysis searches for critical circles along which slope failure would most likely occur. Use of a symmetric crown block allowed the displacement direction to be specified as vertically downward so as to find the results of an analysis as quickly as possible.

Limitations of Stability Analysis Method

The stability solutions provided by the numerical models which implement the equations of Chapter 3 are subject to certain limitations in accuracy because of the sequence in which forces are summed. The selection of two-dimensional versus three-dimensional analysis procedures makes a difference in the results, although it is one that can be anticipated. These factors are in addition to the assumptions about geometry, stiffness, and stresses that were used in developing the equations in Chapter 3.

As described previously, the sequence of operations in evaluating block stability for a given amount of displacement is to first calculate the effect of the displacement on the stresses affecting each block face, and then sum those stresses with the block weight into a resultant force. A possible error that can arise in this
sequence of calculations is the place in the procedure at which the block weight is brought into the equations. An increment of displacement is used to compute a change in normal stress and shear stress on a block face; the shear stress is then constrained by the available shear strength, which is a function of normal stress. A component of the block weight should ideally be included in the normal and shear stress at each grid point. In the analysis models the weight is not brought into the calculations for each displacement increment until the stresses are summed at the end of that increment. This is because of the difficulty of apportioning a component of the block weight to each block face in a generalized solution procedure.

For a typical two-dimensional analysis of the block shown in Figure 5.1a this would make a difference of less than 2% in the normal stress at failure, and a difference of about 3% in the shear stress. Investigators do not usually know shear strength to within 3%, for example, and so the effects of the force summation sequence are overshadowed by uncertainties of the input data.

Another source of small errors in the analysis procedures is the iterative approach used in determining a dilation angle for calculating aperture changes in the nonlinear discontinuity behavior equations. In the linear representation the dilation angle \((i)\) is always a constant, but in the nonlinear representation the dilation angle is:

\[
i = J - \log_{10} \left( \frac{\sigma_{CS}}{\sigma_n} \right)
\]

\( (5.1) \)
from equation 3.3. Since $\sigma_n$ is a function of aperture closure (Figure 3.6), and closure is partly a function of dilatancy, the numerical models use equivalent dilatancy angles from the previous displacement increment rather than iterating to find "current" values of dilatancy angles for use in computing aperture changes. Again, this produces an error in the computed resultant force that is typically of the order of two or three percent. This is not significant for most stability calculations for parametric studies, particularly since the error is systematic.

Recall that $F/W$ is the ratio of resultant support force to block weight. The support force is the force needed to hold the block at equilibrium at the specified level of displacement. Positive $F/W$ ratios thus indicate that an actual supporting force must be applied for the block to be at equilibrium at that displacement, while a negative $F/W$ indicates a pullout force that must be applied to pull the block out to that displacement. Recalling equation 3.26, if $F/W > 0$ then the factor of safety is less than unity, while if $F/W < 0$ the factor of safety is greater than unity.

An unsupported block in a constant stress field could be expected to displace under its own weight until either it fails completely or it reaches a condition of equilibrium at a displacement where $F/W = 0$. Negative $F/W$ values therefore indicate a reserve of stabilizing shear resistance against block failure that can be mobilized by increasing displacements. This presumes that discontinuity properties and loading conditions are constant with time. If displacement continues
past the minimum F/W point, the peak resisting force is surpassed, and the block falls catastrophically as soon as F/W becomes positive. If the block is supported, displacement can continue until the entire block weight is carried by the support (F/W = 1) or equilibrium is reached at an intermediate point.

The selection of two-dimensional versus three-dimensional analysis procedures makes a decided difference in the stability solutions obtained. Figure 5.2 compares block reaction curves for three-dimensional and two-dimensional blocks of the form shown in Figure 5.1. All discontinuities dip at 60° below horizontal, and the problems are equivalent in terms of stiffness, initial stress, and shear strength parameters. Nevertheless, it can be seen that the 2D approach underestimates the stability of the block in comparison to the 3D approach. This is apparently caused by the block confinement and additional strength available in the geometry of the 3D approach. Most parametric studies of this chapter were made with 3D analyses because they are more realistic than 2D calculations.

Keyblock Displacement

As the keyblock of Figure 5.1a displaces from the position that it occupied prior to opening of the excavation, the stresses acting on each block face change continually until the block either fails or becomes stable. Equations for treating these stress'changes in either a nonlinear way or as a linear approximation were developed in Chapter 3. Figure 5.3 shows block reaction curves for typical
Figure 5.2 Block reaction curves for 2D and 3D analyses. Other parameters same in both cases.
Figure 5.3 Block reaction curves of 3D keyblock for linear and nonlinear discontinuity models.
nonlinear and linear analyses of geometrically identical blocks. It is difficult to estimate properties for linear stiffness and shear strength to perfectly match the results of a nonlinear analysis, so the minimum F/W values seen in the two curves do not quite agree.

Two important results can be seen in the general shapes of these curves, however. First, the block reaction curve descends to a minimum F/W ratio somewhat more quickly for the linear behavior. In other words, the linear approximation results in a faster "unloading" of the block towards equilibrium than does the nonlinear model. Second, and more importantly, once the minimum F/W has been passed the linear model indicates a rapid upward trend to where F/W becomes positive and the block fails. The nonlinear model shows a more gradual rise in F/W once the minimum is surpassed, indicating greater possible keyblock displacement before failure occurs. The nonlinear model is more accurate and is used for most of these parametric studies, but the linear approximation is more conservative for design if the assumed linear stiffness values are good estimates.

In general, properties used in either type of analysis affect the block reaction curve in two ways. The shear strength values limit the shear stress on the block faces and thus control the minimum F/W value that can be obtained with block displacement, other things being equal. The stiffness properties in turn control the stress changes that occur with displacement, and hence determine the magnitude of displacement that is necessary to get to the minimum F/W, other things again being equal. In other words, the shear strength affects the
lower limit of the curve, while the stiffness affects the magnitude of permissible block displacement.

Figure 5.4 shows the variation of normal stress and shear stress along the face of the two-dimensional block as displacement occurs. A nonlinear discontinuity model was used and a hydrostatic stress field was assumed for this particular analysis. The stress values at minimum displacement gradually drop and become more evenly distributed as block displacement progresses. The leveling of the distributions reflects the initial variation of stresses with distance away from the tunnel, and the modifying influence of the discontinuity strengths and stiffnesses as displacement occurs. This analysis showed the block to be stable, even though the normal and shear stresses dropped continually with displacement. Such a result is possible because the contribution of the normal stresses to forces that drive the block to failure decrease faster than the shear stresses do. This can be seen in the slopes of the curves of Figure 5.4c, and in the following discussion.

Neglecting keyblock weight for the moment, and examining only one face of a pyramidal keyblock that is subject to falling, the stress on the face that contributes to the forces driving the block towards failure is $\sigma_n \sin \theta$. $\theta$ is the angle between the block face and vertical downward. The forces resisting failure are limited by the available shear strength: $\sigma_n \tan \phi \cos \theta$. If the keyblock is stable, $\sigma_n \tan \phi \cos \theta$ must be sufficiently larger than
Figure 5.4a Variation of normal stress along face with increasing block displacement.
Figure 5.4b  Variation of shear stress along face with increasing block displacement.
Figure 5.4c Variation in stress at a point on block face near apex during displacement.
\( \sigma_n \sin \beta \) to resist a component of the block weight as well as the \( \sigma_n \sin \beta \) component. Rearranging, for the block to be stable:

\[
\tan \phi > \tan \beta \tag{5.2}
\]

Dilatancy and block weight are not included in this equation, but the general trend is apparent.

**Discontinuity Shear Strength**

The shear strength of each discontinuity making up the keyblock contains a dilatancy component and a friction component. Friction is represented by an angle \( \phi_r \), while dilatancy is represented by equation 5.1 for nonlinear behavior. These quantities define the shear strength that is available under a given normal stress using equation 3.3. Figure 5.5 shows the ways in which changes in these three parameters affect keyblock stability.

In Figure 5.5a, the minimum \( F/W \) ratio that can be achieved for the pyramidal block of Figure 5.1a is plotted as a function of joint roughness coefficient (JRC). This plot assumes a hydrostatic stress field prior to excavation, with a stress magnitude of 500 psi in compression. Joint compressive strength (JCS) is 3000 psi and \( \phi_r \) is 25°. The nonlinear curve indicates that block stability is strongly dependent on dilatancy; this becomes more evident mathematically if equation 5.1 is rearranged slightly so that the exponential effect of JRC can be seen:
Figure 6.5a Variation of minimum F/W ratio with Joint Roughness Coefficient (JRC).
\[ \theta = \log_{10}(\left(\frac{JCS}{\sigma_n}\right)^{JRC}) \]  

Increases in JRC then imply that the discontinuity surface is rougher. This in turn makes the available shear strength larger, and also increases the shear stiffness.

Figure 5.5b shows how the minimum F/W is affected by another variable in the nonlinear formulation for dilatancy. JRC is held at 4 in this case, and \( \phi_r = 25^\circ \). F/W ratio is plotted as a function of JCS, and the plot shows that block stability increases as the joint compressive strength increases, although the effect is not as strong as that of JRC. JCS reflects the strength of the asperities that make the discontinuity dilatant; a higher JCS value implies that fewer asperities fail during shear and that correspondingly more must be overridden. Increases in JRC or JCS thus increase the shear strength and slow the decrease in normal stress as the keyblock displaces.

Finally, Figure 5.5c plots the change in minimum F/W ratio as a function of \( \phi_r \). As expected, increased residual friction makes the keyblock more stable. However, the trend is not quite as pronounced as that of JRC because \( \phi_r \) affects only the available shear strength while dilatancy affects both shear strength and the changes in normal stress brought about by aperture opening or closure.
Figure 5.5b Variation of minimum F/W ratio with Joint Compressive Strength (JCS).
Figure 5.5c  Variation of minimum F/W ratio with residual friction angle ($\phi_r$).
Discontinuity Stiffness

It was shown in Chapter 3 that discontinuity normal stiffness is related to the initial aperture of the fracture under negligible normal load, and to the given normal stress acting on the fracture in situ. Conversely, the stiffness must affect the changes in stress that are generated by block displacement. The load-deformation relation has been defined by the maximum possible aperture closure and the initial normal stiffness. Figure 5.6 shows how the initial normal stiffness of a discontinuity would be affected by changes in initial aperture (or maximum closure), as defined by equations 3.11 and 3.9 if the normal stress and normal stiffness are held constant. Smaller apertures mean smaller attainable maximum closures, which imply that load-deformation curves such as that in Figure 3.6 must initially slope upward more steeply for tighter discontinuities.

Figure 5.7 shows the variation of minimum F/W ratio with a range of initial normal stiffnesses. Block stability is seen to decrease as normal stiffness increases. This is because the normal stresses decrease more quickly on discontinuities experiencing a given increment of keyblock displacement. The drop in normal stress brings down the available shear strength before the shear stresses can become large enough to support the block. Although this effect is present, it does not appear to be as pronounced as the shear strength effects on keyblock stability discussed above.
Figure 5.6 Initial normal stiffness versus no-load aperture.
Figure 5.7 Minimum F/W ratio versus initial normal stiffness.
Stresses and Loads

Aside from discontinuity shear strength, the most critical condition affecting the stability of a given keyblock is the stress environment in which it is located. This cannot be studied by simply plugging different stress values into the model and looking at the results because of the effect of normal stiffness variations in situ. As explained in Chapter 3, normal stiffness is a function of normal stress and can be expected to increase as stress magnitudes increase. Conceptually, this should make a rock mass stiffer with depth (Figure 5.8). Discontinuity stiffnesses were adjusted for stress magnitude using the hyperbolic equations of Chapter 3 for single fractures under normal loading, and the $E_m$ values in Figure 5.8 were then calculated using equation 3.4 for various stresses. The plot has a distorted S shape; the early curvature indicates the predominance of discontinuity deformation at low in situ stresses, while the high-stress asymptote is the $1 \times 10^7$ psi elastic modulus of the intact rock. The curve approaches this limit gradually as the discontinuities reach maximum closure in this one-dimensional conceptual model.

Different $E_m$ values corresponding to different levels of confining stress were used in the 3D block behavior model to represent overall stiffness changes so that block stability could be examined for the effects of in situ stress magnitude. Figure 5.9 plots the change in minimum $F/W$ ratio that occurs with stress magnitudes varying from 1000 psi to 5 psi, with other parameters (including lateral stress ratio) held constant at 0.5. The trend is for the block to
Figure 5.8 Variation of $E_{\text{mass}}$ with in situ stress.
Figure 5.9 Minimum F/W ratio versus in situ vertical stress magnitude with 0.5 lateral stress ratio.
become less stable as the initial confining stresses decrease, and the trend accelerates as the stress magnitudes become very small.

Figure 5.10 shows the changes in minimum F/W ratio that result from differing lateral stress ratios at a constant vertical stress of 500 psi. The lateral stresses before excavation are assumed to be equal in magnitude and to act in a horizontal plane. Stability can be seen to decrease as lateral stress ratio decreases; again the trend accelerates as the lateral stress ratio goes below about one-half. Similar calculations were made for horizontal stress anisotropy, with average horizontal stresses being kept constant. Since all four block faces have identical strength and stiffness and the problem geometry was symmetric, the minimum F/W ratios were not affected.

Because confining stresses are so important to block stability, factors that affect the stress magnitudes over the design life of an excavation can become important design considerations. One such factor is stress changes induced by heating or cooling of the rock around an excavation. Equations for evaluating the stress changes caused by temperature variations in an elastic material were given in Chapter 3. Figure 5.11 shows the effect of temperature changes on the stability of the pyramidal block used in the previous calculations. Temperature increases above ambient cause an increase in compressive stresses that have a very slight stabilizing effect on the block. However, temperatures below the pre-excavation ambient conditions have a destabilizing effect that accelerates just as the effects of stress magnitude and stress ratio accelerated in Figures 5.9 and 5.10.
Figure 5.10 Minimum F/W ratio versus lateral stress ratio at constant in situ vertical stress.
Figure 5.11 Variation of minimum F/W ratio caused by temperature change in tunnel.
Block Geometry

So far the parametric studies have involved a keyblock having the geometry shown in Figure 5.1. The tunnel radius, block size, and discontinuity orientations have been constant while properties such as strength and stiffness have been varied. The effects of block geometry on F/W ratio are now examined by varying the geometry of the pyramidal block while holding strengths, stiffnesses, and the in situ stress field constant.

Figure 5.12 shows the effect of keyblock narrowness or slenderness (expressed in terms of included apex angles) on block stability. The radius of the tunnel and the height of the block are constant, and the dips of the discontinuities below horizontal were varied from their values of 60° used in the preceding calculations. The 60° dips resulted in an included angle of 60° at the block apex between opposite block faces. Block slenderness can be seen to have an effect similar in significance to that of changing the discontinuity shear strength. The minimum F/W ratio becomes more negative (more stable) as the block becomes narrower and becomes more positive (less stable) as it becomes broader. This reflects the increased confinement of the block and corresponding increased ability to mobilize shear strength.

Figure 5.13 shows the effect of keyblock size on F/W ratio. In this case the block height was varied while the discontinuity dips and the tunnel radius were constant. Larger blocks are apparently less stable than smaller blocks for two reasons: First, since this block
Figure 5.12 Minimum F/W ratio versus acuteness of apex.
Figure 5.13 Minimum F/W ratio versus keyblock size (ratio of distance between tunnel axis and block apex to the radius of tunnel).
behavior model does not provide for a stress-relieved (damaged) zone of reduced stiffness around the excavation perimeter, smaller blocks are subjected to larger average confining stresses (tangent to the opening) than are larger blocks. Second, and speaking in general terms, the weight of a block increases roughly with the cube of the block height, while the surface area increases in approximate proportion to the square of the height. Since the weight is what moves a block towards failure, and the restraining forces are generated by stresses acting on the block surface area, large keyblocks should be (and are) less stable than small ones.

Figure 5.14 shows how keyblock stability is expected to change as the scale of a problem changes. In these calculations the tunnel radius was varied but the ratio of block height to tunnel radius was kept constant, as were the other parameters. The observed decrease in block stability with increased scale seems to bear out the second reason for size effects outlined in the previous paragraph. The block faces are, in this set of calculations, all subjected to the same initial stress distributions regardless of size because of the form of the elastic stress solution.

Finally, since some fallen keyblocks have been observed to have left a truncated stub of rock in the apex of the cavity which once contained the block, an analysis was performed to see if a mechanism could be found by which block truncation could occur. Figure 5.15 plots tensile stress as a function of radial distance from the tunnel axis at which the pyramidal block was truncated. The apex of the
Figure 5.14 Minimum F/W ratio versus scale of keyblock and tunnel (radius to block apex = 4/3 radius of tunnel).
Figure 6.15 Tensile stress on possible truncated surface near apex of keyblock.
block before truncation occurred was 96 inches from the tunnel axis for this calculation. The plotted curve shows the tensile stress that would be needed on the plane of truncation for this particular block to be stable, even with the shear stresses on faces below the truncation accounted for. The stress would act on a plane at the indicated distance from the tunnel axis, and perpendicular to a line of symmetry running from the tunnel axis to the block apex. Since such a plane becomes smaller as the apex is approached, the stresses become larger with distance from the tunnel axis as shown. If the stub of rock remaining between the truncation and the apex could resist pullout sufficiently, tensile failure would occur at the point along this curve where the tensile strength of the intact rock was exceeded. This explains conceptually why many blocks observed in the field have been truncated.
Chapter 1 described the significance of keyblock stability in underground excavation performance, and broke the problem into two parts: block identification and block stability analysis. Chapter 2 outlined techniques developed by others with which the block identification part of the problem may be solved. Procedures were formulated in Chapter 3 for analyzing the combinations of forces that define keyblock stability as a function of displacement. These procedures contribute to the science of rock mechanics by making advances over previous techniques in two ways: arbitrary, fully three-dimensional in situ stress fields and nonlinear discontinuity deformation behavior can both now be properly treated in an analysis of block stability. These procedures define a reaction curve for block behavior which describes how stability and support requirements change with block displacement.

The procedures of Chapter 3 were implemented in two-dimensional and three-dimensional numerical models for block behavior that are presented in Appendices 1 and 2, respectively. Field observations of keyblocks in tunnels were then described in Chapter 4. These field observations of block stability as well as laboratory tests (by Crawford and Bray, 1983) of keyblock pullout force were back-analyzed to check the validity of the block behavior models. Once the validity of the models was established, sensitivity studies were run to evaluate the effects of parameter changes on keyblock stability. The results of these studies were presented in Chapter 5 for a symmetric
pyramidal block; a relatively simple block was modeled because block geometry itself has an important effect on stability. The intent of this chapter, then, is to draw conclusions about keyblock stability from the work presented in the preceding chapters.

**Design Use of Keyblock Behavior Models**

The use of a keyblock stability analysis in designing underground openings is not necessary if the rock mass to be excavated contains no discontinuities. However, if the rock contains discontinuities, a keyblock analysis may be employed to advantage in design, during construction, or to evaluate certain aspects of post-construction performance. Hamnett and Hoek (1981) described how the failure mechanisms to be considered in excavation design change with depth and in various geologic settings. Important factors included the stress field, strength of intact rock, and nature of discontinuities within the rock mass. These considerations help determine what types of analysis are appropriate for a given design.

The keyblock analysis can be used during the design process with discontinuity set orientations and excavation orientations to evaluate the possibility of encountering potentially unstable rock blocks. Discontinuity set orientations may be estimated from site exploratory drilling using procedures summarized by Yow and Wilder (1983). However, as was shown in Chapter 4, fringe members of joint sets can form keyblocks, and so the observed range of joint orientations of a dispersed set should be checked. The methods outlined in Chapter 2
Identify possibly unstable blocks, and the block behavior models can then evaluate the stability of these blocks from either assumed or measured data for a given excavation size and orientation. The quality of the stability calculation is obviously related to the quality of the data; however, this relationship allows the accuracy needed in the stability calculation to determine the required accuracy of the data.

