77

29

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Gypsy Field Project In Reservoir Characterization

Final Report

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GYPSY FIELD PROJECT IN RESERVOIR CHARACTERIZATION

Objectives

The overall objective of this project is to use the extensive Gypsy Field laboratory and data set as a focus for developing and testing reservoir characterization methods that are targeted at improved recovery of conventional oil.

The Gypsy Field laboratory, as described by Doyle, O'Meara, and Witterholt (1992), consists of coupled outcrop and subsurface sites which have been characterized to a degree of detail not possible in a production operation. Data from these sites entail geological descriptions, core measurements, well logs, vertical seismic surveys, a 3D seismic survey, crosswell seismic surveys, and pressure transient well tests.

The overall project consists of four interdisciplinary sub-projects which are closely interlinked:

1. Modeling depositional environments.
2. Sweep efficiency.
3. Tracer testing.
4. Integrated 3D seismic interpretation.

The first of these aims at improving our ability to model complex depositional environments which trap movable oil. The second is a development geophysics project which proposes to improve the quality of reservoir geological models through better use of 3D seismic data. The third investigates the usefulness of a new numerical technique for identifying unswept oil through rapid calculation of sweep efficiency in large reservoir models. The fourth explores what can be learned from tracer tests in complex depositional environments, particularly those which are fluvial dominated.

Summary

This final report consists of a compendium of three separate reports:

1. Modeling depositional environments.
2. A Time-Space Solution Approach For Prediction of Sweep Efficiency of Waterflooding Using the Cubature Method.
3. Integrated 3D Seismic Interpretation.

Summaries of these reports are contained in "Abstracts" on cover pages within the main body of the Final Report.

In the follow-on Project to the current one, DOE Project DE-FG22-95BC14869, the "Modeling Depositional Environments" and "Tracer Testing" sub-projects are continued. The "Sweep Efficiency" sub-project continues with the same goals, but employs a different approach, multi-grid methods, for improving the speed and accuracy of solving the flow equations.

MODELING DEPOSITIONAL ENVIRONMENTS

Abstract

An output least squares estimation technique is formulated to detect discontinuities on a two dimensional region with sparse pointwise measurements. A method to estimate the location and magnitude of a jump discontinuity is also developed. This technique is presented as a detection tool in the context of well test measurements of pressure for detecting permeability variations. The study is aimed at understanding our ability to detect discontinuities in geological or petrophysical properties due to, for example, channels, as are found in the Gypsy formation. Such discontinuities can have a profound effect on oil recovery. A numerical example demonstrates the use of our method for locating discontinuities in a permeability function in the presence of a background. Conditions are given for uniqueness and differentiability of the optimal estimated permeability function with respect to perturbations of the data.

1. Introduction. We study the determination of possibly discontinuous reservoir parameter functions defined on two dimensional regions from sparse pointwise measurements supplemented with measurements of a nonlinear function of the parameter. The specific application we have in mind is that of determining a permeability function from core measurements and pressure data, cf [6]. Our approach involves two steps. The first is to detect the discontinuous behavior, and the second is to isolate and refine the region containing it. For the first step we use a regularized output least squares procedure in which the reservoir mapping is approximated by linear combinations of bicubic B-splines. The regularization used is the H^1 seminorm that is related to the potential energy functional of an elastic membrane. This regularization gives sufficient compactness to obtain the existence of a solution to the associated minimization problem while implying minimal additional smoothing. Moreover, it seems to be well suited for the detection of the discontinuities and sudden changes so often exhibited by geological mappings [3,6]. Having at least detected an anomaly, we next attempt to isolate it by estimating its magnitude and a region containing it. The result of the procedure is to obtain a discontinuous function. We then essentially subtract this function from the model coefficient thereby, at least intuitively, reducing the discontinuous behavior. Again we consider the detection step to test for further discontinuous behavior. The procedure is repeated for further refinements.

In Section 2 we discuss the estimation of parameters by a regularized output least squares method in which there are data available in the form of measurements of permeability and pressure at locations within a two dimensional reservoir. Of interest here is the detection of the presence of features in the permeability function. The basic existence and approximation results are presented as well as resolution properties of the estimation procedure. If one uses a procedure for the purpose of detecting anomalies, the issue of the sensitivity of the method naturally arises and is discussed in Section 3. In Section 4 we describe a procedure to isolate the anomalies that have been detected. Essentially we seek a piecewise constant function possessing a rectangular subregion containing the feature of interest. Having found such a function, we again consider the regularized output least squares method to detect further anomalies in the permeability function repeating the procedure. Finally, Section 5 is devoted to the reporting the outcome of several numerical experiments.

2. Regularized Output Least Squares Estimation. We study the determination of a spatially dependent permeability mapping from measurements of the permeability at various locations along with pressure measurements. Towards this end, let Ω be an open domain in R^2 representing the reservoir with a Lipschitz boundary Γ . Let $K = K(x, y)$ be a real-valued function defined on Ω denote a permeability function that we wish to estimate. We suppose that measurements $\{K_i\}_{i=1}^{N_o}$ of K are available at N_o locations $\{x_i\}_{i=1}^{N_o}$ along with measurements $\{z_i\}_{i=1}^{N_o}$ of pressure p . We assume that fluid is injected at x_1 . While at x_{N_o} , the condition $p(x_{N_o}) = 0$ holds. The pressure p is a function of K according to Darcy's law

$$(2.1) \quad -\nabla \cdot (K \nabla p) = f \text{ in } \Omega,$$

with the boundary condition

$$(2.2) \quad \frac{\partial p}{\partial n} = 0 \text{ on } \Gamma,$$

and the additional condition that

$$(2.3) \quad \langle \Phi_{N_0}, p \rangle = 0.$$

The problem (2.1)-(2.3) may also be formulated in terms of a minimization problem

$$(2.4) \quad \text{minimize } \int_{\Omega} (K |\nabla p|^2 - 2fp) dx$$

$$\text{subject to } p \in H^1(\Omega) \text{ and } \langle \Phi_{N_0}, p \rangle = 0.$$

A continuous linear functional $\langle \Phi_i, \cdot \rangle$ models the measurement process [4]. For example, let B_i denote a ball of radius ρ_i centered at x_i that is contained entirely in Ω . Let

$$X_i(x, y) = 1 \text{ if } (x, y) \in B_i \text{ and } 0 \text{ otherwise.}$$

For $p \in H^1(\Omega)$, we take

$$\langle \Phi_i, p \rangle = \frac{1}{m(B_i)} \int_{\Omega} X_i(x, y) p(x, y) dx dy$$

where $m(B_i)$ is the measure of B_i . Thus, measurements of the pressure p are represented in terms of the action of certain continuous linear functionals Φ_i on the pressure function.

It is well-known that the problem (2.1)-(2.3) has a unique solution for each function

$$K \in Q'_{ad} = \{K \in H^1(\Omega) : \mu \geq K \geq \nu \text{ almost everywhere in } \Omega\}$$

where μ and ν are positive constants. Moreover, the formulation (2.4) may be solved using the techniques similar to those discussed in [6,7]. Accordingly, the mapping $K \mapsto p(K)$ is well-defined as a mapping from Q'_{ad} to $H^1(\Omega)$. Hence, the measurements on p give rise to the nonlinear constraints on the permeability function $K \in Q$

$$\langle \Phi_i, p(K) \rangle = z_i$$

where $i = 1, \dots, N_0$.

To honor the direct measurements on K , we introduce the linear functional constraints

$$\langle \phi_i, K \rangle = K^{(i)},$$

and define the set

$$Q_{ad} = \{K \in Q'_{ad} : \langle \phi_i, K \rangle = K^{(i)} \text{ for } i = 1, \dots, N_o\}.$$

The linear functionals ϕ_i may be of the type described above.

The approach we use to recover K from the data obtained from measurements $\{K_i\}_{i=1}^{N_o}$ of K and measurements $\{z_i\}_{i=1}^{N_o}$ of p is the so-called regularized output least squares method [7]. Hence, we formulate the following minimization problem

$$(2.5) \quad \text{Find } K_0 \in Q_{ad} \text{ such that } J(K_0) = \text{minimum } J(K) \text{ subject to } K \in Q_{ad}$$

where

$$J(K) = \int_{\Omega} |\nabla K|^2 dx + \gamma \sum_{i=1}^{N_o} (\langle \Phi_i, p(K) \rangle - z_i)^2$$

thereby accomodating the pressure measurements. The linear functional constraints may be included by penalization. Hence, in the penalized case setting

$$(2.6) \quad L(K) = J(K) + \frac{1}{\epsilon} \sum_{i=1}^{N_o} (\langle \phi_i, K \rangle - K_i)^2$$

we have

$$(2.7) \quad \text{minimize } L(K)$$

$$\text{subject to } K \in Q'_{ad}$$

In [7] it is proved that the mapping $K \mapsto p(K)$ is continuous from Q_{ad} with the weak $H^1(\Omega)$ topology into $H^1(\Omega)$ with the weak topology, see also [2]. Existence of a solution follows immediately.

Proposition 2.1. For each ϵ , γ , μ , and ν greater than 0, there exist solutions to problems (2.5) and (2.7). As ϵ approaches zero, weak cluster points $H^1(\Omega)$ of the solutions K_ϵ of problem (2.7) are solutions of (2.5).

Because the mapping $K \mapsto p(K)$ is nonlinear, problem (2.7) is solved numerically by iteration. For example, starting with an initial guess for K , the procedure seeks to update K by means of a descent method. In each step it is necessary to solve a system that approximates (2.1)-(2.3) as well as a gradient system. The approximating system is obtained by the finite element method. Let $\{\alpha_i\}_i^N$ and $\{\psi_i\}_i^M$ be sets of N and M linearly independent functions in $H^1(\Omega)$, respectively. Express p and K as linear combinations

$$(2.8) \quad p = \sum_{i=1}^N c_i \alpha_i$$

and

$$(2.9) \quad K = \sum_{i=1}^M a_i \psi_i.$$

Define the matrices for $k = 1, \dots, M$

$$(2.10) \quad (G^{(k)})_{ij} = \int_{\Omega} \psi_k \nabla \alpha_i \cdot \nabla \alpha_j dx$$

and $i, j = 1, \dots, N$, as well as the column-vector ρ of length N with k -th component

$$(2.11) \quad \rho_k = \int_{\Omega} f \alpha_k dx.$$

The discrete analogue of problem (2.1)-(2.3) is given by

$$(2.12) \quad \sum_{k=1}^M a_k G^{(k)} c = \rho.$$

with the additional condition

$$(2.13) \quad \sum_{k=1}^M c_k \langle \Phi_{N_o}, \alpha_k \rangle = 0.$$

Based on the minimization problem (2.4), we may obtain boundary value problems in which the supplementary condition (2.13) is included by means of a penalization. In this case equations (2.1)-(2.3) are replaced by the boundary value problem

$$(2.14) \quad -\nabla \cdot (K \nabla p) + \frac{1}{\eta} \langle \Phi_{N_o}, p \rangle \Phi_{N_o} = f \text{ in } \Omega$$

$$(2.15) \quad \frac{\partial p}{\partial n} = 0 \text{ on } \Gamma$$

where $\frac{1}{\eta}$ serves as a penalization of the constraint $\langle \Phi_{N_o}, p \rangle = 0$ as $\eta \rightarrow 0$. Thus, introducing the $N \times N$ matrix G_{ϕ} defined by

$$(G_{\phi})_{kj} = \langle \Phi_{N_o}, \alpha_k \rangle \langle \Phi_{N_o}, \alpha_j \rangle$$

and setting

$$G = \sum_{k=1}^M a_k G^{(k)} + \frac{1}{\eta} G_{\phi},$$

equations (2.14) and (2.15) are replaced by the equation

$$(2.16) \quad G c = \rho$$

Therefore, given a vector $a = (a_1, \dots, a_M)$ (the function K), we may calculate the vector $c = (c_1, \dots, c_N)$ (the function p) as the solution to equation (2.16). With these values we may now evaluate the functional $L(a) = L(K)$. The solution of (2.6) is that vector a_0 minimizing the functional $L(a)$ subject to the conditions specifying Q'_{ad} . Define the matrices and vectors

$$(2.17) \quad (H)_{ij} = \sum_{k=1}^{N_o} \langle \Phi_k, \alpha_i \rangle \langle \Phi_k, \alpha_j \rangle$$

$$(2.18) \quad \zeta_i = \sum_{k=1}^{N_o} \langle \Phi_k, \alpha_i \rangle z_i$$

$$C_z = \sum_{k=1}^{N_o} z_i^2$$

for $i, j = 1, \dots, N$, and for $i, j = 1, \dots, M$,

$$(2.19) \quad (G_0)_{ij} = \int_{\Omega} \nabla \psi_i \cdot \nabla \psi_j dx$$

$$(2.20) \quad (H_K)_{ij} = \sum_{n=1}^{N_o} \langle \phi_n, \psi_i \rangle \langle \phi_n, \psi_j \rangle$$

$$(2.21) \quad \kappa_i = \sum_{n=1}^{N_o} \langle \phi_n, \psi_i \rangle K^{(n)}$$

$$(2.22) \quad C_K = \sum_{k=1}^{N_o} (K^{(n)})^2$$

The functional $L(a)$ takes the form

$$(2.23) \quad L(a) = \gamma \{ c(a)^* H c(a) - 2 \zeta^* c(a) + C_z \} + a^* (G_0 + \frac{1}{\epsilon} H_K) a + \frac{1}{\epsilon} (-2 \kappa^* a + C_K)$$

where $c(a)$ is the solution of (2.16) associated with the parameter vector a . The minimization problem is given by

$$(2.24) \quad \begin{aligned} &\text{minimize } L(a) \\ &\text{subject to } K \in Q'_{ad}. \end{aligned}$$

The minimization problem (2.24) is solved as an unconstrained problem assuming essentially that the solution strictly satisfies the inequality constraints specifying Q'_{ad} . This is reasonable from a physical point of view since the bounds μ and ν are posed primarily for the mathematical formulation of the problem. Numerically, a steepest descent method is used decrease the value of $L(a)$ iteratively. In carrying out this procedure it is necessary to compute the gradient of $L(a)$. The derivative of $L(a)$ with increment $h = \text{col}(h_1, \dots, h_M)$ is given by

$$(2.25) \quad DL(a)h = 2\{\gamma(Hc(a) - \zeta)^* Dc(a)h + ((G_0 + \frac{1}{\epsilon}H_K)a - \frac{\kappa}{\epsilon})^* h\}$$

where the derivative of $c(a)$ with increment h , $Dc(a)h$, is the solution of the equation

$$(2.26) \quad GDc(a)h = - \sum_{l=1}^M h_l G^{(l)} c(a)$$

The derivative of $L(a)$ and the derivative of $c(a)$ are an M -vector and an $N \times M$ matrix, respectively.

By introducing the equation

$$(2.27) \quad G\pi = Hc(a) - \zeta,$$

we observe that

$$\pi^* GDc(a)h = (Hc(a) - \zeta)^* Dc(a)h.$$

Hence, it follows that

$$(Hc(a) - \zeta)^* Dc(a)h = - \sum_{k=1}^M h_k (\pi^* G^{(k)} c(a))$$

and (2.25) may be expressed as

$$\frac{1}{2} DL(a)h = -\gamma \sum_{k=1}^M h_k (\pi^* G^{(k)} c(a)) + \{(G_0 + \frac{1}{\epsilon}H_K)a - \frac{1}{\epsilon}\kappa\}^* h.$$

Defining the column M-vector

$$\Xi = [\pi^* G^{(k)} c(a)]_{k=1}^M,$$

we may write $DL(a)h$ as follows

$$(2.28) \quad \frac{1}{2}DL(a)h = \{-\gamma\Xi + (G_o + \frac{1}{\epsilon}H_K)a - \frac{\kappa}{\epsilon}\}^*h$$

and

$$DL(a) = 2[-\gamma\Xi + (G_o + \frac{1}{\epsilon}H_K)a - \frac{\kappa}{\epsilon}].$$

The optimization algorithm starts with an initial guess $a^{(0)}$. On the k-th iteration with the parameter vector $a^{(k)}$ the procedure is as follows

Compute $c(a^{(k)})$ as the solution of (2.16)

Compute $L(a^{(k)})$ by (2.23)

Compute π by solving (2.27)

Compute $DL(a)$ by (2.28)

Update $a^{(k)}$ to find $a^{(k+1)}$ according to the criterion $a^{(k+1)} = a^{(k)} - \beta DL(a^{(k)})$

where β is chosen so that

$$L(a^{(k+1)}) \leq L(a^{(k)}).$$

Finally, we give the optimality system characterizing solutions of (2.24) that belong to the interior of Q_{ad} . These solutions necessarily satisfy the condition $DL(a)h = 0$ for any M-vector h . From equation (2.28) we conclude that

$$(2.29) \quad (G_o + \frac{1}{\epsilon}H_K)a - \frac{\kappa}{\epsilon} - \gamma\Xi = 0$$

The optimality system is recorded in the following.

