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Immersed Interface Methods

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1 Immersed interface methods

1.1 Multigrid methods for Poisson problems

A full multigrid method (FMG) has been developed and implemented by Adams [2], [1] for solving the discrete system of equations that arise from an IIM discretization of elliptic equations. This method has been used to efficiently solve groundwater flow problems with large jumps in the permeability. This was discussed recently by Adams at an invited lecture at Lawrence Livermore National Laboratory and at the DOE PI meeting in Berkeley.

1.2 Domain embedding

Adams and her Ph.D. student, Zhiyun Yang have studying the use of IIM type ideas for elliptic problems that are posed on the inside or outside of a complicated domain. The idea is to immerse the domain in a square computational domain and use a Cartesian grid on the entire domain. In this sense, the approach is similar to Capacitance matrix methods, domain embedding techniques, and fictitious domain methods. These techniques have been used to solve problems with multiple internal interfaces. Yang's thesis[21] and subsequent paper[22] describe these techniques in detail. Yang graduated in 1996 and spent a year as a postdoctoral fellow working with Dr. Mary Brewster at PNL in Richland, WA. She is now employed at Pacificorp in Portland, Oregon.

1.3 Groundwater flow

Applications to groundwater flow continue to be investigated, starting with one-phase saturated flow governed by Darcy's law. Some applications are presented in [2], [3], and [13]. The immersed interface methods for computing the velocity field are coupled with use of the AMRCLAW software to advect a tracer, an adaptive mesh refinement code developed by LeVeque and Marsha Berger. Problems with both nonpermeable objects and permeable objects with large jumps in the problem parameters can be handled with these techniques. LeVeque will discuss this problem in his own talk in a minisymposium at the SIAM Annual meeting in July, organized by Berger and LeVeque, on "Cartesian grid methods for PDEs in complex geometry".

1.4 Solidification

Zhilin Li's thesis[16] contained a preliminary look at one-dimensional solidification problems using the IIM. He has since extended this work[17] and applied it to a glaciology problem in work with Dave McTigue in the Geological Sciences Department at the University of Washington[19].

Donna Calhoun, a graduate student in Applied Mathematics, is now working with LeVeque on solidification problems, and hopes to develop a code that combines fluid dynamics and solidification algorithms on Cartesian grids to study solidification (or melting) coupled with flow in the liquid phase.

She now has a melting code working in two space dimensions that couples motion of the interface with a Crank-Nicholson algorithm for the heat conduction. This code is simplified by assuming that the temperature is constant in the solid phase, so that the heat conduction coefficient can be assumed continuous, but this is suitable for many applications. Latent heat absorption at the interface provides a singular source term that must be handled with the immersed interface method. This is coupled with interface motion by an implicit method similar to that developed for Stokes flow by LeVeque and Li[15], but the motion must now be determined by imposing the constraint that the interface temperature at the end of the time step should be the melting point. This is complicated by the fact that the latent heat sink is proportional to the interface velocity.

Calhoun and LeVeque have started to investigate a stream function-vorticity formulation of incompressible flow that might be useful in the fluid phase on a Cartesian grid. Interestingly, the vorticity generation at a wall, which must be incorporated in such an algorithm to impose the no-slip condition, can be viewed as generating a line of vorticity near the wall that gives a singular source term in the Poisson problem for the stream function. The immersed interface viewpoint gives a new interpretation of boundary conditions for this formulation, which has been the subject of much discussion in the literature. This is being pursued.

Calhoun attended the SIAM conference on Material Sciences in May and will be attending a short course and workshop on "Mathematical Models of Systems Involving Phase Changes" at Heriot-Watts University for several weeks in June. She will also be presenting her work at the Annual SIAM meeting this summer.

1.5 The explicit jump IIM and conductivity inverse problems

Ph.D. student Andreas Wiegmann and Bube have been studying making the IIM more useful in conductivity inverse problems for media with discontinuous conductivities, e.g., locating underground mines. The original IIM and Li's improved fast iterative version of the IIM both yield second-order accurate solutions of the forward problem, an elliptic PDE, but the dependence of the solution on the sub-grid scale alignment of the grid and the interface is not very smooth, making the optimization problem arising from the inverse problem difficult to solve. Work toward this goal of making the solution of the forward problem depend more smoothly on the location of the mine has led to a number of interesting developments. Wiegmann [20] has developed a version of the IIM for linear elliptic problems, called the Explicit Jump IIM (EJIM), with very favorable behavior by choosing the jumps in the Cartesian partial derivatives at intersections of the interface with the grid lines as a set of additional variables. This approach is applicable to a wide variety of problems, including interior and exterior Dirichlet and Neumann problems for regions with boundaries which are not aligned with the Cartesian grid. Second order convergence has been proved in some 1D and 2D problems. Moreover,

EJIM makes the solution of the forward problem for elliptic inverse problems depend significantly more smoothly on the location of the discontinuities than earlier versions of the IIM. Wiegmann has also applied the EJIM strategy successfully to the artificial interface between a coarse grid and a fine grid in a mesh refinement scheme.

