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**STABILITY ISSUES IN
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ADDITION**

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STABILITY ISSUES IN RECONSTITUTION BY WEAPON ADDITION

Gregory H. Canavan

Reconstitution of strategic forces by the unilateral uploading of additional weapons from initially symmetric modest force levels reduces first strike stability. These changes are quantified and traced to changes in first and second strike costs in a model of missile exchanges in which both strikes are optimized analytically.

Reconstitution of strategic forces by the uploading of additional weapons generally reduces first strike stability. This note quantifies those changes in the context of the upload by one side only to augment initially equal forces of small numbers of weapons divided between vulnerable and survivable weapons. It derives the costs and stability indices induced by this unilateral reconstitution, which are based on an aggregated treatment of the interaction of two missile forces, the first and second strikes each could deliver, and their optimal offensive missile allocations based on the analytic minimization of first strike costs, which produces a first strike stability index based on the ratio of first to second strike costs.

Optimal allocations determine both side's missile strikes. The side that uploads has more weapons and allocates a greater fraction of them to his counter value strike. Second strikes do not change when reconstitution is achieved by uploading weapons. The side that does not fractionate sees rising second strike and decreasing first strike costs. The stability indices of both sides fall—less rapidly for the side that uploads. The overall stability index falls significantly as the number of weapons per missile is doubled.

The Exchange Model used is an aggregated, probabilistic treatment of the interaction of two unequal missile forces described in Appendix A,¹ which models exchanges between two missile forces in terms of the first and second strikes each side could deliver. The two sides are denoted by "unprime" and "prime," in accord with the symbols used for their forces and parameters. The unprime side has M vulnerable missiles with m weapons each and N survivable missiles with n weapons each. The prime side has M' vulnerable missiles with m' weapons each and N' survivable missiles with n' weapons each. If unprime strikes first, allocating a fraction f of his total of $W = mM + nN$ weapons at the M' prime vulnerable missiles, the unprime counter force strike is fW and the first strike on value targets is $F = (1 - f)W$.

The counter force portion of the unprime strike delivers an average of $r = fW/M'$ weapons per vulnerable missile, which gives them an average survival probability of $Q' \approx q^r$, where $q = 1 - p$, and p is the single shot probability of kill, which is taken to be 0.6 for all weapons and vulnerable missiles for both sides. The prime second strike is $S' = Q'm'M' + n'N'$.

The prime first strike F' for and the unprime second strike S can be derived by “conjugating” these expressions, i.e., interchanging prime and unprime symbols in them.

The resulting first and second strikes are converted into first and second strike costs through exponential approximations to the fractions of military value targets destroyed, as described in Appendix A. The calculations assume that the unprime and prime sides each have 1,000 value targets, which are denoted below by the symbols $k = k' = 1/1000$ for ease in generalization. The cost of damage to self and incomplete damage to other are joined in a weighted average using a weighting parameter L for unprime and L' for prime, which measure the attacker’s relative preference for damage to the other and prevention of damage to self.² The calculations below use $L = L' = 0.5$ for both sides. Sensitivity to L , which is significant, is studied in earlier notes. The first strikes of each side are optimized by minimizing the cost of executing them through an approach used in the Russian literature³ described in Appendix C,⁴ which has been shown to be sufficiently accurate for the modest force levels treated here.⁵

Forces for the example calculations below are taken to represent nominal post START III levels. Each side initially has 100 vulnerable missiles with 2 warheads each and 100 survivable missiles with 2 weapons each for a total of $W = W' = 2 \times 100 + 2 \times 100 = 400$ weapons. The prime side then increases the number of weapons on each vulnerable missile in unit increments, so that after 8 increments there are 10 weapons on each prime vulnerable missile for a total of $W' = 10 \times 100 + 2 \times 100 = 1,200$ weapons, 83% vulnerable, while unprime remains at $2 \times 100 + 2 \times 100 = 400$ weapons.

Attack allocation. Figure 1 shows f and f' as functions of $\Delta m'$, the number of weapons added to each prime vulnerable missile. For $\Delta m' = 0$ ($m' = 2$), the two attack allocations are equal at $f \approx f' \approx 0.35$. As $\Delta m'$ increases, f increases and f' decreases. By $\Delta m' = 2$ ($m' = 4$), the difference between them is a factor of ≈ 2 . The gap grows for larger increments. By $\Delta m' = 8$ ($m' = 10$), the unprime attack is largely counter force and the prime attack is largely counter value.