Proposition 2.2. The optimality conditions satisfied by interior solutions of (2.24) are given by

$$Gc(a) = \rho$$

$$G\pi = Hc(a) - \zeta$$

$$(G_o + \frac{1}{\epsilon}H_K)a - \frac{\kappa}{\epsilon} - \gamma\Xi = 0$$

where

$$\Xi = [\pi^* G^{(k)} c(a)]_{k=1}^M,$$

3. Uniqueness and Sensitivity In this section we obtain conditions assuring there is at most one interior solution. We will then see that these conditions also imply the differentiability of the solution with respect to the data. The starting point is the system comprising the optimality conditions given in Proposition 2.2. We assume there two sets of data distinguished by sub(super)scripts 1 and 2. Defining the vectors

$$d = c_1 - c_2$$

$$\delta = \pi_2 - \pi_1$$

$$\alpha = a_2 - a_1$$

$$\zeta = \zeta_2 - \zeta_1$$

and

$$\kappa = \kappa_2 - \kappa_1,$$

we obtain the equations

$$\begin{aligned} G_2 d &= - \sum_{k=1}^M \alpha_k G^{(k)} c_1 \\ G_2 \delta &= -\zeta - \sum_{k=1}^M \alpha_k G^{(k)} \pi_1 + H d \\ (G_o + \frac{1}{\epsilon} H_K) \alpha &= \frac{\kappa}{\epsilon} + \gamma \{ \delta^* G^{(l)} c_2 \}_{l=1}^M + \\ &\quad + \gamma \{ \pi_1^* G^{(l)} d \}_{l=1}^M \end{aligned}$$

from the optimality system. To determine estimates, suppose there are positive constants ν_0 , ν , K_0 , and K_1 such that

$$(3.1) \quad d^* G_i d \geq \nu |d|^2,$$

for $i = 1, 2$

$$(3.2) \quad K_0 \geq |G^{(k)}|,$$

for $k = 1, \dots, M$,

$$(3.3) \quad K_1 \geq |H|,$$

and

$$(3.4) \quad d^* (G_o + \frac{1}{\epsilon} H_K) d \geq \nu_0 |d|^2.$$

The conditions (3.1) hold because of the positive definiteness of the matrix on the left side in the underlying model. Condition (3.4) is a consequence of the presence of H_K . With these assignments, we obtain the following estimates for $i = 1, 2$

$$(3.5) \quad |c_i| \leq \frac{|\rho|}{\nu},$$

and thus

$$|G_i^{-1}| \leq \frac{1}{\nu},$$

$$(3.6) \quad |d| \leq \frac{K_0}{\nu^2} |\rho| |\alpha|,$$

$$(3.7) \quad |\delta| \leq \frac{K_0}{\nu^2} \left(\frac{2K_1}{\nu} |\rho| + |\zeta_1| \right) |\alpha| + \frac{|\zeta|}{\nu},$$

and

$$(3.8) \quad \begin{aligned} \nu_0 |\alpha| &\leq \gamma K_0 \frac{|\rho|}{\nu} |\delta| + \\ &+ \gamma K_0 \left(\frac{K_1}{\nu^2} |\rho| + \frac{|\zeta_1|}{\nu} \right) |d| + \frac{|\kappa|}{\epsilon}. \end{aligned}$$

It follows that

$$(3.9) \quad \nu_0 |\alpha| \leq \frac{|\kappa|}{\epsilon} + \frac{\gamma K_0}{\nu^2} |\rho| |\zeta| + \frac{\gamma K_0^2}{\nu^3} |\rho| \left(\frac{3K_1}{\nu} |\rho| + 2|\zeta_1| \right) |\alpha|.$$

Set

$$(3.10) \quad \beta = \nu_0 - \frac{\gamma K_0^2}{\nu^3} \left(\frac{3K_1}{\nu} |\rho| + 2|\zeta_1| \right) |\rho|.$$

Proposition 3.1. If β given by (3.10) is positive where the constants are assigned according to (3.1)-(3.4), then

$$|\alpha| \leq \frac{1}{\beta} \left(\frac{1}{\epsilon} |\kappa| + \frac{K_0}{\nu^2} |\rho| |\zeta| \right).$$

If $\zeta_1 = \zeta_2$ and $\kappa_1 = \kappa_2$, then uniqueness follows.

Corollary 3.2. If $\beta > 0$, then there is at most one solution a belonging to $\text{int } Q_{ad}$.

Remark 3.3 Observe that the condition $\beta > 0$ may be satisfied by choosing the vector ρ associated with the forcing term to be sufficiently small.

In order to detect changes in the optimal parameter obtained by means of the regularized output least squares technique in response to changes in the data, it is useful to examine the differentiability of the optimal estimator with respect to the data. Hence, we next consider conditions under which the mapping associating an optimal estimator a with a data vector ζ is differentiable. Let us suppose then that $a \in Q = \mathcal{R}^M$ and ρ and $\zeta \in X = \mathcal{R}^N$. In fact we suppose that a is a regularized output least squares solution in the interior of Q_{ad} . Thus, define the function $a \mapsto G(a)$ from $\mathcal{R}^M$ to the linear space of $N \times N$ real-valued matrices by

$$G(a) = \sum_{k=1}^M a_k G^{(k)},$$

and set

$$G = G(a) + \frac{1}{\eta} G_\phi.$$

We assume that a is associated with the system of equations given in Proposition 2.2. To examine the differentiability of a with respect to ζ , define the mapping $F : Q \times X \mapsto X$ by

$$F(a, \zeta) = (G_o + \frac{1}{\epsilon} H_K) a - \frac{\kappa}{\epsilon} - \gamma \{ \pi(a, \zeta)^* G^{(k)} c \}_{k=1}^M.$$

By the implicit function theorem [1] the function $\zeta \mapsto a(\zeta)$ is differentiable with respect to ζ if the Frechet partial derivatives of F with respect to a and ζ , $D_a F(a, \zeta)$ and $D_\zeta F(a, \zeta)$ exist and the linear operator $D_a F(a, \zeta) : Q \mapsto Q$ is nonsingular.

To calculate the required derivatives, it is necessary to calculate the derivative of each of the functions $a \mapsto c(a)$ and $(a, \zeta) \mapsto \pi(a, \zeta)$. Setting $G(h) = \sum_{k=1}^M h_k G^{(k)}$, we note that

$$GDc(a)h = -G(h)c(a)$$

so that

$$Dc(a)h = -G^{-1}G(h)c(a).$$

Further,

$$GD_a \pi(a, \zeta)(h) = -G(h)\pi(a, \zeta) + HDc(a)h,$$

and from Proposition 2.2,

$$D_a \pi(a, \zeta)(h) = -G^{-1}[G(h)G^{-1}H + HG^{-1}G(h)]c(a) + G^{-1}G(h)G^{-1}\zeta.$$

Hence, we may write

$$Dc(a)h = -\sum_{i=1}^M h_i G^{-1}G^{(i)}c(a),$$

and

$$D_a \pi(a, \zeta)h = \sum_{i=1}^M h_i \{ G^{-1}G^{(i)}G^{-1}\zeta - G^{-1}[G^{(i)}G^{-1}H + HG^{-1}G^{(i)}]c(a) \}$$

On the other hand the Frechet partial derivative with respect to a is given by

$$D_a F(a, \zeta)h = (G_o + \frac{1}{\epsilon} H_K)h - \{(D_a \pi(a, \zeta)h)^* G^{(k)} c + \pi^* G^{(k)} (Dc(a)h)\}_{k=1}^M.$$

Defining the $M \times M$ matrix

$$\mathcal{K}_{ki} = \{G^{-1} G^{(i)} \zeta - G^{-1} (G^{(i)} G^{-1} H + H G^{-1} G^{(i)}) c\}^* G^{(k)} c - \pi^* G^{(k)} G^{-1} G^{(i)} c,$$

then we may write

$$D_a F(a, \zeta)h = (G_o + \frac{1}{\epsilon} H_K - \gamma \mathcal{K})h.$$

To calculate the partial derivative of F with respect to ζ with increment δ , we first note that

$$D_\zeta \pi(a, \zeta) \delta = -G^{-1} \delta.$$

It follows that the partial derivative with respect to ζ with increment δ is given by

$$(3.11) \quad D_\zeta F(a, \zeta) \delta = \{c^* G^{(k)} G^{-1} \delta\}_{k=1}^M$$

Defining the $M \times N$ matrix $\mathcal{H}_\zeta$ in which the k -th row is given by $c^* G^{(k)} G^{-1}$, we have

$$D_\zeta F(a, \zeta) \delta = \mathcal{H}_\zeta \delta.$$

With these observations, we have the following.

Proposition 3.4. If the matrix $G_o + \frac{1}{\epsilon} H_K - \mathcal{K}$ is nonsingular, then

$$(3.12) \quad Da(\zeta) \delta = -D_a F(a, \zeta)^{-1} D_\zeta F(a, \zeta) \delta.$$

Remark 3.5. Observe that for given $G_o + \frac{1}{\epsilon} H_K$, the nonsingularity of $G_o + \frac{1}{\epsilon} H_K - \mathcal{K}$ may be controlled by the forcing vector ρ .

By applying the estimates previously used, we may obtain sufficient conditions for differentiability. Towards this end, observe from (3.4) that

$$|(G_o + \frac{1}{\epsilon} H_K)^{-1}| \leq \frac{1}{\nu_0}.$$

Thus, it follows by estimating the terms in the expression for $\mathcal{K}_{ki}$ that

$$|\mathcal{K}_{ki}| \leq (\frac{2K_0^2}{\nu^3} |\zeta| |\rho| + \frac{3K_0^2 K_1}{\nu^4} |\rho|^2).$$

We have the following.

Proposition 3.6. If

$$\gamma \frac{K_0^2}{\nu^3} \left(\frac{3K_1}{\nu} |\rho| + 2 |\zeta| \right) |\rho| < \nu_0,$$

then $Da(\zeta)$ exists.

Proof. Under the hypothesis, it follows a norm of $\gamma(G_0 + \frac{1}{\epsilon}H_K)^{-1}K$ is less than one. Hence,

$$(I - \gamma(G_0 + \frac{1}{\epsilon}H_K)^{-1}K)^{-1}$$

exists and

$$G_0 + \frac{1}{\epsilon}H_K - \gamma K$$

is invertible. This implies that $D_a F(a, \zeta)$ is invertible. Since $Da(\zeta)$ exists, the differentiability of a with respect to ζ follows.

Remark 3.7. The conditions of Propositions 3.1 and 3.5 are the same. Hence, it follows that under (3.1) that there is at most one solution and the function $\zeta \mapsto a(\zeta)$ is differentiable.

Remark 3.8. The differentiability of the mapping $\zeta \mapsto a(\zeta)$ provides a tool with which we can investigate the sensitivity of the interior optimal estimators with respect to perturbations in the data ζ . We shall develop these ideas further in a later paper. However, we note that for a perturbation δ of the N-column data vector ζ

$$a(\zeta + \delta) = a(\zeta) + Da(\zeta)\delta + o(|\delta|).$$

Hence, if the $M \times N$ matrix $Da(\zeta)$ has a nontrivial null space $\mathcal{N}$, then it is not possible to detect the perturbation $\delta \in \mathcal{N}$ as a first order effect. Define the N-vectors $\theta_k = c^* G^{(k)} G^{-1}$ for $k = 1, \dots, M$. From (3.11) and (3.12), we see that $\mathcal{N}$ is determined as the orthogonal complement of the span of the vectors $\{\theta_k\}_{k=1}^M$. It follows, for example, that if $M \leq N$ there always are vectors δ that are not detectable.

4. Estimation of the Discontinuity. Having detected a discontinuity, our next step is to isolate and obtain some estimate of it. We proceed by considering an example in which the admissible permeability functions K have a discontinuity determined by two regions within Ω parameterized by 2 real numbers a and b . We denote these two regions by $\Omega(a, b)$ and $\Omega \setminus \Omega(a, b)$. An admissible permeability K is also parameterized constants K_1 and K_2 modelling the magnitude of the discontinuity between the regions. Hence, K takes the form

$$(4.1) \quad K(x, y) = K_0(x, y) + K_e(x, y)$$

where

$$K_e(x, y) = K_1 \text{ if } (x, y) \in \Omega(a, b), \text{ and } K_2 \text{ otherwise.}$$

It is assumed that the function K_0 is known. Introducing the characteristic function Ξ of the set $\Omega(a, b)$, we may write

$$(4.2) \quad K_e(x, y) = (K_1 - K_2)\Xi(x, y) + K_2.$$

and equations (4.1) and (4.2) may be thought of as replacing equation (2.9). To fix ideas let us suppose that $\Omega(a, b)$ is a rectangle of the form $(x_0 - a, x_0 + a) \times (y_0 - b, y_0 + b)$. The stiffness matrix is now given as

$$(G)_{ij} = \int_{\Omega} (K_0 + K_2) \nabla \alpha_i \cdot \nabla \alpha_j dx dy + (K_1 - K_2) \int_{x_0-a}^{x_0+a} \int_{y_0-b}^{y_0+b} \nabla \alpha_i \cdot \nabla \alpha_j dx dy$$

Setting

$$(G_0)_{ij} = \int_{\Omega} \nabla \alpha_i \cdot \nabla \alpha_j dx dy,$$

$$(G_1)_{ij} = \int_{\Omega} K_0 \nabla \alpha_i \cdot \nabla \alpha_j dx dy,$$

and

$$(4.3) \quad (G_2)_{ij}(a, b) = \int_{x_0-a}^{x_0+a} \int_{y_0-b}^{y_0+b} \nabla \alpha_i \cdot \nabla \alpha_j dx dy,$$

we have

$$(4.4) \quad G = G_1 + (K_1 - K_2)G_2(a, b) + K_2G_0.$$

Thus, we obtain an equation analogous to (2.16) given by

$$(4.5) \quad Gc = \rho.$$

with the approximating solution u expressed as $u = \sum_{i=1}^N c_i \alpha_i$.

Define the criterion

$$J(a, b, K_1, K_2) = \sum_{j=1}^{N_o} (\langle \Phi_j, u \rangle - z_j)^2$$

and the functional

$$\mathcal{N}(a, b, K_1, K_2) = \sum_{j=1}^{N_K} (\langle \phi_j, K \rangle - K_j)^2.$$

Under discretization, the functional J takes the form

$$J(a, b, K_1, K_2) = c^* H_P c - 2\zeta^* c + c_z$$

Note that $\mathcal{N}(a, b, K_1, K_2)$ is a discontinuous function and is, in fact, a piecewise constant function of a and b . Hence, we look for a , b , K_1 , and K_2 that minimizes J while keeping $\mathcal{N}(a, b, K_1, K_2)$ at its minimum value, call it $\mathcal{N}_0$. That is, we seek

$$a, b, K_1, \text{ and } K_2 \text{ minimizing } J(a, b, K_1, K_2) \text{ subject to } \mathcal{N} = \mathcal{N}_0.$$

To carryout this program we calculate the partial derivatives of J and c . We find that $d_1 = D_{K_1}c$ and $d_2 = D_{K_2}c$ satisfy the equations

$$(4.6) \quad Gd_1 = G_2(a, b)c$$

and

$$(4.7) \quad Gd_2 = (G_0 - G_2(a, b))c,$$

respectively. To compute $d_a = D_a c$ and $d_b = D_b c$, it is necessary to differentiate the integral in (4.3). Observe that the partial derivatives of the matrix G_2 are matrices whose entries are given by

$$(D_a G_2)_{ij} = \int_{y_0-b}^{y_0+b} \{(\nabla \alpha_i \cdot \nabla \alpha_j)(x_0 + a, y) - (\nabla \alpha_i \cdot \nabla \alpha_j)(x_0 - a, y)\} dy$$

and

$$(D_b G_2)_{ij} = \int_{x_0-a}^{x_0+a} \{(\nabla \alpha_i \cdot \nabla \alpha_j)(x, y_0 + b) - (\nabla \alpha_i \cdot \nabla \alpha_j)(x, y_0 - b)\} dx.$$

Thus, we find that d_a and d_b must satisfy the equations

$$Gd_a = -(K_1 - K_2)D_a G_2 c$$

and

$$Gd_b = -(K_1 - K_2)D_b G_2 c,$$

respectively. The partial derivatives of J now are calculated by the chain rule.

5. A numerical example. We consider a problem in which we specify a coefficient $K(x, y)$ and generate pressure data based on that function by solving the problem (2.1)-(2.3) for p with a specific forcing function f by finite elements. Using this data we then attempt to recover K . Let $\Omega = (0, 1) \times (0, 1)$ and suppose that measurements of pressure and permeability can be made at locations $(0.175, 0.175)$, $(0.835, 0.175)$, $(0.5, 0.5)$, $(0.175, 0.835)$, and $(0.835, 0.835)$. For a test permeability function we use the following

$$K_{test}(x, y) = \begin{cases} 8 + 3.5 \cos(x + y) & \text{for } (x, y) \in (0, 0.3) \times (0, 0.3), \\ 4 + 2.5 \cos(x + y - 2) & \text{for } (x, y) \in (0.75, 1) \times (0.75, 1), \\ 2 + \cos(x + y) & \text{otherwise} \end{cases}$$

shown in Figure 1. Further, we suppose that $p = 0$ at the point $(0.835, 0.835)$ and that fluid is injected at the point $(0.175, 0.175)$. The resulting pressure function obtained by means of a finite element solution is portrayed in Figure 2. For the approximations to the pressure, we use tensor products of cubic B-splines [5] defined on a uniform mesh determined by subdividing $(0,1)$ into seven subintervals. Since imposing Neumann boundary conditions improves accuracy, we use 64 basis functions for approximating pressure adjusted to incorporate the Neumann boundary condition. For approximating the parameter, we again use tensor products of cubic B-splines but defined on a mesh determined by subdividing $(0,1)$ into 5 equal subintervals. Imposing no boundary conditions, we then use 64 basis functions to approximate the parameter. Using data at the observation points, we apply the regularized output least squares method as a detection procedure resulting in Figure 3. Based on this result, we search for a coefficient of the form

$$K_1(x, y) = \begin{cases} K_1 & \text{if } (x, y) \in (0, a) \times (0, b), \\ 2 & \text{otherwise} \end{cases}$$

using the technique discussed in the previous section. The result is illustrated in Figure 4. We then apply further detection by again using the regularized output least squares method to estimate the coefficient K_2 where the permeability has the form

$$K(x, y) = K_1(x, y) + K_2(x, y).$$

The result is portrayed in Figure 5. Based on this computation, we again use the procedure for discontinuous coefficients by searching for discontinuities based on the rectangle $(b, 1) \times (b, 1)$. This yields the Figure 6. Again, we use the regularized output least square method to look for further discontinuities and background with the outcome given in Figure 7. The resulting pressure from the estimated coefficient is given in Figure 8.

6. Conclusions. We have formulated the output least square estimation technique and have discussed its use as a detection tool for problems with pressure data by giving conditions for uniqueness and differentiability of optimal estimated permeability function with respect to perturbations of the data. In addition we introduced a method to estimate the location and magnitude of a jump discontinuity. We also presented a numerical example for the location of discontinuities in a permeability function in the presence of a background. By alternating detection and discontinuity estimation procedures, it seems to be possible to construct coefficients with discontinuities in the presence of a background function.

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STEP I --- A = AO (REG.)