1.6 Numerical methods for acoustic and visco-acoustic inverse problems

In a three-dimensional stratified acoustic medium whose material properties depend only on depth, the surface response to a point source in pressure at the surface can be Radon transformed into plane wave responses. Each plane wave response is the solution of a one-dimensional wave equation. Ph.D. student Jon Jay and Bube have been studying the extension of results of Chaderjian and Bube [8], recovering density, wave speed, and an attenuation coefficient from several plane wave responses in a formal linearization of the problem, to visco-acoustic materials. Jay has proved under some restrictive hypotheses that all coefficients in stratified visco-acoustic model can be recovered from the plane wave responses in the linearized inverse problem. The forward problem has a discontinuity propagating along a characteristic of the same order as the initial discontinuity in the pressure source wavelet. Jay has developed a numerical scheme for the forward problem, and then solved the nonlinear inverse problem numerically to recover these coefficients. The numerical results agree with his theoretical results in that the coefficients governing attenuation and visco-acoustic behavior are more difficult to recover accurately than the density and wave speed.

In [6], Bube has studied the effect of discontinuities in the medium on the order of convergence of numerical methods for recovering the density and wave speed for an acoustic medium without attenuation from several plane wave responses. For media with discontinuities, Bube has proved second order convergence for a special case when the location of the discontinuities satisfies some alignment constraints; when these constraints are violated, convergence reverts to first order. Bube is studying using the IIM to remove these constraints and still obtain second order convergence.

1.7 Finite element version of the IIM

Ph.D. student Thomas Kaupé and Bube have been studying a finite element method version of the IIM. The goal, as with the difference equation versions of the IIM, is to use an underlying Cartesian grid. Most finite-element methods for regions with irregular boundaries or interfaces choose elements which are aligned with the interface, either with unstructured elements or some kind of interface-fitted coordinates. In parallel work, Li has shown second-order convergence in 1D for elliptic problems with homogeneous jump conditions. Kaupé's approach uses a different set of basis functions near the interface. Kaupé has proved second-order convergence for his approach in 1D even with inhomogeneous jump conditions. Work is now in progress extending these ideas to 2D.

1.8 Computation of seismic traveltimes in discontinuous media

Bube has continued his work on computing seismic traveltimes in discontinuous media using finite difference approximations to the eikonal equation

$$(\tau_x)^2 + (\tau_z)^2 = s(x, z)^2,$$

where $\tau(x, z)$ is the first arrival travelttime to be determined and $s(x, z)$ is the given slowness field (the reciprocal of wave speed). As in the other applications of the IIM, a uniform Cartesian grid is used and special finite difference schemes are used near interfaces. Bube [7] has extended a finite difference scheme developed by Vidale for smooth media to be second order accurate when interfaces are present. The nonlinearity of the jump conditions at the interface leads to complications in maintaining second order accuracy near the interface, but these complications are surmountable by using a larger stencil. Bube is also studying the application of the IIM to the system of conservation laws obtained by differentiating the eikonal equation, an approach developed by Van Trier and Symes for smooth media.

2 CLAWPACK and AMRCLAW

LeVeque has continued his work on multi-dimensional wave-propagation methods for hyperbolic systems of conservation laws[14]. The software package `clawpack` (Conservation LAws PACKage) has been further developed and Version 3.0, available from May, 1997 in `netlib`[11], includes algorithms in three space dimensions developed by LeVeque and Jan Olav Langseth [9], [10].

This software has also been extended in joint work with Marsha Berger at the Courant Institute to `AMRCLAW`[5] [4], which implements adaptive mesh refinement. Test Version 0.3 of this code is now available on the web.

These codes have been used in conjunction with work on immersed interface methods in the context of groundwater flow, acoustic and elastic wave propagation [25], [24], [23], and stream function - vorticity formulations of incompressible flow.

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