During construction of an underground opening, structural features such as faults or shears are sometimes encountered unexpectedly. If such structural features apparently intersect to create keyblocks, block stability can be quickly evaluated using the block behavior models. If the trace of the keyblock perimeter has been completely exposed in the excavation surface and block failure has not occurred, the question of stability may or may not be moot, since parameters that may change over the useful life of the excavation could allow a stable block to become unstable. Alternatively, if the keyblock is large, only part of the block may be exposed at first, and a quick analysis may be sufficient before proceeding with the excavation. This would allow the need for additional ground support to be evaluated and remedied before further excavation allowed the block to become unstable.

An analysis of post-construction keyblock stability may be needed in some cases if rock mass temperature or stress condition changes are expected over the life of the excavation. The rock several meters away from the Spent Fuel Test-Climax drifts, for example, experienced
a temperature decrease of over 2°C below ambient in a matter of a few months solely because of the cooling effect of the ventilation airflow. Excavation of adjacent openings can also be important in cases where the stresses would be affected.

Table 6.1 gives a qualitative assessment of the relative significance of various parameters in terms of their influence on the stability of a keyblock in an underground opening. These assessments were developed from the sensitivity studies of Chapter 5, and give a preliminary indication of the importance of each parameter. Following sections amplify some aspects of these parameter influences that may not have been obvious in the figures of Chapter 5.

Excavation Depth and Keyblock Stability

The sensitivity studies of Chapter 5 show that stability of a keyblock in an underground excavation is influenced by the stress magnitudes and lateral stress ratios, among other things. Figure 5.9 illustrated how variations in magnitude of vertical stress affect stability with constant lateral stress ratio, and Figure 5.10 plotted changes in stability caused by changes of lateral stress ratio with constant vertical stress magnitude. In each case, a transition from stable to unstable block behavior is seen where the minimum F/W ratio (support force ratio) goes from negative to positive. At F/W = 0 the block is marginally at equilibrium without any excess forces resisting failure.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Relative Influence on Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>discontinuity shear strength</td>
<td>strong</td>
</tr>
<tr>
<td>discontinuity stiffness</td>
<td>weak (2)</td>
</tr>
<tr>
<td>in situ stress magnitude</td>
<td>strong</td>
</tr>
<tr>
<td>lateral stress ratio</td>
<td>strong influence at low stress magnitudes; weak influence otherwise</td>
</tr>
<tr>
<td>temperature changes*</td>
<td>strong influence if temperatures decrease; weak influence otherwise</td>
</tr>
<tr>
<td>block slenderness (3)</td>
<td>strong</td>
</tr>
<tr>
<td>block size with respect to constant excavation size</td>
<td>moderate</td>
</tr>
<tr>
<td>block and excavation size, dimensions kept in same proportion</td>
<td>moderate</td>
</tr>
</tbody>
</table>

**NOTES:**
(1) The analyses of Chapter 5 did not include dynamic loads, fluid pressures in the discontinuities, or creep or plasticity effects.
(2) Stiffness affects keyblock displacement much more strongly than it does stability.
(3) Block slenderness is the smallest included angle between opposite block faces at the apex.
Similar analyses can be done for many combinations of vertical stress magnitude and lateral stress ratio to determine what stress conditions allow a block to be stable and what conditions lead to failure. Since stability is also heavily influenced by discontinuity shear strength, this type of calculation was run for two blocks having different shear strengths to see if strength changes would make a qualitative difference in the stress effects. Figure 6.1 plots the combinations of vertical stress magnitude and lateral stress ratio necessary for these two blocks to each be at equilibrium with an F/W ratio of zero.

The curves plotted in Figure 6.1 for pyramidal blocks having the two different discontinuity shear strengths are qualitatively the same. A block is stable at relatively low lateral stress ratios if the vertical stress magnitude is large enough. At lower stress magnitudes a larger lateral stress ratio is needed for stability to be attainable, and the effect accelerates as the vertical stress magnitude becomes very small. If vertical stress is converted to excavation depth by dividing by the unit weight of the overburden, the curves agree with observations of tunnel stability in blocky ground. At shallow depths block instability is a problem, but at greater depths the larger in situ stresses make the opening more stable, until stresses are high enough to induce failure of intact rock.

Curves similar to those of Figure 6.1 can be constructed for other combinations of block geometry, shear strength, and in situ stress, but the general trends will be the same. These trends rationally
Figure 6.1  Lateral stress ratio required in situ for F/W ratio of zero.
support the intuition and experience of excavation designers. As an aside, most previous investigators (e.g., Crawford, 1982, and Crawford and Bray, 1983) have examined keyblock behavior only at relatively low horizontal stress magnitudes with zero or negligible vertical stress. These analyses would plot in the upper right-hand corner of Figure 6.1.

**Thermal Effects**

Since stresses play such an important role in keyblock stability, the importance of stress changes induced by thermal loading or unloading is not surprising. Figure 6.2 shows the stress changes that would be induced in the rock at the middle of the block of Figure 5.1 by temperature changes in the adjacent tunnel. This plot assumes that steady-state heat flow conditions have been reached, and thus is a limiting case for transient temperature conditions.

Figure 5.11 showed the effect of temperature changes on a particular hypothetical keyblock. In a general sense, temperature increases result in stress increases caused by thermal expansion of the rock, while temperature decreases have the opposite effect. A rise in temperature in an underground excavation, then, would move the stresses affecting a keyblock to the right and down on Figure 6.1, as all normal compressive stresses would increase. Lower temperatures would move the stresses up and to the left on Figure 6.1, allowing the block to fail once the confining stresses become low enough.
Figure 6.2 Typical linear stress changes around circular tunnel caused by temperature changes within tunnel (compression positive).
Analysis of thermal effects on keyblock stability has obvious application for situations in which excavation ventilation is expected to cool the rock mass, or in which temperature changes will be induced by refrigeration or storage of cold or hot materials in the underground opening. Cold materials such as liquified gaseous hydrocarbons have been stored in underground excavations, but published case histories have not been sufficiently detailed to identify block instability caused by the drop in temperature. Two additional factors that complicate this class of problems, though, are that such extreme cold freezes water in the discontinuities, and that the storage caverns are usually pressurized. Less extreme cases, such as cold food storage, have involved bedded sedimentary rocks in which keyblocks are not formed by the discontinuities. A case of the opposite sort was described by Rehbinder (1984) in which a cavern for hot water storage has been cycled several times between 40°C and 95°C. No block stability problems have been encountered; the temperature never dropped below what it was at the time of excavation and so no thermally-induced keyblock instability would be expected.

This latter situation of temperatures above ambient has particular significance for application to underground openings for nuclear waste disposal. Heat generated by the radionuclide decay of emplaced waste will cause the temperature of the openings to rise; the temperatures will gradually return to pre-emplacement conditions as the waste decay continues. Unless the thermally-induced compressive stresses damage the discontinuity asperities as described in Chapter 3, or cause failure of intact rock, the thermal cycle caused by the radioactive
waste should not by itself cause long term keyblock instability. Stability during construction of the underground openings may be a different matter, though, particularly if the ventilation air cools rock that contains marginally stable keyblocks.

Keyblock Geometry and Discontinuity Strength

The relationship of block geometry to keyblock stability has been considered for a selection of two-dimensional cases by Cording and Mahar (1974) and in a much more generalized three-dimensional sense by Goodman and Shi (1985). The sensitivity studies of Chapter 5 examined the geometric effects of block slenderness, size, and scale for the stability of a simple pyramidal keyblock. Block slenderness can be considered in terms of the angle between opposite block faces where they intersect at the apex, size refers to the block height and volume with respect to a constant excavation size, and scale refers to the size of a block and excavation when the dimensions of the two are maintained in the same proportions.

Increases in size and scale result in a decrease of keyblock stability, other things being equal. This implies that a design for an underground opening should consider first the maximum keyblock size allowed by the excavation dimensions. However, block slenderness has a more pronounced effect on stability than does either size or scale. Basically, if the smallest included apex angle between two opposite faces is larger than the sum of the friction angles and dilatancy angles of the two faces, the block will fail immediately upon
excavation of the opening. If the included apex angle is smaller than
the sum of the friction and dilatancy angles, the keyblock is
potentially stable, but an analysis should be performed as a check on
its behavior. If the block is extremely narrow, though, it may be
assumed as stable unless exceptional circumstances such as extremely
low in situ stresses prevail. Consideration of block narrowness and
discontinuity shear strength must include uncertainties in the data.
A 5° error in block face orientation, for example, can potentially
affect calculated block stability in a way that is similar to a 5°
error in friction angle.

Excavation Sequence and Techniques

The relationship between keyblock stability and the stresses
around an underground excavation suggests that construction sequences
or techniques that affect the stress field can also affect block
stability. Examples of this can be found in the changing pattern of
stresses behind an advancing excavation face, and in the way that
excavation technique can affect the rock mass.

As an excavation advances, the tunnel behind the advancing face
deforms to a position of equilibrium with any support system that is
installed (e.g., Panet and Guenot, 1982). Depending on the initial in
situ stress conditions, the circumferential compressive stresses (hoop
stresses) around the tunnel perimeter may increase behind the
advancing face to reach static conditions as the rock around the
tunnel deforms elastically. Conceptually then, a keyblock in the
perimeter of a cylindrical tunnel would experience its lowest confining stresses directly behind the advancing face. A potentially unstable block in the crown would displace into the excavation either until it failed or until its F/W ratio reached zero. As the working face moved onward, any increase in hoop stresses around the tunnel would increase the confinement of the block and thus drive the F/W ratio to a negative value, indicating an excess of forces resisting failure. If other factors (such as loss of strength with time) do not subsequently affect the stability of the block, the most critical time in the deformation history of the block may be when it has initially been released by the advancing excavation face.

Excavation technique can also affect keyblock stability, even if dynamic loads from blast effects are not involved. Deere (1981) and Hattrup (1981) have both described situations where rock blocks have been dislodged behind the advancing cutterhead of a tunnel boring machine (TBM). The force applied to thrust the cutterhead of a TBM forward is usually provided by hydraulic cylinders acting against shoes or pads that contact the tunnel perimeter. These pads typically are pushed radially outward with stresses up to several hundred pounds per square inch, and must affect the stress field around the tunnel. This perturbation, even though temporary, may aggravate the problems with block stability by reducing confining stresses enough to allow blocks to fall.
Possible Stabilization Measures

The use of shotcrete to support keyblocks in blocky ground has been described by Fernandez-Delgado et al (1979), and the use of rockbolts to stabilize blocks has been described in the papers by Goodman and others (see references). However, other stabilization measures suggest themselves, at least conceptually, because of the impact of stress conditions on keyblock stability. If the thrust of TBM bearing plates against a tunnel perimeter can lower the hoop stresses and allow blocks to become unstable, then artificially induced increases in hoop stresses should increase the stability of the rock. This stabilization of fractured rock has in fact been observed in the effect of tensioned rock bolts on blocky ground (Lang, 1957), where the stresses induced by tensioned bolts create a stabilizing zone of compression around the roof of an opening. Other techniques that could be used in special circumstances to increase compressive stresses for stabilization purposes might include pressurized flat jacks inserted adjacent to a critical keyblock, or temperature increases induced by electrical resistance heaters. The stress changes thus created can be included in the block stability analysis to see if the expected improvements in stability are adequate.

Keyblock Displacement Without Failure

Most of the preceding discussion has been directed towards evaluating keyblock stability. However, keyblock displacement can be important even if failure does not occur. It was shown in Chapter 3
that block displacement increases the shear stresses along the block faces until either the available shear strength is exceeded at all points and the block fails, or enough stress has been mobilized to support the block. Two other phenomena occur simultaneously though, as the keyblock displaces: the normal stress and the normal stiffness on each discontinuity both decrease, and each discontinuity opens. The discontinuity opening may be either slight, because of shear deformation and dilatancy, or somewhat larger, due to a combination of shear deformation, dilatancy, and normal deformation. Block movement thus produces a local zone of increased permeability (caused by increased average discontinuity aperture) and reduced stiffness (caused by decreased discontinuity normal stiffness) in the perimeter of the underground opening. Both phenomena have been observed in the field, and each can be important even if block failure is avoided.
REFERENCES


APPENDIX 1 Two-dimensional keyblock stability analysis program
BSTB2D

The equations developed in Chapter 3 have been implemented into a numerical two-dimensional solution for the stability of a keyblock as a function of block displacement. This solution has been coded in the BASIC computer language as an interactive program named BSTB2D that can be run on a Hewlett-Packard 9816 computer. The user is prompted by the program for data needed for the analysis in a series of queries that are answered from the keyboard. Figure A1.1 illustrates the problem geometry used in the solution; other aspects of the use of the program are stated in the interactive prompts provided to the user.

BSTB2D uses a routine named SILJAK (which is copyrighted by the Hewlett-Packard Company) to solve equation 3.11 of Chapter 3. SILJAK finds all roots of a polynomial to within a specified tolerance without needing the derivative of the equation or an initial guess for a solution. SILJAK is contained in a file named HPBSTAB2D (not included here), which is automatically loaded when BSTB2D is executed. The routine can be purchased from Hewlett Packard, or the user can substitute a similar routine. The steps performed by BSTB2D to analyze keyblock stability are listed below:

1. Prompt user for input data. This data must be in a consistent system of units, and includes:
   -- printing options and a problem header
Figure A1.1  Geometry used in two-dimensional keyblock stability model.
-- rock unit weight, tunnel radius, displacement increment, and number of increments to be solved
-- vertical stress, lateral stress ratio, and internal pressure within tunnel
-- modulus of intact rock, rock mass modulus, Poisson's ratio, and coefficient of thermal expansion
-- temperature change
-- choice of linear or nonlinear joint behavior
-- discontinuity plane locations and orientations

If linear behavior is selected, the following is needed:

-- friction angle, dilatancy angle, and residual friction angle
-- normal stiffness and $k_s/k_n$ ratio

If nonlinear behavior is selected, then the following is required:

-- joint roughness coefficient (JRC), joint compressive strength (JCS), and residual friction angle
-- average spacing and no-load aperture

2. After prompting for input, BSTB2D calculates the volume and weight of the block, assuming unit thickness. The two planes that form the keyblock boundaries are each divided into 20 segments.
3. The Kirsch solution is used to calculate the initial stresses at each of the 21 segment end points on each plane. If the nonlinear stiffness option has been chosen, the initial normal stiffness and aperture closure are computed from pre-excavation stresses and then adjusted for the stress concentrations around the opening.

4. At this time, the user is prompted for the displacement direction as an angle from vertical downward. BSTB2D prints the problem header and echoes the input data. The keyblock is then displaced the specified number of increments; the horizontal and vertical resultant forces and F/W ratio are displayed for each increment.

5. After the specified number of displacement increments have been completed, the user is offered the following options:

-- specify a new displacement magnitude, direction, or both
-- change discontinuity strengths or stiffnesses
-- specify a tunnel temperature change
-- change the far-field vertical and horizontal stresses
-- truncate the block at a selected height
-- entirely restart the program

A listing of BSTB2D follows.
PRINT "BSTAB2D VERSION 1.0 12/12/84"

PRINT "NOTES:

1. ALL UNITS MUST BE CONSISTENT.

2. PLANE 1 IS CW AROUND OPENING FROM BLOCK CENTER; PLANE 2 IS CCW.

3. ALL THETA(PLANE 2) > THETA(APLEX) > ALL THETA(PLANE 1)."

PRINT ""

PRINT *********THIS 2-D PROGRAM CALCULATES SUPPORT FORCES NEEDED TO BRING A BLOCK SUBMITTED TO GRAVITATIONAL LOADING INTO LIMIT EQUILIBRIUM WITH A FACTOR OF SAFETY OF UNITY. THE RESULTS CAN BE EXPRESSED AS A RATIO OF SUPPORT FORCE TO BLOCK WEIGHT; NEGATIVE VALUES INDICATE THE FORCE REQUIRED TO DESTABILIZE AN OTHERWISE STABLE BLOCK. CIRCULAR OPENINGS ARE ASSUMED.

VALUES USED IN THE ANALYSIS:

BLOCK DISPLACEMENT (L)

ROCK UNIT WEIGHT (FL^-3)

NORMAL STIFFNESS OF DISCONTINUITY PLANE (FL^-3)

SHEAR STIFFNESS OF DISCONTINUITY PLANE (FL^-3)

FRICTION ANGLE OF DISCONTINUITY PLANE

DILATENCY ANGLE OF DISCONTINUITY PLANE

PLANE DIPS AND INTERSECTIONS WITH THE TUNNEL

HORIZONTAL AND VERTICAL STRESSES IN SITU (FL^-2)

BLOCK HEIGHT PERPENDICULAR TO EXCAVATION SURFACE (L)

Theta1 IS THE ANGLE CCW FROM RIGHT HORIZONTAL AT WHICH A PLANE INTERSECTS THE TUNNEL; Theta2 IS THE ANGLE AT WHICH THE PLANE ENDS AT THE APEX OR TRUNCATED END OF THE BLOCK.

PRINT *********BSTAB2D REVISIONS: 1.0 12/12/84 ORIGINAL CODE BY JESSE L. YOW, JR.

PRINT ""

PRINT OPTION BASE 1

INTEGER I,J,K

CDM Theta1(2),Theta2(2),Phi(2),Diang(21,2),Normal(2),Nf(2),Shear(2),Sf(2)

CDM Kn(2),Ks(2),Length(2),Alph(2),Dip(2),Dd(2),Head(80),Phires(2)

CDM Inc(21,2),Sinc(21,2),Linc(21,2),Tinc(21,2),Strength(21,2),Rinc(21,2)

CDM Initn(21,2),Inites(21,2),Gamma,Radius,Disinc,Sh,Emass,El,Mu,Calph

CDM Jrc(2),Jcs(2),Weight,Sigratio,Kratio,F#(13),Flc#(13),Flin#(13)

CDM Aperture(2),Space(2),Incl(21,2),Kin(2),Mud

CDM Root(113),Iroot(113),Cof(013),Icof(01)

LOADSUB ALL FROM "HPBSTAB2D" ! LOAD HP PROFITARY SUBROUTINES
INPUT "PROBLEM HEADER (80 CHARACTERS MAX) ", Head

INPUT "RDCK UNIT WEIGHT, TUNNEL RADIUS, DISPLACEMENT INC, AND NO. INCS ", G, Radius, Dim, Inc

INPUT "VERTICAL STRESS, LATERAL STRESS RATIO, AND INTERNAL PRESSURE ", Sv, Sig, E

IF F#="S" THEN 1060

INPUT "MODULUS, MASS MODULUS, POISSON'S RATIO, AND EXPANSION COEF ", E, Mu, T, &

INPUT "DELTA T ", Tdelt

IF F#="H" THEN 1060

Nplanes=2 ! FOR 2-D BLOCKS

FOR 1=1 TO Nplanes

IF F#="F" THEN 800

DISP "INPUT THETA CCW ABOVE RIGHT HORIZONTAL FOR PLANE": I:
INPUT " ", Theta(I)

DISP "INPUT -DIP' CW BELOW RIGHT HORIZONTAL FOR PLANE": I:
INPUT " ", Dip(I)

BOO IF F#="Y" THEN 890

DISP "JRC, JCS, RESIDUAL PHI FOR PLANE": I:
INPUT " ", Jrc(I), Jcs(I), Phires(I)

DISP "AVERAGE SPACING AND APERTURE FOR PLANE": I:
INPUT " ", Space(I), Aperture(I)

MAT Phi= tO>

HAT Bilnng= tO>

GOTO 970

MAT Dilang= (0)

GOTO 970

DISP "INPUT FRICTION ANGLE, DILATANCY ANGLE, & RESIDUAL PHI FOR PLANE": I

INPUT " ", Phi(I), Dilang(I,I), Phires(I)

FOR J=2 TO 21

Dilang(I,J)=Dilang(I,1)

NEXT J

DISP "INPUT NORMAL STIFFNESS K N AND K S/K N STIFFNESS RATIO FOR PLANE": I

INPUT " ", Kn(I), Kratio

Ks(I)=Kratio*Kn(I)

