Hydrodynamic Instability in Strong Media

**S.M. Bakhrakh, O. B. Drennov, N. P. Kovalev, A. I. Lebedev,
E. E. Meshkov, A. L. Mikhailov, N. V. Nevmerzhitsky,
P. N. Nizovtsev, V. A. Rayevsky, G. P. Simonov,
V. P. Solovyev, I. G. Zhidov**

March 5, 1997

DISCLAIMER

This document was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor the University of California nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or the University of California, and shall not be used for advertising or product endorsement purposes.

**Russian Federal Nuclear Center—
VNIIEF**

S.M. Bakhrakh, O.B. Drennov, N.P. Kovalev, A.I. Lebedev,
E.E. Meshkov, A.L. Mikhailov, N.V. Nevmerzhitsky, P.N. Nizovtsev,
V.A. Rayevsky, G.P. Simonov, V.P. Solovyev, I.G. Zhidov

Hydrodynamic Instability in Strong Media

(review of open VNIIEF publications)

Report under LLNL-VNIIEF

Contract No. B307888

Bruce T. Goodwin,
LLNL responsible representative

S.M. Bakhrakh,
Leader of Contract Workers

Table of Contents

ABSTRACT.....	iii
KEY WORDS.....	iii
FOREWORD.....	iv
INTRODUCTION.....	vi
Principal Research Directions and Objectives	vi
<i>Research objectives</i>	vi
<i>Principal research directions and report summary</i>	vii
Selection of Material for Inclusion in This Review	xi
 I. Methods of Experimental Research.....	 1
1.1. The Study of Hydrodynamic Instability in Strong Media Using the Jelly Technique.....	1
1.1.1. <i>Description of the jelly technique</i>	1
1.1.2. <i>Jelly Strength Properties</i>	4
1.2. Experimental Study of Hydrodynamic Instability in Metals	5
1.2.1. <i>Experimental study of Rayleigh-Taylor instability using the x-ray technique</i>	5
1.2.2. <i>Experimental study of Richtmyer-Meshkov instability on preserved samples</i>	13
1.2.3. <i>Experimental study of Kelvin-Helmholtz instability</i>	15
 II. Description of Numerical Methods and Medium Models Used for Research into Hydrodynamic Instability in Strong Media.....	 18
2.1. Medium Models Used for Numerical Simulation and in Analytical Research.....	18
2.1.1. <i>Medium models used in analytical research</i>	18
2.1.2. <i>Medium models used for numerical simulation</i>	20
2.2. Numerical Techniques Used to Study Hydrodynamic Instability Mechanics Problems in Strong Continua	21
2.2.1. <i>Sigma program system</i>	21
2.2.2. <i>2D Lagrangian technique DRAKON-R/2</i>	23
2.3. Methods based on the shell approximation.....	27
2.3.1. <i>Governing equations</i>	27
2.3.2. <i>Numerical method</i>	29
 III. Some Results of Analytical Studies.....	 31
3.1. Elastic Semi-Space Boundary Instability.....	31
3.1.1. <i>Analytical study of perturbation evolution at a liquid-elastic body interface</i>	31
3.1.2. <i>Numerical study of perturbation growth at a fluid-elastic semi-space interface</i>	38

3.2. Instability of the Interface Between Viscous and Ideal Fluids	42
3.2.1. Analytical study of perturbation evolution on the interface between viscous and ideal fluids	42
3.2.2. Numerical study of evolution of perturbations on the interface between viscous and ideal fluids	45
3.2.3. The effect of approximation viscosity in computing Rayleigh-Taylor instability	47
3.3. Elastic-Plastic Layer Instability	48
3.3.1. Equations of motion of a thin ideal elastic layer	48
3.3.2. Solutions to the equations of motion of a thin layer	50
3.3.3. Instability of an arbitrarily thick ideally elastic layer	52
3.3.4. Instability of an elastic-plastic layer	54
3.3.5. Comparison of theoretic estimates with results of experiments and numerical calculations	57
3.4. Effect of Shape and Other Characteristics of the Initial Perturbation	60
3.4.1. Governing equations	61
3.4.2. Perturbations in the layer shape	61
3.4.3. Perturbations in layer particle velocities	63
3.4.4. Perturbations in shape and velocities	64
3.4.5. Perturbations in layer shape and thickness	65
3.4.6. Pulsed acceleration	66
3.4.7. Numerical studies for a thin layer	66
3.4.8. The evolution of boundary perturbations in a compressible fluid semi-space	69
3.5. Instability of a Thin Elastic Layer Under the Action of Perturbations	73
3.5.1. Instability of a thin liquid layer with 3D perturbations	74
3.5.2. Study of 3D perturbations in the Kirchhoff-Love approximation	78

IV. Results of Experiments and Numerical Simulations of Hydrodynamic Instability in Strong Media..... 82

4.1. Experimental Data Obtained Using the Jelly Technique	82
4.1.1. The effect of strength on unstable boundary perturbation evolution. Results of local perturbation evolution research.	82
4.1.2. The effect of strength on the evolution of the turbulent mixing zone in the convergence of a cylindrical ring	85
4.1.3. The effect of turbulent mixing on the acceleration of layers of different strength	94
4.1.4. Using the jelly technique to estimate the critical parameters of instability with strength	99
4.2. Experimental study of hydrodynamic instability in metals	100
4.2.1. Experimental data on Rayleigh-Taylor instability evolution obtained with the pulsed x-ray method	100
4.2.2. Experimental data on Richtmyer-Meshkov instability obtained using preserved samples	107
4.2.3. Experimental data on Kelvin-Helmholtz instability in metals	112
4.3 Numerical simulation of hydrodynamic instability in metals	122
4.3.1. Numerical study of the effect of governing parameters on stability evolution	122
4.3.2. Numerical simulation of experiments on Rayleigh-Taylor instability evolution	135

Abstract

The paper reviews VNIIEF (the All Russian Scientific Research Institute of Experimental Physics) open publications on hydrodynamic instability in strong media.

Key words

- hydrodynamic instability
- Rayleigh-Taylor instability
- Richtmyer-Meshkov instability
- Kelvin-Helmholtz instability
- strength
- experimental studies
- numerical simulation

Foreword

Among the exotic physical processes that occur during the explosion of a nuclear weapon is the relatively little-known process of hydrodynamic instability in solids. The resulting phenomena are similar to the well-studied Rayleigh-Taylor and Kelvin-Helmholz instabilities in liquids; here, the complicated constitutive strength properties of materials are explored as the material is subjected to very high pressures. Research in this field has broad applications since there are other important instances of these phenomena, including explosive metal forming in industrial applications and naturally occurring geophysical flows in planetary interiors. However, the potential for catastrophic failure due to instability growth in nuclear explosives has led to substantial basic, unclassified research in this area within nuclear weapons establishments.

The authors of this report worked within the nuclear weapons establishment of the former USSR, and it is only the end of the Cold War that has allowed this report to be written. This volume is the result of one of the first intellectual exchanges between the national laboratories of Russia and the United States, and it exemplifies the benefits of such an exchange.

The volume surveys the extensive unclassified work on hydrodynamic instabilities in strong media done at the institution now known as the All Russian Scientific Research Institute of Experimental Physics (VNIIEF) in Sarov (formerly Arzamas-16), Russia. It contains results not yet presented in the Western scientific literature: The analytical solutions rely on more limited approximations and thus are more generally applicable than those in the Western scientific literature, and many of the experimental results and numerical simulations have not been reported in the West before.

Beyond the individual results, this volume is a fascinating glimpse into the process of scientific research. It shows the development of a body of work conducted largely separate from, yet parallel to, work done in Western countries; it shows the development of fresh insights and a different perspective on this problem.

These features make for important and fascinating reading. This report serves as an excellent (and perhaps the only) overview of the field, presents new and important results, and provides new insight to the Western reader concerned with hydrodynamic instability in strong media.

Bruce Goodwin

English Editor's Note

It has been a very rewarding task for me to prepare this manuscript for publication, both because I learned much about the subjects covered in this work and because I could contribute to the world-wide distribution of this significant work. The original document was written in Russian and translated by the able translators in Sarov into English. In the US, minor grammatical changes were made to improve readability, and every effort was made to maintain the full meaning and accuracy of the original Russian text. Of course, linguistic connotations do not translate exactly, and cultural nuances may be lost or changed in the translation process. We regret any ambiguity that may have been introduced in the translation process.

Many of the Russian references cited in the bibliography have been translated into English. Where these translations have been located, they are noted after the Russian reference for the convenience of the English reader.

Finally, I would like to thank the Technical Information Department of LLNL, specifically Stephanie Shang, Linda Null, and Dan Moore for the design and formatting of this document.

Elaine A. Chandler

Introduction

Principal Research Directions and Objectives

Research objectives

Hydrodynamic instability phenomena that appear on material interfaces are of great theoretical and practical interest. For this reason, considerable research has been focused on this issue. Traditionally, the following interface instability types are distinguished:

- Rayleigh-Taylor instability (“gravitational instability”) which arises when an interface of different density materials is affected by acceleration whose vector is perpendicular to the interface and directed from the lower density material to the higher density material;
- Richtmyer-Meshkov instability which arises when a stationary shock wave whose propagation direction is perpendicular to the interface passes through the interface of different density materials, i.e., when the interface is accelerated impulsively and then moves without any acceleration (for a time-dependent shock wave the interface moves at an acceleration behind the wave front);
- Kelvin-Helmholtz instability (shear instability) which arises when there is a discontinuity in the tangential component of the velocity field.

To a certain extent, such a classification of hydrodynamic instability types is simply a matter of convention. In nature, various instability types are realized at the same time.

Thus, for example, the evolution of gravitational instability generates a shear instability. When an oblique shock wave is incident on an interface between different materials, a tangential velocity component discontinuity occurs which generates shear instability; but at the same time, shock wave passage due to the normal component generates a Richtmyer-Meshkov instability.

Moreover, the processes causing instability, as a rule, are not simple. Thus, in real systems acceleration is a function of time; a shock wave is followed by unloading or repeated loading, and so on.

There are many such examples, and a list of them would be very long.

It should be emphasized that one encounters hydrodynamic instability literally at every turn. One of the authors of this review often asks his physics students why the atmospheric pressure, which is capable of sustaining a water column 10 meters in height,

cannot sustain a water column ≈ 10 cm in height when a glass filled with water to the rim is turned upside down. A clear explanation of such a phenomenon is a rare affair.

Interest in hydrodynamic instability phenomena grew considerably in the late 1960s in connection with research into inertial confinement thermonuclear fusion.

Hydrodynamic instability phenomena have been quite thoroughly studied for perfect (inviscid) fluids and gases, though results of research revealing important aspects of the fundamental processes in this area are still frequently published.

However, so far there is no clear understanding of the hydrodynamic instability processes in real media possessing strength, compressibility, viscosity, or phase transitions. The literature contains contradictory data on the effect of the principal factors (wavelength and perturbation amplitude, material strength properties, and loading history) affecting instability evolution.

The study of instability in real materials is important for a number of technical applications: processing new materials, molding metal products, explosive welding, etc.

The focus of the collaborative VNIIEF-LLNL effort is theoretic, experimental, and numerical research into interface instability dynamics in real materials subjected both to continuous and pulsed acceleration. The work is directed to the study of both gravitational and shear instability. In addition to the experimental research, we will focus on research into the effect of strength and viscous material characteristics, on medium model specification, and on numerical simulation of the process.

This review of VNIIEF publications on hydrodynamic instability in strong media, which are open but insufficiently known at LLNL, can be considered as the first step of the joint VNIIEF-LLNL research.

Principal research directions and report summary

A characteristic feature of VNIIEF work on hydrodynamic instability in strong media is the complex character of this research: a combination of physical experiments, theoretical treatments, and numerical simulation (numerical experiments). In our opinion, only such an approach will allow progress to be made in the study of so complex a phenomenon as hydrodynamic instability in strong media.

This viewpoint has determined the structure and contents of this review (see the Table of Contents). Below, brief comments are given on the sections of this review.

Section I of the review is devoted to the methods used in experimental studies of hydrodynamic instability in strong media. This section describes experimental device schemes, recording methods, and techniques for experimental result analysis and processing. The studies were performed using three different experimental techniques:

- the jelly technique using gelatin-water solutions of various concentration for the experimental medium and fast filming for recording the process;
- the pulsed x-ray method. This method was used for the experimental study of hydrodynamic instability in metals under dynamic loading;
- the method of preserved sample processing and analysis. The preserved metal samples were used to study Richtmyer-Meshkov and Kelvin-Helmholtz instabilities. Various schemes for setting up the experiments are described: oblique plate collision, the two-stage scheme of metal plate batch loading with an oblique shock wave, and the method of shear flow generation in metals by closure of the wedge-shaped gap under the plate batch.

Section II gives a description of the numerical methods and medium models used in the study of hydrodynamic instability in strong media.

Medium models used for analytic studies are necessarily elementary. This is related to the fact that it is possible to obtain analytic solutions only in the approximation of incompressible perfect fluid, incompressible elastic body, and viscous fluid.

A special class of analytic solutions is for a thin liquid and elastic layer.

The medium models used for numerical simulation are distinguished by their great variety. For numerical simulation a compressible medium model is used. The Mie-Grueneisen form of the equations of state and the elastic-plastic model of strained solid are taken as the governing physical relations. The dynamic yield strength and viscosity factor can be functions of material state and strain rate parameters, permitting the treatment of material behavior factors such as strain hardening and unhardening, transition to the liquid state, etc.

The second part of **Section II** describes the numerical techniques used to study hydrodynamic instability mechanics in strong continua.

VNIIEF has a fairly diverse set of techniques for solving multidimensional continuum mechanics problems. These techniques differ in material reference frame (Lagrangian, Lagrangian-Eulerian, Eulerian coordinates), computational grid type (regular, irregular), techniques for separation of interfaces when these do not coincide with computational grid lines (material concentrations, particles), difference approximations of the laws of conservation, etc. The bulk of these methods were used to model hydrodynamic instability in gaseous and liquid media. Some of the above techniques permit treatment of the strength properties of the medium.

This review includes only the numerical techniques used to study the phenomena of hydrodynamic instability in strong media.

The numerical methods based on the shell approximation which were used to describe the dynamics of thin-walled strong systems contacting liquid or gas are also briefly discussed.

Section III gives some results of analytic studies. The following problems are considered:

1. Elastic semi-space boundary instability: the evolution of small sinusoidal perturbations on the interface between an elastic semi-space and an ideal fluid is considered analytically. Both constant and pulsed media interface accelerations are considered. It is shown that for the case of constant interface acceleration, the perturbation amplitude grows exponentially only if the perturbation wavelength is larger than some critical value dependent on medium parameters and the magnitude of the applied acceleration; behavior of perturbations of wavelengths shorter than critical is oscillatory. At pulsed interface acceleration, the perturbation behavior is shown to be oscillatory for all perturbation wavelengths.
2. Viscous-ideal fluid interface instability. In this case for a constant acceleration directed from light to heavy fluid, the interface is always unstable and the perturbation amplitude grows exponentially with time. For pulsed interface acceleration, the presence of viscosity leads to a cessation of the perturbation amplitude growth in time, doing so faster the shorter the perturbation wavelengths. It is noted that the results of numerical computations of 2D problems are in good agreement with the appropriate analytic relationships.
3. Elastic-plastic layer instability. An analytic solution of the problem of instability growth in a thin elastic plate with 2D perturbations is found in terms of the Kirchhoff-Love shell approximation. Based on a set of simplifying assumptions, a transition was made from the thin layer and semi-space solutions to solution of the problem of arbitrarily thick elastic-plastic layer Rayleigh-Taylor instability evolution.
4. Effect of initial perturbation shape and character. Analytic solutions are found for perturbation evolution as a function of initial perturbation shape and character. The perturbation growth is shown to be affected considerably by tangential mass flows existing at the initial time or by a given initial profile of tangential velocities.

Interface perturbations can either grow exponentially at a certain rate, exhibit periodic oscillations of the initial amplitude, or “turn over” and then exponentially grow “in the opposite direction.” The actual behavior depends on whether the initial tangential velocity profiles are “in phase” or “in anti-phase” with the direction of the velocity normal to the interface and on the ratio of these velocities. It is shown that paradoxical situations are possible where, for some perturbations, the growth rate in a liquid medium turns out to be less than in an elastic medium and that in strong media, in the region of fastest-growing wavelengths, 3D perturbations evolve slower than 2D ones.

5. Instability of a thin elastic layer with 3D perturbations. Analytic solutions to this problem are obtained. The analytic estimations made for a thin incompressible layer show that when perturbations appear in one more dimension, the value of the critical acceleration causing loss of stability increases.

Section IV contains results of the experimental study and numerical simulation of hydrodynamic instability in strong media. The first part of this section reviews the experimental data obtained with the methods mentioned in **Section I**.

The data obtained using the jelly technique to investigate the effect of strength on mixing zone evolution are given (the tests were made in both planar and cylindrical geometries). Approaches to the solution of the problem of evaluating the instability critical parameter in the presence of strength are discussed.

Next, the results of the experimental study of hydrodynamic instability in metals are discussed.

Of greatest interest are the experimental data on Rayleigh-Taylor instability evolution obtained with the pulsed x-ray method. The pulsed x-ray method was used to study the effect on perturbation evolution of:

- perturbation wavelength,
- perturbation amplitude,
- perturbation dimension.

Then the experimental data for Richtmyer-Meshkov instability obtained on preserved samples are presented. In these experiments the data show the effect on hydrodynamic instability of the impacting shock-wave pulse, strength properties, and thermophysical characteristics of several metals.

The conclusion of the experimental data review describes extensive experimental results on Kelvin-Helmholtz instability in metals. The data obtained from the preserved samples show:

- the dependence of the metal pair interface perturbation amplitude on the character of the developing flow (on the Mach number);
- the effect of the physical and chemical state of the interface on the character of the perturbations;
- the effect of the initial material strength, initial temperature, interface finish, and loading profile on the character and geometric shape of the perturbations.