Figure 1

STEP I --- A = A0 (REG.)

Figure 2

STEP I --- A = AO (REG.)

Figure 3

STEP I+1/2----- A = AO (DISC.)

Figure 4

STEP II --- $A = A_0(\text{DISC.}) + A_1(\text{REG.})$

Figure 5

STEP III --- $A = A_0(\text{DISC.}) + A_1(\text{DISC.})$

Figure 6

STEP IV --- $A = A_{0D} + A_{1D} + A_{2R}$

Figure 7

STEP IV --- $A=A00+A1D+A2R$

Figure 8

A TIME-SPACE SOLUTION APPROACH FOR PREDICTION OF SWEEP EFFICIENCY OF WATERFLOODING USING THE CUBATURE METHOD

Abstract

A numerical method for predicting waterflood sweep efficiency is formulated. A unique feature of the cubature method is that the whole of an equation can be discretized instead of the conventional approach of term by term discretization. As a result of including both the temporal and spatial derivative terms in the discretization, a transient-state problem can be solved like a steady-state problem, resulting in potential savings in computational time. This report provides a rigorous derivation of the macroscopic continuity equation. This equation contains a new dispersive fluid transport term for compressible fluids. Integral transformations have been used to develop convenient formulations for oil/water systems with appropriate boundary conditions. Numerical discretization of the governing equations is accomplished using 15, 27, 35, and 75 point time-space computational molecules. These molecules permit the use of significantly large time and space increments without the usual stability, dispersion, and grid orientation problems associated with conventional methods. The new method is shown to be as accurate as the finite difference method in the case of unit mobility flow.

A TIME-SPACE SOLUTION APPROACH FOR
PREDICTION OF SWEEP EFFICIENCY
OF WATERFLOODING USING
THE CUBATURE METHOD

1. EXECUTIVE SUMMARY

Chapter 2 is on the convenient formulations for immiscible displacement in porous media. Although this problem has been investigated by many researchers in the literature, the majority of this work deals with the applications to special cases. In the present study, a generalized and improved formulation of the governing macroscopic differential equations and proper boundary conditions is presented based on the porous media averaging method. The resulting equations are general. Further, these equations are transformed so that the variable properties, such as permeability, porosity, density and viscosity do not appear explicitly. As a result, these equations may lead to numerical solutions of better accuracy with less computational effort.

Chapter 3 describes the work on the application of the cubature method for rapid evaluation of the sweep efficiency in heterogeneous reservoirs. For numerical discretization of the governing equations, the cubature method is applied using the time-space approach by Civan. For this purpose, 15, 27, 35, and 75 point time-space molecules have been selected. These computational molecules can provide highly accurate discretization of the time and space derivatives and therefore permit the use of significantly large time and space increments without the usual stability, dispersion and grid point orientation problems associated with the conventional methods. To demonstrate the time-space approach, the method has been applied to the unit mobility problem concerning a rectangular domain having two adjacent boundaries subject to a uniform injection while the others producing freely. This example, has demonstrated that, even for the time-space approach, the unit mobility flow problem can be solved directly and explicitly. Whereas, the usual course is to resort to iterative methods such as the Gauss-Seidel method when the number of discrete points are too many to avoid the accumulation of the round-off errors. It has also been shown that the time-space approach can be facilitated without requiring the previous time solutions other than those last calculated to avoid storing of these solutions. Whereas, the usual course is to rely on the backward formulae as shown in Chapter 4. The time-space approach is general and applicable with any computational molecule. The details of the cubature formulations for the unit mobility flow and the incompressible two phase flow are presented in Chapter 3.

Chapter 4 describes the application of the time-space approach, using two and three simultaneous time levels, to the unit mobility and non-unit mobility flow cases using the backward difference formulae. The results indicate this method works for both cases.

2. CONVENIENT FORMULATIONS FOR IMMISCIBLE DISPLACEMENT IN POROUS MEDIA

Principal Investigator: Faruk Civan

ABSTRACT

Convenient formulations of the immiscible displacement in porous media for waterflooding are presented. The macroscopic equation of continuity for immiscible displacement is derived by porous media averaging. Richardson's approach and the fractional flow formulation are extended and generalized for anisotropic and heterogeneous porous media. The integral transformations according to Douglas et al and the coordinate transformations presented in this paper lead to differential equations which do not involve the variable fluid and porous media properties explicitly in the differential operators. Therefore, the resulting equations can be discretized and solved more conveniently and accurately than the conventional formulation which require cumbersome discretization formulae for mixed derivatives involving the fluid and porous media properties.

INTRODUCTION

Formulations of the governing equations and boundary conditions for immiscible displacement in porous media is of continuing interest. Neglecting the capillary pressure effect, Buckley and Leverett¹ obtained a simplified formulation. This equation can be solved analytically using Welge's² method in an isotropic homogeneous media for one-dimensional linear, radial, and spherical flow. When the capillary pressure effect is included, the formulation results in a manipulation leading to a convection-dispersion type equation implying that two boundary conditions are necessary. Therefore, the majority of the reported studies have attempted formulating and implementing two boundary conditions. Whereas, as shown in the present study, two boundary conditions are only necessary for compressible fluids and one boundary condition is sufficient for incompressible fluids.

In this study, first, a rigorous derivation of the macroscopic equation of continuity for immiscible displacement in porous media is carried out. The resulting macroscopic equation contains a new term expressing dispersive fluid transport due to hydraulic gradients. This term disappears only for incompressible fluids. Second, the formulations are applied to water/oil systems and the generalized boundary conditions are derived. Third, convenient formulations are presented by transforming the governing equations by means of an integral transformation of the function and a transformation of the coordinates. In the convenient formulations, "the variable properties do not appear explicitly" and numerical solutions of "better accuracy with less computational effort" can be obtained, as stated by Civan and Sliepcevich³. The fractional flow and unit mobility ratio formulations are also presented for specific applications.

MACROSCOPIC EQUATION OF CONTINUITY FOR IMMISCIBLE DISPLACEMENT IN POROUS MEDIA

The derivations given here are based on the modifications of the formulation of waterflooding in naturally fractured reservoirs by Civan^{4,5} for pore surface fluid transfer and pore surface displacement effects.

The microscopic equation of continuity for a phase ℓ in a porous media is given by (Civan^{4,5}):

$$\partial \rho_\ell / \partial t + \vec{\nabla} \cdot (\rho_\ell \vec{v}_\ell) = \dot{m}_\ell \quad (2.1)$$

As shown by Civan^(4,5), for fluids in porous media, a macroscopic equation of continuity can be derived by averaging Eq. 2.1 over a representative elemental volume of porous media and using the following averaging rules, (Slattery⁶, Marle⁷, and Kaviany⁸):

$$\langle \rho_\ell \rangle_r = \frac{1}{V_r} \int_{V_\ell} \rho_\ell dV \quad (2.2)$$

$$\langle \vec{\nabla} \cdot (\rho_\ell \vec{v}_\ell) \rangle_r = \vec{\nabla} \cdot \langle \rho_\ell \vec{v}_\ell \rangle_r + \frac{1}{V_r} \int_{A_s} (\rho_\ell \vec{v}_\ell) \cdot \vec{n}_s dA_s \quad (2.3)$$

and

$$\left\langle \frac{\partial \rho_\ell}{\partial t} \right\rangle_r = \frac{\partial \langle \rho_\ell \rangle_r}{\partial t} - \frac{1}{V_r} \int_{A_s} \vec{v}_s \cdot \vec{n}_s \rho dA_s \quad (2.4)$$

A_s denotes the pore surface, V_r is the representative elemental bulk volume, V_ℓ is the volume of the ℓ^h phase in V_r , $\vec{n}_s$ is the outward normal unit vector, $\vec{v}_s$ is the pore surface displacement velocity. Each quantity is separated into a mean and a spatial deviation from the mean.

The fractional volume of the phase ℓ in porous media is given by

$$\epsilon_\ell = V_\ell / V_r \quad (2.5)$$

The averages over the porous media and the phase ℓ are related by:

$$\langle \rho_\ell \rangle_r = \epsilon_\ell \langle \rho_\ell \rangle_\ell \quad (2.6)$$

and

$$\langle \bar{v}_l \rangle_r = \epsilon_l \langle \bar{v}_l \rangle_l \quad (2.7)$$

Then applying Eq. 2.1-7 and dropping the brackets indicating the volume averages, the macroscopic equation of continuity can be written as (Civan^{4,5}):

$$\frac{\partial}{\partial t}(\epsilon_l \rho_l) + \bar{\nabla} \cdot (\rho_l \bar{v}_l) - \bar{\nabla} \cdot [\epsilon_l \bar{D}_l \cdot \bar{\nabla} \rho_l] + r_l = 0 \quad (2.8)$$

where $\bar{D}_l$ is an empirical hydraulic dispersion coefficient tensor (Civan^{4,5}) and ϵ_l is the volume fraction of phase l in porous media given by

$$\epsilon_l = \phi s_l \quad (2.9)$$

ϕ is the porosity and s_l is the saturation of phase l

$$r_l = \langle \dot{m}_l \rangle_r + \frac{1}{V_r} \int_{A_r} (\rho_l \bar{v}_l) \cdot \bar{n}_s dA_s - \frac{1}{V_r} \int_{A_r} \bar{v}_s \cdot \bar{n}_s \rho dA_s \quad (2.10)$$

The first term on the right represents the fluid loss and the first and the second integral terms deal with the pore surface fluid transfer and pore surface displacement affects, respectively. In many cases, the integral terms are usually negligible.

Adding Eq. 2.8 over all the fluid phases yields the total mass balance equation as:

$$\frac{\partial(\phi \rho)}{\partial t} + \bar{\nabla} \cdot (\rho \bar{v}) - \bar{\nabla} \cdot (\phi \bar{D} \cdot \bar{\nabla} \rho) + r = 0 \quad (2.11)$$

in which the following average quantities have been used:

$$\phi \rho = \sum_l \epsilon_l \rho_l \quad (2.12)$$

$$\rho \bar{v} = \sum_l \rho_l \bar{v}_l \quad (2.13)$$

$$\phi \bar{D} \cdot \bar{\nabla} \rho = \sum_l \epsilon_l \bar{D}_l \cdot \bar{\nabla} \rho_l \quad (2.14)$$

$$r = \sum_l r_l \quad (2.15)$$

For incompressible fluids, Eqs. 2.11 and 2.8 simplify, respectively, as:

$$\frac{\partial \phi}{\partial t} + \bar{\nabla} \cdot \bar{v} + \frac{r}{\rho} = 0 \quad (2.16)$$

and

$$\frac{\partial(\phi s_i)}{\partial t} + \nabla \cdot \bar{v}_i + \frac{r_i}{\rho_i} = 0 \quad (2.17)$$

Further, if $\phi \neq \phi(t)$ is assumed, the Eqs. 2.16 and 2.17 respectively become:

$$\bar{\nabla} \cdot \bar{v} + \frac{r}{\rho} = 0 \quad (2.18)$$

$$\phi \frac{\partial s_i}{\partial t} + \nabla \cdot \bar{v}_i + \frac{r_i}{\rho_i} = 0 \quad (2.19)$$

APPLICATION TO OIL/WATER SYSTEMS

The macroscopic equation of continuity for the water phase in porous media can be obtained from Eqs. 2.8 and 2.9 as (Civan^{4,5}):

$$\frac{\partial}{\partial t}(\phi s_w \rho_w) + \bar{\nabla} \cdot (\rho_w \bar{v}_w) \quad (2.20)$$

$$- \bar{\nabla} \cdot (\phi s_w \bar{D}_w \cdot \bar{\nabla} \rho_w) + r_w = 0$$

t is time, ϕ is the porosity, s_w is the water saturation, ρ_w is the density of water, D_w is the hydraulic dispersion coefficient, r_w is the mass rate of the water lost, and $\bar{v}_w$ is the volumetric flux of the water phase given by:

$$\bar{v}_w = F_w \left(\bar{v} + \frac{k_{r_o}}{\mu_o} \frac{d\bar{p}_c}{ds_w} \bar{k} \cdot \bar{\nabla} s_w - \frac{k_{r_o}}{\mu_o} \Delta \rho g \bar{k} \cdot \bar{\nabla} z \right) \quad (2.21)$$

where

$$\Delta \rho = \rho_w - \rho_o \quad (2.22)$$

and

$$F_w = \left(1 + \frac{k_{r_o} \mu_w}{\mu_o k_{r_w}} \right)^{-1} \quad (2.23)$$

The total volumetric flux is given by:

$$\bar{v} = \bar{v}_w + \bar{v}_o \quad (2.24)$$

$$\begin{aligned} \bar{v} = & - \left(\frac{k_{r_w}}{\mu_w} + \frac{k_{r_o}}{\mu_o} \right) \bar{k} \cdot \bar{\nabla} p_w - \frac{k_{r_o}}{\mu_o} \frac{dp_c}{ds_w} \bar{k} \cdot \bar{\nabla} s_w \\ & - \left(\frac{k_{r_w} \rho_w}{\mu_w} + \frac{k_{r_o} \rho_o}{\mu_o} \right) g \bar{k} \cdot \bar{\nabla} z \end{aligned} \quad (2.25)$$

Based on Eq. 2.20, the water injection and production end boundary conditions can be written respectively, as:

$$\rho_w \bar{v}_w - \phi s_w \bar{D} \cdot \bar{\nabla} \rho_w = (\rho_w \bar{v}_w)_{in} \quad (2.26)$$

and

$$\phi s_w \bar{D} \cdot \bar{\nabla} \rho_w = 0 \quad (2.27)$$

For incompressible fluids, Eq. 2.26 simplifies as:

$$\bar{v}_w = (\bar{v}_w)_{in} \quad (2.28)$$

and Eq. 2.27 is no longer needed.

WATER PHASE SATURATION EQUATION

Assuming incompressible fluids and porous media and substituting Eq. 2.21 into Eq. 2.20 according to Richardson⁹ and Peters et al¹² yields:

$$\phi \frac{\partial s_w}{\partial t} + \nabla \cdot \left[F_w \left(\bar{v} + \frac{k_{r_o}}{\mu_o} \frac{dp_c}{ds_w} \bar{k} \cdot \bar{\nabla} s_w - \frac{k_{r_o}}{\mu_o} \Delta \rho g \bar{k} \cdot \bar{\nabla} z \right) \right] + \frac{r_w}{\rho_w} = 0 \quad (2.29)$$

which, upon expansion and substitution of Eq. 2.18 yields:

$$\begin{aligned} \phi \frac{\partial s_w}{\partial t} + \frac{dF_w}{ds_w} \bar{v} \cdot \bar{\nabla} s_w + \nabla \cdot \left[F_w \frac{k_{r_o}}{\mu_o} \frac{dp_c}{ds_w} \bar{k} \cdot \bar{\nabla} s_w \right] \\ - \nabla \cdot \left[F_w \frac{k_{r_o}}{\mu_o} \Delta \rho g \bar{k} \cdot \nabla z \right] - F_w \frac{r}{\rho} + \frac{r_w}{\rho_w} = 0 \end{aligned} \quad (2.30)$$

When $r = 0$, $r_w = 0$, and an isotropic porous media is considered, Eq. 2.30 simplifies to Eq. 16.101 given by Richardson⁹. Eq. 2.30 is interesting, because it reveals a transient-state, convection, dispersion, with source nature of the water phase transport. Eq. 2.30 can also be written in the following compact form:

$$\begin{aligned} \phi \frac{\partial s_w}{\partial t} + \frac{dF_w}{ds_w} \bar{v} \cdot \bar{\nabla} s_w + \bar{\nabla} \cdot [D_w(s_w) \bar{k} \cdot \bar{\nabla} s_w] \\ - \bar{\nabla} \cdot [T_w(s_w) \bar{k} \cdot \bar{\nabla} z] - F_w \frac{r}{\rho} + \frac{r_w}{\rho_w} = 0 \end{aligned} \quad (2.31)$$

which upon expansion yields the following alternative form:

$$\begin{aligned} \phi \frac{\partial s_w}{\partial t} + \left[\frac{dF_w}{ds_w} \bar{v} + \frac{dD_w}{ds_w} (\bar{k} \cdot \nabla s_w) - \frac{dT_w}{ds_w} (\bar{k} \cdot \bar{\nabla} z) \right] \cdot \bar{\nabla} s_w \\ + D_w \nabla \cdot (\bar{k} \cdot \bar{\nabla} s_w) - T_w \nabla \cdot (\bar{k} \cdot \nabla z) - \frac{F_w r}{\rho} + \frac{r_w}{\rho_w} = 0 \end{aligned} \quad (2.32)$$

in which D_w and T_w can be referred to as the capillary and gravitational dispersion coefficients defined, respectively, as:

$$D_w(s_w) = F_w \frac{k_{r_o}}{\mu_o} \frac{dp_c}{ds_w} \quad (2.33)$$

and

$$T_w(s_w) = F_w \frac{k_{r_o}}{\mu_o} \Delta \rho g \quad (2.34)$$

The boundary conditions at permeable boundaries can be formulated from the continuity of the water phase and total phase fluxes across permeable boundaries, respectively, expressed by:

$$(\vec{v}_w \cdot \vec{n})^- = (\vec{v}_w \cdot \vec{n})^+ \quad (2.35)$$

and

$$(\vec{v} \cdot \vec{n})^- = (\vec{v} \cdot \vec{n})^+ \quad (2.36)$$

Note that - and + denote the inside and outside of the permeable boundary. The outside conditions are assumed to be prescribed. Therefore, substituting Eqs. 2.24 and 2.21 into Eqs. 2.35 and 2.36, the following general boundary condition is obtained:

$$\left[\left(\frac{dp_c}{ds_w} \vec{k} \cdot \vec{\nabla} s_w - \Delta \rho g \vec{k} \cdot \vec{\nabla} z \right) \cdot \vec{n} \right]^- = \left(\frac{\mu_w}{k_{r_w}} \right)^- (\vec{v}_w \cdot \vec{n})^+ - \left(\frac{\mu_o}{k_{r_o}} \right)^- (\vec{v}_o \cdot \vec{n})^+ \quad (2.37)$$

For illustration purposes, consider the classical problem of one-dimensional flow in a homogeneous core. Assume that the core is initially saturated with oil up to the connate water saturation and the flow begins with injecting water at a constant rate to displace oil. For this case, Eq. 2.29 simplifies as:

$$\phi \frac{\partial s_w}{\partial t} + \frac{dF_w}{ds_w} v \frac{\partial s_w}{\partial x} + \frac{\partial}{\partial x} \left(F_w \frac{k_{r_o}}{\mu_o} \frac{dp_c}{ds_w} \frac{\partial s_w}{\partial x} \right) = 0, \quad 0 \leq x \leq L, \quad t > 0 \quad (2.38)$$

The initial condition is given by:

$$s_w = s_{wc}, \quad 0 \leq x \leq L, \quad t = 0 \quad (2.39)$$

At the inlet boundary, because only water is injected, $(v_o)^+ = 0$ and $(v_w)^+ \equiv (v_w)_{in}$. Thus, Eq. 2.37 simplifies to:

$$\frac{dp_c}{ds_w} \frac{\partial s_w}{\partial x} = \frac{\mu_w (v_w)_{in}}{k k_{r_w}}, \quad x = 0, \quad t > 0 \quad (2.40)$$

At the outlet, three separate conditions need to be formulated. Until the water break through time, t^* , $(v_w)^+ = 0$ and the overall volume balance dictates for incompressible fluids that $(v_o)^+ = (v_o)_{out} = (v_w)_{in}$. Thus,

$$\frac{dp_c}{ds_w} \frac{\partial s_w}{\partial x} = - \frac{\mu_o (v_w)_{in}}{k k_{r_o}}, \quad x = L, \quad 0 < t \leq t^* \quad (2.41)$$

Note that Eqs. 2.38, 2.40, and 2.41 can be shown to be the same as Eqs. 25 and 29 of Douglas,

et al¹⁰.