NEXT I

IF F#="F" THEN 1220

IF F#="T" THEN INPUT "BLOCK HEIGHT ", Height
BEGIN CALCULATIONS

FIND MAXIMUM PULLOUT FORCE

GO SUB 2450 ! CALCULATE BLOCK GEOMETRY

FOR I = 1 TO Nplanes

GO SUB 3220 ! CALCULATE THE INITIAL FORCES

NEXT I

IF FLin = "Y" THEN GO SUB 4690 ! CALCULATE INITIAL JOINT CLOSURE

IF F = "S" OR F = "H" THEN 1220

FOR Increment TO Mud

FOR J = 1 TO Nplanes

GO SUB 1780 ! CALCULATE SHEAR AND NORMAL FORCES

NEXT J

GO SUB 1780 ! CALCULATE RESULTANT FORCE

NEXT Increment

GO TO 1540

FOR Incdisp = 1 TO 10

Dis = Disincrd

Headflag = 0

FOR Increment = 1 TO Mud

FOR J = 1 TO Nplanes

GO SUB 1780 ! CALCULATE SHEAR AND NORMAL FORCES

NEXT J

GO SUB 3800 ! PRINT RESULTS

Headflag = 1

Dis = Dis + Disincrd

NEXT Increment

GO TO 1540

FOR Incdisp = 1 TO 10

Dis = Deldisp + Incdisp

FOR J = 1 TO Nplanes

GO SUB 1780

NEXT J

GO TO 1760

GO SUB 2230
SUB 3800 ! PRINT RESULTS

IF F$="D" THEN 1430
IF F$="E" THEN 1760
IF F$="F" THEN 690
IF F$="H" THEN 660
IF F$="M" THEN 1170
IF F$="R" THEN 550
IF F$="S" THEN 620
IF F$="T" THEN 1060

PRINT "OPTIONS AVAILABLE ARE:"
PRINT "D SPECIFY DISPLACEMENT AND DIRECTION"
PRINT "E END"
PRINT "F CHANGE JOINT PLANE PARAMETERS"
PRINT "H TEMPERATURE CHANGE DELTA T"
PRINT "M FIND MAXIMUM PULLOUT FORCE"
PRINT "R RESTART PROGRAM"
PRINT "S CHANGE IN SITU STRESSES"
PRINT "T TRUNCATE BLOCK"
GOTO 1540

STOP

CALCULATE NORMAL AND SHEAR FORCES FOR GIVEN DISPLACEMENT

FOR K=1 TO 21
Ninc(K,J)=Initns(K,J)-Dis*(COS(Dd(J))*SIN(Dd(J))*TAN(0))
IF Ninc(K,J)<0 THEN Ninc(K,J)=0.
Strength(K,J)=Ninc(K,J)*TAN(Phi(J)+Dirang(K,J)) ! CALC AVAILABLE N
Sinc(K,J)=Initss(K,J)+Dis*SIN(Dd(J)) ! CALC DISP GENERATE:
IF Dis=0, THEN 1910
IF Sinc(K,J)<Strength(K,J) THEN 1910
Strength(K,J)=Ninc(K,J)+TAN(Phires(J))
Sinc(K,J)=Strength(K,J)
NEXT K
FOR K=1 TO 21
NEXT K
GOTO 2160

D=Di*SIN(Dd(J)) \* TAN(Dilang(J,J))    ' JOINT OPENING
IF X<0 THEN 2060
Ninc(K,J)=kin(J)/((1/(X))-(1/\text{Aperture}(J)))
GOTO 2080
Ninc(K,J)=0

D=Di*SIN(Dd(J))    ' JOINT SHEAR DISPLACEMENT
Strength(J,J)=Ninc(K,J)*TAN(Dilang(K,J)+\text{Pres}(J))
Sinc(J,J)=Initss(J,J)+Strength(K,J)*100*D/Length(J)
IF Dis=0 THEN 2140
IF Sinc(K,J)<Strength(K,J) THEN 2140
Sinc(K,J)=Strength(K,J)
NEXT J.

EOSUB 3720
N-f lJ)=Nsum
S* <J)=S5um
RETURN

! CALCULATE RESULTANT FORCES
' !
A1=Dip(1)
A2=180-Dip(2)
Fh=W(2)*SIN(A2)+Sf(2)*COS(A2)-Nf(1)*SIN(A1)+Sf(1)*COS(A1)
Fh=-Fh
Fv=Nf(2)*COS(A2)+Sf(2)*SIN(A2)-Sf(1)*COS(A1)+Nf(1)*SIN(A1)+Weight
Fv=-Fv
Alf=90.0-ATN(Fh/Fv)
Fr=SGN(Fv)*SDR(Fv*Fv+Fh*Fh)
Tense=0.
IF Truncw=0. THEN 2420
Tense=Fr*COS(Tpeak-Alf)/Truncw
Tense=SGN(Cos(Tpeak-Alf)*Tense)
Fw*Fr/Weight
RETURN

! CALCULATE BLOCK GEOMETRY
' !
Alph<l)=180-Dip<l)
Alph(2)=180-Dip(2)
Betl=UBO-Alph(1)+Thetal(1)
Beta2=180-Theta(2)+Alpha(2)

IF Beta1>180 OR Beta1<90 THEN PRINT "POSSIBLE PLANE 1 GEOMETRY ERROR"
IF Beta2>180 OR Beta2<90 THEN PRINT "POSSIBLE PLANE 2 GEOMETRY ERROR"

Length(2)=TAN(Alpha(1))*(COS(Theta(2))-COS(Theta(1)))
Length(2)=Radius*SIN(Theta(2))-SIN(Theta(1))-Length(2)
Length(2)=Length(2)/(COS(Alpha(2))*TAN(Alpha(1))-SIN(Alpha(2)))

Length(1)=Radius*SIN(Theta(2))-SIN(Theta(1))
Length(1)=(Length(1)+Length(2)*SIN(Alpha(2)))/SIN(Alpha(1))

Hmax=Length(1)*Length(1)+Radius*Radius
Hmax=Hmax+2*Radius*Length(1)*COS(Alpha(1)-Theta(1))

Hmax=50*Hmax
Hmax=Hmax+Radius

Tpeak=Theta(1)+ASIN((Length(1)/Hmax)*SIN(Alpha(1)-Theta(1)))
Theta2(1)=Tpeak
Theta2(2)=Tpeak

Area=Length(1)*SIN(Alpha(1)-Theta(1))
Area=Area+Length(2)*SIN(Theta(2)-Alpha(2))
Area=Area+Radius*Frac(Theta(2)-Theta(1))/180
Area=Area+Radius/2.0

IF FJ="T" THEN ! CHECK IF BLOCK IS DEFINED AS TRUNCATED
Height=Hmax
Trunch=0.
Truncl=0.
GOTO 2940

Height=Height+Radius
Trunch=Hmax+Radius-Hight
Truncl=Trunch*TAN(Alpha(1)-Tpeak)+TAN(Tpeak-Alpha(2)))
Area=Area-Truncl*Truncl/2.0

Length(1)=Length(1)-Truncl/COS(Alpha(1)-Tpeak)
Length(2)=Length(2)-Truncl/COS(Tpeak-Alpha(2))

Theta2(1)=Tpeak-ATN(Truncl*TAN(Alpha(1)-Tpeak))/H)
Theta2(2)=Tpeak+ATN(Truncl*TAN(Alpha(2)-Tpeak))/H)

FOR I=1 TO Nplanes ! GENERATE INCREMENTS FOR INITIAL STRESSES
Linc(1,I)=0.
Tinc(1,I)=Theta(1)
NEXT I
FOR I=1 TO Nplanes
Din=Length(I)/20.0
3010 FOR J=2 TO 21
3020 Lin(J,1)=Lin(J-1,1)+D1n
3030 NEXT J
3040 NEXT I
3050 FOR J=1 TO 21
3060 Rinc(J,1)=2*Linc(J,1)*Radius*COS(Alpha(1)-Theta(1))
3070 Rinc(J,2)=2*Lin(J,2)+Radius*COS(Theta(2)-Alpha(2))
3080 Rinc(J,2)=2*Lin(J,2)+Radius*COS(Theta(2)-Alpha(2))
3100 NEXT J
3120 !
3130 FOR J=1 TO 21
3140 TLinc(J,1)=Theta(J)+ASIN((Lin(J,1)/Rinc(J,1))*SIN(Alpha(J)-Theta(J))
3150 TLinc(J,2)=Theta(J)-ASIN((Lin(J,2)/Rinc(J,2))*SIN(Theta(J)-Alpha(J))
3160 NEXT J
3170 !
3180 Weight=B2mma*Area*1.0
3190 !
3200 RETURN
3210 ! ****************************
3220 ! SOLVE FOR THE INITIAL FORCES
3230 ! ****************************
3240 FOR J=1 TO 21
3250 Rat=Radius/Rinc(J,1)
3260 Sigr=Cavg*(1-Rat^2)
3270 Sigt=Cavg*Cdev*(1-3*Rat^4)*COS(2*Tinc(J,1))
3280 IF Tdelt=0 THEN 3450
3290 !
3300 !
3310 !
3320 Sigt=Cavg*(1+Rat^2)
3330 Sigt=Cavg*Cdev*(1-3*Rat^4)*COS(2*Tinc(J,1))
3340 Sigt=Cavg*Cdev*(1-3*Rat^4)*COS(2*Tinc(J,1))
3350 IF Tdelt=0 THEN 3450
3360 !
3370 Cin=-(1+Talpha*Tdelt/(2*(1-Mu)*Lna)
3380 Cin=C2/(9)
3390 Cin=10*Rat
3400 Lin=LDS(C3)
3410 !
3440 !
3450 Tau=Cdev*(1+3*Rat^2-3*Rat^4)*SIN(2*Tinc(J,1))
3460 !
3470 Ninc(J,1)=(Sigr+Sigt)/2
3480 Ninc(J,1)=Ninc(J,1)-((Sigr-Sigt)/2)*COS(2*(Alpha(J)-Tinc(J,1)))
3490 Ninc(J,1)=Ninc(J,1)-Tau*SIN(2*(Alpha(J)-Tinc(J,1)))
3500 !
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3510     Sinc(J,I)=Sinc(Sig(J,J)/2)*SIN(2*(Alph(I)-Tinc(J,J)))/2)
3520     Sinc(J,I)=Sinc(J,I)-Tau*COS(2*(Alph(I)-Tinc(J,J)))/2)
3530     IF I=1 THEN Sinc(J,J)=Sinc(J,J)
3540     !
3550     Init*(J,J)=Init*(J,J)
3560     Init*(J,J)=Sinc(J,J)
3570     !
3580     NEXT J
3590     !
3600     J=I
3610     GOSUB 3670
3620     Normal(J)=Nsum
3630     Shear(J)=Ssum
3640     !
3650     RETURN
3660 ! ****************************
3670 ! TRAPEZOIDAL SUMMATION OF STRESSES
3680 ! ****************************
3690     Nsum=0.
3700     Sum=0.
3710     FOR k=1 TO 21
3720     Nsum=Nsum+Ninc(k,J)
3730     Ssum=Ssum+Sinc(k,J)
3740     NEXT K
3750     Nsum=(Nsum-Ninc(1,J)/2-Ninc(21,J)/2)*Length(J)/20
3760     Ssum=(Ssum-Sinc(1,J)/2-Sinc(21,J)/2)*Length(J)/20
3770     !
3780     RETURN
3790     ! ******
3800     PRINT RESULTS
3810     ! ******
3820     IMAGE 4X,D,5D,L,5D,L,6D,L,7D,L,2(X,MD,DE)
3830     IMAGE 5X,D,DD,5D,L,DE,5(X,4D,DE)
3840     !
3850     IF F#="D" OR F#="H" OR F#="M" THEN 4060
3860     !
3870     IF Headflg=1 THEN 4060
3880     PRINT ""  
3890     PRINT Head#
3900     PRINT ""  
3910     PRINT "UNIT WEIGHT, VERT STRESS, HORZ STRESS",Gamma SVC Sh
3920     PRINT "TUNNEL RADIUS, INTERNAL PRESSURE ",Radius Pressure
3930     PRINT "BLOCK HEIGHT, MAXIMUM HEIGHT, WEIGHT ",ROUND(Height,4),ROUND(Hbmax x,4),ROUND(Weight,4)
3940     PRINT "E, E MASS, POISSON'S RATIO, EXP COEF ",E1,Emass,Mu,Talph
3950     !
3960     PRINT ""  
3970     PRINT "PLANE 'DIP' THETA LENGTH (JRC)PHI (JCS)I RES PHI L IN Ks"  
3980     FOR I=1 TO Nplanes
3990     IF Flin="Y" THEN PRINT USING 3820:1,Dip(I),Thetal(I),Length(I),Jrc(I),J
4000     IF Flin="Y" THEN GOTO 4020
4010 PRINT USING 3B20;I,Dip(I),Theta(I),Length(I),Phi(I),Dilang(I),Fires(I),Kn(I),Ks(I)
4020 NEXT I
4030 !
4040 PRINT " "
4050 PRINT " DELTA T DISP. VERT F HORZ F RESULT F TENSILE F/W"
4060 PRINT USING 3B30;Tdelt,Dis,Fv,Fh,Fr,Tense,Fw
4070 !
4080 IF P$<"Y" THEN RETURN
4090 PRINTER IS 701
4100 !
4110 IF Nlines>45 THEN PRINT CHR$(12>
4120 IF Nlines>45 THEN Nlines=0
4130 IF F$="D" OR F$="H" OR F$="M" OR Headflag=1 THEN 4350
4140 !
4150 PRINT " "
4160 PRINT Head#
4170 PRINT " "
4180 PRINT "UNIT WEIGHT, VERT STRESS, HORZ STRESS",Gamma,Sv,Sh
4190 PRINT "TUNNEL RADIUS, INTERNAL F, DISP ALPHA",Radius,Pressure,Alphad
4200 PRINT "BLOC HEIGHT, MAXIMUM HEIGHT, WEIGHT",,GROUND(Height,4),GROUND(Height,4)
4210 PRINT "MODULUS, POISSON'S RATIO, EXP COEF ",E1,Mu,Talph
4220 !
4230 PRINT " "
4240 PRINT "FLANE 'DIP' THETA LENGTH (JRC)PHI (JCS)I RES PHI LIN Kn L IN ks"
4250 FOR I=1 TO Nplanes
4260 IF Flin*="Y" THEN PRINT USING 3B20;I,Dip(I),Theta(I),Length(I),Jrc(I),Jcs(I),Phires(I),Kn(I),Ks(I)
4270 IF Flin*="Y" THEN GOTO 4290
4280 PRINT USING 3B20;I,Dip(I),Theta(I),Length(I),Phi(I),Dilang(I),Fires(I),Kn(I),Ks(I)
4290 NEXT I
4300 !
4310 PRINT " "
4320 PRINT " DELTA T DISP. VERT F HORZ F RESULT F TENSILE F/W"
4330 Nlines=Nlines+13
4340 !
4350 PRINT USING 3B30;Tdelt,Dis,Fv,Fh,Fr,Tense,Fw
4360 Nlines=Nlines+1
4370 !
4380 IF Pinc*<"Y" THEN 4660
4390 !
4400 PRINT " "
4410 PRINT " LENGTH 1 N STRESS S STRESS STRENGTH NS/DISP F ANGLE"
4420 FOR I=1 TO 21
4430 Fang=0
4440 IF Dis=0 THEN 4470
4450 X=Ninc(I,1)/Dis
4460 GOTO 4480
4470 X=0
4480 IF Ninc(I,1)=0 THEN 4500
4490 Fang=ATN(Strength(I,1)/Ninc(I,1))
4500 PRINT USING "7D.2D":Linc(I,1),Ninc(I,1),Sinc(I,1),Strength(I,1),X,Fang
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4510 NEXT I
4520 !
4530 PRINT " "
4540 PRINT " LENGTH 2 N STRESS S STRESS STRENGTH NS/DISP F ANGLE"
4550 FOR I=1 TO 21
4560 Fang=0
4570 IF Di=0 THEN 4600
4580 X=Ninc(I,2)/Dis
4590 GOTO 4610
4600 X=0
4610 IF Ninc(I,2)=0 THEN 4630
4620 Fang=ATN(Strength(I,2)/Ninc(I,2))
4630 PRINT USING "7D.2D";Line(I,2),Ninc(I,2),Strenath(I,2),X,Fang
4640 NEXT I
4650 Nlines=66
4660 PRINTER IS 1
4670 '
4680 RETURN

4690 ! ***************
4700 ! INITIAL JOINT STIFFNESS
4710 ! ***************
4720 FOR I=1 TO Nplanes
4730 Siq=Sv*ABS(COS(Dip(I)))+Sh*Sin(Dip(I))
4740 t.in(I)=(E1/Space(I))/(E1/Emass-1)
4750 X=Siq/(Aperture(I)+t.in(I))
4760 IF X<.25 THEN GOTO 4810
4770 PRINTER IS 1
4780 PRINT "APERTURE SOLUTION WILL NOT CONVERGE"
4790 PAUSE
4800 !
4810 Cof (0)=-Sig/Aperture(I)
4820 Cof(1)=t.in(I)-Cof(0)
4830 Cof(2)=-2*t.in(I)
4840 Cof(3)=t.in(I)
4850 MAT Icof= (0)
4860 MAT Root= (0)
4870 MAT Iroot= (0)
4880 !
4890 CALL Siljak(3,Cof(*),Icof(*),1.E-12,1.E-12,50,Root(*),Iroot(*))
4900 !
4910 MAT SORT Root(*)
4920 Dva=Root(2)
4930 Kin(I)=Kin(I)*(1-Dva)*(1-Dva)
4940 !
4950 NEXT I
4960 !
4970 FOR I=1 TO Nplanes
4980 FOR J=1 TO 21
4990 Incl(J,1)=Initns(J,1)*Aperture(I)/(Kin(I)*Aperture(I)+Initns(J,1))
5000 NEXT J
5010 NEXT I
5020 !
5030 RETURN
5040 END !*ft»#»**#«#«***##«*«**##*####*#«#«»******#«#ft**#*#«****w#«#«*ft*««#
APPENDIX 2 Three-dimensional keyblock stability analysis program

BSTB3D

The equations developed in Chapter 3 have been implemented into a numerical three-dimensional solution for the stability of keyblocks as a function of displacement. This solution has been coded in the BASIC computer language as an interactive program named BSTB3D that can be run on a Hewlett-Packard 9816 computer. The user is prompted by the program for data needed for the analysis in a series of queries that are answered from the keyboard. Figure 5.1 illustrated the problem geometry used in the solution; other aspects of the use of the program are stated in the interactive prompts provided to the user.

The steps performed by BSTB3D to analyze keyblock stability are listed below:

1. Load necessary subroutines from files named "HPBSTAB3D" and "STRESS3DB." File STRESS3DB is listed following the listing of BSTB3D, and contains the coded solutions for three-dimensional stresses at a point near a cylindrical underground excavation. File HPBSTAB3DB contains proprietary subroutines to which the Hewlett-Packard Company holds copyright. Similar subroutines can be substituted by the user, or these codes can be obtained from Hewlett-Packard. These proprietary subroutines are:

   LUDSHT - solves a linear system of equations using triangular decomposition
CMULT - multiplication of two complex numbers
CDIVID - complex number division
CABS - absolute value of a complex number
CSQRT - square root of a complex number
CINV - inverse of a complex number
SILJAK - finds roots of a polynomial without initial guess
or derivative of the equation

2. Prompt user for input data. This data must be in a consistent
system of units, and includes:

   -- printing options and a problem header
   -- number of discontinuity planes, rock mass modulus,
      modulus of intact rock, Poisson's ratio, and coefficient
      of linear thermal expansion
   -- tunnel radius, tunnel axis rise and azimuth, distance
      from tunnel axis to block apex, clockwise angle of line
      between tunnel axis and apex from vertical upward, and
      internal pressure within tunnel
   -- displacement vector dip, azimuth, and increment, number
      of increments, rock unit weight, and changes in tunnel
      temperature
   -- principal stress magnitudes, dips, and azimuths
   -- choice of linear or nonlinear joint behavior

If linear behavior is selected, the following is needed for each
discontinuity forming a block face:
-- dip, dip azimuth, normal stiffness, and shear stiffness
-- friction angle, dilatancy angle, and residual friction angle

If nonlinear behavior is selected, then the following is required:

-- dip, dip azimuth, average spacing, and average aperture at negligible normal stress
-- joint roughness coefficient (JRC), joint compressive strength (JCS), and residual friction angle

3. After prompting for input, BSTB3D echoes the parameters to provide a check to the user.

4. BSTB3D sets up the coordinate systems for the tunnel and rotates the 3D in situ stress tensor into the tunnel system. The rotated stress tensor is listed for inspection.