Feasibility studies are presented for perturbation stabilization and elimination. The mechanisms of instability generation and the role of the cumulative jet are discussed.

The last part of **Section IV** contains the results of numerical simulations of hydrodynamic instability.

The effect of the governing parameters on the Rayleigh-Taylor and Richtmyer-Meshkov instability is studied numerically. The numerical “experiments” investigated the effect of the following parameters on perturbation evolution:

- yield strength,
- shear modulus,
- material compressibility,
- pressure vs. time dependence on the accelerated layer surface,
- shock-wave intensity,
- perturbation wavelength and initial amplitude.

The computations agree with the analytic solutions of **Section III**.

The numerical experiments resulted in the isolation and identification of mechanisms for perturbation evolution in elastic-plastic media.

Finally, results are given for numerical simulations of instability evolution experiments in aluminum plates that were described in **Section IV**. On the basis of numerical simulation of these experiments, a physical model is constructed which allows the description of instability evolution for the whole wavelength and initial perturbation range considered in the experiments. The model accounts both elastic-plastic properties and material viscosity.

Selection of Material for Inclusion in This Review

The review includes open publications on hydrodynamic instability in strong media performed by VNIIEF employees and determined by the terms and conditions of the Contract. The list of 32 publications included in the review was specified at Phase 1 of the Contract.

The review includes only the papers on hydrodynamic instability study which pertain to strong media or are important for understanding hydrodynamic instability in such media. The papers on hydrodynamic instability relating to the problem of inertial thermonuclear fusion and turbulent mixing in gaseous and liquid media are outside the scope of this review. This research should constitute a subject of an independent review.

We also applied these criteria to the description of the experimental methods and numerical techniques. Only those items were included in the review which bear a direct relation to research into hydrodynamic instability in strong media. Other papers were included by the investigators for completeness and integrity of the treatment. Throughout, there are references to appropriate sources in the literature.

In the review, the thematic principle of material statement is taken. In this version of the report, free equation and figure numbering is used (i.e., equations and figures are only numbered when needed for reference to a given chapter or subsection in other sections of the report).

I. Methods of Experimental Research

This section considers descriptions of experimental devices and methods for recording and processing the experimental results.

Our research into hydrodynamic instability in strong media has used three types of basically different experimental methods. To study hydrodynamic instability in metals, two approaches were used:

- Recording with the x-ray method;
- Recording on preserved samples.

The third methodology for studying the effect of strength and hydrodynamic instability evolution stands alone: it is the jelly technique which, through concentration variations, allows control of the medium strength to some extent. In this case, standard methods of fast film recording are used.

1.1. The Study of Hydrodynamic Instability in Strong Media Using the Jelly Technique

1.1.1. Description of the jelly technique

Since 1979, VNIIEF has used the jelly technique [32] to model and study time-dependent hydrodynamic flows. In continuum mechanics, there is a wide class of phenomena which essentially reduces to hydrodynamic flows. This class includes, for example, the problems of cumulative jet generation and evolution, fluid flow about solids, and liquid plate and shell acceleration, including the problem of boundary instability, etc. The common features in these processes are the incompressibility of the condensed medium and the negligible effect of strength properties on the character of the movement.

These shared features allow the modeling of the above processes as time-dependent flows of an ideal fluid. For the construction of a real physical model of such a process, it would be optimal to use a material of minimum strength to imitate the condensed medium under study in order to construct models that can hold a fairly complex initial shape while not affecting the processes under investigation.

Jellies, in particular gelatin-water jelly, can be used for this modeling medium. Here by jellies we mean hardened solutions of polymers in low-molecular fluids [4]. Jellies possess the property that they retain their shape, allowing the production of fairly complex-shaped models from jellies (for example, a cylindrical shell); and this shape is retained for quite a long time.

Time-dependent hydrodynamic flows are realized in such models through the application of a pulsed gas pressure which is considerably higher than the jelly strength. The strength of gelatin-water jelly made by a standard technology for food purposes (solution concentration $\sim 4\text{--}5\%$) is $\sim 10^4$ Pa (0.1 atm). It is this jelly that was most widely used in our experiments.

It is quite suitable to produce pulsed gas pressures through gas explosive mixture (GEM) detonation. For such a detonation in a closed volume, according to estimates, pressures of ~ 10 atm are realized which is about two orders of magnitude higher than the jelly strength; under this condition, jelly (of low gelatin solution concentration) behaves similar to a fluid.

It is essential that the gelatin-based jelly be transparent; this feature allows the use of optical methods for recording the resulting flow. Jelly compressibility is close to water compressibility.

When making samples of jelly, the solution is made first. To do this, a weighed portion of dry gelatin is put in water to swell. Then the water temperature is increased to $\sim 50\text{--}60^\circ\text{C}$. A solution is obtained which is then filtered, cooled down to $T=25\text{--}30^\circ\text{C}$, and poured into a prepared mold. Experimental assembly structural elements are usually used for the mold. The assembly (mold) surfaces contacting the solution are coated with a thin vaseline layer. The solution in the mold is cooled at $T=+6^\circ\text{C}$ for about 2 hours. After solution hardening, the mold (assembly) is kept at room temperature until thermal equilibrium is reached. Then unnecessary mold elements are removed. In this way, samples in the shape of a plane layer, ring, wedge, etc. can be produced. In some cases, a sample (for example, a ring) is manufactured by cutting it from a plane layer of hardened jelly.

In practice, GEM was used in the form of a stoichiometric compound of acetylene-oxygen mixture ($\text{C}_2\text{H}_2+2.5\text{O}_2$) (hereinafter the term GEM denotes this mixture). The GEM chamber is filled by prolonged pumping through gas inlets (the volume of pumped mixture is $\sim 10\text{--}15$ times as large as the chamber volume).

GEM detonation is relatively easily realized by the electric-spark technique. At $T_0=20^\circ\text{C}$ and $P_0=760$ mm Hg, GEM is characterized by the following parameters:

- Density $\rho_0=1.35 \cdot 10^{-3}\text{g/cm}^3$;
- Detonation rate $D=2450$ m/s;
- Detonation wave front pressure $P_j=41$ atm;
- Critical diameter 2 mm.

The characteristic time of pressure equalization in the mixture detonation products (DP) is much less than the characteristic time of jelly model acceleration. Therefore, in the computations when interpreting the experimental results (for example, plane layer acceleration, cylindrical layer expansion or collapse, etc.), one may neglect the time of pressure equalization in DP and assume that the model is accelerated by gas at rest with

$P_0=13.5$ atm pressure, correspondent with the equation of state of an ideal gas with adiabatic exponent $\gamma=1.23$. The sample can also be accelerated by compressed gas (air, helium) or by many other methods.

During the 1980s and 1990s, a number of studies performed using the jelly technique were published. Fig. 1.1-1 gives the setup for these experiments:

Fig. 1.1-1. Typical setup of the experiments for jelly technique measurement: (a) plane jelly layer acceleration in a square cross section channel [33,37], (b) cylindrical shell (ring) scattering under pressure of detonation products of HE gas mixture filling the internal shell cavity [32], (c) cylindrical shell (ring) collapse [35], and (d) angular singularity evolution[34].

Fig. 1.1-2. Flow recording scheme: 1. pulsed flash lamp, 2. experimental assembly, 3. mirror lens ($F=2000$ mm) (a part of the shade device IAB-451), 4. two-objective telescopic system, 5. high-speed framing camera SFR.

Fig. 1.1-2 gives the flow recording scheme. The experimental assembly was located in the field of vision of a long-focus mirror lens ($F=2$ m) (3) (part of the shade facility IAB-451 [49]). The flow was recorded by a high-speed framing camera (5). Optical jointing of the SFR camera and the long-focus lens was with a telescopic optical system composed of two objectives. The use of the long-focus lens within the recording system allowed the image parallax of the jelly layer at its movement along the assembly channel and, correspondingly, the measurement error to be reduced to an acceptable minimum (in the [33,37] type experiments).

1.1.2. Jelly Strength Properties

Jellies exhibit the capability of high recoverable strain at practically zero yield. When stresses exceed a certain limit then jelly decomposes. In contrast to solutions, jelly loses its continuity unrecoverably, i.e., the structure is not re-formed from resultant fragments [4].

Jelly samples were tested for strength. Cylindric jelly samples 20 mm in diameter and height were subjected to compressive force. The ratio between the force, P_r , corresponding to decomposition and the initial sample area characterizes strength. (Note that at decomposition, strains were quite considerable: the area increased more than by a factor of 2. Therefore, the values of pressure causing decomposition are effective, evaluating jelly strength scale). The test results are given in Table 1.1-1. These results are in approximate agreement with the data of paper [76].

Table 1.1-1. Jelly strength dependence on gelatin concentration in water solution.

Concentration, C, %	4.4	8.8	13.2	17.6	35.2
Pr, atm	0.1	0.5	0.9	1.3	2.8

The basic differences between jelly and an ideal fluid are related to strength and viscosity. The effect of these factors on the accuracy of modeling a flow under study, including turbulent mixing evolution at boundaries unstable during acceleration, can be profound.

Jelly strength and viscosity depend on gelatin concentration and temperature. With increasing solution concentration, the jelly models become stronger, for example, at $C \approx 35\%$ the model becomes rubber-like and does not actually decompose when being accelerated by **GEM DP** in [35] type experiments. With decreasing concentration and/or increasing temperature jelly strength and viscosity decrease.

A number of studies, including turbulent mixing at layer and shell boundaries unstable during acceleration, were performed under these conditions [33–37].

Some experiments revealed the effect of jelly strength on instability evolution. They showed that first, the small-scale structure of unstable interface perturbation turns out to be dependent on strength and second, that strength determines dispersed material particle sizes which affects the rate of processes occurring in the turbulent mixing zone.

1.2. Experimental Study of Hydrodynamic Instability in Metals

Study of hydrodynamic instability in metals is of great practical interest. As was mentioned above, two approaches were used for the experimental study of this phenomenon:

- Recording with the x-ray method;
- Recording on preserved samples.

1.2.1. Experimental study of Rayleigh-Taylor instability using the x-ray technique

The experimental studies of Rayleigh-Taylor instability in strong media were performed using metal plate acceleration caused by detonation. At the interface of the accelerated plate and the detonation products (DP), the conditions needed for gravitational instability arose. With this loading technique, the acceleration range reaches 10^9 m/s^2 , while the loading pressure is several tens of GPa. Initial perturbations were applied over the plate surfaces. The evolution of these perturbations during acceleration of the plates was the subject of the research.

The research used the pulsed x-ray method. The images of the plates obtained on a x-ray film were specially read and subjected to digital processing in order to determine plate shapes and to measure perturbation amplitudes. Such a technique for studying gravitational instability in solids was used in papers [50,51]. The experimental studies of gravitational instability in strong media were performed with plates made of annealed aluminum alloy AMg-6. A diagram of the apparatus is given in Fig. 1.2-1.

At a certain time interval after firing of the electric detonator (**ED**) located on the experimental assembly, the x-ray facility actuated and generated an x-ray image of the plate under study on the film. The time interval from **ED** firing to γ -pulse was varied to allow recording of the shape of the plates under study at various times of the instability evolution process.

The radiation source was an x-ray tube with boundary energy of gamma quanta up to 1 MeV, radiation dose up to 1 R at a distance of 1 m, and effective pulse duration ≈ 0.2 microseconds. For sensitive material, Curix or Kodak x-ray films were used in combination with the intensifying screens UFD-A. Resolution of the image generation system used is about 0.3 lines/mm and x-ray penetration capability on the order of 100 mm of steel at a distance of 1 m from the tube anode. The sketch of the experimental assemblies is given in Fig. 1.2-2.

The plane detonation wave generator initiated the **HE** charge. The detonation products generated on **HE** detonation accelerated the plates under study. The plates under study were put at a certain distance from the **HE** charge surface. Introduction of a gap is needed to secure gradual loading of the plates, without generation of shock waves in them.

Fig. 1.2-1. Testing scheme: 1. x-ray source, 2. experimental assembly, 3. x-ray film contained in a protective cassette.

Fig. 1.2-2. Experimental assembly sketch: 1. HE charge, 2. plane detonation wave generator, 3. plate under study, 4. sealing glass.

As was shown by the authors of paper [52], air in the gap leads to accelerated plate surface erosion or, in other words, to the occurrence of additional perturbations. In our case the magnitude of these perturbations turns out to be so high that, evolving during plate acceleration, they lead to complete plate disintegration.

According to [52], when air is removed from the gap erosion is eliminated. Fig. 1.2-3 gives x-ray images of the aluminum plates obtained in two comparative experiments to demonstrate the need of air removal from the gap.

Non-vacuumized gap; S=38 mm.

Vacuumized gap; S=40 mm.

Fig. 1.2-3. X-ray images of aluminum plates obtained in tests with and without gap vacuumization.

Fig. 1.2-4. Loading pressure and aluminum plate displacement vs time.

In the experimental assemblies, air was removed not only from the gap but also from the whole space confined by the glass which hermetically sealed the HE charge. For the plane detonation wave generator, a lens 120 mm in diameter was used to produce generation of a detonation wave with front nonflatness of 0.03 ms in a circle of 80 mm diameter. The cylindric HE charge was a blasting cartridge made of octogen base HE 120 mm in diameter and 90 mm in height. Initial HE density was 1.875 g/cc and detonation rate 8.8 km/s.

The plates under study were disks 66 mm in diameter and 1.5 mm in thickness. The sealing glass was made of low-density material transparent to X radiation. The gap size was 15 mm. The above-described assembly produced aluminum plate loading with the time-dependent loading pressure and displacement given in Fig. 1.2-4. The temporal plots were obtained from 1D computations using the equation of state of detonation products according to [53].

Initial perturbations were placed on the plate surface facing the HE. This was done by making ruffles of triangular cross section. The sketch of the plate with applied perturbations is given in Fig. 1.2-5. The ruffle depth A_0 determined the initial perturbation amplitude, and the period λ , the wavelength.

When the ruffles were made only in one direction perpendicular to the plane XY, longitudinal protrusions triangular in shape were formed on the plate surface. Periodic 2D perturbations were thus generated. When similar ruffles were made in two mutually perpendicular directions, regular tetrahedral pyramids were formed on the plate surface which were 3D perturbations.

Fig. 1.2-5. Sketch of the plate under study.

The experimental assemblies were positioned relative to the x-ray facility in such a way that at the filming time the surface of the plates under study was parallel to the radiation direction, and the radiation direction coincided with the direction of the ripples generating perturbations. During the experiment the plates under study were radiographed. The contour of the plates under study was determined by the images obtained on the x-ray film, and the perturbation amplitude was measured. In order to improve image quality and contour point location accuracy, the plate images were subjected to digital processing. The image digital processing was with the program package [54].

The x-ray images of the plates under study obtained on the x-ray film were scanned (digitized) using the microdensitometer “**FEAG-200**.” Then these were subjected to digital processing: the image field nonuniformity was eliminated; the image was filtered using a second-order differentiating linear filter optimizing the signal/noise ratio in the vicinity of the zero value of the second derivative; and the boundary contour points of the plate under study were determined, which were then used to measure the perturbation amplitude. The boundaries were separated with the gradient method. The boundary criterion was the zero value of Laplacian (the second derivative). This algorithm of digital image processing and the boundary separation criterion allow us to reliably determine the contour indications of objects on x-ray images.

Despite the fact that the plates under study caved in under action of lateral unloading during acceleration and one of the separated plate boundaries became inner, the above-mentioned algorithm was successfully used to determine the contour indications of the boundaries for the plates under study thanks to the following:

1. The algorithm was developed and designed for separation of just inner boundaries.
2. The x-ray images of the plates under study were obtained using “soft” X radiation so that they possess higher contrast and lower granularity of image, with a signal/noise ratio approaching 3. For the same reason radiation scattering is small and boundary smearing due to scattering is minor.
3. The focus size of the x-ray facility is 3 to 4 mm, thanks to which smearing related to x-ray source inaccuracy is no more than 0.5 to 0.8 mm.

4. The absolute value of boundary drift does not essentially affect the perturbation amplitude measurement results as the latter is estimated through computation of the coordinate difference for appropriate boundary points.

The amplitude of perturbations on the accelerated plate surface, as for perturbations in the initial state, is taken to be the difference between abscissas of the most protrusive and the most deepened points of the plate surface (see Fig. 1.2-5).

As with any other measurements, a question arises regarding accuracy of these perturbation amplitude measurements. According to papers [55,56] the total measurement error ΔA can be estimated as the mean-square value of the random, ΔA_{ra} , and systematic, ΔA_{syst} , errors:

$$\Delta A = (\Delta A_{ra}^2 + \Delta A_{syst}^2)^{1/2} \quad (1.2.1)$$

The random error ΔA_{ra} is evaluated by statistics of a limited number of measurements n and selection of the level for the confidence probability α .

The sources of the systematic error ΔA_{syst} can be two factors: the systematic error of the measurement method itself, $\Delta A_{syst.meth}$, and the systematic error introduced by measuring tools, $\Delta A_{syst.tool}$. Here by the measurement method is meant the whole chain of operations from the initial plate image on the x-ray film to determination of the boundary contour for the plate under study: photometry, digital processing, determination of the contour points. The resultant systematic error ΔA_{syst} can be, according to [55,56], also estimated by the formula

$$\Delta A_{syst} = (\Delta A_{syst.meth}^2 + \Delta A_{syst.tool}^2)^{1/2} \quad (1.2.2)$$

The systematic error of a measurement method $\Delta A_{syst.meth}$ was estimated by the x-ray images of a test plate. For these purposes a test plate was manufactured whose shape was made as close to the plate shape as possible for the plates realized in the experiments. The sketch and the x-ray image of the test plate are given in Fig. 1.2-6. The plate magnification factor in the x-ray image is $K=1.26$. On the concave plate surface, as in the experiments, there were perturbations. The actual amplitude of these perturbations was $A_{act} = (0.17 \pm 0.01)$ mm ($\alpha=0.95$).

The mean value of the perturbation amplitude measured using the method used was $A_{mea} = 0.20$ mm. The mean-square deviation of this value was 0.03 mm.