The reason for the different scaling of f and f' with $\Delta m'$ can be seen from Eq. (C2) of Appendix C for optimal attack allocation to vulnerable missiles

$$f_0 = (M'/W \ln q) \ln(-Lk'/km' \ln q), \quad (1)$$

which depends directly on M'/W and logarithmically on m' , L , and the target ratio k'/k . The dependence $f_0 \sim M'/W$ means that the unprime counter force first strike, which is $f_0 W = (M'/\ln q) \ln(-Lk'/km' \ln q)$, is *independent of the number of unprime weapons*. For the optimal unprime f_0 , M' and W are fixed, so $\Delta f_0 \sim \Delta \ln m' = \Delta m'/m'$, which produces the doubling of f_0 seen in Fig. 1 for $\Delta m' = 2$ as well as the saturation of f_0 at large $\Delta m'$. In the conjugate expression for $f_0' \sim M/W'$, M is fixed and $W' \sim m'M'$, so $f_0' \sim 1/m'$, as seen. Thus, the prime counter force first strike $f_0' W' = (M/\ln q) \ln(-L'k/k'm \ln q)$ is independent of m' altogether, and depends instead on the number of unprime vulnerable missiles and their degree of fractionation.

First and second strikes are shown in Fig. 2. The top curve is the prime first strike F' , which grows in proportion to the weapons added. Using the conjugate of Eq. (1) for f_0' gives

$$\begin{aligned} F' &= (1 - f')W' = [1 - (M/W' \ln q) \ln(-L'k/k' m \ln q)]W' \\ &= W' - (M/\ln q) \ln(-L'k/k' m \ln q), \end{aligned} \quad (2)$$

whose second term is independent of m' . Thus, $\Delta F' \sim \Delta W' \sim M' \Delta m'$, as seen in Fig. 2. That the second term is independent of m' reflects the allocation of $(M/\ln q) \ln(-L'k/k' m \ln q)$ weapons to prime's first strike on unprime vulnerable missiles. The unprime first strike on the prime vulnerable missiles involves a fixed number of missiles that is proportional to M and only logarithmically dependent on other attack parameters. It is independent of W —hence, not simply a fraction of unprime total force.

The two middle curves for S and S' have the same, constant value. Substitute the optimal allocation of Eq. (1) into the survivability probability of Eq. (A3) to produce

$$\begin{aligned} Q' &\approx q^r = \exp(\ln q^r) = \exp(r \ln q) = \exp(\ln q f_0 W/M') \\ &= \exp[\ln q (W/M')(M'/W \ln q) \ln(-Lk'/km' \ln q)] = -Lk'/km' \ln q, \end{aligned} \quad (3)$$

which is independent of m' , although it varies bilinearly with L and the ratio of unprime to prime value targets and inversely with the number of weapons per prime missile and the logarithm of their survivability. Thus,

$$S' = Q'm'M' + n'N' = (-Lk'/k \ln q)M' + n'N', \quad (4)$$

which is *independent* of m' because the $1/m'$ variation of Q' cancels the increase in the number of prime warheads in $m'M'$. Thus, the objective of the unprime optimal allocation is to limit and fix the number of vulnerable—and total—weapons in the prime second strike. The conjugate of Eq. (4) is the unprime second strike

$$S = QmM + nN = (-L'k/k' \ln q)M + nN. \quad (5)$$

In this example of unilateral upload from initially equal forces, $L = L'$, $k = k'$, $M = M'$, and $nN = n'N'$; thus, $S = S'$, and the second strikes are constant and equal, as shown. The bottom curve in Fig. 2 is the unprime first strike, F , which falls monotonically. Equation (1) gives

$$F = (1 - f_0)W = W - (M'/\ln q) \ln(-Lk'/km' \ln q), \quad (6)$$

which is the conjugate of Eq. (2). This indicates that the number of weapons in the unprime first strike on vulnerable missiles is proportional to M' and logarithmically dependent on other parameters. In this example W , M' , and parameters other than m' are constant, so $F \sim -\ln m'$, and $\Delta F \sim -\Delta m'/m'$, as seen in Fig. 2.