Between the breakthrough and infinite water injection, both $(v_w)^+ = (v_w)_{out} \neq 0$ and $(v_o)^+ = (v_o)_{out} \neq 0$. Thus, Eq. 2.37 is written as:

$$\frac{dp_c}{ds_w} \frac{\partial s_w}{\partial x} = \frac{\mu_w(v_w)_{out}}{kk_{r_w}} - \frac{\mu_o(v_o)_{out}}{kk_{r_o}}, \quad x = L, \quad t^* < t < \infty \quad (2.42)$$

In addition, the overall balance for the core dictates that:

$$(v_w)_{in} = (v_w)_{out} + (v_o)_{out} \quad (2.43)$$

During the infinite water injection $(v_o)^+ = (v_o)_{out} = 0$ and $(v_w)^+ = (v_w)_{out} = (v_w)_{in}$. Thus, Eq. 2.37 simplifies to:

$$\frac{dp_c}{ds_w} \frac{\partial s_w}{\partial x} = \frac{\mu_w(v_w)_{in}}{kk_{r_w}}, \quad x = L, \quad t \rightarrow \infty \quad (2.44)$$

At later times, Eqs. 2.42 and 2.44 can be replaced by the following simplified boundary condition:

$$s_w = 1 - s_{or} \quad (2.45)$$

Eqs. 2.41, 2.42, and 2.44 can be used to determine the effluent fluid fluxes rather than as the outlet boundary conditions, because for incompressible fluids it is sufficient to only specify the flux at the inlet boundary.

FRACTIONAL FLOW FORMULATION

Civan¹¹ defined a scalar fractional water function, f_w , according to Eq. 21 and

$$\bar{v}_w = f_w \bar{v} \quad (2.46)$$

to obtain the following expression:

$$f_w = F_w \left(1 + \frac{k_{r_o}}{v^2 \mu_o} \frac{dp_c}{ds_w} \bar{v} \cdot \bar{k} \cdot \bar{\nabla} s_w - \frac{k_{r_o}}{v^2 \mu_o} \Delta \rho g \bar{v} \cdot \bar{k} \cdot \bar{\nabla} z \right) \quad (2.47)$$

in which the volumetric flux is given by:

$$v = |\vec{v}| = (v_x^2 + v_y^2 + v_z^2)^{1/2} \quad (2.48)$$

v_x , v_y , and v_z are the components in the x , y , and z - directions.

Assuming incompressible fluids and constant porosity and substituting Eqs. 2.18, 2.46, and 2.47 into Eq. 2.19 for the water phase yields:

$$\phi \frac{\partial s_w}{\partial t} + \vec{v} \cdot \vec{\nabla} f_w - \frac{r}{\rho} f_w + \frac{r_w}{\rho_w} = 0 \quad (2.49)$$

The scalar fractional water function defined by Eq. 2.47 (Civan¹¹) is more appropriate than the vectorial fractional water function defined by Peters, et al¹², because the fractional water is a scalar quantity in the physical problem.

CONVENIENT FORMULATION

As stated by Civan and Sliepcevich³, the transformations of transport equations, in which "the variable properties do not appear explicitly", may lead to numerical solutions of "better accuracy with less computational effort". Therefore, Eq. 2.31 will be converted to a form involving direct applications of the various mathematical operations to the function.

For this purpose, first, define an integral transformation according to Douglas et al¹⁰, as:

$$r(s_w) = \frac{1}{Z} \int_{s_{wc}}^{s_w} D_w(s_w) ds_w \quad (2.50)$$

where

$$Z = \int_{s_{wc}}^{(1-s_{or})} D_w(s_w) ds_w \quad (2.51)$$

Thus, applying Eq. 2.50, Eq. 2.31 can be transformed into:

$$\begin{aligned}
\frac{\phi}{D_w} \frac{\partial r}{\partial t} + \frac{1}{Z} \left[\frac{dF_w}{dr} \bar{\nabla} + \frac{dT_w}{dr} (\bar{k} \cdot \bar{\nabla}_Z) \right] \cdot \bar{\nabla}_r + \bar{\nabla} \cdot (\bar{k} \cdot \bar{\nabla}_r) \\
+ \frac{T_w}{Z} \bar{\nabla} \cdot (\bar{k} \cdot \bar{\nabla}_Z) + \frac{1}{Z} \left(\frac{r_w}{\rho_w} - F_w \frac{r}{\rho} \right) = 0
\end{aligned}
\tag{2.52}$$

Next, consider coordinate transformation from (x, y, z) to (ξ, η, ζ) such that the mapping transformation is given by:

$$\begin{bmatrix} \frac{\partial r}{\partial \xi} \\ \frac{\partial r}{\partial \eta} \\ \frac{\partial r}{\partial \zeta} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\ \frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} & \frac{\partial z}{\partial \zeta} \end{bmatrix} \begin{bmatrix} \frac{\partial r}{\partial x} \\ \frac{\partial r}{\partial y} \\ \frac{\partial r}{\partial z} \end{bmatrix}
\tag{2.53}$$

or simplify

$$\bar{\nabla}_r = \bar{J} \cdot \bar{\nabla}_r
\tag{2.54}$$

where J denotes the Jacobian matrix as shown in Eq. 2.53, and ∇ and $\bar{\nabla}$ are the gradient operators in the original and the transformed coordinates, respectively. To make Eq. 2.52 free of properties in the various operators, define

$$\bar{\nabla}_r = \bar{k} \cdot \bar{\nabla}_r
\tag{2.55}$$

Thus, comparing Eqs. 2.54 and 2.55 leads to the following equation that can be used to develop the mapping functions for the desired transformation:

$$\bar{J} = \bar{k}
\tag{2.56}$$

This method of transformation can be readily extended to other coordinate systems.

Applying Eq. 2.55, Eq. 2.52 is transferred to the following convenient form:

$$\begin{aligned}
\frac{\phi}{D_w} \frac{\partial r}{\partial t} + \frac{1}{Z} \left[\frac{dF_w}{dr} \bar{\nabla} + \frac{dT_w}{dr} \bar{\nabla}_Z \right] \cdot \bar{k}^{-1} \cdot \bar{\nabla}_r + \bar{k}^{-1} \cdot \bar{\nabla} \cdot \bar{\nabla}_r \\
+ \frac{T_w}{Z} \bar{k}^{-1} \cdot \bar{\nabla} \cdot \bar{\nabla}_Z + \frac{1}{Z} \left(\frac{r_w}{\rho_w} - F_w \frac{r}{\rho} \right) = 0
\end{aligned}
\tag{2.57}$$

For example, for horizontal one-dimensional flow in an isotropic homogeneous core, with no source terms, Eq. 2.57 simplifies to:

$$\frac{\phi}{D_w} \frac{\partial r}{\partial t} + \frac{1}{Z} \frac{dF_w}{dr} \tilde{k}^{-1} \frac{\partial r}{\partial \xi} + \tilde{k}^{-1} \frac{\partial^2 r}{\partial \xi^2} = 0 \quad (2.58)$$

Eq. 2.58 is more convenient for numerical discretization than Eq. 2.38 because it involves simple direct derivatives of the function.

UNIT END-POINT MOBILITY RATIO FORMULATION

Inferred by Collins¹³, Craig¹⁴, and Dake¹⁵ the following simplifying assumptions are considered for the unit mobility ratio formulation:

1. The fluid properties (density and viscosity) are assumed constant.
2. The capillary pressure effect is neglected.

$$p_c = 0, \text{ i.e. } p_o = p_w = p \quad (2.59)$$

3. The relative permeabilities are approximated by linear functions of the normalized saturations:

$$k_{rw} = k'_{rw} \hat{s}_w \quad (2.60)$$

$$k_{ro} = k'_{ro} (1 - \hat{s}_w) \quad (2.61)$$

where the normalized mobile water saturation is given by

$$\hat{s}_w = \frac{s_w - s_{wc}}{1 - s_{wc} - s_{or}} \quad (2.62)$$

4. The fluid densities are replaced by the arithmetic average value as:

$$\rho_w = \rho_o = \rho \quad (2.63)$$

where

$$\rho = (\rho_w + \rho_o)/2 \quad (2.64)$$

5. The end-point relative mobilities are replaced by the arithmetic average value as:

$$\frac{k'_{rw}}{\mu_w} = \frac{k'_{ro}}{\mu_o} = \frac{k_r}{\mu} \quad (2.65)$$

where

$$\frac{k_r}{\mu} = \frac{1}{2} \left(\frac{k'_{rw}}{\mu_w} + \frac{k'_{ro}}{\mu_o} \right) \quad (2.66)$$

A flow potential for incompressible fluids is defined as:

$$\Psi = p + \rho g z \quad (2.67)$$

Based on these considerations, Eqs. 2.25 and 2.47 simplify, respectively as:

$$\vec{v} = - (k_r/\mu) \vec{k} \cdot \vec{\nabla} \Psi \quad (2.68)$$

and

$$f_w = \hat{s}_w \quad (2.69)$$

Therefore, substituting Eq. 2.68 into Eq. 2.18 leads to the following flow potential equation:

$$- (k_r/\mu) \vec{\nabla} \cdot (\vec{k} \cdot \vec{\nabla} \Psi) + \frac{r}{\rho} = 0 \quad (2.70)$$

Substituting Eqs. 2.62 and 2.59 into Eq. 2.49 results in the following water saturation equation:

$$(1 - s_{or} - s_{wc}) \phi \frac{\partial \hat{s}_w}{\partial t} + \vec{v} \cdot \vec{\nabla} \hat{s}_w - \hat{s}_w \frac{r}{\rho} + \frac{r_w}{\rho_w} = 0 \quad (2.71)$$

Note that Eq. 2.70 can be more conveniently solved after transforming into the following form by applying the transformation given by Eq. 2.54:

$$-(k_r/\mu)\vec{k}^{-1} \cdot \vec{\nabla} \cdot \vec{\nabla}\psi + \frac{r}{\rho} = 0 \quad (2.72)$$

Note that Eq. 2.72 is solved once because the parameters are independent of time.

FINAL REMARKS

The transformations presented in this study have resulted in convenient formulations of the differential equations in which the differential operators do not involve the variable fluid and porous media properties explicitly. Therefore, these equations can be discretized by substitution of simple first and second order derivative approximation formulae. Besides leading to more accurate numerical solutions, the convenient formulations alleviate the need for using special cumbersome formulae, such as for $\partial/\partial x(a\partial f/\partial x)$ where a is a variable, in the conventional differential equations.

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NOMENCLATURE

A_s	=	solid-fluid contact surface, cm^2
g	=	gravitational acceleration constant, 981 cm/s^2
L	=	core length, cm
n_s	=	unit normal vector
p	=	pressure, atm
t	=	time, seconds
v	=	fluid velocity, cm/s or volume flux, $\text{cm}^3/\text{cm}^2 \cdot \text{s}$
x, y, z	=	distances in the Cartesian coordinate directions, cm

Greek

μ	=	viscosity, cp
ρ	=	density, g/cm^3
ϕ	=	porosity, fraction
ψ	=	flow potential, atm

Operational

∇	=	gradient operator
$\langle \rangle$	=	representative volume average
$\cdot$	=	dot product

Subscripts

ℓ	=	fluid phase
o	=	base conditions
or	=	residual oil
r	=	bulk material
wc	=	connate water

Superscripts

$'$ or $''$	=	spatial deviation
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3. RAPID EVALUATION OF THE SWEEP EFFICIENCY IN HETEROGENEOUS RESERVOIRS USING THE CUBATURE TIME-SPACE APPROACH

3.1 INTRODUCTION

Sweep efficiency is an important factor in determination of the performance of water flooding of oil reservoirs. Thus, development of strategies for successful waterflood operations require methods for prediction of the sweep efficiency for optimum search of well locations and operation conditions among the various alternatives. Such predictive methods are usually based on the simulation of the flow and displacement processes of the oil/water systems in heterogeneous and anisotropic reservoir formations. Frequently, the flow simulators are used to infer for an accurate characterization of the reservoir formation by means of the reservoir history matching method.

Because of the nonlinearities and irregular reservoir characteristics, the solution of the coupled reservoir flow equations are carried out by appropriate numerical methods. For this purpose, the variations of the finite difference and finite elements methods, and, recently, the control volume formulations of these methods have been tried by various investigators, by varying degrees of successes. The main disadvantage of these methods is that their implementation is complicated and the discretization is usually limited to lower order accurate formulations. Therefore, numerical dispersion and instability, and grid point orientation problems are encountered during the simulation of oil-water flow in heterogeneous formations. As a result, a lot of effort has been devoted to the development of numerical fixing methods such as upstream weighing, grid refining, and saturation weighted averaging techniques with uncertain successes. Therefore, there is need for higher order accurate methods which can be implemented without resorting to numerical fixing methods.

In search for a more practical and accurate numerical method, Civan¹⁻³ has developed a new method named as the cubature method and has demonstrated by various small scale problems that the cubature method offers potential advantages over the conventional finite difference and finite elements methods. The most important feature of the cubature method is that its formulation does not get complicated for increasing orders of accuracy. In fact the formulation of the cubature method is universal, i.e. it does not change by going from lower to higher order accurate approximations. Second, the cubature method is equally applicable to differential, integral, and integrodifferential equations. Whereas, the conventional methods inherently demand almost exponentially increasing complexity for higher order formulations which is the basic deterrent for use of high order conventional methods. The cubature method has eliminated this deficiency of the conventional methods. As a result, several orders of higher order accuracy than the

conventional methods can easily be accomplished by the cubature methods even when using relatively larger time steps and grid block sizes. For example, on the solution of water flooding in fractured reservoirs, Civan^{4,5} has demonstrated that even when using 3000 times larger time steps and 10 times greater grid point spacing the cubature method generated better accuracy than the finite difference method. These experiments have proven that the cubature method offers certain advantages. This study investigates and implements the cubature method for field scale waterflood simulation purposes.

Basically the cubature is a numerical approximation method expressed in the form of a weighted linear sum of discrete function values selected from the temporal-spatial computational domain. The details of the method are presented in various publications by Civan¹⁻⁵. The cubature approximation is expressed by the following universal equation for any linear operator and any order of accuracy.

$$L.f(t,x,y,z)_i = \sum_{j=1}^n w_{ij} f(t,x,y,z)_j \quad (3.1)$$

in which L represents a linear operator, $i=1,2,\dots,n$ denote the discrete point locations, $f(t,x,y,z)$ are the discrete function values, w_{ij} are the cubature weights, and n is the total number of discrete point locations considered for numerical approximation. The weighing coefficients for specific operators are defined according to the procedure described by Civan^{2,3}. Briefly, the local functional behavior is approximated by an appropriate function such as multi-variate power series or approximate analytical solutions to the equations of interest. Upon the substitution of such functional, Eq. 3.1 is solved for the weighting coefficients. Then, these weighting coefficients are substituted into Eq. 3.1 for cubature approximation of an operator applied on the function.

A unique feature of the cubature method is the ability of including both the temporal and spatial derivative terms in the discretization leading to time-space methods (Civan^{2,6}). Thus, a transient-state problem can be solved like a steady-state problem. By virtue of this feature the cubature solutions are stable for large time and space increments while producing several orders of higher accurate numerical solutions. In this study, a formulation and application of the cubature method to simulation of waterflooding of oil reservoirs for purposes of the prediction of the sweep efficiency in heterogeneous reservoirs is carried out.

Two special cases are considered for the application of the cubature method:

- (1) A unit mobility type formulation.
- (2) An incompressible two-phase type formulation without and with the capillary pressure effects included.

For each case, cubature based numerical schemes are formulated, programmed, and tested for the sensitivity of the numerical accuracy with respect to the discretization parameters such as the temporal steps and the spatial grid size.

3.2 SWEEP EFFICIENCY

According to Craig⁷ sweep efficiency is defined by

$$E_r = E_d \times E_v \quad (3.2)$$

where E_d is the microscopic oil displacement efficiency and E_v is the volumetric oil displacement efficiency.

For an aerial reservoir model, this equation can be written as:

$$E_r = E_d \times E_A \quad (3.3)$$

Alternatively, the sweep efficiency can be expressed directly in terms of the cumulative water injected per maximum water capacity of the reservoir as:

$$E_r = \frac{\int_A S_w dA}{\int_A (1 - S_{wc} - S_{or}) dA} \quad (3.4)$$

For numerical evaluation purposes, this equation can be written as a sum of the efficiencies over the grid blocks as:

$$E_r = \sum_{Ai} \left[\frac{\int_{Ai} S_w dA}{\int_{Ai} (1 - S_{wc} - S_{or}) dA} \right] \quad (3.5)$$

in which the integrals can be carried out by the application of an appropriate numerical integration method. In this work the cubature integration formulae described in the following section has been used.