Thus, the systematic error value for the measurement method, $\Delta A_{syst.meth}$, estimated with the test is not more than +0.03 mm ($A_{mea} - A_{act} = +0.03$ mm). The systematic error introduced by measuring tools, $\Delta A_{syst.tool}$, was estimated as follows. A measuring tool was a ruler with graduations of 1 mm apart. The systematic error of this tool, ΔA_{tool} , was taken equal to 0.5 mm. To reduce the systematic measurement error introduced by the tool, $\Delta A_{syst.tool}$, the plate image scale was varied. The images with the separated contour of the plates under study obtained after equidensity processing were magnified and used to take

Fig. 1.2-6. Test plate sketch and x-ray image.

measurements. The equidensity processing magnification factor **SCALE** was selected equal to 5. With an x-ray plate image magnification factor **K**, usually close to 1.3 the value of the systematic measurement error introduced by the tool, $\Delta A_{\text{syst.tool}}$, was calculated by the formula

$$\Delta A_{\text{syst.tool}} = \Delta A_{\text{tool}} / (\text{SCALE} * K)$$

and gave ≈ 0.08 mm. As can be seen from (1.2.2), the contribution made by the systematic error of the measurement method, $\Delta A_{\text{syst.meth}} = 0.03$ mm, to the resultant systematic error, ΔA_{syst} , as compared to the contribution made due to the systematic error from the measuring tool, $\Delta A_{\text{syst.tool}} \approx 0.08$ mm, can be neglected. Thus, the resultant systematic measurement error for the perturbation amplitude, ΔA_{syst} , is ≈ 0.1 mm.

Further reduction in the systematic measurement error, ΔA_{syst} , down to < 0.1 mm in order to reduce the resultant error ΔA in dependence (1.2.1) becomes ineffective since for majority of the experiments made the random measurement error, ΔA_{ra} , is, as a rule, > 0.1 mm. This means that the measurement error of the perturbation amplitude mainly depends on measurement statistics.

To demonstrate the algorithm of image digital processing and the measurement technique, Fig. 1.2-7 gives the results for the stages of image digital processing for the plates under study. On images 1–4, the plate magnification factor equals 1.47.

The values of the proper perturbation amplitudes were as follows:

$$A_1 = 1.19 \text{ mm}, A_2 = 1.19 \text{ mm}, A_3 = 1.19 \text{ mm}, A_4 = 1.02 \text{ mm}, A_5 = 0.85 \text{ mm}$$

Then the following was computed:

- The average value of $\langle A \rangle$

$$\langle A \rangle = (\sum A_j) / n = 1.088 \text{ mm}$$

where $n=5$ - the number of measurements.

- The mean-square deviation of a separate measurement S_n

$$S_n = [\sum (<A> - A_j)^2 / (n-1)]^{1/2} = 0.153 \text{ mm}$$
- The mean-square deviation of the average value measurement σ_n

$$\sigma_n = S_n / n^{1/2} = 0.068 \text{ mm}$$
- The random measurement error ΔA_{ra}

$$\Delta A_{ra} = \tau_{\alpha, n} * \sigma_n = 0.190 \text{ mm}$$

where $\tau_{\alpha, n} = 2.8$ —the Student factor for the confidence probability $\alpha = 0.95$ and the number of measurements $n = 5$.

Fig. 1.2-7. Results of plate image digital processing stages: 1. plate image obtained after photometry, 2. image obtained after image field nonuniformity elimination, 3. image obtained after filtering with the second-order differentiating linear filter optimizing the signal/noise ratio in the vicinity of the zero value of the second derivative, 4. the equidensity processing result used as the basis to construct the zero value level lines which are boundary points of the plate under study.

- The systematic measurement error ΔA_{syst}

$$\Delta A_{\text{syst}} = \Delta A_{\text{syst.tool}} = \Delta A_{\text{tool}} / (\text{SCALE} * K) = 0.07 \text{ mm},$$

where $\Delta A_{\text{tool}} = 0.5 \text{ mm}$ —single measurement accuracy equal to the systematic tool error, $K = 1.47$ —the magnification factor of the plate on the image, and $\text{SCALE} = 5$ —the magnification factor of the equidensity image processing used to make measurements.

- The total measurement error, ΔA ,

$$\Delta A = (\Delta A_{\text{ra}}^2 + \Delta A_{\text{syst}}^2)^{1/2} = 0.20 \text{ mm}.$$

The result of the perturbation amplitude measurement was $A = 1.09 \pm 0.20 \text{ (mm)}$. Thus, the above-described experiment version of the x-ray technique using the described algorithm of digital image processing can be used to study gravitational instability in aluminum. The perturbation amplitude measurement error is mainly due to the statistical spread of the value being measured. When the statistical spread of the value being measured is good the measurement error is no more than 0.1 mm.

1.2.2. Experimental study of Richtmyer-Meshkov instability on preserved samples

The experiments used the method of metal sample preservation after intense shock-wave loading. This is achieved using explosion-shielding containers (high-strength preservation ampules) [29]. The ampule safety principle consists in withdrawal of rupture stresses (which arise at rarefaction wave coupling) outside the ampule vessel. For this purpose the ampule is encircled with passive shielding layers where spall ruptures take place. For unambiguous treatment of the results obtained it is necessary to know the wave picture of sample compression and unloading which is most easily calculated in the practically 1D (“plane”) case realized by loading the cylindric ampules from their end side (the ampule diameter is considerably larger than thickness).

A typical experimental scheme for a most common version of the high-strength preservation ampule ($\varnothing 64 \times 14 \text{ mm}$) is given in Fig. 1.2-8 [29].

Fig. 1.2-8.

Vessel (1) and cover (2) are connected through a thread along the surface B-B'. The joint A-A' is further strengthened with welding on an electron-beam tube. The shock-wave pulse of desired amplitude, shape, and duration is generated with a loading device with an HE charge (3). From the side of tensile stress arrival the ampule is surrounded with shielding layers: plate (4) and safety rings (5). The samples are put into ampule payload volume (6) ($V=10\text{-}35\text{cm}^3$). The ampules intended for high loading pressures ($P\sim 100\text{ GPa}$) are made of strong refractory alloyed steels.

In this experiment series, a sample in the form of two disks made of different metals tightly pressed to each other and contacting along either a plane or an appropriately curved interface was put into the free volume of the ampule [28]. The initial perturbations initiating instability evolution were generated using two techniques (see Fig. 1.2-9).

1. "Sinusoidal" concentric curvatures in the interface of a pair of metals composing the sample (see Fig. 1.2-9a);
2. Regular concentric curvatures of the front of the shock waves incident on the plane interface. To do this, a "lens" element composed of two plates made of different metal types contacting along a curved interface was put in front of the sample (from the side of shock wave (SW) entrance).

Fig. 1.2-9a.

Fig. 1.2-9b.

Samples made of Sn, Bi, Pb, Cd, Zn in various combinations were studied in the experiments. The state of the metal pair interface after detonation implosion was studied on a metallographic ground end made in the ampule diametric cross-section plane.

1.2.3. Experimental study of Kelvin-Helmholtz instability

In these experiments the traditional scheme of sample loading with an oblique shock wave is used as a base. Three basic experimental approaches were used.

1.2.3.1. Oblique collision of metal plates

In Fig. 1.2-10, on a massive steel base (1), an immovable plate (2) is put. The cast penetrator plate (3) is fastened above this at a strictly fixed angle α . The penetrator is accelerated by detonation products (DP) of HE charge (4) where a plane sliding detonation wave is realized. The minimum distance between the plates is selected from the condition of penetrator flight time independence before collision.

Fig. 1.2-10.

To prevent spall phenomena in the penetrator material, a thin gasket (5) of low acoustic impedance is put between the penetrator plate and the HE layer, and does not affect the penetrator movement parameters.

In the experiments, the following metals were studied: steel, aluminum alloys, copper, brass, and magnesium in various combinations. After dynamic loading the plates were captured with a layer of porous material. From the sample middle fragments were cut out to manufacture micro-ground ends and make metallographic analysis. The perturbation amplitude and wavelengths given in the text are an averaged value after computation of 10 to 20 neighboring perturbations [21,25,26].

1.2.3.2. Two-stage scheme of metal plate batch loading with oblique shock wave

In Figure 1.2-11, on a massive steel base (1), an immovable plate (2) is placed. Above this, through the gap d , either a cast plate (3) or a batch of plates (3') tightly contacting one another are fixed. The batch plate thickness-to-thickness ratio is selected to preclude their possible exfoliation at movement inside the gap d . Loading samples (3) with the oblique shock wave takes place via the impact of cast plate (4). The condition of the hermetic position of plate (4) relative to element (3), as well as other layers is similar to the above-described scheme (see 1.2.3.1, Fig. 1.2-10, pos.1,2,5). This experiment series used the following metals: copper and aluminum alloys [22,23].

Fig. 1.2-11.

1.2.3.3. Scheme of shear flow realization in metals at closure of wedge-shaped gap under the plate batch

These experiments were performed with the samples pressed into the safety ampules (similar to that described in 1.2.2). The sample consisted of a copper or aluminum disk (3) put in an ampule (1) (see Fig. 1.2-12). Under this disk, another disk (5) of aluminum or copper with a conic hollow was placed. Disks (3) and (5) were separated from the ampule by plates (2) and (6) made of copper foil. In some tests, a ring (4) was put between the disks to increase the flight base of element (3) until impact on element (5). As the wedge-shaped gap collapsed, an oblique disc collision was realized at an angle of $\gamma = 15^\circ$ at the supersonic velocity of the contact point ($V_k > C_0$). The state of the interfaces a-e shown in Fig. 1.2-12 on the shock-wave implosion was analyzed by examining the micro-ground end manufactured on the ampule cross section diametric plane [24].

Fig. 1.2-12.

II. Description of Numerical Methods and Medium Models Used for Research into Hydrodynamic Instability in Strong Media

The content of this section is clear from its title. We emphasize once again that this section considers only those medium models and numerical methods which were used by us for research into hydrodynamic instability in strong media.

2.1. Medium Models Used for Numerical Simulation and in Analytical Research

2.1.1. Medium models used in analytical research

Analytical solutions are found only for examples where the simplest medium models are used: for an incompressible inviscid (ideal) fluid, an incompressible viscous fluid, and an incompressible elastic body.

Representation of the equation of motion in Eulerian or Lagrangian variables is considered by us to be part of the medium model. Most analytical studies of Rayleigh-Taylor interface instability, beginning with those which were historically the earliest, are performed in Eulerian independent variables. Analytical studies in Lagrangian independent variables are performed in a few papers. For the initial stage of instability evolution when strains are not yet very large, the use of the Lagrangian approach is quite justified and more accurate. In strong media the relative weight of this stage of perturbation evolution increases.

The Lagrangian and Eulerian approaches, in principle, are equivalent to each other. However, approximating the basic nonlinear system needed for an analytic treatment, for example by linearization, leads to different equation systems. At linearization, the boundary conditions are set at the current strained interface in the Lagrangian formulation, while in the Eulerian approach the boundary conditions are approximately shifted to the initial interface. It should be particularly emphasized that a linearized system of equations of motion in the Lagrangian form determines the non-linear profile in the space of the observer.

The use of the Lagrangian representation for the equation of motion of thin-wall systems and incompressible fluid allowed us to obtain a number of new results in the analytical study of the non-linear stage of the Rayleigh-Taylor instability [61-64] in two and three dimensions. In Lagrangian coordinates, the equations of motion of an accelerated thin layer turn out, in some special cases, to be linear even at large shears. This feature allows one to analyze them in order to describe the perturbation evolution at a stage non-linear in the space of the observer.

In Rayleigh-Taylor instability of an accelerated interface, the perturbation evolution is localized near the interface. In the analytical description of this phenomenon in Lagrangian independent variables, an important role is played by the terms in the equations which also describe the dynamics of an accelerated thin incompressible liquid layer. It is precisely these terms that involve the source of the perturbation growth. The other terms in the equations of motion that describe the effects of strength, compressibility, and viscosity introduce a stabilizing action to the accelerated interface dynamics.

On linearization of the basic equations of motion of accelerated continuum in the Lagrangian independent variables, the right-hand side of the equations is represented in the form of a sum of the terms describing the accelerated thin incompressible layer dynamics.

We will illustrate this in specific examples.

The linearized equations of motion of the 2D Lagrangian gas dynamics (compressible invicid fluid) can be represented in the form:

$$\begin{aligned}\frac{\partial^2 x_1}{\partial t^2} &= C_0^2 \left(\frac{\partial^2 x_1}{\partial \xi^2} + \frac{\partial^2 y_1}{\partial \xi \partial \eta} \right) - a \frac{\partial y_1}{\partial \xi}; \\ \frac{\partial^2 y_1}{\partial t^2} &= C_0^2 \left(\frac{\partial^2 x_1}{\partial \xi \partial \eta} + \frac{\partial^2 y_1}{\partial \eta^2} \right) + a \frac{\partial x_1}{\partial \xi},\end{aligned}$$

where $x_1 = x - \xi$, $y_1 = y - \eta$; x_1, y_1 are Cartesian coordinates in the space of the observer; t , time; $\xi = x(0)$, $\eta = y(0)$, the initial state coordinates (the Lagrangian coordinates); a , acceleration.

At $a=0$, this system determines a stable oscillatory movement described by a hyperbolic type system. At $C_0^2 = 0$, the system coincides with the equations of the accelerated layer dynamics (see section III) and contains exponentially growing solutions.

For an elastic incompressible body in two dimensions, the linearized equations of motion in the Lagrangian representation can be reduced to the form

$$\begin{aligned}\rho_0 \frac{\partial^2 x_1}{\partial t^2} &= G \left(\frac{\partial^2 x_1}{\partial \xi^2} + \frac{\partial^2 x_1}{\partial \eta^2} \right) - \rho_0 a \frac{\partial y_1}{\partial \xi} \\ \rho_0 \frac{\partial^2 y_1}{\partial t^2} &= G \left(\frac{\partial^2 y_1}{\partial \xi^2} + \frac{\partial^2 y_1}{\partial \eta^2} \right) + \rho_0 a \frac{\partial x_1}{\partial \xi},\end{aligned}$$

where G is the shear modulus. At $a=0$, this system determines a bounded solution; at $G=0$, we obtain the system of thin liquid layer equations having unbounded solutions.

A similar situation takes place for viscous fluid. The linearized equations of motion in the Lagrangian variables contain terms corresponding to the thin liquid layer equations, and it is these terms that result in the possibility of the exponentially growing solutions for the

corresponding problems. This accounts for the great attention paid to the thin liquid layer dynamics in our research.

2.1.2. *Medium models used for numerical simulation*

In contrast to analytical studies which require a very great number of assumptions that simplify the problem, for numerical simulation one can restrict oneself to only minor simplifications of the original problem and take physical models that most adequately describe the process. For numerical simulation in a gas-dynamics approximation, the model of an isotropic compressible, ideal (without viscosity) medium is used. The equation of state of a medium closes the system of differential equations that express the mass, momentum, and energy conservation laws and is usually expressed in the form of tabular or analytic pressure dependencies on density and internal specific energy. The form of the equation of state can be arbitrary (within the framework permitted by thermodynamics). For gaseous media, the equation of state is usually taken in the form of the equation of state of an ideal gas

$$P = (\gamma - 1)\rho E,$$

where P , pressure; γ , Grueneisen factor; ρ , current material density; E , internal energy of the medium. In general, the Grueneisen factor is the medium parameter function.

For condensed media, the Mie-Grueneisen form [3] of the equation of state is most frequently used:

$$P = P_x + P_T = \frac{\rho_0 C_0^2}{n} \left(\left(\frac{\rho}{\rho_0} \right)^n - 1 \right) + \frac{\Gamma}{V} (E - E_x),$$

$$E_x = \int_V^{V_0} P_x dV,$$

where ρ_0 , initial material density; C_0 , volume sound speed; n , specific adiabatic exponent; Γ , Grueneisen factor; V_0 , V , initial and current specific volume, respectively. The parameters in the equation of state characterize particular materials and are usually found from experiment.

To describe elastic-plastic media behavior, the equations of the differential theory of thermoplasticity with complex strengthening [8, 9] are used:

$$\varepsilon = \varepsilon' + \varepsilon^v, \quad \sigma = \sigma' - \sigma^v, \quad \varepsilon' = \varepsilon'^e + \varepsilon'^p, \quad \frac{d\sigma'}{dt} = 2G\varepsilon'^e,$$

$$S = \sigma' - R, \quad R = 2g\varepsilon'^p, \quad S^T C S \leq \frac{2}{3} Y^2, \quad \frac{d\varepsilon'^p}{dt} = \lambda S$$

where $\sigma' = [\sigma'_{rr}, \sigma'_{\theta\theta}, \sigma'_{zz}, \sigma'_{rz}]^T$, $\varepsilon' = [\varepsilon'_{rr}, \varepsilon'_{\theta\theta}, \varepsilon'_{zz}, \varepsilon'_{rz}]^T$, deviatorix stress and strain tensors; $\sigma^v = [P, P, P, 0]^T$, $\varepsilon^v = [e, e, e, 0]^T$, pressure and volume strain; $\varepsilon^e, \varepsilon^p$, elastic and plastic components of the strain deviator; $\mathbf{S}, \mathbf{R}$, components of the tensors of active strains S_{ij} and residual microstresses ρ_{ij} ; $\mathbf{G}$, shear modulus; $\mathbf{g}$, kinematic strengthening modulus; $\mathbf{Y}$, dynamic yield strength which is a function of the parameters of state; δ_1 , parameter identically equal to zero in elastic strain and evaluated from the condition of setting on the yield surface at elastic-plastic strain; d/dt , derivative in the Yawman sense; the symbol “T” means transposition.