5. Equations are set up for each discontinuity face of the keyblock, and coordinates are calculated in the tunnel system for the block corners. If the block is subsequently truncated, corners of the truncation surface are also calculated.

6. The block is partitioned into segments and the volume and weight of the block are computed. Each face is then divided into a grid of points for stress calculations. Face coordinate systems are
set up for rotation of stresses into components acting parallel and perpendicular to the displacement direction.

7. If nonlinear deformation behavior has been chosen for the discontinuities the initial normal stiffnesses are computed using pre-excavation stresses, and subsequently are adjusted for stress concentrations around the tunnel.

8. Stresses are calculated for each of the face grid points and rotated from the tunnel coordinate system into the face coordinate systems.

9. Grid point stresses are adjusted for successive increments of displacement. The stresses are summed into resultant forces affecting the block; these forces and the F/W ratio are printed for each increment.

Once the specified number of displacement increments has been examined, the user is given several options:

-- truncate the block near the apex
-- impose a temperature change in the tunnel
-- specify a new displacement magnitude to be checked
-- specify a new displacement direction
-- change the discontinuity shear parameters
-- restart the program for a new analysis

A listing of BSTB3D follows.
10 PRINT "BSTB3D VERSION 1.3 1/18/85"
20 PRINT """ VALUES USED IN THE ANALYSIS!"
30 PRINT "BLOCK DISPLACEMENT (L) AND DISPLACEMENT DIP AND AZIMUTH"
40 PRINT "ROCK UNIT WEIGHT (FL^-3)"
50 PRINT "NORMAL AND SHEAR STIFFNESSES OF DISCONTINUITY PLANE (FL^-3)"
60 PRINT "FRICITION ANGLE AND DILATENCY ANGLE OF DISCONTINUITY PLANE"
70 PRINT "DISCONTINUITY PLANE DIPS AND DIP AZIMUTHS"
80 PRINT "IN SITU STRESSES AND STRESS ORIENTATIONS (FL^-2)"
90 PRINT "TEMPERATURE CHANGES AND COEF OF THERMAL EXPANSION"
100 PRINT "*********************************************"
110 PRINT "BSTB3D REVISIONS:"
120 PRINT "1.3 1/18/85 ORIGINAL CODE BY JESSE L. YOW, JR."
130 PRINT "*********************************************"
140 DEG
150 OPTION BASE 1
FOR Fprin#=" " ! FLAG FOR LINE PRINTER
Fzz=1.6-6 ! FUZZ FOR ROUNDING TO INTEGER
Foption#=" " ! FLAG FOR OPTIONS
GOSUB 1310 ! PROMPT USER FOR DATA
IF Foption#="F" THEN 670
G1=E1/(2*(1+Mu1))
GOSUB 2230 ! INITIALIZE ARRAYS
GOSUB 2390 ! CALCULATE DIRECTION COSINES WRT REFERENCE COORDINATE SYSTEM
GOSUB 2650 ! ROTATE STRESS FIELD INTO TUNNEL REFERENCE SYSTEM
GOSUB 2880 ! SET UP FLAME EONS AND CORNER COORDINATES
GOSUB 4130 ! PARTITION BLOCK AND COMPUTE VOLUME AND WEIGHT
GOSUB 5140 ! PARTITION BLOCK FACES INTO GRID FOR STRESS COMPUTATIONS
GOSUB 6110 ! ROTATE DISPLACEMENT DIRECTION ONTO FACES
IF Fin#="Y" THEN GOSUB 690 ! CALCULATE INITIAL JOINT STIFFNESSES
IF Foption#="F" THEN GOTO 850
GOSUB 7060 ! CALCULATE STRESSES AT POINTS & ROTATE INTO DISP DIRECTION
GOTO 690

INPUT "DISPLACEMENT DIP AND AZIMUTH ?",Dispdir(*)
GOSUB 2590
GOSUB 6110
GOTO 850

INPUT "TRUNCATED APEX RADIUS ?",Tapexr
GOSUB 3790
GOSUB 4130
GOSUB 5140
GOSUB 850

INPUT "DELTA TEMPERATURE ?",Tdelta
GOSUB 7120 ! CALCULATE THERMAL STRESSES
GOSUB 7460 ! INITIALIZE STRESS MATRICES

FOR Increment=1 TO Nine
Disp=Disp+Dispinc
GOSUB 7600! ADJUST STRESSES FOR DISPLACEMENT
GOSUB 8150! FIND RESULTANT FORCES
GOSUB 8460! PRINT RESULTS
NEXT Increment
GOTO 1090

INPUT "DISPLACEMENT ?",Disp
Deldisp=Disp/10
GOSUB 7460 ! INITIALIZE STRESS MATRICES
191 FOR Incdisp = 1 TO 10
1020 Disp = Incdisp
1030 GOSUB 7600 ! ADJUST STRESSES FOR DISPLACEMENT
1040 GOSUB 8150 ! FIND RESULTANT FORCES
1050 NEXT Incdisp
1060
1070 GOSUB 8460 ! PRINT RESULTS
1080
1090 INPUT "TYPE ONE-LETTER CHOICE OF OPTION ", Foption
1100 IF Foption = "C" THEN 770
1110 IF Foption = "D" THEN 980
1120 IF Foption = "E" THEN 1290
1130 IF Foption = "F" THEN 550
1140 IF Foption = "R" THEN 830
1150 IF Foption = "S" THEN 720
1160 IF Foption = "T" THEN 830
1170
1180 PRINT "  
1190 PRINT "C TRUNCATE BLOCK NEAR APEX"
1200 PRINT "D SPECIFY DISPLACEMENT"
1210 PRINT "E END"
1220 PRINT "F CHANGE JOINT PLANE SHEAR PARAMETERS"
1230 PRINT "R RESTART PROGRAM"
1240 PRINT "S SPECIFY NEW DISPLACEMENT DIRECTION"
1250 PRINT "T SPECIFY TEMPERATURE CHANGE"
1260 PRINT "  
1270 GOTO 1090
1280
1290 STOP
1300 ! ******************************************************
1310 ! INPUT AND ECHO PARAMETERS
1320 ! ******************************************************
1330 PRINTER IS 1
1340 IF Foption = "F" THEN 1530
1350
1360 INPUT "ECHO RESULTS ON PRINTER ", Fprinter
1370 INPUT "PROBLEM HEADER (80 CHARACTERS MAX) ", Head
1380 INPUT "NUMBER OF PLANES, E MASS, E INTACT, POISSON'S RATIO, AND EXPANSION COEF ", Nplanes, Emass, El, Mu, Telph
1390 INPUT "TUNNEL RADIUS, CENTERLINE RISE, AZIMUTH, APEX DISTANCE, THETA, INTERNAL P ", Radius, Trim, Taz, Apexpr, Apexr, Pressure
1400 Tapepr = Apexpr
1410 INPUT "DISPLACEMENT DIP, AZIMUTH, INCREMENT, # INCS, ROCK UNIT WEIGHT, AND DEL T ", Dispdir(*), Dispinc, Ninc, Gamma, Tdelt
1420
1430 ! INPUT IN SITU STRESSES
1440
1450 FOR I = 1 TO 3
1460 DISP "INPUT MAGNITUDE, DIP, AND AZIMUTH OF SIGMA"; I;
1470 INPUT " ", Stin(I), Stindir(I,1), Stindir(I,2)
1480 NEXT I
1490
1500 ! INPUT DISCONTINUITY PLANE DATA
1920 INPUT "NONLINEAR JOINT BEHAVIOR (Y/N) ?", Flin*
1930 FOR I=1 TO Nplanes
1940 IF Flin*="Y" THEN 1620
1950 DISP "INPUT DIP, DIP AZIMUTH, NORMAL STIFFNESS, & SHEAR STIFFNESS FOR PLANE";I;
1960 INPUT "",Jtin(I,1),Jtin(I,2),Kn(I),Ks(I)
1970 DISP "INPUT PHI ANGLE, DILATANCY ANGLE, & RESIDUAL PHI ANGLE FOR PLANE";I;
1980 INPUT "",Phi(I),Di-lang(I,1),Phires(I)
1990 GOTO 1670
2000 DISP "INPUT DIP, DIP AZIMUTH, SPACING, AND APERTURE FOR PLANE";I;
2010 INPUT "",Jtin(I,1),Jtin(I,2),Space(I),Aperture(I)
2020 DISP "INPUT JRC, JCS, AND RESIDUAL PHI ANGLE FOR PLANE";I;
2030 INPUT "",Jrc(I),Jcs(I),Phires(I)
2040 ! ECHO INPUT DATA
2050 PRINT Head*
2060 PRINT "NUMBER OF DISCONTINUITY PLANES ",Nplanes
2070 PRINT "E MASS, E INTACT, POISSON'S RATIO",Emass,Ei,Mul
2080 PRINT "EXPANSION COEF, INTERNAL PRESS",Talph,Pressure
2090 PRINT "TUNNEL RADIUS, RISE, & AZIMUTH ",Radius,Trise,Taz
2100 PRINT "MAX RADIUS & THETA TO BLOCK APES ",&Apex,Apex
2110 PRINT "DISP INCREMENT & ROCK UNIT WEIGHT",Dispinc,Gamma
2120 PRINT "" MAGNITUDE DIP AZIMUTH"
2130 FOR I=1 TO 3
2140 PRINT USING 4200;lab1*,I,Stin(I),Stindir(I,1),Stindir(I,2)
2150 NEXT I
2160 PRINT " "
2170 PRINT "PLANEDIP DIP AZ LIN Kn LIN Ks (JRC)PHI (JCS)PHI RES PHI"
2180 FOR I=1 TO Nplanes
2190 IF Flin*="Y" THEN PRINT USING 4000;I,Jtin(I,1),Jtin(I,2),Kn(I),Ks(I),Jrc(I),Jcs(I),Phires(I)
2200 IF Flin*="Y" THEN 1900
2210 PRINT USING 4000;I,Jtin(I,1),Jtin(I,2),Kn(I),Ks(I),Phires(I),Di-lang(I,1),Phires(I)
2220 NEXT I
2230 !
2240 IF Fprinte#$<"Y" THEN 2200
2250 !
2260 PRINT " NUMBER OF DISCONTINUITY PLANES ",Nplanes
2270 PRINT "E MASS, E INTACT, POISSON'S RATIO",Emass,Ei,Mul
PRINT "EXPANSION COEF, INTERNAL PRESS", T, \nPRINT "TUNNEL RADIUS, RISE, & AZIMUTH", R, \nPRINT "RADIUS AND THETA TO BLOCK APEX", \nPRINT "DISP INCREMENT & ROCK UNIT WEIGHT", \nPRINT "MAGNITUDE DIP AZIMUTH" \nFOR I=1 TO J \nPRINT USING 420sL*b4*, I, Stin(I,1), Stin(I,2) \nNEXT I \nPRINT " FLANE DIP DIP AZ LIN kn LIN k's (JRC) PHI (JC) RES PHI" \nFOR I=1 TO Nplanes \nIF Flin="Y" THEN PRINT USING 400sI, Jtin(I,1), Jtin(I,2), Kn(I), Ks(I), Jrc(I), Jcs(I), Phi(I), RES(I) \nNEXT I \nPRINT " MAT Linter= (0), Corner= (0) \nFOR I=1 TO 3 \nTundir(I,1)=90 \nTundir(I,2)=T+90 \nTundir(2,1)=90-Trise \nTundir(2,2)=Taz \nTundir(3,1)=ABS(Trise) \nIF Trise<0 THEN Tundir(3,2)=Taz \nIF Trise>0 THEN Tundir(3,2)=Taz+180 \nRETURN \nFOR I=1 TO 3 \nStincos(I,1)=FNDircos(Tundir(I,1), Tundir(I,2)) \nStincos(I,2)=FNDircos(Tundir(I,1), Tundir(I,2)) \nStincos(I,3)=FNDircos(Tundir(I,1))
2510 NEXT I
2520 !
2530 FOR I=1 TO Nplanes ! DIRECTION COSINES OF THE JOINTS
2540 Jtcos(I,1)=FNDircos(Jtin(I,1),Jtin(I,2))
2550 Jtcos(I,2)=FNDircos(Jtin(I,1),Jtin(I,2))
2560 Jtcos(I,3)=FNDircos(Jtin(I,1))
2570 NEXT I
2580 !
2590 Ddcos(1)=FNDircos(Dispdir(1)+90,Dispdir(2))
2600 Ddcos(2)=FNDircos(Dispdir(1)+90,Dispdir(2))
2610 Ddcos(3)=FNDircos(Dispdir(1)+90)
2620 !
2630 RETURN
2640 !*****************************************************************************
2650 ! ROTATE STRESSES INTO TUNNEL COORDINATE SYSTEM
2660 !*****************************************************************************
2670 MAT D33= TRN(Stigcos)
2680 MAT D33b= Tuncos*D33a
2690 CALL Transf(D33b(:),Rot:*))
2700 MAT Stig= Rot*Stin
2710 !
2720 PRINT " "
2730 PRINT " SIGMA 1 SIGMA 2 SIGMA 3 TAU 23 TAU 31 TAU 12"
2740 PRINT USING 4I0;Stin(:) ! PRINT OLD STRESS ARRAY ON CRT
2750 PRINT USING 4I0;Stig(:) ! PRINT NEW STRESS ARRAY ON CRT
2760 !
2770 IF Fprinter(">"Y") THEN 2870
2780 !
2790 PRINTER IS 701
2800 PRINT " "
2810 PRINT " SIGMA 1 SIGMA 2 SIGMA 3 TAU 23 TAU 31 TAU 12"
2820 PRINT USING 4I0;Stin(*) ! PRINT OLD STRESS ARRAY
2830 PRINT USING 4I0;Stig(*) ! PRINT NEW STRESS ARRAY
2840 Nlines=Nlines+4
2850 PRINTER IS 1
2860 !
2870 RETURN
2880 !*****************************************************************************
2890 ! PLANE EDNS AND CORNER COORDINATES
2900 !*****************************************************************************
2910 MAT Apecos= (0)
2920 MAT Apex= (0)
2930 MAT Corner= (0)
2940 MAT Jteqn= (0)
2950 MAT Linter= (0)
2960!
2970 Apecos(1)=FNDircos(Apekt,90) ! APEX DIRECTION COSINES
2980 Apecos(2)=FNDircos(Apekt,90)
2990 Apecos(3)=FNDircos(Apekt)
3000 !
FOR I=1 TO 3
    Apex(I)=Apexcos(I)*Apexr  APEX COORDINATES
    NEXT I
FOR I=1 TO Nplanes ! JT EDNS IN TUNNEL COORDINATE SYSTEM
    Jteqn(I,K)=Tuncos(K,1)*Jteqn(I,1)+Tuncos(K,2)*Jteqn(I,2)+Tuncos(K,3)*Jtcos(I,3)
    NEXT K
    Jteqn(I,4)=Apex(I)*Jteqn(I,1)+Apex(2)*Jteqn(I,2)+Apex(3)*Jteqn(I,3)
    NEXT I
    Jteqn(I,4)=Apex(I)*Jteqn(I,1)+Apex(2)*Jteqn(I,2)+Apex(3)*Jteqn(I,3)
    NEXT I
NEXT I
FOR I=1 TO Nplanes ! LINES OF INTERSECTION BETWEEN FACES
    J=I+1
    IF I=Nplanes THEN J=Nplanes
    Linter(1,1)=Jteqn(I,2)*Jteqn(J,3)-Jteqn(I,3)*Jteqn(J,2) ! L DIRECOS
    Linter(I,2)=Jteqn(J,1)*Jteqn(I,3)-Jteqn(I,1)*Jteqn(J,3) ! M DIRECOS
    Linter(I,3)=Jteqn(I,1)*Jteqn(J,2)-Jteqn(J,1)*Jteqn(I,2) ! N DIRECOS
    C=SQR(Linter(I,1)*Linter(I,1)+Linter(I,2)*Linter(I,2)+Linter(I,3)*Linter(I,3)) ! ANGLE IN XZ PLANES BETWEEN INTERSECTION LINE AND LINE THRU APEX
    X=ACS((Linter(I,1)*Apex(I))/C)+(Linter(I,3)*Apex(3))/C) ! ANGLE IN XZ PLANES BETWEEN INTERSECTION LINE AND LINE THRU APEX
    IF X>90 THEN X=180-X ! THETA DIFF BETWEEN APEX AND INT LINE
    Y=ASIN(SIN(X)*Apexr/Radius)-X ! THETA DIFF BETWEEN APEX AND INT LINE
    Linter(I,4)=Y
    Linter(I,5)=I
    Linter(I,6)=J
    NEXT I
FIRST FACE  SECOND FACE
FOR I=Nplanes+1 TO 7
    Linter(I,4)=Apex(I)+180
    NEXT I
FOR I=1 TO Nplanes ! CALCULATE CORNER COORDINATES
    J=Linter(I,5)
    K=Linter(I,6)
    X=Apex(I)+Linter(I,4)
    Test=0
    D33c(1,1)=Jteqn(J,1) ! EDN OF FACE J
    D33c(1,2)=Jteqn(J,2)
    D33c(1,3)=Jteqn(J,3)
    D31a(1,1)=Jteqn(J,4)
    D33c(2,1)=Jteqn(K,1) ! EDN OF FACE K
    D33c(2,2)=Jteqn(K,2)
    D33c(2,3)=Jteqn(K,3)
    D31a(2,1)=Jteqn(K,4)
    D33c(3,1)=FDircosl(X,90) ! PLANE TANGENT TO TUNNEL
    D33c(3,2)=FDircosm(X,90)
    NEXT I
CALL Ludaht(D33c(*),D31a(*),3,1) ! SOLVE FOR X,Y,Z OF CORNER

! IF ABS(1-SQR(D3a(1)+D3a(2)+D3a(3))/Radius)<.01 THEN 3690

X=Apext-Linter(I,4)
Test=Test+1
IF Test<2 THEN 3690

PRINT "PROBLEM WITH CORNER",J,1.
FAUSE

Corner(I,1)=D3a(1) ! X COORDINATE
Corner(I,2)=D3a(2) ! Y COORDINATE
Corner(I,3)=D3a(3) ! Z COORDINATE
Corner(I,4)=J ! FIRST FACE
Corner(I,5)=K ! SECOND FACE
Linter(I,4)=X ! CHANGE TO THETA ANGLE ALONG TUNNEL SURFACE

NEXT I

RETURN

! CALCULATE CORNERS OF TRUNCATED APEX

FOR I=1 TO Nplanes
J=Linter(I,5)
K=Linter(I,6)
X=Apext

D33c(I,1)=Jteqn(J,1) ! EGN OF FACE J
D33c(I,2)=Jteqn(J,2)
D33c(I,3)=Jteqn(J,3)
D31a(I,1)=Jteqn(J,4)

D33c(I,2)=Jteqn(K,1) ! EGN OF FACE K
D33c(I,2)=Jteqn(K,2)
D33c(I,3)=Jteqn(K,3)
D31a(I,2)=Jteqn(K,4)

D33c(I,3)=FNDircosn(X,90) ! PLANE TANGENT TO TUNNEL
D33c(I,3)=FNDircosn(X,90)
D31a(I,3)=Tapekr
CALL Lud@ht(D33a(*),D31a(*),3,1) ! SOLVE FOR X,Y,Z OF CORNER

D3a(1)=D31a(1,1)
D3a(2)=D31a(2,1)
D3a(3)=D31a(3,1)

Tcorn(1,1)=D3a(1) ! X COORDINATE
Tcorn(1,2)=D3a(2) ! Y COORDINATE
Tcorn(1,3)=D3a(3) ! Z COORDINATE