3.3 CUBATURE APPROXIMATION FORMULA

Civan¹⁻⁵ presents the general cubature formula as:

$$Lf(x, y, z)_i \equiv \sum_{j=1}^n w_{ij} f(x, y, z)_j \quad (3.6)$$

where L is a linear operator and w_{ij} are some weighting coefficients. Eq. 3.5 can be written in a simpler way as:

$$Lf_i \equiv \sum_{t=1}^n w_{it} f_t \quad (3.7)$$

The moment equation necessary for determination of the weighting coefficients is given by:

$$\sum_{t=1}^n B_{vt} w_{it} = LB_{vi} \quad \begin{array}{l} v = 0, 1, \dots, \mu \\ i = 1, 2, \dots, n \end{array} \quad (3.8)$$

where the basis functions are given by:

$$B_v = x^{v-\mu} y^{\mu-\lambda} z^\lambda \quad \begin{array}{l} \lambda = 0, 1, \dots, \mu \\ \mu = 0, 1, \dots, v \\ v = 0, 1, \dots \end{array} \quad (3.9)$$

Thus, substituting Eq. 3.8 into 3.7 yields:

$$\sum_{j=1}^n x_j^{v-\mu} y_j^{\mu-\lambda} z_j^\lambda w_{ij} = L \{ x^{v-\mu} y^{\mu-\lambda} z^\lambda \}_i \quad (3.10)$$

in which the locations of the discrete points depend on the computational molecule selected for discretization. Fig. 3.2 shows three molecules having 15, 27, and 37 points. Figs. 3.3 and 3.4 show a 75-point (25 space and 3 time level) time-space computational molecule. Fig. 3.5 depicts a 3 time level quadrature and 5-point space cubature time-space molecule. The time discretization formulae are given at three time levels by:

$$\frac{\partial s_{ij}^n}{\partial t} = \frac{1}{6\Delta t} [-s_{ij}^{n+2} + 6s_{ij}^{n+1} - 3s_{ij}^n - 2s_{ij}^{n-1}] \quad (3.11)$$

$$\frac{\partial s_{ij}^{n+1}}{\partial t} = \frac{1}{6\Delta t} [2s_{ij}^{n+2} + 3s_{ij}^{n+1} - 6s_{ij}^n + s_{ij}^{n-1}] \quad (3.12)$$

$$\frac{\partial s_{ij}^{n+2}}{\partial t} = \frac{1}{6\Delta t} [11s_{ij}^{n+2} - 18s_{ij}^{n+1} + 9s_{ij}^n - 2s_{ij}^{n-1}] \quad (3.13)$$

These formulae are third order accurate.

3.4 UNIT MOBILITY RATIO FLOW PROBLEM

Assumptions (Craig⁷, Dake⁸) :

$$P_c = 0 \quad (3.14)$$

$$\lambda_{r_o} = \lambda_{r_w} = \lambda_r \quad (3.15)$$

where

$$\lambda_{r_o} = k^o_{r_o} / \mu_o \quad (3.16)$$

$$\lambda_{r_w} = k^o_{r_w} / \mu_w \quad (3.17)$$

Normalized saturation:

$$\hat{S}_w = \frac{S_w - S_{w_e}}{1 - S_{w_e} - S_{o_r}} \quad (3.18)$$

$$k_{r_w} = k^o_{r_w} \hat{S}_w \quad (3.19)$$

$$k_{r_o} = k^o_{r_o} (1 - \hat{S}_w) \quad (3.20)$$

Basic Equations

Darcy's Law:

$$\vec{u} = -\lambda_r \vec{k} \cdot \nabla \psi \quad (3.21)$$

Pressure equation:

$$-\lambda_r \vec{\nabla} \cdot (\vec{k} \cdot \vec{\nabla} \psi) + \frac{r}{\rho} = 0 \quad (3.22)$$

Water saturation equation:

$$(1 - S_{o_r} - S_{w_e}) \phi \frac{\partial \hat{S}_w}{\partial t} + \vec{v} \cdot \vec{\nabla} \hat{S}_w - \hat{S}_w \frac{r}{\rho} + \frac{r_w}{\rho_w} = 0 \quad (3.23)$$

Homogeneous Reservoir:

Assume a simplified case as a homogeneous reservoir.

Step 1: Solve (only once because of independency of time)

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{r}{\rho \lambda_w k} \quad (3.24)$$

Step 2: Calculate

$$u_x = -\lambda_r k \frac{\partial \psi}{\partial x} \quad (3.25)$$

} Fixed by time

$$u_y = -\lambda_r k \frac{\partial \psi}{\partial y} \quad (3.26)$$

Step 3: Solve the following as a function of time:

$$\frac{\partial \hat{s}_w}{\partial t} + \frac{u_x}{\emptyset} \frac{\partial \hat{s}_w}{\partial x} + \frac{u_y}{\emptyset} \frac{\partial \hat{s}_w}{\partial y} - \hat{s}_w \frac{r}{\rho} + \frac{r_w}{\rho_w} = 0 \quad (3.27)$$

Heterogeneous Reservoir:

$$\frac{\partial k_x}{\partial x} \frac{\partial \psi}{\partial x} + k_x \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial k_y}{\partial y} \frac{\partial \psi}{\partial y} + k_y \frac{\partial^2 \psi}{\partial y^2} = \frac{r}{\rho \lambda_r} \quad (3.28)$$

$$u_x = -\lambda_r k_x \frac{\partial \psi}{\partial x} \quad (3.29)$$

$$u_y = -\lambda_r k_y \frac{\partial \psi}{\partial y} \quad (3.30)$$

$$\frac{\partial \hat{s}_w}{\partial t} + \frac{u_x}{\emptyset} \frac{\partial \hat{s}_w}{\partial x} + \frac{u_y}{\emptyset} \frac{\partial \hat{s}_w}{\partial y} = \hat{s}_w \frac{r}{\rho} - \frac{r_w}{\rho_w} \quad (3.31)$$

Quadrature - Cubature Solution

Discretize the space derivatives by the cubature and time derivatives by the quadrature.

For example; for a function of $f=f(t,x,y)$

$$\frac{\partial f_i}{\partial t} = \sum_{\ell=1}^{N'} w_{i\ell}^q f_{\ell} \quad \text{quadrature} \quad (3.32)$$

$$\frac{\partial^2 f_i}{\partial \xi^n} = \sum_{\ell=1}^{N'} w_{i\ell}^c f_{\ell} \quad \text{cubature} \quad (3.33)$$

First discretize and solve Eq. 3.1:

$$\begin{aligned} & \left(\sum_{\ell=1}^n w_{i\ell}^x k_{x_i} \right) \sum_{\ell=1}^{n'} w_{i\ell}^x \psi_{\ell} + k_{x_i} \sum_{\ell=1}^{n'} w_{i\ell}^{x^2} \psi_{\ell} + \left(\sum_{\ell=1}^n w_{i\ell}^y k_{y_i} \right) \sum_{\ell=1}^{n'} w_{i\ell}^y \psi_{\ell} \\ & + k_{y_i} \sum_{\ell=1}^{n'} w_{i\ell}^{y^2} \psi_{\ell} = \left(\frac{r}{\rho \lambda_r} \right)_i \end{aligned} \quad (3.34)$$

Solve Eq. 3.34 and then, calculate

$$u_{x_i} = -\lambda_r k_{x_i} \sum_{\ell=1}^{n'} w_{i\ell}^x \psi_{\ell} \quad (3.35)$$

$$u_{y_i} = -\lambda_r k_{y_i} \sum_{\ell=1}^{n'} w_{i\ell}^y \psi_{\ell} \quad (3.36)$$

Solution of Eqs. 3.34-36 are carried out once at the beginning. Then, solve Eq. 3.31 as a function of time:

$$\sum_{\ell=1}^{n'} w_{i\ell}^t \hat{S}_{w_{\ell}} + \left(\frac{u_x}{\phi} \right)_i \sum_{\ell=1}^{n'} w_{i\ell} \tilde{S}_{w_{\ell}} + \left(\frac{u_y}{\phi} \right)_i \sum_{\ell=1}^{n'} w_{i\ell} \hat{S}_{w_{\ell}} = \left(\hat{S}_w \frac{r}{\rho} - \frac{r_w}{\rho_w} \right)_i \quad (3.37)$$

Test Problem

Fig. 3.6 is a schematic of the test problem considered here. Fig. 3.7 illustrates a simple 5-point (2 time level and 3 space) time-space molecule used in the calculations.

The production rate is given by:

$$q_w = 0, t < t_{bt} \quad (\text{until breakthrough}) \quad (3.38)$$

$$q_w = f_w q \neq 0, f_w = \hat{S}_w = \frac{S_w - S_{wc}}{1 - S_{wc} - S_{or}}, t_{bt} < t < \infty \quad (\text{after breakthrough}) \quad (3.39)$$

Step 1. Given k_x , k_y , calculate

$$\left(\frac{\partial k_x}{\partial x}\right)_{i,j} = \frac{(k_x)_{i+1,j} - (k_x)_{i-1,j}}{2\Delta x} \quad (3.40)$$

$$\left(\frac{\partial k_y}{\partial y}\right)_{i,j} = \frac{(k_y)_{i,j+1} - (k_y)_{i,j-1}}{2\Delta y} \quad (3.41)$$

However, spline interpolation might be better.

Step 2. Finite-difference analog of Eq. 3.1.

$$\begin{aligned} \left(\frac{\partial k_x}{\partial x}\right)_{i,j} \frac{P_{i+1,j} - P_{i-1,j}}{2\Delta x} + (k_x)_{i,j} \frac{P_{i+1,j} - 2P_{i,j} + P_{i-1,j}}{(\Delta x)^2} + \\ \left(\frac{\partial k_y}{\partial y}\right)_{i,j} \frac{P_{i,j+1} - P_{i,j-1}}{2\Delta y} + (k_y)_{i,j} \frac{P_{i,j+1} - 2P_{i,j} + P_{i,j-1}}{(\Delta y)^2} = 0 \end{aligned} \quad (3.42)$$

Eq. 3.42 is rearranged for G-S solution of $P_{i,j}$ at $i=1,N$ and $j=1,M$. However, Eq. 3.42 can also be solved directly.

Step 3: Calculate the fluxes:

$$(u_x)_{i,j} = -\lambda_r (k_x)_{i,j} \frac{P_{i+1,j} - P_{i-1,j}}{2\Delta x} \quad (3.43)$$

$$(u_y)_{i,j} = -\lambda_r (k_y)_{i,j} \frac{P_{i,j+1} - P_{i,j-1}}{2\Delta y} \quad (3.44)$$

Step 4: Discretize and solve the saturation equation at the two consecutive time levels shown in Fig. 3.1.

At the n^{th} level:

$$\phi \frac{s^{n+1}_{i,j} - s^{n-1}_{i,j}}{2\Delta t} + (u_x)_{i,j} \frac{s^n_{i,j} - s^n_{i-1,j}}{\Delta x} + (u_y)_{i,j} \frac{s^n_{i,j} - s^n_{i,j-1}}{\Delta y} = 0 \quad (3.45)$$

At the $(n+1)^{\text{th}}$ level:

$$\phi \frac{s^{n+1}_{i,j} - 4s^n_{i,j} + 3s^{n-1}_{i,j}}{2\Delta t} + (u_x)_{i,j} \frac{s^{n+1}_{i,j} - s^{n+1}_{i-1,j}}{\Delta x} + (u_y)_{i,j} \frac{s^{n+1}_{i,j} - s^{n+1}_{i,j-1}}{\Delta y} = 0 \quad (3.46)$$

Rearrange and solve by the Gauss-Seidel method for s_{ij}^n and s_{ij}^{n+1} for $i=1, N$ and $j=1, M$. However, they can be solved directly as shown in the following.

For a direct solution Eqs. 3.45 and 3.46 can be reformulated as:

$$s_{i,j}^{n+1} + (A + B)s_{i,j}^n = \alpha \quad (3.47)$$

$$(c + A + B)s_{i,j}^{n+1} + 6s_{i,j}^n = \beta \quad (3.48)$$

$$A = \left[\frac{2\Delta t(u_x)_{i,j}}{\Delta x} \right] \quad (3.49)$$

$$B = \left[\frac{2\Delta t(u_y)_{i,j}}{\Delta y} \right] \quad (3.50)$$

$$\alpha = s_{i,j}^{n-1} + As_{i-1,j}^n + Bs_{i,j-1}^n \quad (3.51)$$

$$\beta = as_{i,j}^{n-1} - As_{i-1,j}^{n+1} - Bs_{i,j-1}^{n+1} \quad (3.52)$$

Thus, the analytic solution of Eqs. 3.47 and 3.48 yields:

$$\therefore s_{i,j}^{n+1} = [\alpha b - \beta(A + B)] / [b - (A + B)(c + A + B)] \quad (3.53)$$

$$\therefore s_{i,j}^n = [\beta - \alpha(c + A + B)] / [b - (A + B)(c + A + B)] \quad (3.54)$$

Eqs. 3.53 and 3.54 are solved for $i=2,3,\dots,N$ and $j=2,3,\dots,M$.

The initial condition is:

$$s_{i,j}=0 \quad \text{everywhere} \quad (3.55)$$

Assume Dirichlet boundary conditions:

$$s_{i,j}=1.0 : j=1, i=1,2,\dots,N \quad (3.56)$$

$$s_{i,j}=1.0 : i=1, j=1,2,\dots,M \quad (3.57)$$

This exercise demonstrates that the cubature method can solve problems using only the discrete values within the selected computational molecule without needing the discrete

values at the previous times. As a result, the previous time solutions do not need to be stored. In addition, it is shown that a direct solution of the unit mobility flow can be obtained for the time-space formulation to obtain the solutions simultaneously at selected time levels with higher order accuracy. (2nd order in the above example).

3.5 HORIZONTAL TWO-DIMENSIONAL FLOW OF TWO IMMISCIBLE AND INCOMPRESSIBLE FLUIDS

In general, the mathematical description of flow of immiscible and incompressible fluids in porous media is given by Civan^{4,5,9} as:

$$\bar{\nabla} \cdot \bar{v} = 0 \quad (3.58)$$

where

$$\bar{v} = - \left(\frac{k_{rw}}{\mu_w} + \frac{k_{ro}}{\mu_o} \right) \bar{k} \cdot \bar{\nabla} p_w - \frac{k_{ro}}{\mu_o} \frac{dp_c}{ds_w} \bar{k} \cdot \bar{\nabla}_{sw} - \left(\frac{k_{rw}}{\mu_w} \rho_w + \frac{k_{ro}}{\mu_o} \rho_o \right) g \bar{k} \cdot \bar{\nabla} z \quad (3.59)$$

$$\phi \frac{\partial \bar{v}_w}{\partial t} + \bar{\nabla} \cdot \bar{v}_w = 0 \quad (3.60)$$

where

$$\bar{v}_w = - \frac{k_{rw}}{\mu_w} \bar{k} \cdot (\bar{\nabla} p_w + \rho_w g \bar{\nabla} z) \quad (3.61)$$

In general:

$$\bar{k} = \begin{bmatrix} k_x & k_{xy} \\ k_{yx} & k_y \end{bmatrix} \quad (3.62)$$

However, assume $\bar{k} = k$, i.e isotropic but heterogeneous $k=k(x,y)$.

Further, assume the following Kozeny-Carman relationship for $k=k(\phi)$:

$$\frac{k}{k_o} \equiv \left(\frac{\phi}{\phi_o} \right)^3 \left(\frac{1-\phi_o}{1-\phi} \right)^2 \quad (3.63)$$

The problem is to apply point boundary conditions at the injection and production wells as depicted in Fig. 3.9 and calculate the sweep efficiency.

Substituting Eq. 3.59 into 3.58 and expanding yields the following equation of conservation of total mass in the form of a convection-dispersion equation as:

$$\begin{aligned} \lambda_T k_x \frac{\partial^2 P_w}{\partial x^2} + \frac{\partial (\lambda_T k_x)}{\partial x} \frac{\partial P_w}{\partial x} + \lambda_T k_y \frac{\partial^2 P_w}{\partial y^2} + \frac{\partial (\lambda_T k_y)}{\partial y} \frac{\partial P_w}{\partial y} + \\ \delta_o k_x \frac{\partial^2 S_w}{\partial x^2} + \frac{\partial (\delta_o k_x)}{\partial x} \frac{\partial S_w}{\partial x} + \delta_o k_y \frac{\partial^2 S_w}{\partial y^2} + \frac{\partial (\delta_o k_y)}{\partial y} \frac{\partial S_w}{\partial y} = 0 \end{aligned} \quad (3.64)$$

where

$$\lambda_w = \frac{k_{rw}}{\mu_w} \quad (3.65)$$

$$\lambda_o = \frac{k_{ro}}{\mu_o} \quad (3.66)$$

$$\lambda_T = \lambda_w + \lambda_o \quad (3.67)$$

$$\delta_o = \frac{k_{ro}}{\mu_o} \frac{dp_c}{ds_w} \quad (3.68)$$

Expanding Eq. 3.60 yields the following water phase mass balance equation:

$$\phi \frac{\partial s_w}{\partial t} = \frac{\partial (\lambda_w k_x)}{\partial x} \frac{\partial P_w}{\partial x} + \lambda_w k_x \frac{\partial^2 P_w}{\partial x^2} + \frac{\partial (\lambda_w k_y)}{\partial y} \frac{\partial P_w}{\partial y} + \lambda_w k_y \frac{\partial^2 P_w}{\partial y^2} \quad (3.69)$$

Eqs. 3.69 and 3.64 can be written, respectively, as, in the linear form for an iterative solution by evaluating the coefficients at the previous iteration values:

$$\frac{\partial s}{\partial t} + \alpha \frac{\partial p}{\partial x} + \beta \frac{\partial^2 p}{\partial x^2} + \gamma \frac{\partial p}{\partial y} + \delta \frac{\partial^2 p}{\partial y^2} = 0 \quad (3.70)$$

and

$$a \frac{\partial p}{\partial x} + b \frac{\partial^2 p}{\partial x^2} + c \frac{\partial p}{\partial y} + d \frac{\partial^2 p}{\partial y^2} + e \frac{\partial s}{\partial x} + f \frac{\partial^2 s}{\partial x^2} + g \frac{\partial s}{\partial y} + h \frac{\partial^2 s}{\partial y^2} = 0 \quad (3.71)$$

for which the coefficients correspond to those appearing in Eqs. 3.69 and 3.64. The discrete forms of Eqs. 3.70 and 3.71 can be written, respectively, as:

$$\sum a_{ij}s_j + \sum b_{ij}p_j = R_w \quad (3.72)$$

$$\sum w_{ij}s_j + \sum u_{ij}p_j = R \quad (3.73)$$

These equations are solved directly or by the Gauss-Seidel method. Then, the corrections are applied according to Liu and Liu¹⁰ at the center point shown in Fig 3.10:

$$s_c \leftarrow s_c + \varepsilon \quad (3.74)$$

$$p_c \leftarrow p_c + \eta \quad (3.75)$$

At the reference point C:

$$\sum_{\alpha} a_{ij}s_j + a_{ic}(s_c + \varepsilon) + \sum_{\alpha} b_{ij}p_j + b_{ic}(p_c + \eta) = R_w \quad (3.76)$$

Subtracting Eq. 3.72 from 3.76 yields:

$$a_{ic}\varepsilon + b_{ic}\eta = 0 \quad (3.77)$$

Eq. 3.73 yields:

$$\sum_{\alpha} w_{ij}s_j + w_{ic}(s_c + \varepsilon) + \sum_{\alpha} u_{ij}p_j + u_{ic}(p_c + \eta) = R \quad (3.78)$$

Solving Eqs. 3.77 and 3.78 yields the algebraic solutions for the corrections ε and η . Once ε and η values are obtained, then eqs. 3.74 and 3.75 are used to update the values at point C. By sweeping the whole region, the solutions at all the points are obtained.