To simulate the viscosity effect, the tensor components of the linear viscosity (the Navier-Stokes type viscosity), $2\mu\varepsilon'$ are added to the components of the deviatorix stress tensor computed accounting for the Mies condition. In general, the viscosity factor μ is a function of the material state parameters, permitting the simulation of various types of stress tensor dependencies on strain rate. The yield strength can also be a function of the material state parameters and strain rate which lets us take into account material behavior features such as strain hardening and unhardening, material transition to the liquid state, etc.

2.2. Numerical Techniques Used to Study Hydrodynamic Instability Mechanics Problems in Strong Continua

VNIIEF has a large set of numerical methods to solve multidimensional continuum mechanics problems. These techniques differ in the variables in which motion is described (Lagrangian, Lagrangian-Eulerian, Eulerian coordinates), computational grid type (regular, irregular), the techniques used for interface separation when the interfaces do not coincide with the computational grid lines (material concentration, particles), the approximation technique for laws of conservation, etc. Most of these methods were used to model hydrodynamic instability in gaseous and liquid media.

Below we will summarize the numerical techniques which we used to study the phenomenon of hydrodynamic instability in strong and viscous media.

2.2.1. Sigma program system

The Sigma system [57] is designed for computing 2D (plane and axisymmetric) continuum dynamics problems. The Sigma system:

- splits a given problem into computational domains;
- uses the methods of decomposition by physical processes;
- uses arbitrarily-moving computational grids.

The problem being computed is split into a number of computational domains. In each computational domain an independent computational grid is constructed which permits the computation of problems of a practically arbitrary geometry. The grid in the computational domain is quadrangular and regular. Mutual communication across

computational domains is secured by introducing internal boundary conditions. Among other things, splitting problems into computational domains allows one to easily introduce the sliding of materials relative to one another.

The method of splitting by physical processes facilitates the introduction of new physical processes and medium models to be applied to problems. In each computational domain considered in this project, the system programs were used to compute problems in the gas-dynamics approximation, problems which account for the strength properties of media (in terms of the elastic-plastic model), and computations of viscous fluid motion [39, 41].

The difference schemes are constructed using the integro-interpolation technique and are first-order in time and second-order in spatial variables. For gas-dynamics and elastic-plastic computations, as a rule, explicit difference schemes are used, and there are Courant limitations on the size of the time step. To compute shock waves, artificial viscosity is introduced [65].

As was mentioned above, the system uses arbitrarily-moving computational grids, i.e., in the general case it is possible that displacement of the grid nodes does not coincide with material movement. This is particularly true for internal grid nodes, i.e., those which are not at the computational domain boundary. In a particular computational domain, the internal nodes of the computational grid can be Lagrangian and in this case the computation is performed in the Lagrangian variables. However, if a problem with considerable strains is computed, the computational grid can be severely distorted, and this leads to a drastic decrease in the computational time step and sometimes to abnormal exits. To prevent a severe computational grid distortion, the Sigma system allows the computational grid to be reconstructed inside a computational domain at each computational step. The grid reconstruction is done by preset algorithms so that needed grid properties determined by the performer of the computation for a particular problem can be maintained. There is a wide set of such algorithms: a grid that is uniform or quasi-uniform in spacing, uniform in an angle from some center, rectangular, etc.

Thus, in computations with the Sigma system programs, a computational step is composed of three stages:

- At the first stage, motion is computed in a needed approximation (gas dynamics, elasticity-plasticity, etc.) in the Lagrangian variables. The computational grid moves with the medium;
- At the second stage, the computational grid is reconstructed by given algorithms (in so doing, the computational grid boundaries remain Lagrangian);
- At the third stage, the values determined on the computational grid (velocities, internal specific energy, density, computational viscosity, stress tensor deviator components, etc.) are updated from the old grid to the new. In so doing, the laws of conservation of momentum, mass, and energy are used.

The question of the algorithm for updating the stress tensor deviator components was solved through comparison of the computed results with known analytic solutions. Here, as for artificial viscosity, the best agreement between results is achieved by updating each stress tensor component by the algorithm used to update density. Upon updating all the stress tensor components, one checks if the value of the second invariant of the stress tensor exceeds a given yield strength. Otherwise the stress tensor components are modified in conformity with the model of plastic flow being used.

Attempts to model flows with severe strains led to development in the Sigma system framework of the computational methods where boundaries of various materials are not separated as Lagrangian lines, but are followed using concentrations. This has been implemented in the technique EGAK [58] and LEGAK (CLEGD) [59]. These techniques use a special (donor-acceptor) algorithm to compute flows from computational cells containing several materials. Later on, these techniques were executed as individual computational systems [60].

For the problems which are the subject of this review, the best results were obtained with the numerical algorithms using the Lagrangian variables. These computations used, along with the initial difference scheme of the Sigma technique [57], the difference scheme constructed in paper [74] which secures fulfillment of the complete set of conservation laws (including total energy) in the planar case.

The advantage of the methods using the Lagrangian variables for the problems considered in this review is determined by the fact that the approximation for the difference schemes of these methods is usually second-order in spatial variables. In such numerical algorithms, there is no approximation viscosity similar to physical viscosity with a coefficient proportional to the first power of the computational cell linear dimension. The effect of such terms on the hydrodynamic instability problems is analyzed in paper [20].

These issues are discussed in Section III of this review.

2.2.2. 2D Lagrangian technique DRAKON-R/2

For studying Rayleigh-Taylor instability in strong media, good results were obtained using the 2D Lagrangian technique **DRAKON-R/2** (dynamic computation of structures) [14].

The equation system determining the time-dependent elastic-plastic medium strain process is formulated in the Lagrangian variables. Medium motion is described by the general dynamics equation that follows from the d'Alembert-Lagrange principle of possible displacements [5] which, in two dimensions, is of the following form:

$$\int_{\Omega} \delta \dot{\epsilon}^T C \sigma \mathbf{r}^V d\Omega + \int_{\Omega} \rho \delta \dot{\mathbf{u}}^T \ddot{\mathbf{u}} \mathbf{r}^V d\Omega - \int_{\Gamma_{\Omega}} \delta \dot{\mathbf{u}}^T \mathbf{p} \mathbf{r}^V ds = 0, \quad C = \begin{pmatrix} 1 & & \\ & 1 & 0 \\ & 0 & 1 \\ & & & 2 \end{pmatrix} \quad (2.2.1)$$

where Ω , the element cross section in the plane $\mathbf{r}-\mathbf{z}$ (in the axisymmetric case, the element is considered in the cylindrical coordinate system $\langle \mathbf{r} \mathbf{z} \theta \rangle$ whose symmetry axis is $\mathbf{z}$, radial distance depends on the coordinate $\mathbf{r}$, and θ is the angular coordinate; in the case of the plane strained state, $\mathbf{r}$ and $\mathbf{z}$ are axes of the Cartesian coordinate system); Γ_{Ω} , the boundary on which the distributed loading is given; $\epsilon = [\epsilon_{rr}, \epsilon_{\theta\theta}, \epsilon_{zz}, \epsilon_{rz}]^T$, $\sigma = [\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}, \sigma_{rz}]^T$, vectors composed of the strain and stress tensor components; $\mathbf{u} = [u_r, u_z]^T$, the displacement vector; $\mathbf{p} = [p_r, p_z]^T$, the distributed loading vector; ρ , material density; v , the symmetry parameter equal to 1 in the axisymmetric case and 0 in the plane strained-state case; the point above the subscript denotes differentiation in time, the letter “T” denotes transposition. From the Lagrangian variational equation, the complete system of continuum equations follows [5]: the equations of momentum balance; the equations of energy; the equations of entropy balance, as well as boundary conditions.

For the physical relations, the Mie-Grueneisen form of the equations of state [3] and the equations of the differential theory of thermoplasticity with combined strengthening [8, 9] are taken. For viscous media, Navier-Stokes-type viscosity is used with the viscosity factor being a function of material state parameters. For shock wave computation, artificial viscosity is used.

Strain rates are estimated using the Cauchy relations [6] as the basis in the current state metrics, which allows us to take into account the geometric non-linearity

$$\dot{\epsilon} = \mathbf{D} \dot{\mathbf{u}} \quad (2.2.2)$$

where $\mathbf{D}$, the differential operator matrix:

$$\mathbf{D} = \begin{bmatrix} \partial_r & v/r & 0 & 0.5\partial_z \\ 0 & 0 & \partial_z & 0.5\partial_r \end{bmatrix}^T \quad (2.2.3)$$

where $\partial_r = \partial / \partial r$, $\partial_z = \partial / \partial z$.

The governing equation system discretization is based on the “natural” approximation of partial derivatives by spatial variables and on the “crest” type explicit scheme of time integration [10, 38]. The formulas for the natural approximation of the scalar function gradient ϕ follow from the theorem of gradient and the theorem of mean. In two dimensions

$$\nabla \varphi = \lim_{\omega \rightarrow 0} \frac{1}{\omega} \int_{\eta} \varphi d\eta, \quad \nabla = [\partial_r, \partial_z]^T, \quad d\eta = [d\eta_r, d\eta_z]^T, \quad (2.2.4)$$

where η , the boundary of the element ω in area; $d\eta$, the projection of the element boundary on the coordinate axes. Estimating the value of the function at the computational element (cell) edge center as the arithmetic mean of its values at approximate vertices from (2.2.4), we obtain the formula for approximate computation of $\nabla \varphi$

$$\nabla \varphi = A^{-1} S \Phi, \quad A = 2\omega, \quad (2.2.5)$$

here: $\Phi = [\varphi_1, \varphi_2, \dots, \varphi_N]^T$, the vector of function node values; N , the number of vertices in a cell,

$$S = \begin{bmatrix} S_{z1} & S_{z2} & \dots & S_{zN} \\ -S_{r1} & -S_{r2} & \dots & -S_{rN} \end{bmatrix}, \quad (2.2.6)$$

where S_{ij} , the sum of projections on the i -th coordinate axis of the polygon sides adjacent to vertex j .

Using (2.2.5.), Cauchy relations (2.2.2) for the N -angular cell number j will be written in the form

$$\dot{\varepsilon}_l = A^{-1} B_l \dot{U}_l, \quad (2.2.7)$$

where $B_l = [b_{l1}, b_{l2}, \dots, b_{lN}]$, the differential operator difference analog matrix;

$\dot{U}_l = [\dot{u}_1, \dot{u}_2, \dots, \dot{u}_N]^T$, the vector of cell vertex velocities. The elements of the matrix B_l and the vector $\dot{U}_l$ are of the form

$$b_{jl} = \begin{bmatrix} S_{zj} v / (\langle r \rangle N) & 0 & -0.5 S_{rj} \\ 0 & 0 & -S_{rj} & 0.5 S_{zj} \end{bmatrix}^T, \quad (2.2.8)$$

$$\dot{u}_j = [\dot{u}_{rj}, \dot{u}_{zj}]^T. \quad (2.2.9)$$

For stresses σ_l which are constant in the cell l , the first addend in (2.2.1), with substitution of (2.2.7), is expressed through $\sigma \dot{U}_l$ by the formula

$$\int_{\Omega_l} \delta \dot{\varepsilon}^T C \sigma r^V d\Omega = \delta \dot{U}_l^T B_l^T C \sigma_l \langle r \rangle_l^V / 2 = -\delta \dot{U}_l^T F_l. \quad (2.2.10)$$

Applying the theorem of the mean for the second and third addends in (2.2.1), we obtain

$$\int_{\Omega_1} \delta \dot{\mathbf{u}}^T \rho \ddot{\mathbf{u}} d\Omega = \delta \dot{\mathbf{U}}_1^T \rho \frac{\langle \mathbf{r} \rangle_1^V \Omega_1}{N} \ddot{\mathbf{U}}_1 = \delta \dot{\mathbf{U}}_1^T \mathbf{m} \ddot{\mathbf{U}}_1, \quad (2.2.11)$$

$$\int_{\Gamma_{\Omega_1}} \delta \dot{\mathbf{u}}^T \mathbf{p} \mathbf{r}^V ds = \delta \dot{\mathbf{U}}_1^T \mathbf{Q}_1. \quad (2.2.12)$$

In (2.2.11) and (2.2.12) $\mathbf{m}$ is the fraction cell mass accounting for one cell vertex; $\mathbf{Q}_1 = [q_1, q_2, \dots, q_N]$ is the vector of node forces whose components are derived similar to (2.2.9)

$$q_j = (\mathbf{p}_{j-1} \langle \mathbf{r} \rangle_{j-1}^V \Delta s_{j-1} + \mathbf{p}_j \langle \mathbf{r} \rangle_j^V \Delta s_j) / 2;$$

here $\mathbf{p}_j$, $\langle \mathbf{r} \rangle_j^V$, Δs_j are the value of surface loading, mean radius, and the length of the j -th side of the computational cell, respectively. Introducing continuous numbering for the difference grid nodes and summing expressions (2.10)–(2.12) over all cells, we obtain

$$\delta \dot{\mathbf{U}}^T (\mathbf{M} \ddot{\mathbf{U}} - \mathbf{F} - \mathbf{Q}) = 0, \quad (2.2.13)$$

where $\mathbf{M}$ is the diagonal mass matrix obtained by the cell mass distribution by the difference grid nodes. From (2.13), owing to arbitrary $\delta \dot{\mathbf{U}}$, the system of ordinary differential equations follows:

$$\mathbf{M} \ddot{\mathbf{U}} = \mathbf{F} + \mathbf{Q}, \quad (2.2.14)$$

which is a difference analog of equation (2.2.1).

To calculate time derivatives, the explicit two-step, second-order “crest” type scheme is used. The time step is evaluated from the Courant stability condition.

The numerical solution is constructed on a quadrangular difference grid. To preclude “non-physical” grid element distortions, which appear when the element diagonals move parallel to themselves without any change in their length (in this case the strain rate tensor components equal zero), in addition to the quadrangular cell, two pairs of triangular cells are considered which result from the division of a quadrangular cell by its diagonals [38]. Then the expression for the virtual work of internal forces (the first addend in (2.2.1)) for the quadrangular cell can be written in the form

$$A = A_4 + \alpha \left[0.5 (\overline{A}_3^1 + \overline{A}_3^2 + \overline{A}_3^3 + \overline{A}_3^4) - \overline{A}_4 \right] \quad \alpha \in [0,1],$$

$$A_4 = \int_{\Omega_4} \delta \dot{\epsilon}^T \sigma r^V d\Omega, \quad \overline{A}_4 = \int_{\Omega_4} \delta \dot{\epsilon}^T \overline{\sigma} r^V d\Omega, \quad \overline{A}_3^i = \int_{\Omega_3^i} \delta \dot{\epsilon}^T \overline{\sigma} r^V d\Omega, \quad (2.2.15)$$

where Ω_4 , Ω_3^i are areas of the quadrangular and i -th triangular cells and $\overline{\sigma}$ is the vector composed of the stress tensor components computed in an appropriate cell. Discretization of (2.2.15) is performed similar to (2.12.10). When the Mie-Grueneisen form of the equation of state is used, it is sufficient to consider only the cold pressure component in the sector $\overline{\sigma} = [-P_x, -P_x, -P_x, 0]^T$. In this case, the computational algorithm is essentially simplified and at the same time the efficiency of non-physical distortions of the difference grid is preserved.

The contact coupling is computed on inconsistent difference grids which permits the description of the medium strain processes at any relative displacements of the interfaces. The condition on the solid interfaces accounts for break-off, slipping with friction, and the rigid bond. They are formulated as the conditions of non-penetration on the surface portions which are in contact at a given time and the conditions of free boundaries on other portions. The contact criterion is the geometric intersection of free surfaces of the bodies. In effect, the contact coupling computation reduces to the problem of estimation of contact pressure which is a component in the distributed loading vector (2.2.1).

2.3. Methods Based on the Shell Approximation

Reference [42] describes a method for computing the dynamics of thin, strong rotation shells in contact with fluid, which is the ROB technique, based on the Timoshenko-type approximation extended to large shears. The implementation of this method in the framework of the multi-domain system “Sigma” [57] allowed us to solve complex 2D problems of thin-wall structure strains where, alongside the thin shell, there are liquid media and non-thin strong bodies which interact with it. Reference [42] gives some computed results for explosions in shielding closed chambers.

2.3.1. Governing equations

The equations of motion are written in Lagrangian independent variables with respect to an infinitesimal rotation shell element which is, at the same time, of finite (though small) dimensions along the direction normal to the median shell surface. Inside the element, the stressed-strained state is assumed to be homogeneous along the directions tangential to the medium surface and inhomogeneous in the direction of its normal. The layer elements have a homogeneous velocity profile $\tilde{w}$ in the normal direction to the layer median surface line and a linear velocity profile $\tilde{u}$ in the tangential direction:

$$\tilde{w}(s, \xi, t) = \tilde{w}(s, t) \Big|_{\xi=0}, \quad \tilde{u}(s, \xi, t) = \tilde{u}(s, t) \Big|_{\xi=0} + \xi \omega(s, t)$$

where ξ , the distance from the median line along its normal, $-h/2 < \xi < h/2$;
 s , the length along the median layer line; ω , the angular velocity of normal revolution
(rotation as a solid ensemble).

The shear and velocity vector components are also estimated in the cylindrical coordinate system of the observer, $\mathbf{x0y}$ ($y = 0$, the axis of revolution). Alongside this, the Lagrangian coordinate system rigidly connected with the strained median surface is used in which the components of the stress tensor, $\sigma_\theta, \sigma_\varphi, \sigma_{\theta\xi}$, and the strained tensor, $\varepsilon_\theta, \varepsilon_\varphi, \varepsilon_{\theta\xi}$, are estimated.