NEXT 1

RETURN

PARTITION BLOCK AND COMPUTE VOLUME AND WEIGHT

MAT D74=Linter
FOR I=1 TO 7 ! GROUP THETA INTERCEPtS AROUND Apex
  IF D76(1,4):180+Apex THEN D76(1,4)=D76(1,4)-360
END FOR

MAT D76=Linter
MAT SRT D76(:,4) TO D76 ! FIND ORDER OF THETA VALUES
MAT D76=Linter
MAT REORDER D76 BY D76 ! ORDER INTERSECT LINES BY THETA
FOR I=1 TO 7
  17(I)=D76(1,4)
END FOR

! FIND THETA FROM Apex

OUT=2*Radius*SIN(0.05) ! "WIDTH" OF A 0.1 DEGREE ARC (SEGMENT BASE WIDTH)
ITmax=MIN(17(*))
Jmax=MAX(17(*))
J=INT(D76(1,5)+Fz2) ! INITIAL PLANES BOUNDING SEGMENT
K=INT(D76(1,6)+Fz2) ! INITIAL PLANES BOUNDING SEGMENT
L=2
Vol=0

FOR Iv=Itmin TO Itmax STEP .1 ! COMPUTE BLOCK VOLUME BY SEGMENTS
  IF Iv>180 THEN 4730 CI CHECK IF DONE WITH VOLUME
  IF Iv<17(I) THEN 4570 ! CHECK IF PASSED INTERSECT LINE L
  IF J>DO76(L,5) THEN 4450 ! REVISE J
    J=INT(D76(L,6)+Fz2)
  GOTO 4470
  IF J<DO76(L,6) THEN 4470
  J=INT(D76(L,5)+Fz2)
  IF K>DO76(L,5) THEN 4500 ! REVISE K
  K=INT(D76(L,6)+Fz2)
  GOTO 4470
  IF K<DO76(L,6) THEN 4520
198

2410 K=INT(D76(L,5)+F22)
2420 L=L+1
2430 IF L>Nplanes THEN 4730
2440 IF J<K THEN 4400 ! IF J=K THEN VOLUME CALCULATION IS FINISHED
2450 GOTO 4710
2460 !
2470 A=SIN((IV+Apex)*)
2480 C=COS((IV+Apex)*)
2490 D=Radius
2500 !
2510 X=A*SPI+Ap*K!
2520 ! Z=D
2530 !
2540 Y1=(Jteqn(J,1)+x+Jteqn(J,3)+Jteqn(J,4))/Jteqn(J,2)
2550 Y2=(Jteqn(K,1)+x+Jteqn(K,3)+Jteqn(K,4))/Jteqn(K,2)
2560 Del=ABS(Y1-YC) ! LENGTH OF SEGMENT BASE ALONG TUNNEL
2570 B=A=ABS((Y1-YC) ! AREA OF SEGMENT BASE
2580 Ht=ABS((A+Apex)+C*Apex-S)/SRD(A+A+C)
2590 ! HEIGHT OF SEGMENT
2600 Dvol=BS*Ht/3 ! VOLUME OF SEGMENT
2610 Vol=Vol+Dvol ! VOLUME OF BLOCK
2620 NEXT Iv
2630 !
2640 Tarec=0
2650 FOR I=3 TO Nplanes
2660 Dx=Tcorn(I,1)-Tcorn(I-1,1)
2670 Dy=Tcorn(I,2)-Tcorn(I-1,2)
2680 Dz=Tcorn(I,3)-Tcorn(I-1,3)
2690 L1=SQR(D:)+Dx+Dy*Dy+Dz*Dz)
2700 !
2710 D:=(Tcorn(I,1)-T corn(I-1,1)
2720 Dy=Tcorn(I,2)-Tcorn(I-1,2)
2730 Dz=Tcorn(I,3)-Tcorn(I-1,3)
2740 L2=SQR(D:)+Dx+Dy*Dy+Dz*Dz)
2750 !
2760 Dz=Tcorn(I,1)-Tcorn(I-1,1)
2770 Dy=Tcorn(I,2)-Tcorn(I-1,2)
2780 Dz=Tcorn(I,3)-Tcorn(I-1,3)
2790 L3=SQR(D:)+Dx+Dy*Dy+Dz*Dz)
2800 !
2810 S=(L1+L2+L3)/2
2820 Tarea=Tarea+SRD(S*(S-L1)*(S-L2)*(S-L3))
2830 NEXT I
2840 !
2850 Tvol=Tarea*(Apexr-Tapexr)/2
2860 Vol=Vol-Tvol
2870 !
2880 Weight=Vol*Gamma
2890 !
PARTITION BLOCK FACES INTO GRID FOR STRESS COMPUTATIONS

MAT Face n= 101
PRINT " "
PRINT "BLOCK VOLUME & WEIGHT: " , DROUND (Vol, 6), DROUND (Weight, 4)
PRINT " "
IF Printer<"Y" THEN 5120

PRINTER IS 701
PRINT " "
PRINT "BLOCK VOLUME & WEIGHT: " , DROUND (Vol, 6), DROUND (Weight, 4)
PRINT " "
Nlines=Nlines+2
PRINT " "
PRINTER IS 1
PRINT " "
RETURN

FOR i=1 TO Nplanes
  Tra=Apex+1.E-6
  FOR j=1 TO Nplanes; FIND EDGES AND CORNERS OF FACE
    IF Corner (i, 4) .GE. 1 THEN 5220
  NEXT 1
  IF Corner (i, 4) .LT. 1 THEN 5250
  IF Corner (i, 5) .GE. 1 THEN 5270
  NEXT 1
  IF Corner (i, 5) .LT. 1 THEN 5290
  NEXT 1
  BETACCS: Linter (Ke1, 1) + Linter + e2. 11 = Linter (Ke1, 2) + Linter (Ke2, 1) + Linter (Ke2, 2) + Linter (Ke2, 3) + Linter (Ke1, 3) ! INCLUDED ANGLE IN FACE AT Apex

  Dx1=(Corner:Ke1,1)-Apex(1)/10
  Dy1=(Corner:Ke1,2)-Apex(2)/10
  Dx2=(Corner:Ke2,1)-Apex(1)/10
  Dy2=(Corner:Ke2,2)-Apex(2)/10
  Dx3=(Corner:Ke2,3)-Apex(3)/10
  Dy3=(Corner:Ke2,3)-Apex(3)/10

  IF Options="C" THEN 5450
  X=(Tcorn(Ke1,1)+Tcorn(Ke2,1))/2
  Z=(Tcorn(Ke1,2)+Tcorn(Ke2,2))/2
  PRINT " "
  PRINTER IS 1
  PRINT " "
  RETURN