For solution according to Liu and Liu¹⁰, the distributive relaxation scheme is carried out by two steps:

- 1.) Freezing p_c , perform point Gauss-Seidel relaxation on Eq. 3.72.
- 2.) Modify p_c and s_c according to Eqs. 3.74 and 3.75.

For implementation of this method, the first, second, higher order backward formula can be used at all time levels. For example, consider:

$$\frac{\partial s_c^{n+1}}{\partial t} = \frac{s_c^{n+1} - s_c^n}{\Delta t} + o(\Delta t) \quad (3.79)$$

$$\frac{\partial s_c^{n+1}}{\partial t} = \frac{f_c^{n+1} - 4f_c^n + 3f_c^{n-1}}{2\Delta t} + o(\Delta t^2) \quad (3.80)$$

$$\frac{\partial s_c^{n+1}}{\partial t} = \frac{2f_c^{n+2} + 3s_c^{n+1} - 6s_c^n + s_c^{n-1}}{6\Delta t} + o(\Delta t^3) \quad (3.81)$$

However, as depicted in Fig. 3.10, unless the number of time steps taken is equal or greater than the order of accuracy, the backward points beyond the time period considered for the time-space domain will have to be used and therefore, extra storage for the previous solutions will be necessary. Therefore, it is not practical to use backward points beyond the time period for which the solutions are being searched. Civan¹² alleviated this problem by confining the grid points to points existing within the time period considered for the solution.

For example, for the two-time step period shown in Fig. 3.10, the following second order central and backward difference formula can be used, respectively, at the η and $(\eta + 1)$ time levels:

$$\frac{\partial f_c^n}{\partial t} = \frac{f_c^{n+1} - f_c^{n-1}}{2\Delta t} + o(\Delta t^2) \quad (3.82)$$

$$\frac{\partial f_c^{n+1}}{\partial t} = \frac{f_c^{n+1} - 4f_c^n + 3f_c^{n-1}}{2\Delta t} + o(\Delta t^2) \quad (3.83)$$

Note the values f_c^{n-1} are known as the initial condition for each period.

Eqs. 3.82 and 3.83 can be replaced by multiple point cubature approximations for higher order accuracy.

For the space derivatives use a nine-point central difference approximation obtained by the cubature method.

$$\frac{\partial f_c}{\partial x} = \sum_{j=1}^9 a_{ij} f_j + o(\Delta x^3, \Delta y^3) \quad (3.84)$$

$$\frac{\partial^2 f_c}{\partial x^2} = \sum_{j=1}^9 b_{ij} f_j + o(\Delta x^3, \Delta y^3) \quad (3.85)$$

$$\frac{\partial f_c}{\partial y} = \sum_{j=1}^9 c_{ij} f_j + o(\Delta x^3, \Delta y^3) \quad (3.86)$$

$$\frac{\partial^2 f_c}{\partial y^2} = \sum_{j=1}^9 d_{ij} f_j + o(\Delta x^3, \Delta y^3) \quad (3.87)$$

3.6 REFERENCES

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Fig. 3.1 Sketch of waterflooding of an oil reservoir

Fig. 3.2 Typical time-space molecules

Fig. 3.3

A 25 space and 3 time points computational molecule

Fig. 3.4 Display of a 25 point computational molecule
(Modified after Liu et al¹¹)

Fig. 3.5 Three-time step and five-space points computational molecule

Fig. 3.6 Test problem

Fig. 3.7 Two time steps and three space points computational molecule

Fig. 3.8 Iso-saturation contours

Fig. 3.9 Test problem

Fig. 3.10 Two time step and nine space points computational molecule

4. SWEEP EFFICIENCY PREDICTION USING HIGH ORDER FINTE DIFFERENCE TIME-SPACE APPROACH

4.1 INTRODUCTION

Development of accurate methods towards an in depth understanding of reservoir structure and petrophysics is essential for successful characterization of petroleum reservoirs. In addition to other ingredients of the overall development effects, accurate modeling of the reservoir architecture and fluid flow through porous media, and efficient numerical solution of the resulting model constitutes the major task. Based on the previous test studies carried out by Civan¹, the cubature method appears to be an advantageous method because this method provides accurate and rapid numerical solutions over relatively small number of grid blocks.

In this report, formulation of the cubature numerical scheme for the unit mobility ratio flow case and general case, and results are presented.

4.2 UNIT MOBILITY RATIO CASE

The complete mathematical model for two-phase fluid flow in porous media are as follows:

$$\nabla \left(\frac{\rho_w k k_{rw}}{\mu_w} \nabla p_w \right) + q_w = \frac{\alpha \phi \rho_w s_w}{\alpha} \quad (4.1)$$

$$\nabla \left(\frac{\rho_o k k_{ro}}{\mu_o} \nabla p_o \right) + q_o = \frac{\alpha \phi \rho_o s_o}{\alpha} \quad (4.2)$$

$$\text{I.C. } p_o = p_{oi} \text{ at } t = 0 \quad (4.3)$$

$$s_w = s_{wi} \text{ at } t = 0 \quad (4.4)$$

$$\text{B.C. } \frac{\partial p}{\partial n} \Big|_t = 0 \text{ or } p \Big|_t = p_e \quad (4.5)$$

The following assumptions were made in order to reduce the complexity of the model to test problem-unit mobility ratio flow case:

1. The mobility of water equals the one of oil
2. No capillary pressure
3. Rock and fluids are incompressible
4. Isotropic, homogeneous reservoir

After the above assumptions have been made, the following two simplified equations for pressure and saturation are obtained.

$$\frac{\partial}{\partial x} \left(k_x \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(k_y \frac{\partial p}{\partial y} \right) + \frac{q_{wv}}{\lambda_r} + \frac{q_{ov}}{\lambda_r} = 0 \quad (4.6)$$

$$\phi \frac{\partial s_w}{\partial t} + \frac{u_x}{1 - s_{wc} - s_{or}} \frac{\partial s_w}{\partial x} + \frac{u_y}{1 - s_{wc} - s_{or}} \frac{\partial s_w}{\partial y} - q_{wv} = 0 \quad (4.7)$$

The injection rate is taken as positive and the production rate as negative. The directional fluxes are calculated by:

$$u_x = -\lambda_r k_x \frac{\partial p}{\partial x} \quad (4.8)$$

$$u_y = -\lambda_r k_y \frac{\partial p}{\partial y} \quad (4.9)$$

where the mobility is given by

$$\lambda_r = \frac{k_{ro}}{\mu_o} = \frac{k_{rw}}{\mu_w} \quad (4.10)$$

A quarter of a five-spot pattern was taken as the base to calculate pressure and water saturation distributions. There is no time term in the pressure equation (4.6). Thus, pressure is independent of time. Therefore, equation (4.6) is solved only once using the finite difference method. First and second derivatives of pressure are approximated by second order central finite difference.

$$\frac{\partial p}{\partial x} = \frac{p_{i+1,j} - p_{i-1,j}}{2\Delta x} + O(\Delta x^2) \quad (4.11)$$

$$\frac{\partial p}{\partial y} = \frac{p_{i,j+1} - p_{i,j-1}}{2\Delta y} + O(\Delta y^2) \quad (4.12)$$

$$\frac{\partial^2 p}{\partial x^2} = \frac{p_{i+1,j} - 2p_{i,j} + p_{i-1,j}}{\Delta x^2} + O(\Delta x^2) \quad (4.13)$$

$$\frac{\partial^2 p}{\partial y^2} = \frac{p_{i,j+1} - 2p_{i,j} + p_{i,j-1}}{\Delta y^2} + O(\Delta y^2) \quad (4.14)$$

Then, the resulting finite difference equation is solved by using the Gauss-Seidel iteration method.

For saturation equation (4.7), two methods, finite difference scheme and time-space scheme, are used and good results are obtained.

4.2.1 Finite Difference Formulation (One Time Step at One Time)

Using first order backward difference to discretize time and space leads to a finite difference equation. Solution is obtained by using the Gauss-Seidel iteration method to solve the finite difference equation.

$$\frac{\partial s_w}{\partial t} = \frac{s_{wij}^{n+1} - s_{wij}^n}{\Delta t} + O(\Delta t) \quad (4.15)$$

$$\frac{\partial s_w}{\partial x} = \frac{s_{wij}^{n+1} - s_{wi-1,j}^{n+1}}{\Delta x} + O(\Delta x) \quad (4.16)$$

$$\frac{\partial s_w}{\partial y} = \frac{s_{wij}^{n+1} - s_{wij-1}^{n+1}}{\Delta y} + O(\Delta y) \quad (4.17)$$

4.2.2 Time-Space Formulation

The basic idea for the time-space method is that the equation of unknowns are solved simultaneously for more than one time step (two, three, four etc.) instead of just one time step at one time which the conventional finite difference method does.

4.2.2.1 Two time steps at one time

As depicted in Fig. 4.1, there are two time steps, one from n time level to $n+1$ time level and one from $n+1$ time level to $n+2$ time level. Second order backward difference is used to discretize the time derivative and first order backward difference is used to discretize the space derivative.

Fig. 4.1 Illustration of time-space method
(two time steps at one time)

Fig. 4.2 Illustration of time-space method
(three time steps at one time)

At the $n+1$ time level:

$$\frac{\partial s_{wij}^{n+1}}{\partial t} = \frac{s_{wij}^{n+1} - 4s_{wij}^n + 3s_{wij}^{n-1}}{2\Delta t} + O(\Delta t^2) \quad (4.18)$$

$$\frac{\partial s_{wij}^{n+1}}{\partial x} \frac{s_{wij}^{n+1} - s_{wi-1,j}^{n+1}}{\Delta x} + O(\Delta x) \quad (4.19)$$

$$\frac{\partial s_{wij}^{n+1}}{\partial y} \frac{s_{wij}^{n+1} - s_{wi,j-1}^{n+1}}{\Delta y} + O(\Delta y) \quad (4.20)$$

At the $n+2$ time level:

$$\frac{\partial s_{wij}^{n+2}}{\partial t} = \frac{s_{wij}^{n+2} - 4s_{wij}^{n+1} + 3s_{wij}^n}{2\Delta t} - O(\Delta t^2) \quad (4.21)$$

$$\frac{\partial s_{wij}^{n+2}}{\partial x} \frac{s_{wij}^{n+2} - s_{wi-1,j}^{n+2}}{\Delta x} + O(\Delta x) \quad (4.22)$$

$$\frac{\partial s_{wij}^{n+2}}{\partial y} \frac{s_{wij}^{n+2} - s_{wi,j-1}^{n+2}}{\Delta y} + O(\Delta y) \quad (4.23)$$

Thus, two sets of difference equations, one set for $n+1$ time level, one set for $n+2$ time level, are obtained and solved at one time (simultaneously) using the Gauss-Seidel iteration method.

4.2.2.2 Three time -steps at one time

As depicted in Fig. 4.2, there are three time steps, $n+1$, $n+2$, and $n+3$. Third order backward difference is used to discretize the time derivative and first order backward difference (or second order central difference) is used to discretize the space derivative.

$$\frac{\partial s_{wi,j}^k}{\partial t} = \frac{1}{6\Delta t} (11s_{wi,j}^k - 18s_{wi,j}^{k-1} - 9s_{wi,j}^{k-2} - 2s_{wi,j}^{k-3}) + O(\Delta t^3) \quad (4.24)$$

$$\frac{\partial s_{wi,j}^k}{\partial x} = \frac{s_{wi+1,j}^k - s_{wi-1,j}^k}{2\Delta x} + O(\Delta x^2) \quad (4.25)$$

$$\frac{\partial s_{wi,j}^k}{\partial y} = \frac{s_{wi,j+1}^k - s_{wi,j-1}^k}{2\Delta y} + O(\Delta y^2) \quad (4.26)$$

$$k = n+1, n+2, n+3 \quad (4.27)$$

Three sets of difference equations for these three time level are obtained and solved at one time using the Gauss-Seidel iteration method.

4.3 GENERAL CASE

General case refers to less assumptions than the unit mobility ratio flow case although the following assumptions are still made:

1. No capillary pressure
2. No gravity force
3. Isotropic, homogeneous reservoir

Same as the unit mobility ratio flow case, a quarter of a five-spot pattern was taken as the base to calculate pressure and water saturation distributions and, hence, for the sweep efficiency calculation using an IMPES method. The bottom hole pressures of the injection and production wells were fixed at some prescribed values.

4.3.1 Finite Difference Formulation

4.3.1.1 Second order in space, first order in time

A second order central difference in space is applied to discretize equations (4.1) and (4.2). Using

$$\frac{\partial^2 p_{ij}^{n+1}}{\partial x^2} = \frac{p_{i+1,j}^{n+1} - 2p_{ij}^{n+1} + p_{i-1,j}^{n+1}}{\Delta x^2} + O(\Delta x^2) \quad (4.28)$$

$$\frac{\partial^2 p_{ij}^{n+1}}{\partial y^2} = \frac{p_{i,j+1}^{n+1} - 2p_{ij}^{n+1} + p_{i,j-1}^{n+1}}{\Delta y^2} + O(\Delta y^2) \quad (4.29)$$

and first order backward difference in time

$$\frac{\partial p}{\partial t} = \frac{p_{ij}^{n+1} - p_{ij}^n}{\Delta t} + O(\Delta t) \quad (4.30)$$

to discretize space and time terms in equation (4.1) following difference scheme is obtained:

$$T_{oyj-1/2} p_{oi,j-1} + T_{oxi-1/2} p_{i-1,j} - (T_{oxi-1/2} + T_{oxi+1/2} + T_{oyj-1/2} + T_{oyj+1/2} - \frac{V_{ij} \beta_{oi,j}}{\Delta t}) p_{ij}^n + \quad (4.31)$$

$$T_{oxi+1/2} p_{i+1,j} + T_{oyj+1/2} p_{i,j+1} = V_{ij} \phi \rho_o \frac{s_{oi,j}^{n+1} - s_{oi,j}^n}{\Delta t} - Q_{oi,j} - \frac{V_{ij} \beta_{oi,j}}{\Delta t} p_{ij}^n$$

$$\text{where } T_{oxi-1/2} = \frac{2\Delta y_j h \lambda_{oxi-1/2}}{\Delta x_{i-1} + \Delta x_i} \quad (4.32)$$

$$T_{oxi+1/2} = \frac{2\Delta y_j h \lambda_{oxi+1/2}}{\Delta x_{i+1} + \Delta x_i} \quad (4.33)$$

$$T_{oyj-1/2} = \frac{2\Delta x_i h \lambda_{oyj-1/2}}{\Delta y_{j-1} + \Delta y_j} \quad (4.34)$$

$$T_{oyj+1/2} = \frac{2\Delta x_i h \lambda_{oyj+1/2}}{\Delta y_{j+1} + \Delta y_j} \quad (4.35)$$

$$\lambda_{ox} = \frac{\rho_o k_x k_{ro}}{\mu_o} \quad (4.36)$$

$$\lambda_{oy} = \frac{\rho_o k_y k_{ro}}{\mu_o} \quad (4.37)$$

$$V_{ij} = \Delta x_i \Delta y_j h \quad (4.38)$$

$$Q_{oij} = q_o \Delta x_i \Delta y_j h \quad (4.39)$$

A similar equation for the water phase has been written. These two equations were then combined by making use of the relation $S_o + S_w = 1$, to get the difference equation in the following form for the oil phase pressure.

$$c_{ij} p_{oij-1} + a_{ij} p_{oi-1j} + e_{ij} p_{oij} + b_{ij} p_{oi+1j} + d_{ij} p_{oij+1} = f_{ij} \quad (4.40)$$

This equation is rearranged for the Gauss-Seidel iteration method as:

$$p_{oij} = [f_{ij} - (c_{ij} p_{oij-1} + a_{ij} p_{oi-1j} + b_{ij} p_{oi+1j} + d_{ij} p_{oij+1})] / e_{ij} \quad (4.41)$$