In the general case, the shell element is affected both by internal forces and moments and the external pressure P directed along the following directions and integrated over layer thickness:

$$\begin{aligned} \text{Forces:} \quad N_\theta &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_\theta d\xi, \quad N_\varphi = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_\varphi d\xi, \quad Q = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{\theta\xi} d\xi, \\ \text{Moments:} \quad M_\theta &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_\theta \xi d\xi, \quad M_\varphi = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_\varphi \xi d\xi. \end{aligned}$$

As is usually assumed in the theory of thin shells, we will neglect the force normal to the median surface. Then the governing equations of motion reflecting the laws of variation in layer element momentum and angular momentum can be written in the form [66,76]

$$\rho h y \frac{d^2 x}{dt^2} = \frac{d}{ds} (N_\theta y \sin \theta + Q y \cos \theta) - p \cos \theta y,$$

$$\rho h y \frac{d^2 y}{dt^2} = \frac{d}{ds} (N_\theta y \cos \theta + Q y \sin \theta) - N_\varphi + p \sin \theta y,$$

$$\frac{\rho h^3}{12} y \frac{d^2 \varphi}{dt^2} = \frac{\partial}{\partial s} (M_\theta y) - M_\varphi \cos \theta - Q y,$$

$$\text{where } \sin \theta = \frac{\partial x}{\partial s}, \quad \cos \theta = \frac{\partial y}{\partial s}.$$

These equations determine the translational movement and accelerations of the center of mass of an element and the angular acceleration of the particles in rotation about the elemental center of mass.

$$\text{Let } \frac{dx}{dt} = u, \quad \frac{dy}{dt} = v, \quad \frac{d\phi}{dt} = \omega.$$

In the selected coordinates, the relation between the velocity components and the strain rate tensor components is as follows:

$$\frac{d\ell_\theta}{dt} = \frac{\partial u}{\partial s} \sin \theta + \frac{\partial v}{\partial s} \cos \theta + \xi \frac{\partial \omega}{\partial s}, \quad \frac{d\ell_\phi}{dt} = \frac{v}{y} + \xi \frac{\omega}{y} \cos \theta,$$

$$\frac{d\chi}{dt} = 2\ell_{\xi\xi} = -\left(\frac{\partial v}{\partial s} \sin \theta - \frac{\partial u}{\partial s} \cos \theta - \omega\right) \left[1 - \left(\frac{2\xi}{h}\right)^2\right],$$

$$\frac{\partial \omega}{\partial s} = \frac{\partial \chi_\theta}{\partial t}, \quad \frac{\omega}{y} \cos \theta = \frac{\partial \chi_\phi}{\partial t},$$

where χ_θ and χ_ϕ are variations in the shell curvatures. Hooke's law for an elastic thin layer is written in the form [7]

$$\sigma_\theta = \frac{E}{1-\nu^2} (\ell_\theta + \nu \ell_\phi), \quad \sigma_\phi = \frac{E}{1-\nu^2} (\ell_\phi + \nu \ell_\theta), \quad \sigma_{\xi\xi} = G\gamma.$$

In the plastic case, the stress intensity limitations $\tilde{\sigma}$ take place:

$$\tilde{\sigma} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_\theta - \sigma_\phi)^2 + \sigma_\theta^2 + \sigma_\phi^2 + 6\sigma_{\xi\xi}^2}.$$

The system of equations above is integrated under certain initial boundary conditions. Two types of shell end boundary conditions are possible: (1) kinematic, where limitations on displacements are imposed, and (2) static, where acting forces and moments are given.

Along the main direction, the pressure from fluid surrounding the shell is set as the boundary condition. In its turn, the current shell position serves as the boundary condition at numerical integration of the shell written in the Lagrangian–Eulerian coordinates.

2.3.2. Numerical method

The above system of equations is integrated numerically by a “crest”-type explicit difference scheme where velocities and coordinates are specified at the integer points of the difference grid and stresses at the half integer. The difference scheme satisfies the following property: if movement has to be spherically-symmetric by virtue of the setup of the physical problem, then it also remains spherical in the difference solution, i.e., no tensile strains arise

in this case. The stability analysis performed for the planar case by the Neumann method revealed the necessary conditions on the computational step:

$$\tau < c_1 \frac{\Delta s}{c_{ob}} \quad \tau < c_2 \frac{h}{C_{sd}}, \text{ where } c_1, c_2 < 1.$$

We mention a special case important for the Rayleigh-Taylor instability problem, namely where one can neglect the layer material strength. Then the equations of motion of the layer elements, also valid for severe mixing, will be of the form

$$\rho h \frac{\partial^2 x}{\partial t^2} = -p \cos \theta, \quad \rho h \frac{\partial^2 y}{\partial t^2} = p \sin \theta.$$

Here θ is the angle of layer particle tangent inclination to the axis $\mathbf{y}$; ρ, h , current layer element density and thickness.

Then, using the law of conservation of layer particle mass, $dm = \rho_0 h_0 d\xi = \rho h ds$, the considered equation system can be written in the form

$$\frac{\partial^2 x}{\partial t^2} = -a \frac{\partial y}{\partial \xi}, \quad \frac{\partial^2 y}{\partial t^2} = a \frac{\partial x}{\partial \xi}, \quad \text{and} \quad a = \frac{p}{\rho h}.$$

In the simplest case of $\mathbf{a} = \mathbf{const}$, this system is linear with constant coefficients. Therefore, one can obtain the solutions by the method of separation of variables as was done in [61].

III. Some Results of Analytical Studies

This section gives the results of analytical studies of the Rayleigh-Taylor instability problems. Some solutions are obtained by linearizing the equations written in Eulerian variables. These are the solutions for an elastic semi-space (3.1) and a semi-space filled with viscous fluid (3.2). The analytical solutions given in other items of this section are obtained for the equations written in Lagrangian variables.

Particular attention is paid to the solutions for a thin liquid layer. These solutions are used to study the effect of initial perturbation type and shape on instability evolution.

The analytical solutions given in this section can only be obtained using a number of simplifying assumptions (linearization, incompressibility, infinitesimal perturbation amplitudes, etc.). In this connection it is of interest to compare the analytical solutions with the computed results obtained using numerical techniques implementing the complete set of the laws of conservation for a continuum.

The following objectives are achieved in this work:

- Testing the numerical techniques in regimes in which the approximations used to obtain the analytic solutions are valid;
- Determining new laws using numerical calculations and researching the effect of the governing parameters on the process.

In pursuing these objectives, one also tests the limitations under which the analytical solutions were obtained.

3.1. Elastic Semi-Space Boundary Instability [12,13]

3.1.1. Analytical study of perturbation evolution at a liquid-elastic body interface

1. Constant acceleration. Assume that one of the media (the upper medium in Fig. 3.1-1) is an ideal incompressible fluid of density ρ_1 and that the lower medium is an elastic body of density ρ_2 and shear modulus $\mathbf{G}$. The system moves at constant acceleration $\mathbf{g}$ directed along the axis OY. The movement of the upper fluid is described by the following linearized hydrodynamic equations (the fluid is incompressible):

$$\begin{aligned}
u_x + v_y &= 0, \\
u_t + \frac{1}{\rho_1} P_x &= 0 \\
v_t + \frac{1}{\rho_1} P_y + g &= 0.
\end{aligned} \tag{3.1.1}$$

Here $\mathbf{u}, \mathbf{v}$ are the components of the velocity vector along the axes $\mathbf{X}$ and $\mathbf{Y}$, respectively, and $\mathbf{P}$ is pressure; the subscript denotes differentiation with respect to the appropriate variable.

Fig. 3.1-1.

These equations have the solution

$$\begin{aligned}
u &= -\phi_x, \quad v = -\phi_y, \\
P &= P_0 - \rho_1 g y + \rho_1 \phi_t,
\end{aligned} \tag{3.1.2}$$

where P_0 is the average pressure on the media interface, and the function $\phi(\mathbf{x}, \mathbf{y}, t)$ satisfies the equation

$$\phi_{xx} + \phi_{yy} = 0. \tag{3.1.3}$$

Under the assumption that the solution is periodic in $\mathbf{X}$ and that at $\mathbf{y} \rightarrow \infty$ the perturbations should tend to zero, the general solution of Eq. (3.1.3) is of the form

$$\phi = A \cos nx e^{-ny} f(t),$$

where $\mathbf{A}$ is a constant and $\mathbf{f(t)}$ is an arbitrary time function; let $\mathbf{f(t)}=\mathbf{ch}\omega\mathbf{t}$, where ω is constant. For the upper fluid we then have

$$\begin{aligned}\varphi_1 &= Ae^{-ny} \cos(nx) \cdot \text{ch}(\omega t), \\ P_1 &= P_0 - g\rho_1 y + \rho_1 (\varphi_1)_t, \\ u &= -(\varphi_1)_x, v = -(\varphi_1)_y.\end{aligned}\tag{3.1.4}$$

The motion of an elastic body is described by the following system of linearized equations:

$$\begin{aligned}u_x + v_y &= 0, \\ u_t + \frac{1}{\rho_2} P_x &= \frac{G}{\rho_2} (U_{xx} + U_{yy}), \\ v_t + \frac{1}{\rho_2} P_y + g &= \frac{G}{\rho_2} (V_{xx} + V_{yy}),\end{aligned}\tag{3.1.5}$$

where $\mathbf{U}$ and $\mathbf{V}$ are strains along the $\mathbf{X}$ and $\mathbf{Y}$ axes, respectively, and

$$U_t = u, V_t = v.$$

We seek the solution to this system in the form

$$\begin{aligned}u &= -\varphi_{xt} - \psi_{yt}, v = -\varphi_{yt} + \psi_{xt}, \\ P &= P_0 - g\rho_2 y + \rho_2 \varphi_{tt},\end{aligned}$$

where the functions $\varphi(\mathbf{x}, \mathbf{y}, \mathbf{t})$, $\psi(\mathbf{x}, \mathbf{y}, \mathbf{t})$ satisfy the following equations:

$$\begin{aligned}\varphi_{xx} + \varphi_{yy} &= 0, \\ \psi_{tt} &= \frac{G}{\rho_2} (\psi_{xx} + \psi_{yy}).\end{aligned}\tag{3.1.6}$$

The solution to equations (3.1.6) must be periodic in $\mathbf{x}$ and tend to zero as $\mathbf{y} \rightarrow \infty$. The following functions satisfy these requirements:

$$\begin{aligned}\varphi_2 &= Be^{my} \cos(nx) \cdot \text{sh}(\omega t), \\ \psi_2 &= Ce^{my} \sin(nx) \cdot \text{sh}(\omega t),\end{aligned}$$

where $\mathbf{B}$ and $\mathbf{C}$ are constants, and $m^2 = n^2 + \frac{\rho_2 \omega^2}{G}$.

Assume that for $t > 0$ the media interface is described as follows: $y = a(t)\cos nx$, i.e., only the perturbation amplitude varies with time while the wavelength remains invariant.

From the infinitesimal perturbation condition it follows that $\left. \frac{dy}{dt} \right|_{y \approx 0} = v|_{y \approx 0}$.

Hence,

$$\left. \frac{da(t)}{dt} \cos nx \right|_{y \approx 0} = A n e^{-ny} \cos(nx) \cdot \text{ch}(\omega t) \Big|_{y \approx 0} = A n \cos(nx) \cdot \text{ch}(\omega t).$$

Finally,

$$\frac{da}{dt} = A n \cdot \text{ch}(\omega t) \quad \text{or} \quad \frac{d^2 a}{dt^2} = \omega^2 a. \quad (3.1.7)$$

Thus, variation in amplitude of a sinusoidal perturbation on the interface between an elastic body and an ideal fluid is described by a differential equation of the same form as for two ideal fluids; and knowledge of the value ω completely determines the law of growth in the perturbation amplitude.

Determination of the value ω involves the boundary conditions on the media interface; these are the equality of velocities normal to the interface and the equality of the stress tensor components

$$\begin{aligned} v_1 \Big|_{y \approx 0} &= v_2 \Big|_{y \approx 0}, \\ -P_1 \Big|_{y \approx 0} &= \left(-P_2 + 2G \frac{\partial V_2}{\partial y} \right) \Big|_{y \approx 0}, \\ 0 &= 2G \left(\frac{\partial V_2}{\partial x} + \frac{\partial U_2}{\partial y} \right) \Big|_{y \approx 0}. \end{aligned} \quad (3.1.8)$$

From the first equality in system (3.1.8) it follows that:

$$A n \cdot \cos(nx) \cdot \text{ch}(\omega t) = -B \omega n \cdot \text{ch}(\omega t) \cdot \cos(nx) + C n \omega \cdot \text{ch}(\omega t) \cdot \cos(nx).$$

Hence,

$$A = -B \omega + C \omega \quad (3.1.9)$$

Next,

$$\begin{aligned}\frac{\partial V_2}{\partial y} &= (-Bn^2 e^{ny} + Cmne^{my}) \cos(nx) \cdot \text{sh}(\omega t), \\ -P_0 + g\rho_1 y - \rho_1 A e^{-ny} \cos(nx) \omega \cdot \text{sh}(\omega t) &= \\ -P_0 + g\rho_2 y - \rho_2 B \omega^2 e^{ny} \cos(nx) \cdot \text{sh}(\omega t) + 2G[-Bn^2 e^{ny} + Cmne^{my}] \cos(nx) \cdot \text{sh}(\omega t).\end{aligned}$$

Differentiating the last equality with respect to t and using the fact that

$$\frac{dy}{dt} = An \cos(nx) \cdot \text{ch}(\omega t),$$

we obtain

$$A[gn(\rho_1 - \rho_2) - \rho_1 \omega^2] + B[\rho_2 \omega^3 + 2Gn^2 \omega] - C \cdot 2Gmn\omega = 0. \quad (3.1.10)$$

And, finally, the last equality in system (3.1.8) gives

$$B \cdot 2n^2 - C(n^2 + m^2) = 0. \quad (3.1.11)$$

We obtain a system of linear homogeneous equations (3.1.9–3.1.11) in three unknowns $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$.

Setting the determinant of this system equal to zero we have

$$-4Gn^3 m + (n^2 + m^2)(\rho_2 \omega^2 + 2Gn^2) + [gn(\rho_1 - \rho_2) - \rho_1 \omega^2](n^2 - m^2) = 0.$$

After simple transformations and using the fact that $m^2 = n^2 + \frac{\rho_2 \omega^2}{G}$,

we obtain the following equation in $\omega : z = \omega^2$,

$$\left(z + \frac{2G}{\rho_2} n^2\right)^2 = -\frac{\rho_1}{\rho_2} z^2 + \frac{4G^2}{\rho_2^2} n^3 m - gnz \frac{\rho_2 - \rho_1}{\rho_2}. \quad (3.1.12)$$

In the general case Eq. (3.1.12) is of the third degree in $\mathbf{z}$ (one root of the equation is known: $\mathbf{z}=0$).

To determine the law of growth of the perturbation amplitude, it is necessary to study the real roots of this equation. For any given media parameters $\rho_1, \rho_2, \mathbf{G}$ and perturbation wavelength, the real root of Eq. (3.1.12) can be found numerically.

Assume that $\rho_1 \ll \rho_2$. Then, analyzing the roots of Eq.(3.1.12), we find that for

$$-gn \frac{\rho_2 - \rho_1}{\rho_2} > \frac{2G}{\rho_2} n^2 \quad (3.1.13)$$

this equation has a positive root $z_1 > 0$; otherwise the real root is negative.

Thus, in contrast to the situation for ideal incompressible fluids where a perturbation amplitude grows exponentially for any perturbation wavelength if $g(\rho_2 - \rho_1) < 0$, in this case the amplitude grows exponentially only if the perturbation wavelength is greater than some critical wavelength that depends on media parameters and applied acceleration. From Eq. (3.1.13) we write the wavelength condition

$$\lambda > \lambda_{cr} = \frac{4\pi G}{g(\rho_1 - \rho_2)}. \quad (3.1.14)$$

Behavior of perturbations with wavelengths less than critical will be of an oscillatory character.

2. Pulsed acceleration. We assume that the media interface acceleration is of the following form:

$$g(t) = \begin{cases} g_1 \neq 0 & 0 \leq t \leq t_1, \\ 0 & t > t_1, \end{cases} \quad (3.1.15)$$

with g being sufficiently large (tends to ∞ with $t_1 \rightarrow 0$).

Within the segment $0 \leq t \leq t_1$ in Eq. (3.1.12) one can neglect the terms which do not contain g (g_1 is very large). We obtain

$$z = \omega^2 = -g_1 n \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1},$$

and the solution to Eq. (3.1.7) when $g(\rho_2 - \rho_1) < 0$ is of the form

$$a_1(t) = a_0 \cdot \text{ch}(\omega t), \omega = \sqrt{\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}} g_1 n.$$

If $t > t_1$ Eq. (3.1.12) reduces to the following (consider the case $\rho_1 \ll \rho_2$):

$$\left(z + \frac{2G}{\rho_2} n^2 \right)^2 = \frac{4G^2}{\rho_2^2} n^4 \sqrt{1 + z \frac{\rho_2}{G n^2}}.$$

The real root of this equation is $z = -0.9126 \frac{G}{\rho_2} n^2$. Thus, in this case, we have $z = \omega^2 < 0$ and the general solution to Eq.(3.1.7) at $t > t_1$ is of the form

$$a_{II}(t) = c_1 \cos \omega_1 t + c_2 \sin \omega_1 t,$$

where

$$\omega_1 = \sqrt{0.9126 \frac{G}{\rho_2} n^2}.$$

To estimate the constants c_1 and c_2 we equate the values of the amplitudes and their first derivatives at $t = t_1$:

$$\begin{aligned} a_I(t_1) &= a_{II}(t_1), \\ \left. \frac{da_I}{dt} \right|_{t_1} &= \left. \frac{da_{II}}{dt} \right|_{t_1}. \end{aligned}$$

We obtain:

$$\begin{aligned} C_1 &= a_0 \left[\cosh \omega_1 t_1 \cos \omega_1 t_1 - \frac{\omega}{\omega_1} \sinh \omega_1 t_1 \sin \omega_1 t_1 \right], \\ C_2 &= a_0 \left[\cosh \omega_1 t_1 \sin \omega_1 t_1 + \frac{\omega}{\omega_1} \sinh \omega_1 t_1 \cos \omega_1 t_1 \right]. \end{aligned}$$

Now, making $g_1 \rightarrow \infty$ and $t_1 \rightarrow 0$, as $g_1 t_1 = v$ remains constant, we obtain

$$c_1 = a_0, c_2 = a_0 \frac{nv}{\omega_1}.$$

Finally, we have

$$a(t) = a_0 \left[\cos \omega_1 t + \frac{vn}{\omega_1} \sin \omega_1 t \right], \quad (3.1.16)$$

which we rewrite as

$$a(t) = \frac{a_0}{\cos \varphi_0} \cos(\omega_1 t - \varphi_0), \quad (3.1.17)$$

where

$$\varphi_0 = \arctg \frac{vn}{\omega_1}, \quad \omega_1 = n \sqrt{\frac{0.9126G}{\rho_2}}.$$

Thus, when one of the media is an elastic body, pulsed acceleration of the interface between two media causes perturbation behavior that is of an oscillatory character for any initial perturbation wavelength; the oscillation period and amplitude depend on the media parameters and perturbation wavelength as given in relation (3.1.17).