  FOR k=1 TO 10; COMPUTE COORDINATES OF EDGE POINTS
  Facexyz(I, k, 1)=Corner(Ke1,1)-Dy1+Dx1*(k-1)
  Facexyz(I, k, 2)=Corner(Ke1,2)-Dy1+Dx1*(k-1)
  Facexyz(I, k, 3)=Corner(Ke2,1)-Dy2+Dx2*(k-1)
  Facexyz(I, k, 4)=Corner(Ke2,2)-Dy2+Dx2*(k-1)
  Facexyz(I, k, 5)=Corner(Ke2,2)-Dy2+Dx2*(k-1)
200

```
5510  Facexyz(I,K+1,3)=Corner(Kc2,3)-Dz2*(K-1)
5520  NEXT K
5530  !
5540  FOR K=1 TO 3
5550    Facexyz(I,1,K)=Apex(K)
5560  NEXT K
5570  !
5580  L=2!
5590  FOR J=1 TO 9 ! ASSIGN COORDINATES TO POINTS WITHIN FACE EDGES
5600    Dvd=I-J
5610    Dv=(Facexyz(I,J,1)-Facexyz(I,J+1,1))/Dvd
5620    Dv=(Facexyz(I,J,2)-Facexyz(I,J+1,2))/Dvd
5630    Dv=(Facexyz(I,J,3)-Facexyz(I,J+1,3))/Dvd
5640    FOR I=1 TO Dvd=1
5650      L=L+1
5660      Facexyz(I,L,1)=Facexyz(I,J,1)-Dx*K
5670      Facexyz(I,L,2)=Facexyz(I,J,2)-Dy*K
5680      Facexyz(I,L,3)=Facexyz(I,J,3)-Dz*K.
5690  NEXT K
5700  NEXT J
5710  !
5720  Dh=0 ! CALCULATE AREAS FOR FACE SEGMENTS
5730  Dw=0
5740  FOR J=1 TO 3
5750    Dh=Dh+(Facexyz(I,J,1)-Facexyz(I,J,2))^2
5760    Dw=Dw+(Facexyz(I,J,2)-Facexyz(I,J,3))^2
5770  NEXT J
5780  Dh=SDR(Dh)
5790  Dw=SDR(Dw)
5800 !
5810  Ar1=Dh*Dw*SIN(Ept)
5820  Ar2=Ar1/2
5830  Ar4=Ar1/4
5840 !
5850  Facexyz(I,1,4)=Ar4 ! AREAS FOR SEGMENTS AT CORNERS AND APEX
5860  Facexyz(I,1,4)=Ar4
5870  Facexyz(I,1,4)=Ar4
5880 ! FOR J=2 TO 10 ! AREAS FOR SEGMENTS ALONG FACE EDGES
5890    Facexyz(I,J,4)=Ar2
5900    Facexyz(I,J+1,4)=Ar2
5910    Facexyz(I,J+2,4)=Ar2
5920  NEXT J
5930 !
5940 ! FOR J=31 TO 66 ! AREAS FOR SEGMENTS WITHIN FACE EDGES
5950    Facexyz(I,J,4)=Ar1
5960  NEXT J
5970 !
5980 ! FOR J=1 TO 66 ! SET AREA TO ZERO IF WITHIN TUNNEL OR TRUNCATED
5990    R=SDR(Facexyz(I,J,1)-2+Facexyz(I,J,2)-2)
6000```

IF R<Radius-Fz THEN Fxyz(I,J,4)=0
IF R>Tr THEN Fxyz(I,J,4)=0
NEXT J
FOR J=1 TO 66 : AREA OF FACES
  Farea(I)=Farea(I)+Fxyz(I,J,4)
NEXT J
NEXT I
RETURN
!
********************
*
******
!
SET UP DISP. DIRECTIONS FOR FACES
MAT D= Dcos  
! ROTATE DISP DIRCOS INTO TUNNEL COORDINATE SYSTEM
MAT Dcos= Tuncos*D
!
FOR I=1 TO Nplanes  
  PROJECT INTO FACES (DOUBLE X PRODUCT)
  Slider(I,1,1)=Jteqn(I,1)*Dcos(3)-Dcos(2)*Jteqn(I,1)  ! XL
  Slider(I,1,2)=Dcos(1)*Jteqn(I,1)-Jteqn(I,2)*Dcos(3)  ! XM
  Slider(I,1,3)=Jteqn(I,2)*Dcos(1)-Dcos(3)*Jteqn(I,2)  ! XN
  C=SQR(Slider(I,1,1)^2+Slider(I,1,2)^2+Slider(I,1,3)^2)
  FOR J=1 TO 3
    Slider(I,1,J)=Slider(I,1,J)/C  
    Slider(I,1,J)=Slider(I,1,J)/C  
  NEXT J
  FOR J=1 TO 3  
    Slider(I,3,J)=Slider(I,3,J)/C  
  NEXT J
  FOR J=1 TO 3
    Check=Check+Jteqn(I,J)*Slider(I,3,J)
  NEXT J
  IF ABS(Check)<Fz THEN S5000
  PRINT "DISP DIRECTION IS NOT IN PLANE FOR FACE",I
  PAUSE
  NEXT I
FOR I=1 TO Nplanes

! FIND ANGLE BETWEEN FACE AND DISPLACEMENT DIRECTION
X=0
FOR J=1 TO 3
X=X+Slider(I,J)+Dcos(J)
NEXT J
Data(I)=45/10
IF Data(I)>90 THEN Data(I)=180-Data(I)
NEXT I

FOR I=1 TO Nplanes

! SET UP STRESS ROTATION MATRIX FOR EACH FACE
FOR J=1 TO 3
FOR K=1 TO 3
t33a(J,K)=Slider(I,J,K)
NEXT K
NEXT J
CALL Trans(D33a(1:10,1:3),Rot(1:10,1:3)) ! CREATE ROTATION MATRIX FOR PLANE I

MAT D6 Rot Stig

Sin(I)=D6(1,2)
! NORMAL STRESS ON PLANE BEFORE EXCAVATION

FOR J=1 TO 6
FOR K=1 TO 6
Strot(I,J,1)=Rot(J,K)
NEXT K
NEXT J
NEXT I
RETURN

! COMPUTE INITIAL JOINT STIFFNESSES
FOR I=1 TO Nplanes
Kin(I)=(E1/Space(I))/(E1/Emass-1)
X=Sin(I)/(Aperture(I)*Kin(I))
IF X<0.25 THEN GOTO 6920

CO*(0)=Sin(I)/Aperture(I)
CO*(1)=Kin(I)-COF*(0)
COF(2)=-2*Kin(I)
COF(3)=Kin(I)
MAT ICOF=(0)

CALL Silljak(3,COF(1:3),ICOF(1:3),1E-12,1E-12,50,Root(1:3),IROOT(1:3))
MAT SORT Root(1:3)
Dva=Root(2)

\[ K\text{(I)} = K\text{(I)}(1-D\text{va})(1-D\text{va}) \]

NEXT I

RETURN

! ********************************************

! COMPUTE STRESSES AT GRID POINTS ON EACH FACE

! ********************************************

MAT Initial= (0)

MAT Closure= (0)

Froot=0 ! FLAG FOR FIRST USE OF STRESS3D

FOR 1=1 TO Nplanes

FOR J=1 TO 66

DISP I,J

IF False\text{xy}(I,J,4)=0 THEN 7430 ' POINT IS WITHIN TUNNEL

Ra=SQR(Face\text{xy}(I,J,1)^2+Face\text{yx}(I,J,3)^2)/Radius ! RADIUS FOR AMADEI

IF Face\text{xy}(I,J,1)=0 THEN 7210

X=ATN(Face\text{yx}(I,J,3)/Face\text{xy}(I,J,1))

GOTO 7230

X=90

GOTO 7230

X=90-X

IF Face\text{xy}(I,J,1)<0 THEN X=X+180

X=X-90

Tet=360-X

IF Tet>360 THEN Tet=Tet-360

CALL Stress3d(Ra,Tet) ! COMPUTE STRESSES IN TUNNEL SYSTEM AT POINT

FOR k=1 TO 6

FOR L=1 TO 6

Rot(k,L)=Slirot(I,k,L)

NEXT L

NEXT K

MAT D6= Rot*Stpt ! ROTATE STRESSES ONTO FACE I OF BLOCK

Initial(I,J,1)=D6(2) ! NORMAL STRESS (Y)

Initial(I,J,2)=D6(4) ! SHEAR STRESS ALONG DISPLACEMENT DIRECTION (YZ)

Initial(I,J,3)=D6(6) ! SHEAR STRESS ACROSS DISPLACEMENT DIRECTION (XY)

NEXT J

NEXT I

DISP "

RETURN

! ********************************************

! INITIALIZE STRESS AND CLOSURE MATRICES

! ********************************************

FOR 1=1 TO Nplanes
FOR J=1 TO 66
IF Flin<>"V" THEN 7540
Closure(I,J)=Initial(I,J,1)*Aperture(I)/(Kin(I)*Aperture(I)+Initial(I,J,1))
Final(I,J,1)=Initial(I,J,1)
Final(I,J,2)=Initial(I,J,2)
NEXT J
NEXT I
RETURN

! ADJUST STRESSES FOR DISPLACEMENT
D=Disp
IF Flin="Y" THEN 7870 ! CHECK FOR NON-LINEAR JOINT BEHAVIOR
FOR I=1 TO Nplanes ! LINEAR JOINT BEHAVIOR
FOR J=1 TO 66
IF Facexyz(I,J,4)<0 THEN 7830 ! POINT IS WITHIN TUNNEL
X=Kn(I)*D*(SIN(Deta(I))-COS(Deta(I))*TAN(Dilang(I,J))
Final(I,J,1)=Initial(I,J,1)-X ! NORMAL STRESS CHANGED BY DISPLACEMENT
IF Final(I,J,1)<0 THEN Final(I,J,1)=0 ! DISALLOW TENSION
X=Kn(I)*D*COS(Deta(I))
Final(I,J,2)=Initial(I,J,2)-X ! SHEAR STRESS CHANGED BY DISPLACEMENT
X=Final(I,J,1)*TAN(Phi(I)+Dilang(I,J)) ! COMPUTE MAXIMUM FRICTION
IF ABS(Final(I,J,2))<X THEN 7850
X=Final(I,J,2)*TAN(Phi(I)) ! COMPUTE RESIDUAL FRICTION
Final(I,J,2)=X*SGN(Final(I,J,2))
NEXT J
NEXT I
GOTO 8120

FOR I=1 TO Nplanes ! NON-LINEAR JOINT BEHAVIOR
FOR J=1 TO 66
X=Final(I,J,1)
IF X<1 THEN X=1
Dilang(I,J)=Jrc(I)*LGT(Jcs(I))/X
Dope=0+(SIN(Deta(I))-COS(Deta(I))*TAN(Dilang(I,J))) ! JOINT OPENING
X=Closure(I,J)-Dope
IF X<Closure(I,J) THEN 8000
X=Closure(I,J)
NEXT J
NEXT I
GOTO 8120
205

8010 Final(I,J,1)=Sin(I)/(1/(X))-(1/Aperture(I))
8020 GOTO 8050
8030 Final(I,J,1)=0
8040 ! Dear=D*COS(Dea(I))
8050 ! Strength=Final(I,J,1)*TAN(Dilang(I,J)+Phi(I,J))
8060 ! Final(I,J,2)=Initial(I,J,2)-Strength*100*Dear*COS(Dea(I))#/Aperture(I)
8070 IF ABS(Final(I,J,2))<Strength THEN 8100
8080 Final(I,J,2)=Strength*SGN(Final(I,J,2))
8090 NEXT J
8100 NEXT I
8110 DISP " "
8120 RETURN
8130 ! *****************************************
8140 ! SUM RESULTANT FORCES ACTING ON BLOCK
8150 ! *****************************************
8160 MAT Force= (0)
8170 FOR I=1 TO Nplanes ! SUM STRESSES INTO FORCES ON PLANE I
8180 FOR J=1 TO 66
8190 Force(I,J)=Force(I,J)+Final(I,J,1)*Face(I,J,4)
8200 Force(I,J)=Force(I,J)+Final(I,J,2)*Face(I,J,4)
8210 NEXT J
8220 NEXT I
8230 !
8240 NEXT I
8250 MAT Fb= (0)
8260 FOR I=1 TO Nplanes ! SUM FORCES IN BLOCK COORDINATE SYSTEM
8270 FOR J=1 TO 5
8280 Fb(I,J)=Fb(I,J)-Force(I,J)*Slider(1,2,3)-Force(I,J)*Slider(1,3,2)
8290 NEXT J
8300 NEXT I
8310 !
8320 !
8330 MAT D33b= TRN(Tuncos)
8340 MAT Fr= D33b*Fb ! ROTATE FORCE VECTORS INTO REFERENCE SYSTEM
8350 Fr(3)=Fr(3)*Weight ! ADD WEIGHT TO FORCE IN Z DIRECTION
8360 Fw=SGN(Fr(1)*Fr(1)+Fr(2)*Fr(2)+Fr(3)*Fr(3))
8370 Fw=Fw*SGN(Fr(3))/Weight
8380 !
8390 !
8400 Tense=0
8410 IF Tarea<=0 THEN RETURN
8420 IF Foption="C" THEN Tense=Fr(3)*COS(Apext)/Tarea
8430 !
8440 RETURN
8450 ! *****************************************
8460 ! PRINT OUT RESULTS
8470 ! *****************************************
8480 IF Foption="D" OR Foption="T" OR Foption="S" OR Fheader=1 THEN 8510
8490 PRINT " "
8500 PRINT "DEL T DISP. DIF AT Az Fx Fy Fz Fr/W"
8510 NT TENSILE"
8510 PRINT USING 430; Tdelt, Disp, Dispdir(*), Fr(*), Fw, TapeDir, Tense
8520 !
8530 IF Fprnter"<>"Y" THEN B660
8540 !
8550 PRINTER IS 701
8560 IF Foption"<>"D" OR Foption"<>"T" OR Foption"<>"S" OR FHeader=1 THEN B610
8570 PRINT "O
8580 PRINT "DEL T DISP. DIP AZ Fa Fy Fa Fr/W HT TENSILE"
8590 FHeader=1
8600 Nlines=Nlines+2
8610 PRINT USING 430; Tdelt, Disp, Dispdir(*), Fr(*), Fw, TapeDir, Tense
8620 Nlines=Nlines+1
8630 IF Nlines<(Npages*66-6) THEN B660
8640 PRINT CHR*(12)
8650 Npages=Npages+1
8660 PRINTER IS 1
8670 !
8680 RETURN
8690 END !**********************************************************************
8700 DEF FNDCos(L(Alph, Bet)) ! DIRECTION COSINE L
8710 Del=SIN(Alph)*SIN(Bet)
8720 RETURN Del
8730 FNEND !**********************************************************************
8740 DEF FNDCosM(Alph, Bet) ! DIRECTION COSINE M
8750 Dcm=SIN(Alph)*COS(Bet)
8760 RETURN Dcm
8770 FNEND !**********************************************************************
8780 DEF FNDCosN(Alph) ! DIRECTION COSINE N
8790 Dcn=COS(Alph)
8800 RETURN Dcn
8810 FNEND !**********************************************************************
8820 SUB Transf(A(*),B(*)) ! CONSTRUCT 6X6 ROTATION MATRIX
8830 OPTION BASE 1
8840 INTEGER I, J
8850 FOR J=1 TO 3
8860 FOR J=1 TO 3
8870 B(I,J)=A(I,J)*A(I,J)
8880 NEXT J
8890 B(I,4)=2*A(I,2)*A(I,3)
8900 B(I,5)=2*A(I,3)*A(I,1)
8910 B(I,6)=2*A(I,1)*A(I,2)
8920 NEXT I
8930 FOR J=1 TO 3
8940 B(4,J)=A(2,J)*A(3,J)
8950 B(5,J)=A(3,J)*A(1,J)
8960 B(6,J)=A(1,J)*A(2,J)
8970 NEXT J
8980 B(4,4)=A(2,2)*A(3,3)+A(3,2)*A(2,3)
8990 B(4,5)=A(2,3)*A(3,1)+A(3,3)*A(2,1)
9000 B(4,6)=A(2,1)*A(3,2)+A(3,1)*A(2,2)
<table>
<thead>
<tr>
<th>Line</th>
<th>Equation</th>
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</thead>
<tbody>
<tr>
<td>9010</td>
<td>$B(5,4) = A(1,2) \cdot A(3,3) \cdot A(3,2) \cdot A(1,1)$</td>
</tr>
<tr>
<td>9020</td>
<td>$B(5,5) = A(1,3) \cdot A(3,1) \cdot A(3,3) \cdot A(1,1)$</td>
</tr>
<tr>
<td>9030</td>
<td>$B(5,6) = A(1,1) \cdot A(3,2) \cdot A(3,1) \cdot A(1,2)$</td>
</tr>
<tr>
<td>9040</td>
<td>$B(6,4) = A(1,2) \cdot A(2,3) \cdot A(2,2) \cdot A(1,3)$</td>
</tr>
<tr>
<td>9050</td>
<td>$B(6,5) = A(1,3) \cdot A(2,1) \cdot A(2,3) \cdot A(1,1)$</td>
</tr>
<tr>
<td>9060</td>
<td>$B(6,6) = A(1,1) \cdot A(2,2) \cdot A(2,1) \cdot A(1,2)$</td>
</tr>
<tr>
<td>9070</td>
<td><strong>SUBEND</strong></td>
</tr>
</tbody>
</table>
SUB Str3d(Ra,Tet)

CODED BY JESSE YOW IN NOVEMBER, 1984

PART OF THIS CODING IS MODIFIED FOR USE ON AN hp©816 COMPUTER FROM A PROGRAM CALLED BERNIC IN BERNARD AMADEII'S THESIS (1982)

THE THERMAL STRESSES ARE CALCULATED WITH EONS 3.86 FROM ZUDANS, YEN, AND STEIGELMANN (1965)

OPTION BASE 1

INTEGER I,J

COM /Data2/ Stin<>,Stindir<>,El,E2,E3,Mu1,Mu2,Mu3,G1,G2,G3

COM /Data3/ Radius,Apexr,Apexl,Tris,Taz,Tundir<>,Head<>

COM /Data4/ Disp,Dispdir<>,Dispinc,Farea<>,Gamma,Teleph,Tdelt

COM /Data5/ Phi(a<>,Jrc<>,Jcs<>,Fpressue,Viewvec<>,Viewcos<>)

COM /Flags/ Fprinters,Froot,Nlines,Npages,Header,Option,Loaded

COM /Stress/ Stig<>,Stp<>,Mu<>,Imu<>,Flb<>,Fr<>,Beta<>,Aa<>

DIM Hp(6,6),Ts<6,6),Tts<6,6),Th(6,6),Te(6,6),Tte(6,6),Ah<3,3),Sf<6>

DIM T<6,6),T2<6,6),T3<6,6),A<3,3),Heat<6),T6<6>

DIM F<6,6),Fq<6),Aea<7),TS<6),T4<6),R<6),Io<6>

DIM L4<3),Lua<3),Landa<3),L2<3),L2<3),L3<3),L3<3),L4<3),L5<3)

DIM Dum<6>,Idum<6>,Dumr<6>,Idumr<6>,Sol<6),Rsol<6),Isol<6>

INITIALIZE MATRICES

MAT Heat= (0)

Sfo<1)<Stig<1>
Sfo<2)<Stig<2>
Sfo<3)<Stig<3>
Sfo<4)<Stig<4>
Sfo<5)<Stig<5>
Sfo<6)<Stig<6>

IF Froot=1 THEN 1370

MAT A= (0)
MAT Ah= (0)
MAT Hp= (0)

SET UP COMPLIANCE MATRIX Hp

FOR I=1 TO 3
  Hp(I,I)=1/E1
  Hp(I+3,I+3)=1/G1
NEXT I

Hp(I,2)<Mu1/E1
CALL Orien2(Tris,Taz,A(*))
CALL Transf(A(*),Ts(*))
MAT Tts= TRN(Ts)
!
CALL Orienh(Tris,Taz,An(*))
CALL Transf(An(*),Th(*))
FOR I=1 TO 6
  FOR J=1 TO 6
    Te(I,J)=TMI,J>
    IF (I<=3 AND J>3>) THEN Te(I,J)=5+Th(I,J)
    IF (I>3 AND J'<=3>) THEN Te(I,J)=2+Th(I,J)
    NEXT J
  NEXT I
MAT Tts= TRN(Te)
!
CALL STRAIN-STRESS MATRIX AA IN TUNNEL COORDINATE SYSTEM
MAT Tl= Te*Tts
MAT T2= Tl»Hp
MAT T3= T2«Ts
MAT Aa= T3*Tte
!
CALC COEFFICIENTS IN BETA MATRIX
FOR I=1 TO 6
  FOR J=1 TO 6
    Beta(I,J)=Aa(I,J)-Aa(I,3)*Aa(J,3)/Aa(3,3)
  NEXT J
NEXT I
!
CALC COEFFICIENTS OF 6TH DEGREE POLYNOMIAL
Aaa(1)=Beta(1,1)+Beta(5,5)-Beta(1,5)*Beta(1,5)
Aaa(2)=2*Beta(1,5)*Beta(1,4)+Beta(5,6))
Aaa(3)=Aaa(2)-2*(Beta(1,6)+Beta(5,5)*Beta(1,1)*Beta(4,5))
Aaa(4)=4*Beta(5,5)*Beta(1,2)+Beta(6,6)+Beta(1,1)*Beta(4,5)+Beta(4,5)+Beta(1,1)
Aaa(5)=Beta(4,4)
Aaa(6)=Aaa(5)+Beta(1,4)*Beta(5,6))
Aaa(7)=2*Beta(1,5)*Beta(2,5)+Beta(2,5)*Beta(4,6))
Aaa(8)=2*Beta(2,6)*Beta(5,5)-2*Beta(4,5)*2*Beta(1,2)+Beta(6,6))-2*Beta(1,6)+Beta(4,4)
Aaa(9)=Aaa(4)+2*Beta(1,5)*Beta(2,4)+2*Beta(1,4)+Beta(5,6))
Aaa(10)=Aaa(5)+Beta(2,2)+Beta(4,5)+Beta(4,5)+Beta(2,2)+Beta(4,4)+2*Beta(2,4)*(Beta(2,5)+Beta(4,6))
Aaa(11)=Beta(2,2)+Beta(4,4)+2*Beta(2,4)+Beta(4,4)+2*Beta(2,4)+Beta(4,4)+2*Beta(2,4)+Beta(4,4)}
Aaa(7) = Beta(2, 2) * Beta(4, 4) - Beta(2, 4) * Beta(2, 4)

! FIND ROOTS OF POLYNOMIAL

FOR I = 1 TO 7
   Dum(I-1) = Aaa(B-1)
   Idum(I-1) = 0
   NEXT I

Tola = 1, E-18
Tolf = 1, E-18
Itmax = 50
CALL Sijel(6, Dum(*), Idum(*), Tola, Tolf, Itmax, Dumr(*), Idumr(*))

FOR I = 1 TO 6
   Root(I) = Dumr(I-1)
   Iroot(I) = Idumr(I-1)
   NEXT I

Mu(I) = Root(2)
Mu(2) = Root(2)
Mu(3) = Root(1)
Imu(I) = Iroot(3)
Imu(2) = Iroot(2)
Imu(3) = Iroot(1)

FOR I = 1 TO 6
   Ri = Root(I)
   Ii = Iroot(I)
   CALL Cmult(Ri, Ii, R1, R2, R3, I3)
   CALL Cmult(Ri, Ii, R2, R3, R4, I4)
   CALL Cmult(Ri, Ii, R3, R4, R5, I5)
   CALL Cmult(Ri, Ii, R4, R5, R6, I6)
   Rsol(I) = Aaa(I) * R6 + Aaa(2) * R5 + Aaa(3) * R4 + Aaa(4) * R3 + Aaa(5) * R2 + Aaa(6) * R1 + Aa
   Sol(I) = Rsol(I) + Isol(I)
   NEXT I

CALL COEFFICIENTS Landa AND DELTA

FOR J = 1 TO 3
   CALL Cmult(Mu(J), Imu(J), Mu(J), Imu(J), R2, I2)
   CALL Cmult(Mu(J), Imu(J), Mu(J), Imu(J), R3, I3)
   CALL Cmult(Mu(J), Imu(J), Mu(J), Imu(J), R4, I4)