Unprime costs. Figure 3 shows the unprime first and second cost, together with their component costs as described in Appendix B. At the right side of the figure, the bottom curve is the cost for damage to self by the prime second strike following an unprime first strike, which is constant. The second line is the cost for incomplete damage to prime by a unprime second strike. The third curve is the cost for incomplete first strike damage to prime. The fourth curve is the

cost of damage to self when prime strikes first, which is the most rapidly varying of the component costs for reasons discussed below. The fifth curve is the first strike cost, which is the sum of the first and third lines. The sixth (top) curve is the second strike cost, which is the sum of the second and fourth lines.

The gap between the first and second strike costs is zero at $m' = 2$, but grows rapidly for larger m' , due largely to the increase in the cost of damage to unprime when prime strikes first, which increases from equality with the cost for damage to self by a prime second strike at $m' = 2$ to a 2-fold larger value at $m' = 10$.

The variation of these costs can be studied with the analytic optimizations above. The cost of retaliatory damage to self for the unprime first strike is

$$C_{1s} = (1 - e^{-kS'})/(1 + L) \approx kS'/(1 + L), \quad (7)$$

where the final form holds for $S' \ll 1/k$, which is the case for this example. For $L = 1/2$ and the $S' \approx 250$ of Fig. 2, this gives $C_{1s} \approx 0.25 / 3/2 \approx 0.17$, in accord with the bottom line of Fig. 3. As a result of the optimal unprime attack allocation, S' does not change with prime upload; thus, neither does C_{1s} . The cost due to incomplete damage by the first strike is

$$C_{1o} = Le^{-k'F}/(1 + L) \approx L(1 - k'F)/(1 + L), \quad (8)$$

which for $L = 1/2$ increases from $\approx 1/2 (1 - k'F)/(1 + L) / 3/2 \approx (1 - 0.35 \times 0.4) / 3 \approx 0.29$ at $m' = 2$ to $\approx (1 - 0.1 \times 0.4) / 3 \approx 0.32$ at $m' = 10$. For $F, S' \ll 1/k$, the results of Eqs. (7) and (8) can be combined to give

$$(1+L)(C_{1s} + C_{1o}) = (1+L)C_1 = (1 - e^{-kS'} + Le^{-k'F}) \approx kS' + L(1 - k'F), \quad (9)$$

which indicates that for moderate forces, the unprime first strike cost is dependent on the difference between the prime second and unprime first strikes, the latter weighted by the unprime attack preference parameter L . Because C_{1s} and C_{1o} vary slowly with m' , so does C_1 . This expression can be simplified in terms of the optimal allocations to

$$(1+L)C_1 \approx k[(-Lk'/k \ln q)M' + n'N'] + L\{1 - k'[W - (M'/\ln q) \ln(-Lk'/km' \ln q)]\}, \quad (10)$$

in which the first term for C_{1s} depends linearly on M' but is independent of m' because the $\sim 1/m'$ scaling of Q' offsets the increase in W' . The second term corresponds to an attack of magnitude $(M'/\ln q) \ln(-Lk'/km' \ln q)$ on prime vulnerable missiles. Thus, $C_1 \sim \ln m'$, and $\Delta C_1 \sim \Delta m'/m'$, as seen in the curve next to the top of Fig. 3.

The cost of damage to unprime as a result of a first strike by prime is given by Eq. (B4)

$$C_{2s} = (1 - e^{-kF})/(1 + L) \approx kF/(1 + L), \quad (11)$$

in which the final form is accurate only for modest m' . For $L = 1/2$, C_{2s} increases from $\approx (1 - f')kW'/(1 + L) \approx 0.65 \times 0.4 / 3/2 \approx 0.17$ at $m' = 2$, to $\approx 0.9 \times 1.2 / 3/2 \approx 0.7$, in rough accord with the fourth curve of Fig. 3. The difference between this estimate and the ≈ 0.4 from the curve is the saturation due to the nonlinear term in Eq. (11), which becomes important at about $m' = 4-6$,

or $W' \approx 600-800$ weapons. The cost to unprime due to incomplete damage to prime by the unprime second strike that follows the prime first strike is given by Eq. (B5)