3.1.2. Numerical study of perturbation growth at a fluid-elastic semi-space interface

The computations discussed in this section were performed using the numerical codes Sigma and Sigma VII (see Section II) [57, 40, 41].

The numerical study of the effect of the elastic property of the medium on the interface perturbation growth was performed on the following problem:

The band **ABCD** [see Fig. 3.1-1 above Eq. (3.1.2)] is composed of two materials of different density; one volume, **DCMP**, is filled with an elastic medium of density $\rho_{02}=10 \text{ g/cm}^3$ and shear modulus **G**; the other volume, **ABMP**, is filled with an ideal fluid of density $\rho_{01}=1 \text{ g/cm}^3$. Initially on the boundary **MP** there is a sinusoidal surface perturbation given by

$$y = a_0 \cos nx, \quad a_0 = 0.04 \text{ cm}, \quad n = \frac{2\pi}{\lambda},$$

$$\lambda = 2 \text{ cm}, \quad n = 3.14 \frac{1}{\text{cm}}.$$

The surfaces **AD** and **BC** are rigid walls, i.e., the velocities normal to them are equal to zero. The band width (the distance **BA**) equals one-half of the perturbation wavelength.

The equations of state of materials were taken in the form $\mathbf{P} = \mathbf{C}_0^2 (\rho - \rho_0)$, where $\rho_0 = \rho_{01}$ for the upper and $\rho_0 = \rho_{02}$ for the lower medium. Analytical solutions were obtained in the previous subsection under the assumption of incompressibility. In the computations, the material incompressibility was simulated by setting sound speeds c_0 high compared to the mass velocities characteristic of the problem ($c_0 \gg u$). The value $c_0=40 \text{ km/s}$ was used.

3. Constant acceleration. Acceleration was set equal to 10^{11} cm/s^2 . A series of problems were computed where the elastic medium shear modulus **G** was varied; the following variants were computed:

$$G=\{50,100,200,500,1500\} \text{ kbar}.$$

In 3.1.1, it was shown that for this case the perturbation amplitude growth is described by Eq. (3.1.7) where $z=\omega^2$ is the real root of Eq. (3.1.12).

Table 3.1-1 gives the real roots of this equation z_{an} found by the segment-halving method using the parameters $\rho_1=1 \text{ g/cm}^3$, $\rho_2=10 \text{ g/cm}^3$, $n=3.14 \text{ 1/cm}$, and the various values of the shear modulus G . We find that for $G=\{50,100\}$ kbar, Eq. (3.1.12) has a positive root z and, hence, at these values of the shear modulus G , the perturbation amplitude grows exponentially with time according to the equation

$$a(t) = a_0 \cdot \text{ch}(\omega_{an} t), \quad \omega_{an} = \sqrt{z_{an}}$$

At $G=\{200, 500, 1500\}$ kbar, Eq.(3.1.12) has a negative root, i.e., in this case the interface oscillates according to the law

$$a(t) = a_0 \cos(\omega_{an} t), \quad \omega_{an} = \sqrt{-z_{an}}.$$

Table 3.1-1 gives the values of the oscillation period ($T_{an} = 2\pi/\omega_{an}$), corresponding to various values of the shear modulus. The results of the numerical computations are presented in Figs. 3.1-2 and 3.1-3 and in Table 3.1-1. Fig. 3.1-2 shows the time dependence of the ratio between the amplitude $a(t)$ and the initial perturbation amplitude a_0 . In the computations it was found that for shear moduli $G=\{50,100\}$ kbar, the perturbations are unstable and the perturbation amplitude grows exponentially (curves 2 and 3).

Table 3.1-1.

G [kbar]	z_{an}	ω_{an} [10^5 1/s]	ω_{comp} [10^5 1/s]	T_{an} [10^{-5} s]	T_{comp} [10^{-5} s]
50	13.84	3.72	3.88	—	—
100	5.71	2.39	2.45	—	—
200	-6.57	2.56	2.51	2.45	2.5
500	-35.87	5.99	5.87	1.049	1.07
1500	-125.47	11.2	11.02	0.56	0.57

Fig. 3.1-2.

Fig. 3.1-3.

Table 3.1-1 gives the values of ω_{comp} (calculated as the tangent of $\text{arcch}(a(t)/a_0)$ graph straight line slope (Fig. 3.1-3) to the axis t) found in the computations with these shear modulus values.

At $G=\{200, 500, 1500\}$ kbar, oscillations of the media interface were observed in the computations (curves 4, 5, 6 in Fig. 3.1-2). Table 3.1-1 also gives the values of the periods T_{comp} of these oscillations.

4. Pulsed acceleration. A plane shock wave of 400 kbar amplitude falls onto the curved boundary PM of the elastic body DCMP. The initial conditions in the region are $\rho_0=10 \text{ g/cm}^3$, $u_0=0$, $P_0=0$, and the equation of state is

$$P = c_0^2 (\rho - \rho_0), c_0 = 10 \text{ km/s.}$$

In the analytic treatment, the variation in the perturbation amplitude is described by Eq. (3.1.17), i.e., in the case under consideration the boundary is predicted to oscillate for any value of shear modulus G ; the oscillation period is

$$T_{\text{an.}} = \frac{2\pi\sqrt{\rho_0}}{\sqrt{0.9126Gn}}.$$

Table 3.1-2 gives the values T_{an} for various shear modulus values.

Figure 3.1-4 presents the time-dependent values $a(t)/a_0$ obtained in the computations with various values of the shear modulus G . Table 3.1-2 gives the values of the oscillation period $T_{\text{comp.}}$ found in the computations.

Table 3.1-2.

G [kbar]	$T_{\text{an.}} [10^{-5} \text{ s}]$	$T_{\text{comp}} [10^{-5} \text{ s}]$
50	2.96	—
100	2.09	—
500	0.936	0.94
700	0.79	0.78
1500	0.54	0.56

Fig. 3.1-4.

By comparing these results, one concludes that there is good agreement between the analytic solutions and the data obtained from numerical solution of the general continuum mechanics equations.

3.2. Instability of the Interface Between Viscous and Ideal Fluids

3.2.1. Analytical study of perturbation evolution on the interface between viscous and ideal fluids

The interface instability for constant acceleration and small perturbation amplitudes was studied in paper [75], where both media were assumed to be viscous incompressible fluids. In this case the perturbation amplitude satisfies the equation

$$\frac{d^2 a}{dt^2} = \omega^2 a, \quad a(0) = a_0, \quad \left. \frac{da}{dt} \right|_{t=0} = 0. \quad (3.2.1)$$

Paper [75] gives the tenth-degree equation for the value ω , whose roots are hard to study. In contrast to paper [75], we are interested in the evolution of small initial perturbations upon shock wave passage through the interface. Thus we restrict ourselves to the case where one of the media is viscous and the second is an ideal fluid. For constant acceleration under these assumptions, it is possible to show that ω satisfies the fourth-degree algebraic equation

$$\left(\omega + \frac{2\mu n^2}{\rho_2} \right)^2 - \frac{4\mu^2}{\rho_2^2} n^3 z + \frac{\rho_1}{\rho_2} \omega^2 - \frac{\rho_1 - \rho_2}{\rho_2} g n = 0, \quad (3.2.2)$$

where $z^2 = n^2 + \frac{\omega}{v}$, and $v = \mu/\rho_2$ is the kinematic viscosity. To find the roots of Eq. (3.2.2), one may use numerical methods.

Consider the case where $g(\rho_2 - \rho_1) < 0$, which corresponds to the unstable situation for ideal fluids. It is easy to show that Eq. (3.2.2) always has a positive real root ω_1 with $0 < \omega_1 < \sqrt{gn \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}}$. Let the function $f(\omega)$ equal the l.h.s. of (3.2.2.). When $\omega = 0$, this expression equals $-\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}gn$; i.e., in the case of $g(\rho_2 - \rho_1) < 0$ we have $f(0) < 0$. The function $f(\omega)$ is continuous and $f(\omega) > 0$ if $\omega > 0$. It is easy to show that $f(\omega_1) > 0$ if $\omega_1 = \sqrt{gn \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}}$. It follows then that the above statement about the existence of a positive real root for Eq.(3.2.2) is valid.

Thus, when one of the media is a viscous fluid, at $g(\rho_2 - \rho_1) < 0$ exponential growth in the perturbation amplitude will take place for any perturbation wavelength, but the growth rate will be less than when both fluids are ideal.

Next, let the acceleration be pulsed, that is be of the following form:

$$g(t) = \begin{cases} g \neq 0 & 0 \leq t \leq t_1; \\ 0 & t > t_1. \end{cases}$$

Within the segment $0 \leq t \leq t_1$ in Eq. (3.2.2), one can neglect the terms which do not contain g (g is very high). Then we have $\omega^2 = Rgn$, where $R = (\rho_1 - \rho_2)/(\rho_1 + \rho_2)$ and the solution to Eq.(3.2.1) is of the form (the case of $\omega^2 > 0$ being considered):

$$\begin{aligned} a_1(t) &= a_0 \cosh \omega t; \\ \omega &= \sqrt{\frac{\rho_2 - \rho_1}{\rho_1 + \rho_2}} gn. \end{aligned} \tag{3.2.3}$$

Let $\rho_1 \ll \rho_2$, then at $t > t_1$, upon simple rearrangement, Eq. (3.2.2) becomes of the form

$$(\omega + 2vn^2)^2 - 4v^2n^4 \sqrt{1 + \frac{\omega}{n^2v}} = 0.$$

The real roots of this equation are $\omega_1 = 0$, $\omega_2 = -0.9126vn^2$. Hence, at $t > t_1$ the solution to Eq. (3.2.1) is of the form

$$a_{\parallel}(t) = c_1 + c_2 e^{-0.9126\omega n^2 t}. \quad (3.2.4)$$

The constants c_1 and c_2 are found from the condition of solution smoothness at $t=t_1$:

$$\begin{aligned} a_1(t_1) &= a_{\parallel}(t_1), \\ \left. \frac{da_1}{dt} \right|_{t_1} &= \left. \frac{da_{\parallel}}{dt} \right|_{t_1}. \end{aligned} \quad (3.2.5)$$

We have then

$$\begin{aligned} c_1 &= a_0 \operatorname{ch}(\omega t_1) + a_0 \omega \operatorname{sh}(\omega t_1) \frac{1}{0.9126\omega n^2} \\ c_2 &= -\frac{a_0 \omega}{0.9126\omega n^2} \operatorname{sh}(\omega t_1) \end{aligned} \quad (3.2.6)$$

where

$$\omega^2 = gnR.$$

Assume that the acceleration is of δ -function character, i.e., in the segment $0 \leq t \leq t_1$: $g \rightarrow \infty$ and the segment $[0, t_1]$ itself tends to 0 ($t_1 \rightarrow 0$), but the complete increment in velocity imparted in the segment $0 \leq t \leq t_1$ remains finite and equal to velocity $u = gt_1$.

We obtain

$$c_1 = a_0 \left(1 + \operatorname{Run} \frac{1}{0.9126\omega n^2} \right), \quad c_2 = -\frac{a_0 \operatorname{Run}}{0.9126\omega n^2}.$$

Finally we have

$$\begin{aligned} a(t) &= a_0 \left\{ 1 + \frac{1 - e^{-0.9126\omega n^2 t}}{0.9126\omega n^2} \operatorname{Run} \right\}, \\ \frac{da}{dt} &= a_0 \operatorname{Run} e^{-0.9126\omega n^2 t}. \end{aligned} \quad (3.2.7)$$

Formulas (3.2.7) give the solution of the problem. By comparing formulas (3.2.7) with the formula obtained by Richtmyer in paper [2] for two inviscid incompressible gases

$$\frac{da}{dt} = a_0 \operatorname{Run}, \quad a(0) = a_0,$$

one can analyze the effect on the interface instability of having viscosity in one of the media during shock wave passage. Evidently, at $t \rightarrow \infty$ viscosity completely stabilizes the perturbation growth $\left(\frac{da}{dt} \rightarrow 0\right)$ with the stabilization degree growing with physical viscosity ν (i.e., in more viscous fluids the perturbation growth ceases faster). Apparently, the perturbation stabilization increases with increasing wavenumber n (shorter wavelength perturbations die away faster).

3.2.2. Numerical study of evolution of perturbations on the interface between viscous and ideal fluids

The computations were performed with the Sigma and Sigma-VII techniques [57, 40, 41]. The problem formulated in 3.1 was reconsidered with the region MNDC assumed to be viscous fluid with the dynamic viscosity factor μ ; the region MNAB was ideal fluid.

The following variants were computed: $\mu = \{2, 4, 7\} 10^3$ pz.

1. Constant acceleration. Acceleration g was taken to be equal to 10^{11}cm/s^2 . According to the conclusions made above in the case under consideration, the perturbation amplitude behavior is described by differential equation (3.2.1) where ω is a real root of Eq. (3.2.2). Table 3.2-1 gives the values of the real root of Eq.(3.2.2) for the problem parameters used and for various values of the viscosity factor μ .

The results of the numerical computations are presented in Fig. 3.2-1 and Table 3.2-1. Fig. 3.2-1 shows the time dependence of $a(t)/a_0$. As in the analytic treatment, we found that when one of the media is viscous the perturbation amplitude grows exponentially (at $g(\rho_2 - \rho_1) < 0$), but the amplitude growth rate is less than for two ideal fluids. Table 3.2-1 also gives the computed values of ω_{ras} .

Table 3.2-1.

$\mu [10^3 \text{pz}]$	$\omega_{\text{an}} [10^5 1/\text{s}]$	$\omega_{\text{ras}} [10^5 1/\text{s}]$
0	5.07	5.02
2	3.31	3.42
4	2.45	2.6
7	1.72	1.85

Fig. 3.2-1.

2. Pulsed acceleration. A plane shock wave of 400 kbar arrives at the surface PM of the viscous fluid filling the region DCMP (Fig. 3.1-1). The initial conditions in the region are $\rho_0 = 10 \text{ g/cm}^3$, $u_0 = 0$, $P_0 = 0$, and the equation of state is $P = c_0^2 (p - \rho_0)$, $c_0 = 10 \text{ km/s}$.

The computed results are presented in Fig. 3.2-2. In the computations it was found that with time the presence of viscosity leads to cessation of the perturbation amplitude growth, doing so faster for higher values of the physical viscosity.

Fig. 3.2-2.

3.2.3. The effect of approximation viscosity in computing Rayleigh-Taylor instability

A difference scheme approximation is known to be equivalent to the original differential equation system to within the terms $O(h^\alpha, \tau^\beta)$; as compared to the initial differential equations, the difference scheme contains “superfluous” terms whose effect on the solution is similar to the effect of viscosity. The effect of this “approximation” viscosity on the numerical solution can be analyzed using the solution which was found above. For this purpose we considered the evolution of a sinusoidal perturbation ($y = a_0 \cos nx$; $\lambda = 4$ cm; $a_0 = 0.2$ cm) on the interface between helium ($\rho = 0.167 \cdot 10^{-3} \text{ g/cm}^3$) and air ($\rho = 1.205 \cdot 10^{-3} \text{ g/cm}^3$) after a plane shock wave of $u = 0.349$ km/s mass velocity propagating over helium passed through the interface.

Fig. 3.2-3.

Figure 3.2-3 gives the time dependence of the perturbation amplitude obtained from the computations in the gas-dynamics approximation with the Sigma technique in the Lagrangian variables (curve 1) and with the EGAK technique in the Eulerian variables (curve 2). In the computations the number of the points is the same: 20 points per wavelength.

The solution obtained with the Sigma technique practically coincides with the Richtmyer solution [2] for inviscid fluids. Curve (3) is the solution obtained by formula (3.2.7) when air is assigned the viscosity $\mu = \nu\rho = 1$ pz (real air viscosity is 10^4 times lower). By comparing curves (2) and (3), one can conclude that the value of the approximation viscosity in the computation with the EGAK technique at a given number of points is on the order of 1 pz. The same estimate results from the analysis of the first differential approximation of the EGAK technique difference scheme. The approximation viscosity factor turns out to be equal

$$\text{to } \mu = \frac{h\rho u}{4} = 1.3 \text{ pz, where } h \text{ is the linear size of the computational cell.}$$

Figure 3.2-3 also gives the result (curve 4) of the 2D computation with the Sigma technique where air was assigned a viscosity of 1 pz.

This treatment is presented to illustrate the statements at the beginning of Section III concerning the usefulness of the simplest analytic solutions for testing numerical techniques. The utility arises from the fact that solutions of the hydrodynamic instability problems are in essence sensitive to various kinds of perturbations, including errors of the difference scheme approximation.

3.3. Elastic-Plastic Layer Instability

In some cases it is possible to describe **R-T** instability evolution in terms of the elastic-plastic model [44,67]. To obtain an exact analytic solution in the elastic-plastic approximation seems to be impossible even for a semi-space.

The approximate solution of the problem of arbitrarily thick incompressible elastic-plastic layer **R-T** instability presented below is obtained on the basis of a set of simplifying assumptions [15].