   CALL Cmult(Beta(J, 1), 0, R3, I3, Rd1, Id1)
   CALL Cmult(Beta(1, 4) + Beta(5, 6), 0, R2, I2, Rd2, Id2)
   CALL Cmult(Beta(2, 5) + Beta(4, 6), 0, Mu(J), Imu(J), Rd3, Id3)
   L3(J) = Rd1 - Rd2 + Rd3 - Beta(2, 4)
   Il3(J) = Id1 - Id2 + Id3
   CALL Cmult(Beta(5, 5), 0, R2, I2, Rd1, Id1)
CALL Cmult(2*B*t<(4,5),0,Mu(J),Imu(J),Rd2,Id2)
L2(J)=Rd1-Rd2+B*t<(4,4)
I12(J)=Id1-Id2

CALL Cmult(Beta(1,1),0,R4,14,Rd1,Id1)
CALL Cmult(2*Beta(1,6),0,R3,13,Rd2,Id2)
CALL Cmult(2*Beta(1,2)+Beta(6,6),0,R2,12,Rd3,Id3)
CALL Cmult(2*Beta(2,6),0,Mu(J),Imu(J),Rd4,Id4)
L4(J)=Rd1-Rd2+Rd3-Rd4+B*t<(2,2)
114(J)=Id1-Id2+Id3-Id4

CALL Cdivid(L3(J),113(J),L2(J),I12(J),Land(J),Ilanda(J))
IF J=3 THEN CALL Cdivid(L3(J),113(J),L4(J),114(J),Land(J),Ilanda(J))
Landa(J)=-Ilanda(J)
Ilanda(J)=-Ilanda(J)
NEXT J

R1=Mu(1)-Mu(J)
I1=Imu(1)-Imu(J)
CALL Cmult(Landa(3),Ilanda(3),R1,11,R2,12)
CALL Cmult(Landa(2),Ilanda(2),R2,12,R1,11)
R3=Mu(3)-Mu(J)
I3=Imu(3)-Imu(J)
CALL Cmult(Landa(3),Ilanda(3),R3,13,R4,14)
CALL Cmult(Landa(1),Ilanda(1),R4,14,R3,13)
Delta=Mu(2)-Mu(1)+R1+R3
Idelta=Imu(2)-Imu(1)+I1+I3

CALL CALCULATE THE NON-THERMAL STRESSES AT A POINT RA,TET
CALL FORMAT(Mu(*),Imu(*),Landa(*),Ilanda(*),Delta,Idelta,Tet,Ra,Aa(*),Fx(*),Fy(*),Frk(*))

MAT T4= Fx*Sf ! ROTATE THE NON-THERMAL STRESSES AT THE POINT
FOR I=1 TO 6
T5(I)=T4(I)+Pressure*Frk(I)
NEXT I

CALL CALCULATE THE THERMAL STRESSES AT RA,TET

IF Tdelta<>0 THEN 2100

Lnr=LOG(10)
C1=1+T*taul*Tdelt/(2*(1-Mu)*Lna)
C2=1/99
C3=10/Ra
Lnr=LOG(C3)

Sig=C1*(-Lnr-C2*(1+C3+C3)*Lna) ! RADIAL STRESS
Sig=C1*(1-Lnr-C2*(1+C3+C3)*Lna) ! TANGENTIAL STRESS
Sig=C1*(1-2*Lnr-2*C2*Lna) ! AXIAL STRESS
2060 ! CALL Tetset (Tet, T2(*)): ROTATE STRESSES INTO XYZ SYSTEM
2070 MAT T3 = INV (T2)
2080 MAT Heat = T3*T6
2100 ! SUM THERMAL AND NON-THERMAL COMPONENTS
2110 Stpt(1) = T5(1)+Heat(1)
2110 Stpt(2) = T5(2)+Heat(2)
2120 Stpt(2) = T5(2)+Heat(2)
2130 Stpt(4) = T5(4)+Heat(4)
2140 Stpt(6) = T5(6)+Heat(6)
2150 ! Froot = 1
2160 SUBEND !******************************************************************************************
2190 SUB Or1enh (D, B, Ah(*))
2200 OPTION BASE 1
2210 !
2220 IF ABS (D) = 90 THEN 2420
2230 !
2240 Bx = B + 90
2250 By = B + 180
2260 Bz = B
2270 !
2280 D = 0
2290 Dy = 90 - D
2300 Dz = D
2310 !
2320 Ah(1,1) = COS (Dz) * COS (Bx)
2330 Ah (1, 2) = SIN (Dz) * COS (Bx)
2340 Ah(1, 3) = SIN (Bx)
2350 Ah(2, 1) = COS (Dy) * COS (By)
2360 Ah(2, 2) = SIN (Dy)
2370 Ah(2, 3) = COS (Dy) * SIN (By)
2380 Ah (3, 1) = COS (Bz) * COS (Dz)
2390 Ah (3, 2) = SIN (Dz)
2400 Ah (3, 3) = COS (Dz) * SIN (Bz)
2410 SUBEXIT
2420 Ah(1, 3) = -1
2430 Ah (2, 1) = -1
2440 Ah (3, 2) = 1
2450 SUBEND !******************************************************************************************
2460 SUB Or1enh2 (D, B, Ah(*))
2470 OPTION BASE 1
2480 Tet = 90 - D
2490 St = B + 90
2500 IF St > 360 THEN St = St - 360
2510 IF St>180 THEN St=St-180
2520 IF B<90 AND B>»0 THEN 2550
2530 IF B<180 AND B>90 THEN 2590
2540 IF B<270 AND B>-180 THEN 2590
2550 Betax=St+180
2560 Betay=St
2570 Betaz=St+270
2580 GOTO 2620
2590 Betax=St
2600 Betay=St+180
2610 Betaz=St+90
2620 Deltax=90-Tet
2630 Deltay=Tet
2640 Deltaz=0
2650 A(1,1)=COS(Deltax)*COS(Betax)
2660 A(1,2)=SIN(Deltax)
2670 A(1,3)=COS(Deltax)*SIN(Betax)
2680 A(2,1)=COS(Deltay)*COS(Betay)
2690 A(2,2)=SIN(Deltay)
2700 A(2,3)=COS(Deltay)*SIN(Betay)
2710 A(3,1)=COS(Deltaz)*COS(Betaz)
2720 A(3,2)=SIN(Deltaz)
2730 A(3,3)=COS(Deltaz)*SIN(Betaz)
2740 SUBEND
2750 SUB Tetss(Tet,Tss)
2760 OPTION BASE 1
2770 MAT Tss= (0)
2780 Tet2=2*Tet
2790 Tss(1,1)=COS(Tet)*COS(Tet)
2800 Tss(2,2)=Tss(1,1)
2810 Tss(1,2)=SIN(Tet)*SIN(Tet)
2820 Tss(2,1)=Tss(1,2)
2830 Tss(3,3)=1
2840 Tss(1,6)=SIN(Tet2)
2850 Tss(2,6)=-Tss(1,6)
2860 Tss(6,1)=-S*Tss(1,6)
2870 Tss(6,2)=Tss(6,1)
2880 Tss(6,6)=COS(Tet2)
2890 Tss(4,4)=COS(Tet)
2900 Tss(5,5)=Tss(4,4)
2910 Tss(4,5)=-SIN(Tet)
2920 Tss(5,4)=-Tss(4,5)
2930 SUBEND
2940 SUB Fxmat(Mu(*),Imu(*),Landa(*),llanda(*),Delta,Idelta,Tet,fa,Aa(*),Fx(*),
2950 Fqu(*))
2960 OPTION BASE 1
2970 INTEGER I
2980 DIM ksi(3),lksi(3),sxi(3),1sxi(3),gama(3),1gama(3)
2990 !
IF Tet=0 THEN Tet=.001
IF Tet=90 THEN Tet=89.999
IF Tet=180 THEN Tet=179.999
IF Tet=270 THEN Tet=270.001
MAT Fx=(0)
MAT Fy=(0)
FOR i=1 TO 3
CALL Cmult(SIN(Tet),0,Mu(i),Imu(i),Z4,Z4)
Z4=Z4+COS(Tet)
CALL Cmult(Z4,14,R0,0,R1,11)
CALL Cmult(R1,11,R1,11,R2,12)
CALL Cmult(Mu(i),Imu(i),Mu(i),Imu(i),R3,13)
R2=R2+1-R3
I2=I2+13
CALL Csqrt(R2,I2,Sq(i),Isq(i))
IF Tet>90 AND Tet<270 THEN Sq(i)=-Sq(i)
IF Tet>90 AND Tet<270 THEN Isq(i)=-Isq(i)
R1=Sq(i)+R1
I1=Isq(i)+I1
CALL Cmult(0,1,Mu(i),Imu(i),R2,12)
R2=R2+1-R2
I2=I2-R2
CALL Csqrt(R1,11,R2,12,Ksi(i),Iksi(i))
CALL Cmult(Delta,Idelta,ksi(i),Iksi(i),R1,11)
CALL Cmult(R1,11,Sq(i),Isq(i),R2,12)
CALL Csqrt(R2,12,Gama(i),Igama(i))
Gama(i)=Gama(i)
Igama(i)=-Igama(i)
NEXT i
CALL Cmult(0,1,Gama(i),Igama(i),R1,11)
CALL Cmult(R1,11,Mu(i),Imu(i),R2,12)
CALL Cmult(R2,12,Mu(i),Imu(i),R1,11)
CALL Cmult(Landa(2),Ilando(2),Landa(3),Ilando(3),R2,12)
R2=R2-1
CALL Cmult(R1,11,R2,12,R4,14)
CALL Cmult(0,1,Gama(2),Igama(2),R1,11)
CALL Cmult(R1,11,Mu(2),Imu(2),R2,12)
CALL Cmult(R2,12,Mu(2),Imu(2),R1,11)
CALL Cmult(Landa(1),Ilando(1),Landa(3),Ilando(3),R2,12)
R2=R2-1
R2=-I2
CALL Cmult(R1,11,R2,12,R3,13)
R4=R4+R3
CALL Cmult(0,1,Gama(3),Igama(3),R2,12)
CALL Cmult(R2,12,Mu(3),Imu(3),R1,11)
CALL Cmult(R1,11,Mu(3),Imu(3),R2,12)
CALL Cmult(R2,12,Landa(3),Ilanda(3),R1,11)
R2=Landa(1)-Landa(2)
I2=Ilanda(1)-Ilanda(2)
CALL Cmult(R1,11,R2,12,R3,13)
R4=R4+R3
I4=I4+I3
Fx(1,1)=-R4

CALL Cmult(Gama(1),Igama(1),Mu(1),Imu(1),R2,12)
CALL Cmult(R2,12,Mu(1),Imu(1),R1,11)
CALL Cmult(Mu(3),Imu(3),Landa(3),Ilanda(3),R3,13)
CALL Cmult(R3,13,Landa(2),Ilanda(2),R2,12)
R3=Mu(2)-R2
I3=Imu(2)-I2
CALL Cmult(R1,11,R3,13,R4,14)
CALL Cmult(R2,12,Mu(2),Imu(2),R1,11)
CALL Cmult(R2,12,Landa(1),Ilanda(1),Landa(3),Ilanda(3),R3,13)
CALL Cmult(R3,13,Mu(3),Imu(3),R2,12)
R2=R2+Mu(1)
12=I2-Imu(1)
CALL Cmult(R1,11,R2,12,R3,13)
R4=R4+R3
I4=I4+I3
Fx(1,2)=-R4

CALL Cmult(Gama(1),Igama(1),Mu(1),Imu(1),R2,12)
CALL Cmult(R2,12,Mu(1),Imu(1),R1,11)
CALL Cmult(R1,11,Mu(3),Imu(3),R2,12)
CALL Cmult(R2,12,Landa(3),Ilanda(3),R1,11)
CALL Cmult(Mu(1),Imu(1),Landa(2),Ilanda(2),R2,12)
CALL Cmult(Mu(2),Imu(2),Landa(2),Ilanda(1),R3,13)
R2=R2-R3
I2=I2-13
CALL Cmult(R1,11,R2,12,R3,13)
R4=R4+R3
I4=I4+I3
Fx(1,2)=-R4

CALL Cmult(Gama(1),Igama(1),Mu(1),Imu(1),R2,12)
CALL Cmult(R2,12,Mu(1),Imu(1),R1,11)
CALL Cmult(R1,11,Mu(3),Imu(3),R2,12)
CALL Cmult(R2,12,Landa(2),Ilanda(2),Landa(3),Ilanda(3),R2,12)
R2=R2-R3
R2=R2+R3
R2=R2+R3
I2=I2+13

CALL Cmult(0,1,Mu(3),Imu(3),R5,15)
CALL Cmult(R5,15,Landa(3),Ilanda(3),R5,15)
CALL Cmult(R5,15,Landa(3),Ilanda(3),R5,15)
R2=R2-R3
I2=I2-I3
CALL Cmult(R1, I1, R2, I2, R4, I4)
CALL Cmult(Gama(2), Igama(2), Mu(2), Imu(2), R2, I2)
CALL Cmult(R2, I2, Mu(2), Imu(2), R1, I1)
CALL Cmult(Landa(1), Ilanda(1), Landa(3), Ilanda(3), R2, I2)
R2=R2+R3
I2=I2+I3
CALL Cmult(0, 1, Mu(3), Imu(3), R3, I3)
CALL Cmult(R3, I3, Landa(1), Ilanda(1), R5, I5)
R2=R2+R3
CALL Cmult(R5, I5, Landa(2), Ilanda(2), R3, I3)
R2=R2+R3
I4=I4+I3
CALL Cmult(Gama(3), Igama(3), Mu(3), Imu(3), R1, I1)
CALL Cmult(R1, I1, Mu(3), Imu(3), R2, I2)
CALL Cmult(R2, I2, Landa(3), Ilanda(3), R1, I1)
R2=R2-R3
I2=I2-I3
CALL Cmult(0, 1, Mu(1), Imu(1), R5, I5)
CALL Cmult(R5, I5, Landa(2), Ilanda(2), R3, I3)
R2=R2+R3
I2=I2+I3
CALL Cmult(0, 1, Mu(2), Imu(2), R5, I5)
CALL Cmult(R5, I5, Landa(1), Ilanda(1), R3, I3)
R2=R2+R3
I4=I4+I3
R4=R4+R3
Fx(1, 2) = R4
CALL Cmult(Mu(1), Ilanda(1), Mu(1), Imu(1), R1, I1)
CALL Cmult(R1, I1, Gama(1), Igama(1), R2, I2)
CALL Cmult(R2, I2, Landa(3), Ilanda(3), R1, I1)
R2=Mu(3)-Mu(2)
I2=Imu(3)-Imu(2)
CALL Cmult(R1, I1, R2, I2, R4, I4)
CALL Cmult(Mu(2), Imu(2), Mu(2), Imu(2), R1, I1)
CALL Cmult(R2, I2, Landa(3), Ilanda(3), R1, I1)
R2=Mu(1)-Mu(3)
I2=Imu(1)-Imu(3)
CALL Cmult(R1, I1, R2, I2, R3, I3)
R4=R4+R3
CALL Cmult(Mu(3), Imu(3), Mu(3), Imu(3), R1, I1)
CALL Cmult(R1, I1, Gamma(3), Igamma(3), R2, I2)
CALL Cmult(R2, I2, Land(3), Iland(3), R1, I1)
R2 = Mu(2) - Mu(1)
I2 = Imu(2) - Imu(1)
CALL Cmult(R1, I1, R2, I2, R3, I3)
R4 = R4 + R3
I4 = I4 + I3
Fx(1, 5) = R4
CALL Cmult(0, 1, Gamma(1), Igamma(1), R1, I1)
CALL Cmult(R1, I1, Gamma(2), Igamma(2), R2, I2)
CALL Cmult(R2, I2, Gamma(3), Igamma(3), R1, I1)
R2 = Mu(1) = Mu(2)
I2 = Imu(1) - Imu(2)
CALL Cmult(R1, I1, R2, I2, R4, I4)
R4 = R4 + R3
I4 = I4 + I3
CALL Cmult(R1, I1, R2, I2, R3, I3)
CALL Cmult(R1, I1, R2, I2, R3, I3)
R4 = R4 + R3
I4 = I4 + I3
Fx(1, 5) = R4
CALL Cmult(0, 1, Gamma(3), Igamma(3), R2, I2)
CALL Cmult(R1, I1, R2, I2, R4, I4)
CALL Cmult(0, 1, Gamma(2), Igamma(2), R1, I1)
CALL Cmult(R1, I1, Land(1), Iland(1), R2, I2)
R2 = R2 - R1
I2 = I2 - R2
CALL Cmult(R1, I1, R2, I2, R3, I3)
R4 = R4 + R3
I4 = I4 + I3
CALL Cmult(0, 1, Gamma(3), Igamma(3), R2, I2)
5010 CALL Cmult(R2, I2, Landa(3), Ilanda(3), R1, I1)
5020 R2=Landa(1)-Landa(2)
5030 I2=Ilanda(1)-Ilanda(2)
5040 CALL Cmult(R1, I1, R2, I2, R3, I3)
5050 R4=R4+R3
5060 J4=J4+13
5070 Fx(2, 1) = -R4
5080 CALL Cmult(Mu(3), Imu(3), Landa(3), Ilanda(3), R2, I2)
5100 CALL Cmult(R2, I2, Landa(2), Ilanda(2), R1, I1)
5110 R1=Mu(2)-R1
5120 I1=Imu(2)-I1
5130 CALL Cmult(Gama(1), Igama(1), R1, I1, R4, I4)
5140 CALL Cmult(Lenda(1), Ilanda(1), Landa(3), Ilanda(3), R2, I2)
5150 CALL Cmult(R2, I2, Mu(3), Imu(3), R1, I1)
5160 R1=Mu(3)-R1
5170 I1=Imu(3)-I1
5180 CALL Cmult(Gama(2), Igama(2), R1, I1, R3, I3)
5190 R4=R4+R3
5200 I4=I4+13
5210 CALL Cmult(Mu(1), Imu(1), Landa(2), Ilanda(2), R1, I1)
5220 CALL Cmult(Mu(2), Imu(2), Landa(1), Ilanda(1), R2, I2)
5230 R1=R1-R2
5240 I1=I1-I2
5250 CALL Cmult(Gama(3), Igama(3), R2, I2, R3, I3)
5260 CALL Cmult(R2, I2, R1, I1, R3, I3)
5270 R4=R4+R3
5280 I4=I4+13
5290 Fx(2, 2) = -R4
5300 CALL Cmult(0, 1, Mu(3), Imu(3), R1, I1)
5320 CALL Cmult(R1, I1, Landa(3), Ilanda(3), R2, I2)
5330 CALL Cmult(R2, I2, Landa(2), Ilanda(2), R1, I1)
5340 CALL Cmult(0, 1, Mu(2), Imu(2), R2, I2)
5350 CALL Cmult(Landa(2), Ilanda(2), Landa(3), Ilanda(3), R3, I3)
5360 R3=R3-1+R2-R1
5370 I3=I3+I2-I1
5380 CALL Cmult(Gama(1), Igama(1), R3, I3, R4, I4)
5390 CALL Cmult(Landa(1), Ilanda(1), Landa(3), Ilanda(3), R1, I1)
5400 CALL Cmult(0, 1, Mu(3), Imu(3), R2, I2)
5410 CALL Cmult(R2, I2, Landa(1), Ilanda(1), R3, I3)
5420 CALL Cmult(R3, I3, Landa(3), Ilanda(3), R2, I2)
5430 CALL Cmult(0, 1, Mu(1), Imu(1), R3, I3)
5440 R1=R1+R2-R3
5450 I1=I1+I2-I3
5460 CALL Cmult(Gama(2), Igama(2), R3, I3, R4, I4)
5470 R4=R4+R3
5480 I4=I4+13
5490 CALL Cmult(0, 1, Mu(1), Imu(1), R2, I2)
5500 CALL Cmult(R2, I2, Landa(2), Ilanda(2), R1, I1)
CALL Cmult(0, l, Mu(2), I mu(2), R3, R3)
5520 CALL Cmult(R3, R3, Landa(1), I landa(1), R2, R2)
5530 R1=Landa(1)-Landa(2)+R1-R2
5540 I1=landa(1)-landa(2)+I1-12
5550 CALL Cmult(Gama(3), Igama(3), Landa(3), I landa(3), R2, I2)
5560 CALL Cmult(R2, R2, R2, R2, R3, R3)
5570 R4=R4+R3
5580 I4=I4+I3
5590 Fx(2, 6)=R4
5600 CALL Cmult(Gama(1), Igama(1), Landa(3), I landa(3), R1, R1)
5620 R2=Mu(3)-Mu(2)
5630 I2=Imu(3)-Imu(2)
5640 CALL Cmult(R1, R1, R2, R2, R4, I4)
5650 CALL Cmult(Gama(2), Igama(2), Landa(3), I landa(3), R1, R1)
5660 R2=Mu(1)-Mu(2)
5670 I2=Imu(1)-Imu(2)
5680 CALL Cmult(R1, R1, R2, R2, R3, R3)
5690 R4=R4+R3
5700 I4=I4+I3
5710 CALL Cmult(Gama(3), Igama(3), Landa(3), I landa(3), R1, R1)
5720 R2=M u(2)-Mu(1)
5730 I2=Imu(2)-Imu(1)
5740 CALL Cmult(R1, R1, R2, R2, R3, R3)
5750 R4=R4+R3
5760 I4=I4+I3
5770 Fx(2, 4)=R4
5780 CALL Cmult(0, l, Gama(1), Igama(1), R2, R2)
5800 CALL Cmult(R2, R2, Landa(2), I landa(2), R1, R1)
5810 R2=Mu(3)-Mu(2)
5820 I2=Imu(3)-Imu(2)
5830 CALL Cmult(R1, R1, R2, R2, R4, I4)
5840 CALL Cmult(0, l, Gama(2), Igama(2), R2, R2)
5850 CALL Cmult(R2, R2, Landa(3), I landa(3), R1, R1)
5860 R2=Mu(1)-Mu(3)
5870 I2=Imu(1)-Imu(3)
5880 CALL Cmult(R1, R1, R2, R2, R3, R3)
5890 R4=R4+R3
5900 I4=I4+I3
5910 CALL Cmult(0, l, Gama(3), Igama(3), R2, R2)
5920 CALL Cmult(R2, R2, Landa(3), I landa(3), R1, R1)
5930 R2=Mu(2)-Mu(1)
5940 I2=Imu(2)-Imu(1)
5950 CALL Cmult(R1, R1, R2, R2, R3, R3)
5960 R4=R4+R3
5970 I4=I4+I3
5980 Fx(2, 5)=R4
5990 CALL Cmult(0, l, Gama(1), Igama(1), R2, R2)
6010 CALL Cmult(R2,12,Mu(1),Imu(1),R1,11)
6020 CALL Cmult(Landa(2),Ilanda(2),Landa(3),Ilanda(3),R2,12)
6030 R2=R2-1
6040 CALL Cmult(R1,11,R2,12,R4,14)
6050 CALL Cmult(Gama(2),Igamma(2),R2,12)
6060 CALL Cmult(R2,I2,Mu(2),Imu(2),R1,11)
6070 CALL Cmult(Landa(1),Ilanda(1),Landa(3),Ilanda(3),R2,12)
6080 R2=1-R2
6090 I2=I2
6100 CALL Cmult(R1,11,R2,12,R3,13)
6110 R4=R4+R3
6120 I4=I4+I3
6130 CALL Cmult(0,1,Gama(3),Igamma(3),R1,11)
6140 CALL Cmult(R1,11,Mu(3),Imu(3),R2,12)
6150 CALL Cmult(R2,12,Landa(3),Ilanda(3),R1,11)
6160 R2=Landa(1)-Landa(2)
6170 I2=Ilanda(1)-Ilanda(2)
6180 CALL Cmult(R1,11,R2,12,R3,13)
6190 R4=R4+R3
6200 I4=I4+I3
6210 FN(6,1)=R4
6220 CALL Cmult(Gama(1),Igamma(1),Mu(1),Imu(1),R1,11)
6230 CALL Cmult(R2,12,Mu(3),Imu(3),Landa(2),Ilanda(2),R3,13)
6240 CALL Cmult(R1,11,Mu(3),Imu(3),R2,12)
6250 CALL Cmult(R3,13,Landa(3),Ilanda(3),R2,12)
6260 R2=Mu(2)-R2
6270 I2=Imu(2)-I2
6280 CALL Cmult(R1,11,R2,12,R4,14)
6290 CALL Cmult(Gama(2),Igamma(2),Mu(2),Imu(2),R1,11)
6300 CALL Cmult(Landa(1),Ilanda(1),Landa(3),Ilanda(3),R3,13)
6310 CALL Cmult(R3,13,Mu(3),Imu(3),R2,12)
6320 R2=R2-Mu(1)
6330 I2=I2-Imu(1)
6340 CALL Cmult(R1,11,R2,12,R3,13)
6350 R4=R4+R3
6360 I4=I4+I3
6370 CALL Cmult(Gama(3),Igamma(3),Mu(3),Imu(3),R2,12)
6380 CALL Cmult(R2,12,Landa(3),Ilanda(3),R1,11)
6390 CALL Cmult(Mu(1),Imu(1),Landa(2),Ilanda(2),R2,12)
6400 CALL Cmult(R2,12,Landa(3),Ilanda(3),R1,11)
6410 R2=R2-R3
6420 I2=I2-13
6430 CALL Cmult(R1,11,R2,12,R3,13)
6440 R4=R4+R3
6450 I4=I4+I3
6460 FN(6,2)=R4