$$C_{20} = Le^{-k'S}/(1+L) \approx L(1-k'S)/(1+L), \quad (12)$$

which for optimal allocation does not change because S does not change. The results of Eqs. (11) and (12) can be combined for modest F and S into

$$(1+L)(C_{2s} + C_{20}) = (1+L)C_2 = 1 - e^{-kF'} + Le^{-k'S} \approx kF' + L(1 - k'S), \quad (13)$$

in which the final form is accurate only for modest m' . This second strike cost depends on the difference between the unprime second and prime first strikes, weighted by the unprime attack preference L . The approximate form can be further simplified using the optimal allocations to

$$(1+L)C_2 \approx k[W' - (M/\ln q) \ln(-L'k/k'm \ln q)] + L\{1 - k'[M(-L'k/k' \ln q) + nN]\}, \quad (14)$$

in which the second term is independent of m' and the first varies as $C_2 \sim -W'$ so that $\Delta C_2 \sim \Delta m'$ as seen in the curve next to the top on Fig. 3. For $\Delta m' = 0$, C_1 and C_2 are nearly equal, but by $m' = 10$, C_1 is about 35% below C_2 .

Prime costs follow by conjugation. From Eq. (7). The cost of the retaliatory damage to unprime after his first strike is

$$C_{1s'} = (1 - e^{-k'S})/(1+L') \approx k'S/(1+L'), \quad (15)$$

which is approximately linear in the unprime second strike S . For optimal attack allocation, S does not change, so $C_{1s'} \approx 0.25 / 3/2 \approx 0.17$ for all m' . From Eq. (8) the cost to prime due to his incomplete damage to unprime in his first strike is

$$C_{10'} = L'e^{-kF'}/(1+L') \approx (1 - kF')/(1+L'), \quad (16)$$

which decreases from $1/2 \times e^{-(1-0.35) \times 0.4} / 3/2 \approx 0.26$ at $m' = 2$ to $\approx e^{-0.9 \times 1.1} / 3 \approx 0.12$ at $m' = 10$. For $F, S' \ll 1/k$, the two results of Eqs. (7) and (8) can be combined into

$$(1+L')(C_{1s'} + C_{10'}) = (1+L')C_1' \approx k'S + L'(1 - kF'), \quad (17)$$

which can be simplified in terms of the optimal allocations above to

$$(1+L')C_1' \approx k'[-(L'k/k' \ln q)M + nN] + L'\{1 - k[W' - (M/\ln q) \ln(-L'k/k' \ln q)]\}, \quad (18)$$

in which the first term does not vary with m' and the second varies as $-W'$, giving $\Delta C_2 \sim -\Delta m'$, as seen in the curve next to the top on Fig. 4. The cost of damage to prime from an unprime first strike is the conjugate of Eq. (11)

$$C_{2s'} = (1 - e^{-k'F})/(1+L') \approx k'F/(1+L'), \quad (19)$$

which for $L' = 1/2$ increases from $\approx (1 - 0.35) \times 0.4 / 3/2 \approx 0.17$ at $m' = 2$ to $\approx (1 - 0.8) \times 0.4 / 3/2 \approx 0.05$, in rough accord with the bottom curve of Fig. 4, the difference from this estimate being due to saturation due to the nonlinear term for $m' > 4-6$. The cost to prime due to incomplete damage his second strike is given by the conjugate of Eq. (12)

$$C_{20'} = L'e^{-kS'}/(1+L') \approx L'(1 - kS')/(1+L'), \quad (20)$$

which does not change because S' does not change. The results of Eqs. (19) and (20) can be combined for modest F and S into

$$(1+L')(C_{2s}' + C_{2o}') = (1+L')C_2' = 1 - e^{-k'F} + L'e^{-k'S} \approx k'F + L'(1 - k'S'), \quad (21)$$

in which the final expression for small forces again depends on the difference between the unprime second and prime first strikes, with the former weighted by the unprime attack preference L . The approximate form can be further simplified with the optimal allocations to

$$(1 + L')C_2' \approx k'[W - (M'/\ln q) \ln(-Lk'/km'\ln q)] + L'\{1 - k'[M'(-Lk'/k\ln q) + n'N']\}, \quad (22)$$

in which the second term is independent of m' and the first varies as $C_2' \sim -\ln m'$, so that $\Delta C_2 \sim -\Delta m' / m'$, as seen on Fig. 3. For $\Delta m' = 0$, C_1' and C_2' are nearly equal, but by $m' = 10$, C_1' is about 20% below C_2' .