3.3.1. Equations of motion of a thin ideal elastic layer

Assume that a material layer is incompressible and possesses ideal elasticity. The thickness of the layer **H** is small compared to both the characteristic wavelength λ and the perturbation amplitude. Select the coordinate system with the origin at some point of the median surface and the axis **OY** directed along its normal. The plane **XOZ** coincides with the median surface of the non-strained layer. Oscillations of such a layer about the equilibrium position can be described by the equation for thin shells in the Kirchhoff-Love approximation [4]. For plane strain and in the absence of pressure the equations are of the following form (we assume the plate point displacements to be small compared to the plate thickness):

$$\begin{aligned} \rho H \frac{\partial^2 U_x}{\partial t^2} &= \frac{EH}{1-\nu^2} \frac{\partial^2 U_x}{\partial x^2}, \\ \rho H \frac{\partial^2 U_y}{\partial t^2} &= -\frac{EH^3}{12(1-\nu^2)} \frac{\partial^4 U_y}{\partial x^4}, \end{aligned} \tag{3.3.1}$$

where: U_x, U_y , displacements along the axes **OX** and **OY**; **E**, Young modulus; ν , Poisson factor equal to 0.5 in this case.

Fig. 3.3-1.

Consider the equations of motion of the layer in the frame of reference moving at acceleration g . In doing so, take into account the force of gravity and the constant pressure $P = \rho g H$ applied to the layer, Fig. 3.3-1.

Then in the direction OX there is additional loading

$$F_x = P_x = P \sin(\alpha) \approx -P \frac{\partial U_y}{\partial x} = -\rho g H \frac{\partial U_y}{\partial x}. \quad (3.3.2)$$

In the direction OY the additional loading equals

$$F_y = P \cos(\alpha) - \rho g (H + dH) \approx -\rho g \cdot dH. \quad (3.3.3)$$

In Eqs. (3.3.2) and (3.3.3) we neglected the terms higher than first order infinitesimal. By virtue of layer incompressibility,

$$dH \approx -H \frac{\partial U_x}{\partial x}. \quad (3.3.4)$$

Finally, the system of equations of motion of the layer accounting for (3.3.2-3.3.4) can be written in the following form:

$$\begin{aligned} \frac{\partial^2 U_x}{\partial t^2} &= 4C^2 \frac{\partial^2 U_x}{\partial x^2} - g \frac{\partial U_y}{\partial x}, \\ \frac{\partial^2 U_y}{\partial t^2} &= g \frac{\partial U_x}{\partial x} - \frac{(CH)^3}{3} \frac{\partial^4 U_y}{\partial x^4} \end{aligned} \quad (3.3.5)$$

where $C = (G/\rho)^{1/2} = (E/(3\rho))^{1/2}$, sound speed of shear waves for the incompressible layer ($\nu=0.5$); G , shear modulus.

Assume that the initial perturbation source is the layer thickness variation given by

$$H=H_0+\Delta H\cdot\sin(Kx), \quad (3.3.6)$$

$$(\Delta H \ll H \ll \lambda = 2\pi/K).$$

The constant pressure applied to the layer is equal to

$$P=\rho \, g \, H_0 . \quad (3.3.7)$$

Accounting for (3.3.6-3.3.7) in (3.3.2-3.3.4) and neglecting the terms higher than first order infinitesimal, the equations of motion of the layer (3.3.5) can then be written:

$$\begin{aligned} \frac{\partial^2 U_x}{\partial t^2} &= 4C^2 \frac{\partial^2 U_x}{\partial x^2} - g \frac{\partial U_y}{\partial x} \\ \frac{\partial^2 U_y}{\partial t^2} &= g \frac{\partial U_x}{\partial x} - \frac{(CH)^2}{3} \frac{\partial^4 U_y}{\partial x^4} - g \frac{\Delta H}{H} \sin(K \cdot x) \end{aligned} \quad (3.3.8)$$

The initial conditions of the problem are as follows:

$$\begin{aligned} U_x(0) &= U_y(0) = 0, \\ \dot{U}_x(0) &= \dot{U}_y(0) = 0. \end{aligned} \quad (3.3.9)$$

It should be noted that if in expression (3.3.8) the sound speed of the flexural waves is taken to be zero, we obtain the classic **Ott** equation for small perturbations [61].

Equations of motion for other perturbation sources can be obtained similarly. However, this does not lead to basic changes in the problem.

3.3.2. *Solutions to the equations of motion of a thin layer*

We seek the solution to equations (3.3.8) with initial conditions (3.3.9) in the form

$$\begin{aligned} U_x &= \left(a_{x_1} (\cos(\omega_1 t) - 1) + a_{x_2} (\cos(\omega_2 t) - 1) \right) \cos(Kx), \\ U_y &= \left(a_{y_1} (\cos(\omega_1 t) - 1) + a_{y_2} (\cos(\omega_2 t) - 1) \right) \sin(Kx). \end{aligned} \quad (3.3.10)$$

Apparently for (3.3.10) conditions, (3.3.9) holds automatically. Substituting (3.3.10) in (3.3.8) and solving for constants a_{x_i} , a_{y_i} we obtain

$$\begin{aligned}
 U_x &= \frac{\Delta H}{H} \frac{K g^2}{\omega_1^2 - \omega_2^2} \left[\frac{\cos(\omega_1 t) - 1}{\omega_1^2} - \frac{\cos(\omega_2 t) - 1}{\omega_2^2} \right] \cos(Kx), \\
 U_y &= \frac{\Delta H}{H} \frac{g}{\omega_1^2 - \omega_2^2} \left[\left(1 - \frac{4(KC)^2}{\omega_1^2} \right) (\cos(\omega_1 t) - 1) \right. \\
 &\quad \left. - \left(1 - \frac{4(KC)^2}{\omega_2^2} \right) (\cos(\omega_2 t) - 1) \right] \sin(Kx)
 \end{aligned} \tag{3.3.11}$$

Natural oscillation frequencies are estimated from the solution to equation system (3.3.5) in the form

$$\begin{aligned}
 U_x &= \cos(Kx) \cdot \cos(\omega t), \\
 U_y &= \sin(Kx) \cdot \cos(\omega t).
 \end{aligned} \tag{3.3.12}$$

Substituting (3.3.12) into (3.3.5) we obtain

$$\begin{aligned}
 (4(KC)^2 - \omega^2) a_x + K g a_y &= 0, \\
 K g a_x + \left(\frac{(KH)^2}{3} (KC)^2 - \omega^2 \right) a_y &= 0.
 \end{aligned} \tag{3.3.13}$$

Equating the discriminant of system (3.3.13) to zero, we arrive at the equation for natural frequencies

$$\omega^4 - \omega^2 (KC)^2 \left(4 + \frac{(KH)^2}{3} \right) + \frac{4(KC)^4 (KH)^2}{3} - (K g)^2 = 0. \tag{3.3.14}$$

The roots of this equation are as follows:

$$\begin{aligned}
 \omega_{1,2}^2 &= \frac{1}{2} \left[(KC)^2 \left(4 + \frac{(KH)^2}{3} \right) \right. \\
 &\quad \left. \pm \sqrt{(KC)^4 \left(4 + \frac{(KH)^2}{3} \right)^2 + 4 \left((K g)^2 - \frac{4(KC)^4 (KH)^2}{3} \right)} \right].
 \end{aligned} \tag{3.3.15}$$

Thus, two oscillation frequencies, ω_1 and ω_2 , are realized of which the smallest can become equal to zero or turn out to be imaginary. The imaginary value of ω_2 corresponds to the solution that tends to infinity with time:

$$U_x \sim U_y \sim \exp(|\omega_2|t) .$$

The wavelength correspondent with the zero oscillation frequency determines the stability boundary and is called a critical wavelength, λ_{cr} . Thus, the condition of ideally elastic thin layer instability can be written in the following form:

$$\lambda > \lambda_{cr}, \quad K < K_{cr} .$$

From expression (3.3.14) it follows that the frequency ω equals zero if the following equation holds:

$$(Kg)^2 - \frac{4}{3}(KC)^4(KH)^2 = 0. \quad (3.3.16)$$

From this result we find:

$$\begin{aligned} \lambda_{cr} &= 2\pi \left(\frac{4}{3}\right)^{1/4} \sqrt{\frac{C^2 H}{g}}, \\ K_{cr} &= \left(\frac{3}{4}\right)^{1/4} \sqrt{\frac{g}{C^2 H}}. \end{aligned} \quad (3.3.17)$$

Thus, $\lambda_{cr} \sim H^{1/2}$, i.e., the critical wavelength decreases with decreasing layer thickness.

3.3.3. *Instability of an arbitrarily thick ideally elastic layer*

Apparently it is impossible to obtain an accurate analytic solution of the problem of the instability of an arbitrarily thick layer even with the simplifying assumptions taken in this paper. As is known, for the gravitational instability of an ideal fluid only a part of the layer adjacent to the interface of heavy and light materials is involved in the motion. The thickness of this layer is comparable to the perturbation wavelength $H_{ef} \sim \lambda$. The material displacement distribution over the thickness is of exponential character:

$$U_y(x, y, t) = \exp(-Ky) \cdot \text{ch}(\sqrt{Kg} t) \cdot \cos(Kx). \quad (3.3.18)$$

Therefore, one may assume that for the acceleration of a strong material layer the character of the instability evolution is affected not by the whole layer thickness but only by a small thickness H_{ef} (the “effective” thickness of the layer involved in motion). Evidently, as the thickness of the layer decreases, H_{ef} tends to its full thickness, i.e., the following asymptotic relations must hold for H_{ef} :

$$H_{ef} \rightarrow H \text{ if } \lambda/H \rightarrow \infty, \quad (3.3.19)$$

$$H_{ef} \rightarrow \lambda \text{ if } \lambda/H \rightarrow 0.$$

Introducing the effective layer thickness $\mathbf{H}_{\text{ef}}$, we extend the solution for a thin plate obtained in the previous subsection to the case of an arbitrarily thick layer. Taking into account the exponential character of the material displacement distribution over depth (3.3.18), we reasonably consider the following dependence as the expression for $\mathbf{H}_{\text{ef}}$:

$$\mathbf{H}_{\text{ef}} = \frac{(1 - \exp(-\alpha \mathbf{K} \mathbf{H}))}{\alpha \mathbf{K}} \quad (3.3.20)$$

where α , some constant. It is evident that for this dependence, conditions (3.3.19) are satisfied. We use equation (3.3.16) to determine the critical wave number for a thin layer. Substituting expression (3.3.20) for $\mathbf{H}_{\text{ef}}$ instead of $\mathbf{H}$ and making simple rearrangements we obtain

$$\frac{\mathbf{g}}{2\mathbf{C}^2} - \frac{\mathbf{K}(1 - \exp(-\alpha \mathbf{K} \mathbf{H}))}{\sqrt{3}\alpha} = 0.$$

The value of α is determined by the requirement that with increasing $\mathbf{H}$ the wave number $\mathbf{K}$ should approach its exact value corresponding to the solution for the semi-space [12,13] $\mathbf{H}_{\infty} = \mathbf{g} \cdot (2\mathbf{C})^{-1}$. Hence, $\alpha = (1/3)^{1/2}$. Thus, the expression for $\mathbf{H}_{\text{ef}}$ is finally of the form

$$\mathbf{H}_{\text{ef}} = \frac{\sqrt{3}}{\mathbf{K}} \left(1 - \exp\left(-\frac{\mathbf{K} \mathbf{H}}{\sqrt{3}}\right) \right), \quad (3.3.21)$$

and for the critical parameters λ_{cr} and $\mathbf{K}_{\text{cr}}$ we obtain the transcendental equations

$$\begin{aligned} \mathbf{K}_{\text{cr}} \left(1 - \exp\left(-\frac{\mathbf{K}_{\text{cr}} \mathbf{H}}{\sqrt{3}}\right) \right) &= \mathbf{K}_{\infty} \equiv \frac{\mathbf{g}}{2\mathbf{C}^2}, \\ \lambda_{\text{cr}} &= \lambda_{\infty} \left(1 - \exp\left(-\frac{2\pi \mathbf{H}}{\sqrt{3}\lambda_{\text{cr}}}\right) \right) \equiv \frac{4\pi \mathbf{C}^2}{\mathbf{g}} \left(1 - \exp\left(-\frac{2\pi \mathbf{H}}{\sqrt{3}\lambda_{\text{cr}}}\right) \right). \end{aligned} \quad (3.3.22)$$

Figure 3.3-2 gives in dimensionless coordinates the critical wavelength dependence on layer thickness expressed in (3.3.22) and the results for numerical computations of instability in an ideally elastic incompressible layer using the 2D Lagrangian technique. In the computations, layer incompressibility was approximated by using a 10-fold increase in the volume sound speed relative to the average value for a given material. As can be seen from Fig. 3.3-2, the obtained solution (3.3.22) agrees with the results of numerical computations in the approximation of ideal elasticity.

Fig. 3.3-2. Critical wavelength vs layer thickness.

3.3.4. Instability of an elastic-plastic layer

In many cases, the elastic-plastic material model provides a satisfactory description of solid body behavior in the act of instability [14,67]. Therefore it would be of interest to find the instability criterion in the elastic-plastic approximation.

Evidently, at $\lambda > \lambda_{cr}$ the system is absolutely unstable. In the instability region ($\lambda < \lambda_{cr}$) the layer oscillation amplitude and, hence, the strain and stress level depend on the initial perturbation amplitude and wavelength. In real materials, stresses can not increase without limit. Therefore, the necessary condition for instability evolution is the transition to the plastic strain region. Let us assume that if the stress level is higher than the material yield strength, then the perturbation growth continues unlimitedly and estimate the stress σ_i on the layer surface in the maximum flexion region. In this case the strain tensor components $\epsilon_{xy} = \epsilon_{yz} = \epsilon_{xz} = \epsilon_z = 0$ and, due to incompressibility, $\epsilon_x + \epsilon_y + \epsilon_z = 0$, so that

$$\sigma_i = 2\sqrt{3} G |\epsilon_x|.$$

As is known [68], in thin plates at small deflections the determining factor is the flexion strain. From (3.3.11) it follows that, in the region under consideration, the maximum strain on the plate surface due to flexion is

$$|\epsilon_x| = 0.5H \left| \frac{\partial^2 U_y}{\partial x^2} \right| \approx \frac{4(KC)^2 K^2 \Delta H g}{\omega_1^2 \omega_2^2}. \quad (3.3.23)$$

From the natural frequency equation (3.3.14) it follows that

$$\omega_1^2 \omega_2^2 = 4(\mathbf{KC})^4 \left(\frac{(\mathbf{KH}_{ef})^2}{3} - \left(\frac{\mathbf{K}_\infty}{\mathbf{K}} \right)^2 \right). \quad (3.3.24)$$

Accounting for (3.3.20) in expression (3.3.24) gives

$$\omega_1^2 \omega_2^2 = 4(\mathbf{KC})^4 \left(\left(1 - \exp\left(-\frac{\mathbf{KH}}{\sqrt{3}}\right) \right)^2 - \left(\frac{\mathbf{K}_\infty}{\mathbf{K}} \right)^2 \right). \quad (3.3.25)$$

Thus,

$$|\varepsilon_x| \approx \frac{\Delta \mathbf{H} \mathbf{g}}{\mathbf{C}^2} \left(\left(1 - \exp\left(-\frac{\mathbf{KH}}{\sqrt{3}}\right) \right)^2 - \left(\frac{\mathbf{K}_\infty}{\mathbf{K}} \right)^2 \right)^{-1},$$

and

$$\sigma_i \approx \frac{2\sqrt{3} \mathbf{G} \Delta \mathbf{H} \mathbf{g}}{\mathbf{C}^2} \left(\left(1 - \exp\left(-\frac{\mathbf{KH}}{\sqrt{3}}\right) \right)^2 - \left(\frac{\mathbf{K}_\infty}{\mathbf{K}} \right)^2 \right)^{-1}. \quad (3.3.26)$$

In conformity with the assumption about the transition to the instability region at $\lambda < \lambda_{cr}$ in the event of plastic deformations, the stability criterion can be formulated as a condition that the stress intensity not exceed the stress intensity associated with the material yield strength $\sigma_i < \sigma_T$. According to (3.3.26), this condition can be written with respect to the initial thickness difference in the form

$$\Delta \mathbf{H} < \Delta \mathbf{H}_{cr},$$

$$\Delta \mathbf{H}_{cr} \approx \frac{\sigma_T}{2\sqrt{3}\rho \mathbf{g}} \left(\left(1 - \exp\left(-\frac{\mathbf{KH}}{\sqrt{3}}\right) \right)^2 - \left(\frac{\mathbf{K}_\infty}{\mathbf{K}} \right)^2 \right). \quad (3.3.27)$$

Here $\Delta \mathbf{H}_{cr}$ is the maximum value of the initial thickness difference at which the system is yet stable.

Consider some limiting cases.