6470 CALL Cmult(Landa(2),Ilanda(2),Landa(3),Ilanda(3),R1,11)
6480 CALL Cmult(0,1,Mu(2),Imu(2),R2,12)
6490 CALL Cmult(0,1,Mu(3),Imu(3),R3,13)
CALL Cmult(R3,I3,Landa(2),Ilanda(2),R5,I5)
6520 CALL Cmult(R5,I5,Landa(3),Ilanda(3),R3,I3)
6530 R4=R4+R3
6540 14=14+13
6550 CALL Cmult(0,1,Mu(2),Imu(2),R2,I2)
6560 CALL Cmult(R2,I2,Landa(2),Ilanda(2),R1,11)
6570 CALL Cmult(R3,I3,Landa(3),Ilanda(3),R1,11)
6580 CALL Cmult(R2,I2,Landa(1),Ilanda(1),R2,12)
6590 CALL Cmult(R3,13,Landa(3),Ilanda(3),R2,12)
6600 CALL Cmult(R3,13,Mu(3),Imu(3),R2,12)
6610 CALL Cmult(R3,13,Landa(3),Ilanda(3),R3,13)
6620 R4=R4+R3
6630 14=14+13
6640 CALL Cmult(Gama(2),Igamma(2),Mu(2),Imu(2),R2,12)
6650 CALL Cmult(R2,12,R1,11,R3,13)
6660 R4=R4+R3
6670 14=14+13
6680 CALL Cmult(0,1,Mu(1),Imu(1),R2,12)
6690 CALL Cmult(R2,12,Landa(2),Ilanda(2),R1,11)
6700 CALL Cmult(R3,I3,Landa(3),Ilanda(3),R1,11)
6710 CALL Cmult(R2,12,Landa(1),Ilanda(1),R2,12)
6720 R2=Landa(1)-Landa(2)+R1-R2
6730 12=Ilanda(1)-Ilanda(2)+11-12
6740 CALL Cmult(Gama(3),Igamma(3),Mu(3),Imu(3),R3,13)
6750 CALL Cmult(R3,13,Landa(3),Ilanda(3),R1,11)
6760 CALL Cmult(R3,13,Landa(3),Ilanda(3),R1,11)
6770 R4=R4+R3
6780 14=14+13
6790 Fx(6,5)=R4
6800 CALL Cmult(Mu(1),Imu(1),Gama(1),Igamma(1),R2,12)
6810 CALL Cmult(R2,12,Landa(3),Ilanda(3),R1,11)
6820 R2=Mu(3)-Mu(2)
6830 R2=Mu(3)-Mu(2)
6840 12=Imu(3)-Imu(2)
6850 CALL Cmult(R1,I1,R2,12,R4,14)
6860 CALL Cmult(R2,12,Landa(2),Igamma(2),R2,12)
6870 CALL Cmult(R2,12,Landa(3),Ilanda(3),R1,11)
6880 R2=Mu(1)-Mu(3)
6890 12=Imu(1)-Imu(3)
6900 CALL Cmult(R1,I1,R2,12,R3,13)
6910 R4=R4+R3
6920 14=14+13
6930 CALL Cmult(Mu(3),Imu(3),Gama(3),Igamma(3),R2,12)
6940 CALL Cmult(R2,12,Landa(3),Ilanda(3),R1,11)
6950 R2=Mu(2)-Mu(1)
6960 12=Imu(2)-Imu(1)
6970 CALL Cmult(R1,I1,R2,12,R3,13)
6980 R4=R4+R3
6990 14=14+13
7000 Fx(6,4)=R4
CALL Cmult(0,1,Mu(1),Imu(1),R1,11)
7020 CALL Cmult(R1,11,Gama(1),Igama(1),R2,12)
7040 CALL Cmult(R2,12,Landa(3),Ilanda(3),R1,11)
7050 R2=Mu(3)-Mu(2)
7060 12=Imu(3)-Imu(2)
7070 CALL Cmult(R1,11,R2,12,R4,14)
7080 CALL Cmult(0,1,Mu(2),Imu(2),R1,11)
7090 CALL Cmult(R1,11,Gama(2),Igama(2),R2,12)
7100 CALL Cmult(R2,12,Landa(3),Ilanda(3),R1,11)
7110 R2=Mu(1)-Mu(3)
7120 12=Imu(1)-Imu(3)
7130 CALL Cmult(R1,11,R2,12,R3,13)
7140 R4=R4+R3
7150 14=14+13
7160 CALL Cmult(0,1,Mu(3),Imu(3),R1,11)
7170 CALL Cmult(R1,11,Gama(3),Igama(3),R2,12)
7180 CALL Cmult(R2,12,Landa(3),Ilanda(3),R1,11)
7190 R2=Mu(2)-Mu(1)
7200 12=Imu(2)-Imu(1)
7210 CALL Cmult(R1,11,R2,12,R3,13)
7220 R4=R4+R3
7230 14=14+13
7240 Fx(6,5)=R4
7250
7260 CALL Cmult(0,1,Gama(1),Igama(1),R1,11)
7270 CALL Cmult(R1,11,Landa(1),Ilanda(1),R2,12)
7280 CALL Cmult(R2,12,Mu(1),Imu(1),R1,11)
7290 CALL Cmult(Landa(2),Ilanda(2),Landa(3),Ilanda(3),R2,12)
7300 R2=R2-1
7310 CALL Cmult(R1,11,R2,12,R4,14)
7320 CALL Cmult(0,1,Gama(2),Igama(2),R1,11)
7330 CALL Cmult(R1,11,Landa(2),Ilanda(2),R2,12)
7340 CALL Cmult(R2,12,Mu(2),Imu(2),R1,11)
7350 CALL Cmult(Landa(1),Ilanda(1),Landa(3),Ilanda(3),R2,12)
7360 R2=1-R2
7370 12=12
7380 CALL Cmult(R1,11,R2,12,R3,13)
7390 R4=R4+R3
7400 14=14+13
7410 CALL Cmult(0,1,Gama(3),Igama(3),R2,12)
7420 CALL Cmult(R2,12,Mu(3),Imu(3),R1,11)
7430 R2=Landa(1)-Landa(2)
7440 12=Ilanda(1)-Ilanda(2)
7450 CALL Cmult(R1,11,R2,12,R3,13)
7460 R4=R4+R3
7470 14=14+13
7480 Fx(5,1)=R4
7490
7500 CALL Cmult(Gama(1),Igama(1),Landa(1),Ilanda(1),R2,12)
CALL Cmult(R2, 12, Mu(1), Imu(1), R1, 11)
7520 CALL Cmult(Mu(3), Imu(3), Landa(3), Ilanda(3), R3, 13)
7530 CALL Cmult(R3, 13, Landa(2), Ilanda(2), R2, 12)
7540 R2 = Mu(2) - R2
7550 I2 = Imu(2) - I2
7560 CALL Cmult(R1, 11, R2, 12, R4, 14)
7570 CALL Cmult(Imu(2), Landa(2), Landa(3), Ilanda(2), R2, 12)
7580 CALL Cmult(R2, 12, Mu(2), Imu(2), R1, 11)
7590 CALL Cmult(Landa(1), Ilanda(1), Landa(3), Ilanda(3), R3, 13)
7600 CALL Cmult(R3, 13, Mu(3), Imu(3), R2, 12)
7610 R2 = R2 - Mu(1)
7620 I2 = I2 - Imu(1)
7630 CALL Cmult(R1, 11, R2, 12, R3, 13)
7640 R4 = R4 + R3
7650 I4 = I4 + I3
7660 CALL Cmult(Gama(3), Igama(3), Mu(3), Imu(3), R1, 11)
7670 CALL Cmult(Mu(1), Imu(1), Landa(2), Ilanda(2), R2, 12)
7680 CALL Cmult(Mu(2), Imu(2), Landa(1), Ilanda(1), R3, 13)
7690 R2 = R2 - R3
7700 I2 = I2 - I3
7710 CALL Cmult(R1, 11, R2, 12, R3, 13)
7720 R4 = R4 + R3
7730 I4 = I4 + I3
7740 Fk(5, 2) = -R4
7750
7760 CALL Cmult(Landa(2), Ilanda(2), Landa(3), Ilanda(3), R1, 11)
7770 CALL Cmult(0, 1, Mu(2), Imu(2), R2, 12)
7780 CALL Cmult(0, 1, Mu(3), Imu(3), R3, 13)
7790 CALL Cmult(R2, 12, Landa(2), Ilanda(2), R5, 15)
7800 CALL Cmult(R1, 11, R2, 12, R3, 13)
7810 R1 = R1 - R2 - R3
7820 I1 = I1 - I2 - I3
7830 CALL Cmult(Gama(1), Igama(1), Mu(1), Imu(1), R2, 12)
7840 CALL Cmult(R2, 12, Landa(1), Ilanda(1), R5, 15)
7850 CALL Cmult(R5, 15, R1, 11, R4, 14)
7860 CALL Cmult(Landa(1), Ilanda(1), Landa(3), Ilanda(3), R1, 11)
7870 CALL Cmult(0, 1, Landa(1), Ilanda(1), R2, 12)
7880 CALL Cmult(R2, 12, Landa(3), Ilanda(3), R3, 13)
7890 CALL Cmult(R5, 15, Mu(3), Imu(3), R2, 12)
7900 CALL Cmult(0, 1, Mu(1), Imu(1), R3, 13)
7910 R1 = 1 - R1 + R2 - R3
7920 I1 = I1 - I2 - I3
7930 CALL Cmult(Gama(2), Igama(2), Mu(2), Imu(2), R3, 13)
7940 CALL Cmult(R3, 13, Landa(2), Ilanda(2), R2, 12)
7950 CALL Cmult(R2, 12, R1, 11, R3, 13)
7960 R4 = R4 + R3
7970 I4 = I4 + I3
7980 CALL Cmult(0, 1, Mu(1), Imu(1), R2, 12)
7990 CALL Cmult(R2, 12, Landa(2), Ilanda(2), R1, 11)
8000 CALL Cmult(0, 1, Mu(2), Imu(2), R3, 13)
CALL Cmult(R3, I3, Landa(1), Ilanda(1), R2, I2)
B020 R2 = Landa(1) - Landa(2) - R1 - R2
B030 I2 = Ilanda(1) - Ilanda(2) + I1 - 12
B040 CALL Cmult(Same(3), Igama(3), Mu(3), Imu(3), R1, I1)
B050 CALL Cmult(R1, I1, R2, I2, R3, I3)
B060 R4 = R4 + R3
B070 I4 = I4 + I3
B080 Rx(5, 6) = R4
B090
B100 CALL Cmult(Mu(1), Imu(1), Gama(1), Igama(1), R1, I1)
B110 CALL Cmult(R1, I1, Landa(1), Ilanda(1), R2, I2)
B120 CALL Cmult(R2, I2, Landa(3), Ilanda(3), R1, I1)
B130 R2 = Mu(3) - Mu(2)
B140 I2 = Imu(3) - Imu(2)
B150 CALL Cmult(R1, I1, R2, I2, R4, I4)
B160 CALL Cmult(Mu(2), Imu(2), Gama(2), Igama(2), R1, I1)
B170 CALL Cmult(R1, I1, Landa(2), Ilanda(2), R2, I2)
B180 CALL Cmult(R2, I2, Landa(3), Ilanda(3), R1, I1)
B190 R2 = Mu(1) - Mu(3)
B200 I2 = Imu(1) - Imu(3)
B210 CALL Cmult(R1, I1, R2, I2, R3, I3)
B220 R4 = R4 + R3
B230 I4 = I4 + I3
B240 CALL Cmult(Mu(3), Imu(3), Gama(3), Igama(3), R1, I1)
B250 R2 = Mu(2) - Mu(1)
B260 I2 = Imu(2) - Imu(1)
B270 CALL Cmult(R1, I1, R2, I2, R3, I3)
B280 I1 = R4 + R3
B290 I1 = I4 + I3
B300 Rx(5, 4) = R4
B310
B320 CALL Cmult(0, 1, Gama(1), Igama(1), R2, I2)
B330 CALL Cmult(R2, I2, Mu(1), Imu(1), R1, I1)
B340 CALL Cmult(R1, I1, Landa(1), Ilanda(1), R2, I2)
B350 CALL Cmult(R2, I2, Landa(3), Ilanda(3), R1, I1)
B360 R2 = Mu(3) - Mu(2)
B370 I2 = Imu(3) - Imu(2)
B380 CALL Cmult(R1, I1, R2, I2, R4, I4)
B390 CALL Cmult(0, 1, Gama(2), Igama(2), R2, I2)
B400 CALL Cmult(R2, I2, Mu(2), Imu(2), R1, I1)
B410 CALL Cmult(R1, I1, Landa(2), Ilanda(2), R2, I2)
B420 CALL Cmult(R2, I2, Landa(3), Ilanda(3), R1, I1)
B430 R2 = Mu(1) - Mu(3)
B440 I2 = Imu(1) - Imu(3)
B450 CALL Cmult(R1, I1, R2, I2, R3, I3)
B460 R4 = R4 + R3
B470 I4 = I4 + I3
B480 CALL Cmult(0, 1, Gama(3), Igama(3), R2, I2)
B490 CALL Cmult(R2, I2, Mu(3), Imu(3), R1, I1)
B500 R2 = Mu(2) - Mu(1)
CALL Cmult(R1, I1, R2, I2, R3, I3)
R4=R4+R3
I4=I4+I3
Fx(5, 3) = R4

CALL Cmult(0, 1, Gama(1), Igamma(1), R2, I2)
CALL Cmult(R2, I2, Landa(1), llanda(1), R1, I1)
CALL Cmult(Landa(2), llanda(2), Landa(3), llanda(3), R2, I2)
R2=R2-1

CALL Cmult(R1, I1, R2, I2, R3, I3)
R4=R4+R3
I2=-I2

Fx(4, 1) = R4

CALL Cmult(Gama(1), Igamma(1), Landa(1), llanda(1), R1, I1)
CALL Cmult(Mu(3), Imu(3), Landa(3), llanda(3), R2, I2)
R2=Mu(2)-R2
I2=Imu(2)-I2

CALL Cmult(R1, I1, R2, I2, R3, I3)
R4=R4+R3
I2=-I2

CALL Cmult(Mu(1), Imu(1), Landa(2), llanda(2), R2, I2)
CALL Cmult(0, 1, Gama(2), Igamma(2), R2, I2)
CALL Cmult(R2, I2, Landa(2), llanda(2), R1, I1)
CALL Cmult(Landa(1), llanda(1), Landa(3), llanda(3), R2, I2)
R2=R2-1

CALL Cmult(R1, I1, R2, I2, R3, I3)
R4=R4+R3
I2=-I2

CALL Cmult(Gama(3), Igamma(3), R2, I2, R3, I3)
R4=R4+R3
I2=I4+I3

Fx(4, 2) = R4
CALL Cmult(Landa(2), Landa(2), Landa(3), llanda(3), R1, I1)

CALL Cmult(0, 1, Mu(2), Imu(2), R2, I2)

CALL Cmult(0, 1, Mu(3), Imu(3), R3, I3)

CALL Cmult(R3, I3, Landa(2), llanda(2), R5, I5)

CALL Cmult(R5, I5, Landa(5), llanda(5), R3, I3)

R1 = R1 + R2 - R3

I1 = I1 + I2 - I3

CALL Cmult(Gama(1), Iqama(1), Landa(1), llanda(1), R2, I2)

CALL Cmult(R2, I2, R1, I1, R4, I4)

CALL Cmult(Landa(1), Landa(1), Landa(3), llanda(3), R1, I1)

CALL Cmult(R1, I1, Landa(1), llanda(1), R2, I2)

CALL Cmult(R3, I3, Mu(3), Imu(3), R2, I2)

CALL Cmult(R5, I5, Mu(3), Imu(3), R3, I3)

R1 = R1 + R2 - R3

I1 = I1 + I2 - I3

CALL Cmult(Gama(2), Igama(2), Landa(2), llanda(2), R2, I2)

CALL Cmult(R2, I2, R1, I1, R4, I4)

R4 = R4 + R3

I4 = I4 + I3

CALL Cmult(Gama(3), Igama(3), Landa(3), llanda(3), R2, I2)

CALL Cmult(R2, I2, R1, I1, R4, I4)

R2 = Landa(1) - Landa(2) + R1 - R2

I2 = Landa(1) - Landa(2) + I1 - I2

CALL Cmult(Gama(3), Igama(3), R2, I2, R3, I3)

R4 = R4 + R3

R3 = llanda(4) - Mu(3)

I2 = llanda(4) - Imu(3)

CALL Cmult(R1, I1, R2, I2, R3, I3)

R4 = R4 + R3

I4 = I4 + I3

R2 = Mu(2) - Mu(1)

I2 = Mu(2) - Imu(1)

CALL Cmult(Gama(3), Igama(3), R2, I2, R3, I3)

R4 = R4 + R3

I4 = I4 + I3

Fx(4, 4) = -R4

Fx(4, 6) = R4
9510 CALL Cmult\(0,1,Gama(1),Igama(1),R1,11\)
9520 CALL Cmult\(R1,11,Landa(1),Llanda(1),R2,12\)
9530 CALL Cmult\(R2,12,Landa(3),Llanda(3),R1,11\)
9540 \(R2=Mu(3)-Mu(2)\)
9550 \(I2=Imu(3)-Imu(2)\)
9560 CALL Cmult\(R1,11,R2,12,R4,14\)
9570 CALL Cmult\(0,1,Gama(2),Igama(2),R1,11\)
9580 CALL Cmult\(R1,11,Landa(2),Llanda(2),R2,12\)
9590 CALL Cmult\(R2,12,Landa(3),Llanda(3),R1,11\)
9600 \(R2=Mu(1)-Mu(3)\)
9610 \(I2=Imu(1)-Imu(3)\)
9620 CALL Cmult\(R1,11,R2,12,R3,13\)
9630 \(R4=R4+R3\)
9640 \(I4=I4+I3\)
9650 CALL Cmult\(0,1,Gama(3),Igama(3),R1,11\)
9660 \(R2=Mu(2)-Mu(1)\)
9670 \(I2=Imu(2)-Imu(1)\)
9680 CALL Cmult\(R1,11,R2,12,R3,13\)
9690 \(R4=R4+R3\)
9700 \(I4=I4+I3\)
9710 \(F(4,5)=R4\)

9720 FOR I=1 TO 6
9730 IF I=3 THEN 9760
9740 \(F(3,1)=\frac{(Aa(3,1)\times F(1,1)+Aa(3,2)\times F(2,1)+Aa(3,4)\times F(4,1)+Aa(3,5)\times F(5,1)+Aa(3,6)\times F(6,1))}{Aa(3,3)}\)
9750 NEXT I
9760 NEXT I

9770 FOR I=1 TO 6
9780 \(F(1,1)=1+\frac{F(1,1)}{F(1,1)}\)
9790 NEXT I
9800 NEXT I

9810 CALL Cmult\(Mu(3),Imu(3),Landa(2),Llanda(2),R2,12\)
9820 CALL Cmult\(R2,12,Landa(3),Llanda(3),R1,11\)
9830 CALL Cmult\(Landa(2),Llanda(2),Landa(3),Llanda(3),R3,13\)
9840 \(R3=R3-1\)
9850 CALL Cmult\(0,1,R3,13,R2,12\)
9860 \(R1=Mu(2)-R1+R2\)
9870 \(I1=Imu(2)-I1+I2\)
9880 CALL Cmult\(Gama(1),Igama(1),R1,11,21,1;1\)
9890 CALL Cmult\(Landa(1),Llanda(1),Landa(3),Llanda(3),R2,12\)
9900 CALL Cmult\(R2,12,Mu(3),Imu(3),R1,11\)
9910 \(R2=1-R2\)
9920 \(I2=12\)
9930 CALL Cmult\(0,1,R2,12,R3,13\)
9940 \(R1=R1-Mu(1)+R3\)
9950 \(I1=Imu(1)+I3\)
9960 CALL Cmult\(Gama(2),Igama(2),R1,11,22,1;2\)
9970 CALL Cmult\(Landa(2),Llanda(2),R1,11\)
9980 CALL Cmult\(Mu(1),Imu(1),Landa(2),Llanda(2),R1,11\)
CALL Cmult(Mu(2),Imu(2),Landa(1),Ilanda(1),R2,12)
10020  R3=Landa(1)-Landa(2)
10030  I3=Ilanda(1)-Ilanda(2)
10040  CALL Cmult(0,1,R3,I3,R4,14)
10050  R1=R1-R2+R3
10060  I1=I1-I2+I3
10070  CALL Cmult(Gama(3),Igama(3),f1,f1,f3,f1)
10080  CALL Cmult(Mu(1),Imu(1),Mu(1),Imu(1),R4,14)
10100  CALL Cmult(R4,14,f1,f1,R1,I1)
10110  CALL Cmult(R4,14,f2,f2,R2,12)
10120  CALL Cmult(R4,14,f2,f2,R2,12)
10130  CALL Cmult(Mu(2),Imu(2),Mu(2),Imu(2),R4,14)
10140  CALL Cmult(R4,14,Landa(3),Ilanda(3),R5,15)
10150  CALL Cmult(R5,15,f3,f3,f3,f3)
10160  Fqx(4)=R1+R2+R3
10170  CALL Cmult(Landa(3),Ilanda(3),f1,f1,f3,f1)
10190  Fqx(2)=Z1+Z2+R1
10200  CALL Cmult(Mu(1),Imu(1),Z1,f1,R1,11)
10220  CALL Cmult(Mu(2),Imu(2),Z2,f2,R2,12)
10230  CALL Cmult(Landa(3),Ilanda(3),Mu(3),Imu(3),R4,14)
10240  CALL Cmult(R4,14,Z3,f3,f3,f3)
10250  Fqx(6)=R1+R2+R3
10260  CALL Cmult(Landa(1),Ilanda(1),Mu(1),Imu(1),R4,14)
10280  CALL Cmult(R4,14,Z1,f1,R1,11)
10290  CALL Cmult(Landa(2),Ilanda(2),Mu(2),Imu(2),R4,14)
10300  CALL Cmult(R4,14,Z2,f2,R2,12)
10310  CALL Cmult(Mu(3),Imu(3),Z3,f3,f3,f3)
10320  Fqx(5)=R1+R2+R3
10330  CALL Cmult(Landa(1),Ilanda(1),Z1,f1,R1,11)
10350  CALL Cmult(Landa(2),Ilanda(2),Z2,f2,R2,12)
10360  Fqx(4)=R1+R2+Z3
10370  Fqx(3)=Fqx(1)*Aa(3,1)+Fqx(2)*Aa(3,2)+Fqx(4)*Aa(3,4)+Fqx(5)*Aa(3,5)+Fqx(6)*Aa(3,6)
10390  !SUBEND
PREFACE

This report develops three-dimensional techniques to assess the behavior of underground openings in cases where opening stability is controlled by rock mass structures such as joints. These techniques for evaluating stability complement rather than replace other excavation design approaches. The techniques are suitable for cylindrical openings like waste package emplacement holes and are good approximations for certain other excavation geometries as well. Emplacement hole stability is important for waste package integrity and retrieval and can also be a factor in emplacement hole construction.

A subsequent report will use currently available site geologic information from exploratory drilling with the methods developed herein to examine the stability of waste package emplacement holes that may be constructed at Yucca Mountain, NV. Example solutions in this subsequent report will illustrate the effects of emplacement hole orientation and data variability on opening stability. A final analysis of stability is planned for when fracture characterization data are available from the Exploratory Shaft facilities. This work is funded in the Nuclear Waste Management Group of Lawrence Livermore National Laboratory as part of the Nevada Nuclear Waste Storage Investigations Project of the U.S. Department of Energy.