Simplified expressions for the costs result for substituting the expressions for the optimal values of f , r , and Q and their conjugates from above, which reduce Eqs. (10) and (14) to

$$(1+L)C_1 \approx k[Q'm'M' + n'N'] + L[1 - k'(W - rM')], \quad (23)$$

$$(1 + L)C_2 \approx k[W' - r'M] + L[1 - k'[QmM + nN]], \quad (24)$$

which are simpler to manipulate, and should lead to no confusion as long as it is remembered that the survival probability $Q' \sim 1/m'$, so that the effective number of weapons per prime missile in second strike $Q'm'$ is independent of m' , and that r varies logarithmically with m' .

Stability indices. Figure 5 shows the stability index for unprime, $I = C_1'/C_2'$, for prime, $I' = C_1/C_2$, and their composite index, $I \times I'$. The index for prime falls little until $m' \approx 4$, after which it falls about 10% in the next $\Delta m' = 4$. The index for unprime begins falling immediately, falling about 30% by $m' = 8$. The composite index largely tracks I , although it falls more rapidly for $m' > 6$ where both I and I' are falling. The overall drop in the composite index is about 40%, which is significant. The reason for the drop can be seen in the unprime and prime costs presented above. Figure 3 shows that both C_1 and C_2 increase with m' , but that C_2 increases more rapidly, so the ratio of first to second strike costs falls rapidly, and first strike stability with it. Conversely, Fig. 4 shows that C_1' and C_2' both fall with m' , but remain parallel for small m' , so initially the fall in stability is slight. For larger m' the curves diverge, so stability also falls for prime. The product composite index takes the worst of each, so it falls faster than either.

It is possible to interpret these trends further using the approximate optimizations discussed above. That is facilitated by noting that for small F and S , the prime stability index, which is the ratio of Eqs. (10) and (13) is approximately

$$C_1 / C_2 \approx [L + kS' - Lk'F] / [L + kF' - Lk'S] \approx 1 + (k/L)(S' - F') - k'(S - F), \quad (25)$$

which indicates that the unprime index is the weighted sum of the differences between the prime and unprime second and first strike. The difference between Eqs. (10) and (13) is

$$(1 + L)(C_1 - C_2) \approx [L + kS' - Lk'F] - [L + kF' - Lk'S] \approx k(S' - F') + Lk'(S - F); \quad (26)$$

thus, in this limit $C_1/C_2 \approx 1 + (C_1 - C_2)(1 + L)/L$, so the difference of costs in Eq. (24) is a rescaling of the ratio index of Eq. (25) by the attack parameter L . In terms of the simplified expressions for C_1 and C_2 in Eqs. (23) and (24), the stability index becomes

$$\begin{aligned}
(I - 1)L &\approx k(Q'm'M' + n'N') - Lk'(W - r'M') - [k(W' - r'M) - Lk'(QmM + nN)] \\
&= (-Lk'm + kr' + Lk'mQ)M + (kQ'm' + Lk'r - km')M' \\
&= [Lk'm + k\ln(-L'k/k'm\ln q)/\ln q - LL'k/\ln q]M \\
&\quad + [-Lk'/\ln q + Lk'\ln(-Lk'/km'\ln q)/\ln q - km']M', \tag{27}
\end{aligned}$$

which shows the dependence of I on all parameters explicitly. I depends only on M , M' , and the number of weapons each carries. That is also the case for I' . The stability indices do not depend on the number or fractionation of the survivable forces, although those forces do affect the cost of first and second strikes, as indicated in Eqs. (23) and (24). Thus, if the decision to strike is based on C_1 , that decision is not affected by the survivable forces, although the damage done by and the costs of those strikes is increased by those forces. Equation (27) provides a rapid assessment of the impact of changes in the number of missiles and weapons on each side. for example, the sensitivity of I to m' for $k = k' = 0.001$ is given by

$$L\partial I/\partial m' = [-(Lk'/\ln q)/m' - k]M' = [-0.5/\ln 0.4 \times 1/m' - 1]kM' \approx [0.55/m' - 1]0.1, \tag{28}$$

which for $m' = 2$ gives $\partial I/\partial m' \approx -0.073 / 1/2 = -0.15$, in accord with the scaling seen in Fig. 5.