After calculating the pressure values, the water saturation is calculated explicitly. For the no flow boundary condition, the first and second order difference schemes have been tried:

$$\text{First order} \quad \frac{\partial p}{\partial x} = \frac{p_{ij} - p_{i+1j}}{\Delta x} + O(\Delta x), \quad \frac{\partial p}{\partial y} = \frac{p_{ij} - p_{ij+1}}{\Delta y} + O(\Delta y) \quad (4.42)$$

$$\text{Second order} \quad \frac{\partial p}{\partial x} = \frac{p_{i+1j} - p_{i-1j}}{2\Delta x} + O(\Delta x^2), \quad \frac{\partial p}{\partial y} = \frac{p_{ij+1} - p_{ij-1}}{2\Delta y} + O(\Delta y^2) \quad (4.43)$$

4.3.1.2 Fourth order in space, second order in time

The forms of the fourth order difference formulae in space can be written as:

$$\frac{\partial p_{ij}}{\partial x} = \frac{-p_{i+2j} + 8p_{i+1j} - 8p_{i-1j} + p_{i-2j}}{12\Delta x} + O(\Delta x^4) \quad (4.44)$$

$$\frac{\partial^2 p_{ij}}{\partial x^2} = \frac{-p_{i+2j} + 16p_{i+1j} - 30p_{ij} + 16p_{i-1j} - p_{i-2j}}{12\Delta x^2} + O(\Delta x^4) \quad (4.45)$$

Second order backward difference formula is used in the time direction:

$$\frac{\partial p}{\partial t} = \frac{3p_{ij}^{n+1} - 4p_{ij}^n + p_{ij}^{n-1}}{2\Delta t} + O(\Delta t^2) \quad (4.46)$$

Using the above difference formulae, equation (4.1) is discretized as:

$$BO1_{ij} p_{oi,j-2} + BO2_{ij} p_{oi,j-1} + BO3_{ij} p_{oi-2,j} + BO4_{ij} p_{oi-1,j} - BO5_{ij} p_{oi,j} + BO6_{ij} p_{oi+1,j} + BO7_{ij} p_{oi+2,j} + BO8_{ij} p_{oi,j+1} + BO9_{ij} p_{oi,j+2} = BO10_{ij} + \frac{V_{ij} \phi p_o}{2\Delta t} (3s_{oi,j}^{n+1} - 4s_{oi,j}^n + s_{oi,j}^{n-1}) \quad (4.47)$$

The equation is rearranged for the Gauss-Seidel iteration method as:

$$p_{oi,j} = [C10_{ij} - (C1_{ij} p_{oi,j-2} + C2_{ij} p_{oi,j-1} + C3_{ij} p_{oi-2,j} + C4_{ij} p_{oi-1,j} + C6_{ij} p_{oi+1,j} + C7_{ij} p_{oi+2,j} + C8_{ij} p_{oi,j+1} + C9_{ij} p_{oi,j+2})] / C5_{ij} \quad (4.48)$$

This is convenient because this scheme is somewhat similar to those reported before by others. The nine points in this scheme are in four directions, whereas they are in eight directions in other models (see Fig. 4.3 and Fig. 4.4).

The numerical solution of the pressure equation is accomplished by the Gauss-Seidel iteration method while the saturations are calculated explicitly. This scheme will be particularly advantageous when the above mentioned simplifying assumptions are removed for general applications.

Fig. 4.3 Nine-Point Formulation
Used in This Research

Fig. 4.4 Nine-Point Formulation
Used by Others

4.3.2 Time -Space Formulation (Two Time Steps at One Time)

Similar to the unit mobility ratio case, two sets of difference equations for two time level $n+1$ and $n+2$ are obtained by using second order central difference to discretize the time term for $n+1$ time level and second order backward difference to discretize the time term for $n+2$ time level. Second order central difference are used to discretize the space term for both time levels.

Time:

$$\text{At } n+1 \text{ level } \frac{\partial s_{wi,j}^{n+1}}{\partial t} = \frac{s_{wi,j}^{n+2} - s_{wi,j}^n}{2\Delta t} + O(\Delta t^2) \quad (4.49)$$

$$\text{At } n+2 \text{ level } \frac{\partial s_{wi,j}^{n+2}}{\partial t} = \frac{s_{wi,j}^n - 4s_{wi,j}^{n+1} + 3s_{wi,j}^{n+2}}{2\Delta t} + O(\Delta t^2) \quad (4.50)$$

$$\text{Space: } \frac{\partial s_{wi,j}^k}{\partial x} = \frac{s_{wi+1,j}^k - s_{wi-1,j}^k}{2\Delta x} + O(\Delta x^2) \quad (4.51)$$

$$\frac{\partial s_{wi,j}^k}{\partial y} = \frac{s_{wi,j+1}^k - s_{wi,j-1}^k}{2\Delta y} + O(\Delta y^2) \quad (4.52)$$

$$k=n+1, n+2 \quad (4.53)$$

These two sets of equations are solved using the Gauss-Seidel iteration method.

4.4 RESULTS

4.4.1 Unit Mobility Ratio Case

The parameters used in this case are shown in table 4.1.

Table 4.1 Parameters used in the unit mobility ratio flow case

Grid Number: 18 by 18	Grid Length: 8.33 m by 8.33 m
Permeability: 0.2d	Porosity: 0.3
Initial Oil Saturation: 0.8	Irreducible Oil Saturation: 0.2
Connate Water Saturation: 0.2	Injection Rate: 0.06 m ³ /d
Mobility: 0.1	

Fig. 4.5 shows the pressure profile. The diagonal line is a straight line as expected. Fig. 4.6 shows the saturation profile calculated by the finite difference. The saturation profile calculated by the time-space methods, two time steps at one time and three time step at one time, are shown as Fig. 4.7 and Fig. 4.8. Figs. 4.6-8 are the results calculated using the first order backward difference in space and show no difference between them. Whereas Fig. 4.9 is the result calculated using second order central difference in space. Therefore, Fig. 4.9 is different from Fig. 4.8.

4.4.2 General Case

Table 4.2 lists the parameters used in this general case.

Table 4.2 Parameters used in the general case

Grid Number: 18 by 18	Grid Length: 8.33 m by 8.33 m
Permeability: 0.2 d	Porosity: 0.3
Initial Oil Saturation: 0.8	Irreducible Oil Saturation: 0.2
Connate Water Saturation: 0.2	Formation Thickness: 20.00 m
Oil Viscosity: 6.00 μm^2	Water Viscosity: 6.00 μm^2
Injection Pressure: 15.00 Mpa	Production Pressure: 10.00 Mpa
Compressibility: 10^{-4} 1/Mpa	

Figs. 4.10-13 are the pressure and saturation profiles for the finite difference and the time-space methods. An interesting case having a block inside with different permeability from outside was also solved. The pressure and saturation pfiles for this case are shown in Fig. 4.14 and Fig. 4.15.

Fig. 4.5 Pressure Profile for Unit Mobility Ratio Case

**Fig. 4.6 Saturation Profile for Unit Mobility Ratio Case
PV Injected=0.4, Finite Difference Method (First
Order Backward Difference in Space)**

**Fig. 4.7 Saturation Profile for Unit Mobility Ratio Case
PV Injected=0.4, Time Space Method (Two Time Steps
at One Time, First Order Backward Difference in
Space)**

Fig. 4.8 Saturation Profile for Unit Mobility Ratio Case
 PV Injected=0.4, Time Space Method (Three Time
 Steps at One Time, First Order Backward Difference in
 Space)

Fig. 4.9 Saturation Profile for Unit Mobility Ratio Case
 PV Injected=0.4, Time Space Method (Three Time
 Steps at One Time, Second Order Central Difference in
 Space)

**Fig. 4.10 Pressure Profile for General Case, PV
Injected=4.3, Finite Difference Method (Second Order
Central Difference in Space)**

Fig. 4.11 Saturation Profile for General Case, PV
 Injected=4.3, Finite Difference Method (Second Order
 central Difference in Space)

Fig. 4.12 Pressure Profile for General Case, PV
 Injected=4.3, Time-Space Method (Second Order
 central Difference in Space)

Fig. 4.13 Saturation Profile for General Case, PV
 Injected=4.3, Time-Space Method (Second Order
 Difference in Space)

Fig. 4.14 Pressure Profile for General Case, PV
 Injected=2.0, Finite Difference Method, A Block Inside,
 $k_1/k_2=10$ (Second Order central Difference in Space)

Fig. 4.15 Saturation Profile for General Case, PV Injected=2.0, Finite Difference Method, A Block Inside, $k_1/k_2=10$ (Second Order central Difference in Space)

4.5 CONCLUSIONS (COMMENTS)

1. The time-space method is shown to work.
2. The results of the time-space methods(two time step, three time step) have the same order of accuracy as the finite difference method for the unit mobility ratio case.
3. The time-space method is expected to select larger time step than the finite difference with much relaxed stability limits, although the former takes longer computational time for one time step than the later. At present, it is hard to say whether the total calculational time for the time-space method would be less than the finite difference method. Much more work needs to be done to address this point.

4.6 REFERENCES

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5. PROGRAM LISTING FOR THE UNIT MOBILITY RATIO FLOW CASE

```

C *****
C *      THIS PROGRAM IS USED FOR SIMULATION OF UNIT MOBILITY RATIO CASE      *
C *                                     U1.FOR, 5/31/1995                          *
C *****
COMMON/ DATA/ P(20,20), PN(20,20), SO(20,20), SON(20,20), SW(20,20),
& SWN(20,20), AKX(20,20), AKY(20,20), T2(20,20), T4(20,20),
& TT1(20,20), TT2(20,20), TT3(20,20), TT4(20,20), TT5(20,20),
& QO(20,20), QW(20,20), DP(20,20), DSW(20,20), UX(20,20), UY(20,20),
& C1(20,20), C2(20,20), C3(20,20), SWK(20,20)

C-----INPUT DATA-----
OPEN (5, FILE='data.in', STATUS='OLD')

READ (5, 101) NX, NY, NT
101  FORMAT (3I5)
C    NX--THE NUMBER OF X GRID
C    NY--THE NUMBER OF Y GRID
C    NT--THE NUMBER OF TIME STEP

READ (5, *) DD, AT, AM, PI, SOI, SOR, SWC, PHI
C    DD--THE LENGTH OF GRID (DX=DY=DD)
C    AT--THE TIME STEP
C    AM--MOBILITY
C    PI--THE INITIAL PRESSURE
C    SOI--THE INITIAL OIL SATURATION
C    SOR--THE RESIDUAL OIL SATURATION
C    SWC--THE CONNATE WATER SATURATION
C    PHI--THE VALUE OF POROSITY

READ (5, *) Q, A1, A2, A3, B1, B2, B3
C    Q--INJECTION RATE
CLOSE(5)

C-----PRINT INPUT DATA-----
OPEN (7, FILE='data.out', STATUS='UNKNOWN')
OPEN (8, FILE='plot.dat', STATUS='UNKNOWN')
WRITE (7, 111) NX, NY, NT
111  FORMAT (/20X, 'THE NUMBER OF X GRID NX=', I3/
&        20X, 'THE NUMBER OF Y GRID NY=', I3/
&        20X, 'THE NUMBER OF TIME STEP NT=', I3/)

WRITE (7, 222) DD, AT, AM, PI
222  FORMAT (/20X, 'THE LENGTH OF GRID DD=', F6.2, 1X, 'm'/
&        20X, 'THE TIME STEP AT=', F6.2, 1X, 'day'/
&        20X, 'THE MOBILITY AM=', F6.2/
&        20X, 'THE INITIAL PRESSURE PI=', F6.2, 1X, 'MPa'//)

WRITE (7, 444) SOI, SOR, SWC, PHI
444  FORMAT (/20X, 'INITIAL OIL SATURATION SOI=', F5.2/
&        20X, 'RESIDUAL OIL SATURATION SOR=', F5.2/
&        20X, 'CONATE WATER SATURATION SWC=', F5.2/
&        20X, 'POROSITY PHI=', F5.2//)

QI=Q*0.0864*100*100*DD*DD
WRITE (7, 445) QI, A1, A2, A3, B1, B2, B3
445  FORMAT (/20X, 'WATER INJECTION RATE=', E14.3, 2X, 'm^3/d'/
&        20X, 'COEFFICIENT OF PERMEABILITY A1=', F8.4/
&        20X, 'COEFFICIENT OF PERMEABILITY A2=', F8.4/

```

```

& 20X, 'COEFFICIENT OF PERMEABILITY A3=', F8.4/
& 20X, 'COEFFICIENT OF PERMEABILITY B1=', F8.4/
& 20X, 'COEFFICIENT OF PERMEABILITY B2=', F8.4/
& 20X, 'COEFFICIENT OF PERMEABILITY B3=', F8.4/

```

----- UNIT CONVERSION -----

```

AT=AT*86400
DD=DD*100
PI=PI*10

```

-----INPUT INITIAL DATA TO EACH GRID-----

```

DO 33 I=1, NX
DO 33 J=1, NY
P(I,J)=PI
PN(I,J)=PI
SO(I,J)=SOI
SON(I,J)=SOI
SW(I,J)=1-SOI
SWN(I,J)=1-SOI
CONTINUE

```

33

-----INPUT INITIAL DATA TO EACH GRID-----

```

DX=DD
DY=DD
T1=A2/(2*DX)
T3=B3/(2*DY)
DO 500 I=1, NX
DO 500 J=1, NY
X=(I-0.5)*DX
Y=(J-0.5)*DY
AKX(I,J)=A1+A2*X+A3*Y
AKY(I,J)=B1+B2*X+B3*Y
T2(I,J)=AKX(I,J)/(DX*DX)
T4(I,J)=AKY(I,J)/(DY*DY)
TT1(I,J)=-T3+T4(I,J)
TT2(I,J)=-T1+T2(I,J)
TT3(I,J)=-2*T2(I,J)-2*T4(I,J)
TT4(I,J)=T1+T2(I,J)
TT5(I,J)=T3+T4(I,J)

```

500

```

CONTINUE
DO 550 I=1, NX
DO 550 J=1, NY
QO(I,J)=0.
QW(I,J)=0.

```

550

CONTINUE

```

QW(2,2)=Q
QW(NX-1,NY-1)=0
QO(NX-1,NY-1)=-Q

```

503

```

DO 501 I=2, NX-1
DO 501 J=2, NY-1
PN(I,J)=(-TT1(I,J)*P(I,J-1)-TT2(I,J)*P(I-1,J)-TT4(I,J)*
& P(I+1,J)-TT5(I,J)*P(I,J+1)-QW(I,J)/AM-QO(I,J)/AM)/TT3(I,J)
DP(I,J)=PN(I,J)-P(I,J)
P(I,J)=PN(I,J)
WRITE(*,*) P(I,J)
CONTINUE

```

501

```

DO 504 I=1, NX
  P(I,1)=P(I,2)
  P(I,NY)=P(I,NY-1)
504 CONTINUE
DO 505 I=1, NY
  P(1,I)=P(2,I)
  P(NX,I)=P(NX-1,I)
505 CONTINUE

DO 502 I=2, NX-1
DO 502 J=2, NY-1
  IF (ABS(DP(I,J)).GE.0.001) GOTO 503
502 CONTINUE

```

```

C-----PRINT PRESSURE RESULT-----
WRITE (7, 701)
WRITE (8, 701)
DO 801 J=NY-1, 2, -1
  WRITE (7, 888) (P(I,J)/10., i=2, nx-1)
  DO 801 I=2, NX-1
    WRITE (8,890) I-1, J-1, P(I,J)/10.
801 CONTINUE

```

```

C-----
DO 506 I=2, NX-1
DO 506 J=2, NY-1
  UX(I,J)=-AM*AKX(I,J)*(P(I+1,J)-P(I-1,J))/(2*DX)
  UY(I,J)=-AM*AKY(I,J)*(P(I,J+1)-P(I,J-1))/(2*DY)
  write(*,*) ux(i,j), uy(i,j)
506 CONTINUE
  pause
DO 507 I=1, NX
  UX(I,1)=UX(I,2)
  UX(I,NY)=UX(I,NY-1)
  UY(I,1)=UY(I,2)
  UY(I,NY)=UY(I,NY-1)
507 CONTINUE
DO 508 I=1, NY
  UX(1,I)=UX(2,I)
  UX(NX,I)=UX(NX-1,I)
  UY(1,I)=UY(2,I)
  UY(NX,I)=UY(NX-1,I)
508 CONTINUE

```

```

C-----PRINT UX RESULT-----
WRITE (7, 704)
WRITE (8, 704)
DO 811 J=NY-1, 2, -1
  WRITE (7, 888) (UX(I,J)*100000, i=2, nx-1)
  DO 811 I=2, NX-1
    WRITE (8,890) I-1, J-1, UX(I,J)*100000
811 CONTINUE
704 FORMAT(/28X, 'UX DISTRIBUTION (E-05 cm/s)'/)

```

```

C-----PRINT UY RESULT-----
WRITE (7, 705)
WRITE (8, 705)
DO 812 J=NY-1, 2, -1

```



```

WRITE (7, 888) (UY(I,J)*100000, i=2, nx-1)
DO 812 I=2, NX-1
WRITE (8,890) I-1, J-1, UY(I,J)*100000
812 CONTINUE
705 FORMAT(/28X, 'UY DISTRIBUTION (E-05 cm/s)'/)

```

```

SS=1-SWC-SOR
C4=PHI/AT
DO 509 I=1, NX
DO 509 J=1, NY
  C1(I,J)=-UY(I,J)/DY/SS
  C2(I,J)=-UX(I,J)/DX/SS
  C3(I,J)=-C4-C2(I,J)-C1(I,J)
509 CONTINUE

```

```

KKKN=86400*30/(1*AT)
DO 1111 IT1=1, NT
TT=1*IT1*AT*KKKN/86400
WRITE (7, 123) TT
WRITE (8, 123) TT

```

```

123 & FORMAT (////20X, 40('**')/20X, ' * TOTAL TIME IS', F8.2, 1X,
& ' DAYS **'/20X, 40('**')//)

```

```

DO 9999 KKK=1, KKKN
512 DO 510 I=2, NX-1
DO 510 J=2, NY-1
  SWK(I,J)=SWN(I,J)
  SWN(I,J)=(-C4*SW(I,J)+C1(I,J)*SWN(I,J-1)+C2(I,J)*SWN(I-1,J)-
& QW(I,J))/(-C4+C1(I,J)+C2(I,J))
  IF (SWN(2,2).GT.0.8) SWN(I,J)=0.8
  DSW(I,J)=SWN(I,J)-SWK(I,J)