Summary and conclusions. This note derives the costs and stability indices induced by unilateral reconstitution of missile forces by the uploading of additional weapons from an initially symmetrical configuration with low force levels. They are based on an aggregated, probabilistic treatment of the interaction of two sides' missile forces, approximations to the first and second strikes each could deliver, and optimal offensive missile allocations between missiles and value based on analytic minimization of first strike costs. Stability indices for each side are taken to be the ratio of their first and second strike costs. The composite index is the product of their individual indices.

Optimal allocation gives a first strike proportional to the product of the opponent's missiles and the logarithm of his fractionation. The side that uploads allocates fewer weapons to counter force and more to counter value. The side that does not upload allocates more to counter force at the expense of his counter value strike. Second strikes do not change when reconstitution is achieved by fractionation due to the objectives of both side's optimal allocation. The side that does not fractionate sees rising second strike costs, primarily because of rapidly rising costs of damage to self. The side that fractionates sees decreasing first and second strike costs because of decreasing cost of damage to other in first strikes and of decreasing damage to self in second strikes. The stability indices of both sides fall—less rapidly for the side that fractionates. The overall index falls about 30% as the number of weapons per missile doubles.

APPENDIX A. EXCHANGE MODEL

It is possible to model exchanges between equal missile forces in terms of the first, F, and second, S, strikes one side could deliver. That analysis can be extended to unequal forces by treating the strikes F' and S' that the second side forces (denoted by primes for simplicity) could deliver. In it, the symbol convention is that unprime has M vulnerable missiles with m weapons each and N survivable missiles with n weapons each, and prime has M' vulnerable missiles with m' weapons each and N' survivable missiles with n' weapons each. If unprime is the first striker and a fraction f of his weapons is directed at prime's vulnerable missiles, unprime's first strike on value targets is

$$F = (1 - f)(mM + nN). \quad (A1)$$

The average number of weapons delivered on each of prime's vulnerable missile is

$$r = f(mM + nN)/M'. \quad (A2)$$

For r large, their average probability of survival is approximately⁶

$$Q' \approx q^r \approx e^{fW \ln q / M'}, \quad (A3)$$

where $q = 1 - p$, and p is the attacking missile's single shot probability of kill, which is taken to be $p = 0.6$ for all missiles. Prime's second strike is

$$S' = m'M'Q + n'N' \approx m'M'q^r + n'N', \quad (A4)$$

which is delivered on value, as missiles remaining at the end of the exchange are taken to have no value in this two strike engagement. The corresponding equations for prime's strike can be derived either by repeating the logic from his perspective or simply by conjugating the equations above, i.e., interchanging primed and unprime symbols in Eqs. (A1) - (A4).

APPENDIX B. STRIKE COSTS AND STABILITY INDICES.

These first and second strike magnitudes can be converted into the costs of striking first and second through exponential approximations to the fractions of value targets destroyed. The cost of damage to unprime when he strikes first is approximated by

$$C_{1s} = (1 - e^{-kS'})/(1 + L), \quad (B1)$$

where the constant $k \approx 1/1000$ is roughly equal to the inverse of the size of unprime military value target set that prime wishes to hold at risk,⁷ and L is a parameter. For a small prime second strike, $C_{1s} \sim kS'/(1 + L)$, which is small; for a large prime second strike, $C_{1s} \sim 1/(1 + L)$. The cost to unprime of incomplete damage to prime is approximated as

$$C_{10} = Le^{-k'F}/(1 + L), \quad (B2)$$

where $k' \approx 1/1000$ is roughly the inverse of unprime value that unprime wishes to hold at risk. C_{10} is small for F large and large for F small. C_{1s} and C_{10} are formally incommensurate, as

they represent damage to different parties, but a conventional approximation to a total cost for striking first is their weighted sum⁸

$$C_1 = C_{1s} + C_{1o} = (1 - e^{-kS'} + Le^{-k'F})/(1 + L), \quad (B3)$$

where L is a constant that represents the attacker's relative preference for inflicting damage on the other and preventing damage to self. L small means that as a first striker, unprime is primarily concerned about denying damage; L large means he is more concerned about inflicting damage on the other. The conventional assumption that $L \leq 1$ and construction of C_1 as a weighted average is plausible but not unique.⁹ Second strike costs are also composed of damage to self and other. The former is approximated for unprime by

$$C_{2s} = (1 - e^{-kF})/(1 + L); \quad (B4)$$

the latter by

$$C_{2o} = Le^{-k'S}/(1 + L); \quad (B5)$$

and the total cost for unprime striking second by

$$C_2 = C_{2s} + C_{2o} = (1 - e^{-kF'} + Le^{-k'S})/(1 + L), \quad (B6)$$

The first and second strike costs for prime can be obtained either by rederiving these results from prime's viewpoint or by conjugating Eqs. (B3) and (B6), which introduces a second constant L' , which reflects prime's attack preference¹⁰

There is some arbitrariness in converting C_1 and C_2 into stability indices.¹¹ It is conventional to use the ratio of first and second strike costs, $I = C_1/C_2$, as a stability index for unprime, and $I' = C_1'/C_2'$, as a stability index for prime. When this index is large, there is no advantage to striking first, and when it is small, there is an advantage, which may be perceived as an incentive to first attack in a crisis. For unequal forces, the product of the stability indices of the two sides is used as a compound index

$$\text{Index} = I \times I' = (C_1/C_2)(C_1'/C_2'). \quad (B7)$$

APPENDIX C. OPTIMAL ATTACK ALLOCATION.

For unprime, optimal attack allocation amounts to choosing f that minimizes his first strike cost C_1 , which is accomplished by differentiating Eq. (B3) with respect to f , setting the result to zero, and solving for f . For large forces the resulting equation is transcendental, but the optimal f for small forces ($F, S \ll 1/k$) holds sufficiently accurately for moderate forces ($F, S < 1/k$), for which Eq. (B3) reduces to

$$(1 + L) C_1 \approx k(m'M'e^{fW \ln q/M'} + n'N') + L[1 - k'(1 - f)W], \quad (C1)$$

whose derivative with respect to f has a minimum at

$$f_0 = (M'/W \ln q) \ln(-Lk'/km' \ln q). \quad (C2)$$

The equation for prime's first strike allocation is the conjugate of Eq. (C2). f_0 scales directly on the opponent's vulnerable missiles M' and inversely on one's own total weapons $W = mM + nN$. In a first strike, the distinction between vulnerable and survivable missiles is not significant, so the degree of fractionation of each is unimportant, only W matters. It is plausible that the number of weapons allocated to missiles should be proportional to the number of missiles, i.e., $Wf_{\text{opt}} \sim M'$, so that the vulnerable missiles are roughly covered. If in addition, the number of vulnerable weapons mM is proportional to the number of survivable weapons, $mM \sim nN$, then $f_0 \propto M'/mM$, i.e., f_0 scales in proportion to the relative number of the opponent's vulnerable missiles and inversely with one's own weapon inventory.

If the number of vulnerable missiles on each sides change proportionally, $M \sim M'$, then $f_0 \sim 1/m$. If the number of weapons per missile does not change, f_0 is constant. For $m = n$, i.e., equal fractionation of vulnerable and survivable weapons, $W = m(M + N)$, so that the allocation only depends on the total number of missiles, not on whether they are vulnerable or survivable. To first order it is unprime's weapons per missile that determines his allocation; prime's weapons per missile m' enters only logarithmically, as do L , k , and k' . The allocation decreases with L ; it is insensitive to prime's attack parameter L' , which does not enter C_1 .

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f vs t

Fig. 1. First strike allocations to missiles versus added weapons.

F & S vs t

Fig. 2. First and second strikes versus added weapons per missile.

Unprime 1 & 2 costs

Fig. 3. Unprime first and second strike and component costs versus added weapons per missile.

Fig. 4. Prime costs versus added weapons per missile.

indices vs t

Fig. 5. Stability indices versus added weapons per missile.