```

```

510 WRITE(*,*) SW(I,J)
CONTINUE

```

```

IF ((SWN(NX-1,NY-1)-0.2).LT.0.00001) GOTO 515
FW=(SWN(NX-1,NY-1)-SWC)/SS
QW(NX-1,NY-1)=-Q*FW

```

C -----BOUNDARY CONDITION-----

```

515 DO 513 I=1, NX
  SWN(I,1)=SWN(I,2)
  SWN(I,NY)=SWN(I,NY-1)

```

```

513 CONTINUE
DO 514 I=1, NY
  SWN(1,I)=SWN(2,I)
  SWN(NX,I)=SWN(NX-1,I)

```

```

514 CONTINUE

```

```

DO 511 I=2, NX-1
DO 511 J=2, NY-1
  IF (ABS(DSW(I,J)).GE.0.001) GOTO 512
511 CONTINUE

```

```

DO 516 I=1, NX
DO 516 J=1, NY
  SW(I,J)=SWN(I,J)
516 CONTINUE

```

9999 CONTINUE

```
C -----PRINT RESULTS-----  
  WRITE (7, 702) QW(nx-1,ny-1)*0.0864*DD*DD  
  WRITE (7, 703)  
  WRITE (8, 703)  
  
  DO 803 J=NY-1, 2, -1  
    write (7,888) (sw(i,j), I=2, NX-1)  
    DO 803 I=2, NX-1  
      WRITE (8,890) I-1, J-1, SW(I,J)  
803  CONTINUE  
  
701  FORMAT(/28X, 'PRESSURE DISTRIBUTION (MPa)'/)  
702  FORMAT(/16X, 'WATER PRODUCTION RATE QW=',E14.3,1X,'m^3/d'/)  
703  FORMAT(/24X, 'WATER SATURATION DISTRIBUTION'/)  
888  FORMAT(6F12.4)  
890  FORMAT(2I4,6F12.4)  
1111 CONTINUE  
     CLOSE (7)  
     CLOSE (8)  
     CLOSE (9)  
     STOP  
     END
```

----- THE RESULTS OF SIMULATION -----

THE NUMBER OF X GRID NX= 8
 THE NUMBER OF Y GRID NY= 8
 THE NUMBER OF TIME STEP NT= 30

THE LENGTH OF GRID DD= 25.00 m
 THE TIME STEP AT= 0.01 day
 THE MOBILITY AM= 0.10
 THE INITIAL PRESSURE PI= 12.00 MPa

INITIAL OIL SATURATION SOI= 0.80
 RESIDUAL OIL SATURATION SOR= 0.20
 CONATE WATER SATURATION SWC= 0.20
 POROSITY PHI= 0.30

WATER INJECTION RATE= 0.540E-01 m³/d
 COEFFICIENT OF PERMEABILITY A1= 0.2000
 COEFFICIENT OF PERMEABILITY A2= 0.0000
 COEFFICIENT OF PERMEABILITY A3= 0.0000
 COEFFICIENT OF PERMEABILITY B1= 0.2000
 COEFFICIENT OF PERMEABILITY B2= 0.0000
 COEFFICIENT OF PERMEABILITY B3= 0.0000

PRESSURE DISTRIBUTION (MPa)

12.1860	11.9904	11.5991	10.9868	10.0526	8.4901
12.3816	12.1860	11.8200	11.3087	10.6807	10.0526
12.7729	12.5520	12.1860	11.7474	11.3087	10.9868
13.3851	13.0632	12.6246	12.1860	11.8200	11.5991
14.3192	13.6912	13.0632	12.5520	12.1860	11.9904
15.8816	14.3192	13.3851	12.7729	12.3816	12.1860

UX DISTRIBUTION (E-05 cm/s)

0.7825	2.3477	4.0142	6.1860	9.9868	6.2498
0.7825	2.2467	3.5091	4.5572	5.0246	2.5124
0.8835	2.3476	3.2186	3.5091	3.0423	1.2876
1.2874	3.0418	3.5089	3.2186	2.3477	0.8836
2.5121	5.0240	4.5567	3.5089	2.2467	0.7825
6.2493	9.9859	6.1853	4.0138	2.3476	0.7825

UY DISTRIBUTION (E-05 cm/s)

0.7825	0.7825	0.8836	1.2876	2.5124	6.2498
2.3476	2.2467	2.3477	3.0423	5.0246	9.9868
4.0138	3.5089	3.2186	3.5091	4.5572	6.1860
6.1853	4.5567	3.5089	3.2186	3.5091	4.0142
9.9859	5.0240	3.0418	2.3476	2.2467	2.3477
6.2493	2.5121	1.2874	0.8835	0.7825	0.7825

*****5.7*****

* TOTAL TIME IS 30.00 DAYS *

WATER PRODUCTION RATE QW= 0.000E+00 m³/d

WATER SATURATION DISTRIBUTION

0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
0.2014	0.2002	0.2000	0.2000	0.2000	0.2000
0.2227	0.2032	0.2002	0.2000	0.2000	0.2000
0.3851	0.2350	0.2032	0.2002	0.2000	0.2000
0.8000	0.3851	0.2227	0.2014	0.2000	0.2000

* TOTAL TIME IS 60.00 DAYS *

WATER PRODUCTION RATE QW= 0.000E+00 m³/d

WATER SATURATION DISTRIBUTION

0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
0.2008	0.2002	0.2000	0.2000	0.2000	0.2000
0.2138	0.2034	0.2006	0.2001	0.2000	0.2000
0.3028	0.2307	0.2052	0.2006	0.2000	0.2000
0.5667	0.3484	0.2307	0.2034	0.2002	0.2000
0.8000	0.5667	0.3028	0.2138	0.2008	0.2000

* TOTAL TIME IS 90.00 DAYS *

WATER PRODUCTION RATE QW= 0.000E+00 m³/d

WATER SATURATION DISTRIBUTION

0.2001	0.2000	0.2000	0.2000	0.2000	0.2000
0.2042	0.2014	0.2003	0.2001	0.2000	0.2000
0.2426	0.2157	0.2039	0.2007	0.2001	0.2000
0.3995	0.2875	0.2231	0.2039	0.2003	0.2000
0.6688	0.4710	0.2875	0.2157	0.2014	0.2000
0.8000	0.6688	0.3995	0.2426	0.2042	0.2001

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*****
*          TOTAL TIME IS 120.00 DAYS          *
*****
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WATER PRODUCTION RATE QW= 0.000E+00 m³/d

WATER SATURATION DISTRIBUTION

0.2004	0.2002	0.2000	0.2000	0.2000	0.2000
0.2116	0.2050	0.2017	0.2005	0.2001	0.2000
0.2849	0.2401	0.2131	0.2032	0.2005	0.2000
0.4901	0.3615	0.2573	0.2131	0.2017	0.2000
0.7263	0.5729	0.3615	0.2401	0.2050	0.2002
0.8000	0.7263	0.4901	0.2849	0.2116	0.2004

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*****
*          TOTAL TIME IS 150.00 DAYS          *
*****
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WATER PRODUCTION RATE QW= 0.000E+00 m³/d

WATER SATURATION DISTRIBUTION

0.2012	0.2006	0.2002	0.2001	0.2000	0.2000
0.2241	0.2125	0.2051	0.2019	0.2005	0.2000
0.3358	0.2761	0.2309	0.2094	0.2019	0.2001
0.5665	0.4397	0.3058	0.2309	0.2051	0.2002
0.7587	0.6487	0.4397	0.2761	0.2125	0.2006
0.8000	0.7587	0.5665	0.3358	0.2241	0.2012

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*****
*          TOTAL TIME IS 180.00 DAYS          *
*****
```

WATER PRODUCTION RATE QW= 0.000E+00 m³/d

WATER SATURATION DISTRIBUTION

0.2026	0.2015	0.2007	0.2004	0.2002	0.2000
0.2418	0.2246	0.2119	0.2051	0.2018	0.2002
0.3902	0.3211	0.2578	0.2210	0.2051	0.2004
0.6272	0.5133	0.3636	0.2578	0.2119	0.2007
0.7769	0.7018	0.5133	5.90.3211	0.2246	0.2015

0.8000

0.7769

0.6272

0.3902

0.2418

0.2026

 * TOTAL TIME IS 210.00 DAYS *

WATER PRODUCTION RATE QW= -0.115E-04 m³/d

WATER SATURATION DISTRIBUTION

0.2049	0.2031	0.2018	0.2011	0.2007	0.2001
0.2641	0.2418	0.2229	0.2113	0.2046	0.2007
0.4444	0.3712	0.2931	0.2393	0.2113	0.2011
0.6738	0.5775	0.4250	0.2931	0.2229	0.2018
0.7871	0.7374	0.5775	0.3712	0.2418	0.2031
0.8000	0.7871	0.6738	0.4444	0.2641	0.2049

 * TOTAL TIME IS 240.00 DAYS *

WATER PRODUCTION RATE QW= -0.364E-04 m³/d

WATER SATURATION DISTRIBUTION

0.2081	0.2055	0.2036	0.2025	0.2018	0.2004
0.2902	0.2636	0.2386	0.2213	0.2098	0.2018
0.4958	0.4231	0.3350	0.2644	0.2213	0.2025
0.7087	0.6307	0.4854	0.3350	0.2386	0.2036
0.7929	0.7607	0.6307	0.4231	0.2636	0.2055
0.8000	0.7929	0.7087	0.4958	0.2902	0.2081

INTEGRATED 3D SEISMIC INTERPRETATION

Abstract

Near-surface velocity variations are shown to be the cause of the large discrepancies in velocity measurements obtained with logging tools, crosswell seismic and surface seismic surveys for the Gypsy Subsurface Site. This interpretation is made using a dispersion model incorporating effects due to scattering, attenuation, and ray path. The model emphasizes the importance of scale, rather than anisotropy, on reconciling such diverse measurements of velocity as are available in the Gypsy data set. This report addresses problems encountered in correlating well control with surface seismic data in three-dimensional geological models. It develops concepts for a process-based approach to predicting geological length scales.

Integrated 3-D Seismic Interpretation

Accomplishments

The accomplishments associated with the 3-D seismic interpretation at the Gypsy test site include the following:

- Recognition of near-surface velocity variations as the cause of the large discrepancies (as much as 50 msec) between VSP measurements at the Gypsy subsurface site.
- Computer programs written to compute (a) the velocity limits of dispersion caused by elastic scattering in a layered sequence and (b) the velocity dispersion caused by attenuation (used in the interpretation described next).
- Interpretation of differences between diverse velocity measurements at the Gypsy test site in terms of velocity dispersion caused by scattering, attenuation, and volume or path effects (see paper submitted to *Geophysics* in Appendix A). Since the interpretation was accomplished without having to resort to using an intrinsic anisotropy, this study emphasizes the importance of scale effects upon crosswell studies and their relationship to surface seismic studies. Application of velocity dispersion opens up new frontiers in reservoir characterization and logging approaches for reservoirs using velocity dispersion measurements.
- Developed concepts for a process-based approach to predicting geological length scales, i.e., the small scale variations within a depositional environment that are required to predict wave response at different frequencies (see paper by Brown 1996a, in Appendix B).
- Work accomplished on integrating scaling considerations into 1-D synthetic seismograms (Brown, 1996b, paper in progress). Initial portion of a computer program has been written for a reflectivity modeling program to account for offset-dependent and thin-layer effects upon 1-D synthetic seismograms.

Discussion

The tasks proposed for this project were designed to evaluate the data available and to identify problem areas within these tasks that confront the petroleum industry. The exploratory tasks were:

1. Construct 1-D synthetic seismograms at each well to tie well and seismic data.
2. Use 3-D Gypsy data, processed by Western Geophysical to construct isopach maps and seismic attribute maps of the Gypsy interval.
3. Invert seismic data to obtain 2-D estimates of the acoustic impedance along selected traverses.

The first two tasks listed above were completed as a part of the M.S. thesis work of Seifert (1994) and Seifert et al. (1994). During that study important problems were encountered correlating the well control with the surface seismic data. As a result of these velocity problems, the seismic contribution to the final maps was highly questionable. The bulk of the interpretation rested upon well control and interpolation between wells using 3-D modeling software because of the insecurities caused by the velocity problems and the poor match of the synthetic seismograms to the actual data. Task 3 (inversion of the seismic) could not be accomplished without a satisfactory understanding of the well control.

At this point the project seemed to be stuck on the issue of velocity contradictions. However, as luck would have it, the time spent on the solution to the velocity problems at the Gypsy test site were well worth the effort. The work on the apparent velocity contradictions led to an understanding of the seismic response of a reservoir which is suggestive of new approaches to reservoir characterization and well logging via dispersion measurements. Although the concept that scattering and attenuation cause velocity dispersion is not a new one, the magnitude and importance of these effects have not been generally recognized. It is this recognition of the relative roles of scattering and attenuation as causes of velocity dispersion that represents the major accomplishment of this project. This report reviews the progress made in utilizing and understanding velocity dispersion during the first and only year of funding by the Department of Energy and during the period after the end of the first year which was funded by the Oklahoma Geological Survey.

When 1-D synthetic seismograms do not give a satisfactory fit to the seismic data, the well logs (usually sonics and density logs) are often modified in practice so that a match can be made. Without additional data to constrain the problem, such poetic license has been justified in the industry for years. However, the Gypsy test site was blessed (or cursed depending upon the point of view) with an abundance of complementary velocity measurements. None of the velocity measurements seemed to agree with any other. The velocity measurements appeared so contradictory that even the question of the reflection coefficients off the top and bottom of the Gypsy sandstone interval could not be answered with confidence. In summary, the additional

velocity measurements made in and between the wells at the Gypsy test site only added to the lack of confidence in the initial interpretation.

One of the velocity problems found during this study caused large differences (as much as 50 msec) in the predicted traveltimes to the top of the Gypsy formation at an approximate depth of 900 feet. The explanation in terms of near-surface stratigraphy and structure were presented as a portion of the SEG presentation by Brown et al. (1994).

The second series of velocity contradictions occurred around the interval containing the Gypsy sandstone, the reservoir rock for this study. Here VSP, sonic logs, Bender logs (a low-frequency sonic) and crosswell seismic all seemed to give a different velocity picture. The differences were reconciled in terms of scattering and attenuation in the formations of interest. Software was written to compute the expected limits of dispersion due to elastic scattering and the contribution to dispersion due to attenuation. A paper on this work by Brown and Seifert (1995) has been submitted to Geophysics (SEG) for publication (see Appendix A).

The work described in the paper in Appendix A led to the recognition that geological length scales, e.g., bed thickness for horizontally layered media, that control the mode of propagation through a formation and intrinsic attenuation both act to control the amount of velocity dispersion within a formation. If the separate contributions to the total observed velocity dispersion of scattering and attenuation can be identified, then important reservoir parameters can be determined. The scattering portion of dispersion is related to the geological environment of a formation because the length scales (both horizontal and vertical) within an environment determine how seismic energy is propagated (and scattered). The attenuation portion of velocity dispersion is believed by some to be related to important reservoir properties. Gas reservoirs in particular are expected to have an anomalously high intrinsic attenuation (low Q). Thus velocity dispersion as a logging measurement potentially could be used to identify geological environments as well as reservoir fluids. This is a very important point because there are many areas of the world where conventional logging methods cannot be used to identify pay zones. The problem in using dispersion measurements is one of separately identifying the respective contributions of scattering and attenuation.

One approach to estimating the amount of the dispersion caused by elastic scattering is to model elastic-wave propagation through a formation with length scales appropriate to the geological environment being studied. High-resolution well logs can be used to identify small scale variations in a vertical sense, but fail to give any idea of the lateral scale, i.e., the lateral distance over which a velocity (or a lithofacies) is approximately constant. Geologists have been making lateral predictions for years of the large scale stratigraphic changes between wells based upon well logs. However, their efforts have not been pointed to the detailed distribution of facies within an environment at the scale required to understand the seismic response for a crosswell study. To solve this problem, 3-D computerized modeling concepts have been proposed for fluvial environments. These ideas were presented in a paper by Brown and Levey (1994) and later expanded in the paper by Brown (1996a) (see Appendix B). Although there are a number of commercial stratigraphic modeling packages available, the ideas discussed in these papers addressed the issue of predicting geological length scales that affect the seismic response.

The conclusion from this work is that geological length scales related to depositional environments cause the elastic scattering contribution to velocity dispersion. For example, simple modeling of the frequency of flooding within a fluvial system (such as the Gypsy sandstone interval) can be tied to velocities and ultimately to the seismic response of different facies within the fluvial system. In this way seismic studies accounting for the dispersive properties of a formation can be used to map reservoir properties.

Since the first goal of any seismic project is matching the well control to the seismic data, the last portion of the year funded by DOE was spent working toward a method of producing 1-D synthetic seismograms which account for the scaling effects described above. The second year of support for this project came from the Oklahoma Geological Survey and will continue during the completion of a paper-in-progress on the subject of incorporating scale into 1-D synthetic seismograms. A portion of a computer program for reflectivity modeling of the full-wave response from a layered medium has been written. The reflectivity modeling program will play an integral part in the building of 1-D synthetic seismograms designed to account for scaling effects. Questions to be examined during this continuation of the project include:

- how to account for the bandwidth of the seismic signal and the distribution of scales (e.g., bed thicknesses) [rather than using the approximation of a single frequency and a single bed thickness]
- how to account for scale effects, e.g., thin layering, upon reflections (as well as the transmitted signals studied thus far).

The ultimate goal then is to be able to incorporate scaling concepts into 1-D synthetic seismograms at the well control. Once well control can be matched by accounting for scaling effects, prediction away from the well control can be accomplished using the same scaling concepts and geological modeling to invert the seismic response. Some of these problems will be discussed in the upcoming presentation at the SEG by Brown and Seifert (1995). The final result will be a reservoir mapping tool based upon a new understanding of the relationship between the seismic response, the length scales of the geology (which are directly related to the depositional environments), and the reservoir fluid content.

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5 *Preprints removed and
processed separately*