If $\mathbf{KH} \gg 1$, expression (3.3.27) takes the following form:

$$\Delta \mathbf{H}_{cr} \approx \frac{\sigma_T}{2\sqrt{3}\rho \mathbf{g}} \left(1 - \left(\frac{\mathbf{K}_\infty}{\mathbf{K}} \right)^2 \right) \equiv \frac{\sigma_T}{2\sqrt{3}\rho \mathbf{g}} \left(1 - \left(\frac{\lambda}{\lambda_\infty} \right)^2 \right). \quad (3.3.28)$$

In the short wavelength region ($\lambda \ll \lambda_\infty$), Eq. (3.3.28) transforms within a constant to the Drucker criterion [69]

$$\Delta H_{cr} \approx \frac{\sigma_T}{2\sqrt{3}\rho g}.$$

For thin plates where $\mathbf{KH} \ll 1$, we obtain from expression (3.3.27) with (3.3.22)

$$\Delta H_{cr} \approx \frac{\sigma_T (\mathbf{KH})^2}{6\sqrt{3}\rho g} \left(1 - \left(\frac{\mathbf{K}_{cr}}{\mathbf{K}} \right)^4 \right). \quad (3.3.29)$$

Taking into consideration that the condition for transition to instability given in (3.3.27) in the short wavelength region is approximate, and mainly describes the qualitative picture of the process, we write this condition in the following form:

$$\Delta H_{cr} \approx \frac{\beta \sigma_T}{\rho g} \left(\left(1 - \exp\left(-\frac{\mathbf{KH}}{\sqrt{3}} \right) \right)^2 - \left(\frac{\mathbf{K}_\infty}{\mathbf{K}} \right)^2 \right),$$

where β is the correction factor which should secure consistency of (3.3.27) with Drucker solution [69]. By analogy with (3.3.21), we consider an expression for β in the form of the exponential dependence on the relative wavelength:

$$\beta = 2 \left(1 - \frac{4\sqrt{3}-1}{4\sqrt{3}} \exp\left(-\frac{\mathbf{KH}}{\sqrt{3}} \right) \right) \approx 2 \left(1 - 0.856 \exp\left(-\frac{\mathbf{KH}}{\sqrt{3}} \right) \right).$$

Then the condition of the transition to instability (3.3.27) takes the following form:

$$\Delta H_{cr} \approx \frac{2(1 - 0.856 \exp(-\mathbf{KH}/\sqrt{3}))\sigma_T}{\rho g} \left(\left(1 - \exp\left(-\frac{\mathbf{KH}}{\sqrt{3}} \right) \right)^2 - \left(\frac{\mathbf{K}_\infty}{\mathbf{K}} \right)^2 \right). \quad (3.3.30)$$

It is evident that for $\mathbf{KH} \gg 1$, expression (3.3.30) transforms to the Drucker criterion, while at $\mathbf{KH} \ll 1$ we obtain the condition for thin plates (3.3.29). From (3.3.30) we deduce that the Miles instability criterion [70] $\lambda > \lambda_\infty = 4\pi C^2/g$ is valid only in the short wavelength region, $\mathbf{H} \gg \lambda_\infty$, and for small initial perturbation amplitude, $\Delta H \ll 2\sigma_T/(\rho g)$.

The relationship between the instability region boundaries corresponding to the Drucker and Miles solutions and the solution presented in this paper is shown in Fig. 3.3-3. As this figure illustrates, the layer stability region is sensitive to the layer thickness. Note that in the solution obtained in [67] there is no such dependence.

Fig. 3.3-3. Stability boundaries for an elastic-plastic layer of various thickness.

3.3.5. Comparison of theoretic estimates with results of experiments and numerical calculations

To compare the estimates obtained on the basis of the approximate solution found in this paper with the experimental and numerical data, publications [50,51,67] were used. Figure 3.3-4 gives the computed stability boundary for an aluminum plate 2.54 mm thick accelerated by pressure $P=10$ GPa.

To construct the instability boundary, the following parameter values were used: $\rho=2.71$ g/cm³; $G=30$ GPa; $\sigma_T=0.33$ GPa. This figure also shows the stability boundary obtained from numerical calculations using the Lagrangian technique [14]. In these computations, as in the previous case, layer incompressibility was approximated by a 10-fold increase in the volume sound speed. As Fig. 3.3-4 demonstrates, Eq. (3.3.30) satisfactorily agrees with the results of numerical calculations. To illustrate the material compressibility effect on the stability boundary position, Fig. 3.3-4 presents the computed curve corresponding to the stability boundary for a compressible layer. In these calculations, the pressure acting on the plate surface during the time $t=1.5$ μ s increased up to $P=10$ GPa and then remained constant. As can be seen from Fig. 3.3-4, the stability boundary for the incompressible layer is located somewhat higher than for the compressible layer. This effect is accounted for by the influence of the compression and rarefaction waves arising at layer loading.

The experimental results known to us are discussed in papers by Barnes [50,51]. In the Barnes experiments, an aluminum plate 2.54 mm thick was accelerated by chemical HE detonation products producing a pressure pulse of maximum amplitude $P_{\max}=10$ GPa on the plate surface. The loaded surface had sinusoidal perturbations with wavelengths $\lambda=2.5$ and $\lambda=5$ mm, respectively. At the initial perturbation amplitude $\Delta H=0.05$ mm for $\lambda=2.5$ and $\lambda=5$ mm there was practically no perturbation growth, while for $\Delta H=0.1$ mm and $\lambda=5$ mm by

$t \approx 8 \mu\text{s}$, the perturbation amplitude had increased by a factor of about 7 from its initial value. Figure 3.3-5 presents the stability boundary computed by Eq. (3.3.30).

Fig. 3.3-4. Stability boundary for the aluminum plate under pressure $P=10$ GPa. 1. approximate solution (3.3.30), 2. numerical solution for the incompressible layer, 3. numerical solution for the compressible layer.

Fig. 3.3-5. Stability boundary of the aluminum plate under action of pressure pulse $P_{\text{max}}=10$ GPa. 1. analytic solution (3.3.30), 2. numerical solution [67], 3. analytic solution [67].

The following data were used in the computation: $\rho=2.71 \text{ g/cm}^3$; $G=30 \text{ GPa}$; $\sigma_T=0.33 \text{ GPa}$ (this yield strength value was used in paper [67] for the numerical simulation of the Barnes experiments). The same figure shows the computed dependence of the stability boundary on the relative wavelength obtained in paper [67] and the stability region boundary corresponding to the analytic solution presented in [67]. As in the previous case, the stability boundary constructed using expression (3.3.30) is located somewhat higher than the

computed curve [67]. However, it should be noted that Eq. (3.3.30) agrees with the numerical and experimental results much better than the analytic solution of paper [67]. Moreover, the solution obtained in [67] for a semi-space differs from the exact result [12,13] by a factor of 2. In this connection, the authors of paper [67] used a shear modulus that was a factor of 2 too small ($G=15$ GPa; while in reality $G=30$ GPa) in their calculations to make the solution consistent for the limiting case of short wavelengths.

Here it is worthwhile to discuss the results of paper [70] by Miles and paper [67] by Robinson concerning the estimation of critical wavelength. According to these papers, λ_{cr} does not depend on layer thickness. This erroneous result is due to the method used in [67] where the strain distribution characteristic of an ideal fluid layer undergoing instability was formally substituted into the variational equation of motion accounting for the deviatoric stress tensor. For an ideal fluid the character of the perturbation growth and the perturbation distribution over thickness do not depend on layer thickness, and therefore the expressions for the critical wavelength obtained on the basis of the modal technique also do not depend on thickness. Indeed, in a thin elastic plate accelerated by constant pressure, the instability evolution process mainly depends on flexion rather than displacement in the layer plane. In this connection the layer stiffness determined by the ratio H/λ is of decisive significance.

Figure 3.3-6 shows the curves determining the stability region boundary for an aluminum layer 2.5 mm thick at various pressure levels obtained by Eq. (3.3.30) of this paper and in the numerical calculations of [67].

As can be seen from Fig. 3.3-6, at pressure $P=20$ GPa there is a satisfactory agreement between the approximate analytic and the numerical solutions. At $P=50$ GPa, the analytic and numerical solutions agree only in the critical wavelength which determines the stability boundary at small initial perturbation amplitudes. This discrepancy results from the use of the material incompressibility assumption in the derivation of (3.3.30); at a pressure of 50 GPa, material compression in the aluminum plate at shock-free loading reaches $\rho/\rho_0 \sim 1.4$. In the layer, compression and rarefaction waves arise which periodically transfer the material into the plastic state resulting in the fact that resistance to instability growth decreases [14]. The intensity of these waves also depends on the character of the layer loading. Thus, for example, paper [14] shows that a variation in the pressure rise interval duration can lead to a transition from the stable state to instability. Therefore the stability criterion obtained in this paper is valid only when it is possible to neglect such processes. In this sense the problem considered in this paper is equivalent to the classic problem of the gravitational (Rayleigh-Taylor) instability. Otherwise the approximate solution overestimates the thickness difference value ΔH_{cr} limiting the stability region, but nevertheless provides a proper estimate of the maximum wavelength, λ_{cr} , above which the system is absolutely unstable.

Fig. 3.3-6. Stability region boundaries for an aluminum layer 2.5 mm thick at various pressure levels. 1,2. approximate solution (3.3.30), 3,4. numerical calculation [67].

3.4. Effect of Shape and Other Characteristics of the Initial Perturbation

Evidently, the evolution of a Rayleigh-Taylor instability is largely dependent on the shape and character of initial perturbations. These dependencies manifest themselves most clearly in a thin liquid layer.

Traditionally the papers on gravitational instability considered primarily the evolution of sinusoidal perturbations of the interface shape. Below we consider more general perturbation shapes and study the evolution of such perturbations. The perturbation growth is also affected by tangential mass flows which take place at the initial time or by a given initial profile of tangential velocities. Depending on whether the initial profiles of the tangential velocity are “in phase” or “in anti-phase” with the direction of the velocity normal to the interface, as well as on the ratio of these velocities, the interface perturbation can exponentially grow at a certain rate, exhibit periodic oscillations with the initial amplitude, or “turn over” and then exponentially grow “in the opposite direction.”

Paradoxical situations are possible where for some perturbations the growth rate in a liquid medium turns out to be less than in an elastic medium.

The analysis made in [61,63] does not completely exhaust the whole variety of situations possible in the problem of accelerated thin liquid layer perturbation evolution. A complete investigation can be made if all roots of the dispersion equation, including those which also provide stable solutions, are considered. It is also important to assess the validity of the dependencies in more general cases. Going beyond the incompressibility conditions allows us to consider a more general set of initial data in the problem of ideal fluid semi-space interface instability.

3.4.1. Governing equations

Plane motion of thin liquid plate particles is described by equations [61]:

$$\frac{\partial^2 x}{\partial t^2} = -a \frac{\partial y}{\partial \xi}, \quad \frac{\partial^2 y}{\partial t^2} = a \frac{\partial x}{\partial \xi} - g \quad . \quad (3.4.1)$$

Here $\mathbf{x}, \mathbf{y}$ are Cartesian coordinates of the observer's space, $\mathbf{t}$, time, ξ , the Lagrangian coordinate of the particle initially at coordinate $\mathbf{x}_0$,

$$a = \frac{p}{\rho h_0} \quad a > 0,$$

where ρ is density, $\mathbf{h}_0$, initial layer thickness, $\mathbf{p}$, external pressure applied “from below,” $\mathbf{g}$, mass acceleration whose direction is opposite to the $\mathbf{Y}$ axis.

Next, we assume that $\mathbf{a}=\mathbf{g}$. At the initial time, the thin layer occupies the position $\mathbf{y}=\mathbf{0}$ on which the initial perturbations are applied. The further motion of the layer particles is described by Eqs. (3.4.1), which are a corollary of the laws of conservation of layer element mass and momentum. These should be complemented with the equation for conservation of layer particle mass, $\mathbf{h}d\mathbf{s}=\mathbf{h}_0d\xi=\mathbf{const}$, relating the current particle thickness $\mathbf{h}$ and current particle length $d\mathbf{s}$.

Consider the case where $\mathbf{a}=\mathbf{const}$. System (3.4.1) is a fourth-order linear system with constant coefficients. It is convenient to introduce a new variable $\mathbf{x}_1=\mathbf{x}-\xi$. Then the basic system in the variables $\mathbf{x}_1$ will take the form:

$$\frac{\partial^2 x_1}{\partial t^2} = -a \frac{\partial y}{\partial \xi}, \quad \frac{\partial^2 y}{\partial t^2} = a \frac{\partial x_1}{\partial \xi} \quad . \quad (3.4.2)$$

System (3.4.2) describes a physical model in which the external pressure causes the acceleration normal to the current strained layer surface; the pressure itself does not depend on the motion of the layer particles.

3.4.2. Perturbations in the layer shape

Consider a particular solution to system (3.4.1) corresponding to setting the initial velocities to zero, and with the perturbation in the layer shape determined by relations:

$$x=\xi +r_1 A \cos k\xi, \quad y=A \sin k\xi \quad , \quad (3.4.3)$$

where $\mathbf{A}$ is perturbation amplitude, $\mathbf{r}_1$, dimensionless parameter, $\mathbf{k}=2\pi/\lambda$, wave number. We call curve (3.4.3) a **hypercycloid**.

One can directly verify that these initial conditions correspond to a solution of Eq. (3.4.1) of the form

$$x = \xi + \left(\frac{r_1 - 1}{2} \text{ch}(t\sqrt{ka}) + \frac{1 + r_1}{2} \cos(t\sqrt{ka}) \right) A \cos k\xi \quad (3.4.4)$$

$$y = \left(\frac{1 - r_1}{2} \text{ch}(t\sqrt{ka}) + \frac{1 + r_1}{2} \cos(t\sqrt{ka}) \right) A \sin k\xi.$$

The perturbations in the shape of the median surface of the layer of form (3.4.3) entail appropriate changes in the distribution of linear mass density in the observer's space and in the thickness of the Lagrangian particles composing the layer. Depending on the value of the parameter r_1 , the changes in the thickness which arise will promote or counteract the growth in the initial perturbations of the layer median line shape.

We describe the dependence of the initial perturbation evolution on the value of the dimensionless parameter r_1 which plays the key role in this problem. The following cases are possible:

1. Let $r_1 = -1$; then system (3.4.3) determines the cycloid given at the initial time which faces “downward with its cusps”, i.e., the most cusped vertices of the cycloid are at the bottom and the most slanting at the top. On setting the perturbations $ds_0, d\xi$, the layer thickness increases at the bottom vertices and decreases at the top vertices. In this case the initial mass flow promotes further growth in the layer shape perturbations. At large t the cycloid perturbation amplitude is $y \sim A \text{ch}(t\sqrt{ka})$. This case was treated in [61]. However, this in no way exhausts the whole variety of the layer behavior.
2. $r_1 = 0$; the initial perturbation constitutes the sinusoid

$$x = \xi, \quad y = A \sin k\xi.$$

According to formulas (3.4.4), at high t , $y \sim (A/2) \text{ch}(t\sqrt{ka})$. Thus, the sinusoidal perturbations grow twice as slowly as the cycloidal. The relative growth rate is the same, while the prefactor is twice as small. This is qualitatively accounted for by the practical absence of initial variations in the layer thickness for sinusoidal-shaped perturbations.

3. $r_1 = 1$. Curve (3.4.3) constitutes a cycloid facing “upwards with its cusps.” In contrast to the case of $r_1 = -1$, in this case mass inflow takes place at the top vertices and mass outflow at the bottom vertices. Thus, the perturbations in the layer median line shape and relating perturbations in the thickness turn out to be “in counter-phase.” For such an initial perturbation, the layer particles make periodic oscillatory movements about the position $y = 0$ at frequency $\omega = \sqrt{ka}$.

Note that the sinusoid is a half-sum of appropriate cycloids facing downward and upward with their cusps. Therefore, the sinusoidal perturbations grow twice as slowly as the perturbations in the shape of the cycloid facing downward with its cusps.

4. $r_1 < -1$. In this case we obtain a “pointed” cycloid facing downward with its cusps. The perturbation grows faster than for the cycloid.
5. $r_1 > 1$. The initial layer shape in this case is a hypercycloid facing upward with its cusps. The initial perturbations turn over and exponentially grow in the opposite direction.

3.4.3. Perturbations in layer particle velocities

Consider a particular solution corresponding to the absence of initial perturbations in the layer shape,

$\mathbf{x}(0)=\xi$; $\mathbf{y}(0)=\mathbf{0}$, and the following initial velocities:

$$V_x(0) = r_2 B \cos k\xi, \quad V_y(0) = B \sin k\xi \quad (3.4.5)$$

where r_2 is a dimensionless parameter.

One can directly verify that these initial conditions correspond to the solution

$$\begin{aligned} x &= \xi + \left(\frac{r_2 - 1}{2} \text{sh}(t\sqrt{ka}) + \frac{1 + r_2}{2} \sin(t\sqrt{ka}) \right) \frac{B}{\sqrt{ka}} \cos k\xi \\ y &= \left(\frac{1 - r_2}{2} \text{sh}(t\sqrt{ka}) + \frac{1 + r_2}{2} \sin(t\sqrt{ka}) \right) \frac{B}{\sqrt{ka}} \sin k\xi \end{aligned} \quad (3.4.6)$$

Similar to the case described above, we have:

At $r_2 = -1$, system (3.4.6) determines the solution which at high t grows as:

$$y \sim B \text{sh}(t\sqrt{ka}) / \sqrt{ka} \quad .$$

At $r_2 = 0$, at the initial time the sinusoidal perturbations occur only in the vertical velocity. At large t the perturbation amplitude is

$$y \sim B \text{sh}(t\sqrt{ka}) / 2\sqrt{ka} \quad .$$

At $r_2 = 1$, the given initial velocity perturbations produce a limited oscillatory solution. This is accounted for by the fact that the initial velocity $\mathbf{V}_x$ is “not in phase” with velocity $\mathbf{V}_y$

and causes mass outflow from the bottom vertices and mass inflow to the top vertices which inhibits the growth of the perturbation amplitude.

At $r_2 < -1$, the perturbations grow at a higher rate than at $r_2 = -1$. At $r_2 > 1$, the initial perturbation “turns over” and then exponentially grows in the opposite direction.

3.4.4. Perturbations in shape and velocities

Initially, let the layer shape have the appearance of hypercycloid (3.4.3) and the velocity be perturbed according to (3.4.5). Then one can introduce a dimensionless parameter r_3 determining the initial perturbation amplitude ratio:

$$r_3 = \frac{B}{\sqrt{ka}} / A \quad . \quad (3.4.7)$$

The solution corresponding to such initial perturbations is of the form

$$\begin{aligned} x = & \xi + \left[\left(\frac{r_1 - 1}{2} \text{ch}(t\sqrt{ka}) + \frac{1 + r_1}{2} \cos(t\sqrt{ka}) \right) + \right. \\ & \left. + \left(\frac{r_2 - 1}{2} \text{sh}(t\sqrt{ka}) + \frac{1 + r_2}{2} \sin(t\sqrt{ka}) \right) r_3 \right] A \cos k\xi, \\ y = & \left[\left(\frac{1 - r_1}{2} \text{ch}(t\sqrt{ka}) + \frac{1 + r_1}{2} \cos(t\sqrt{ka}) \right) + \right. \\ & \left. + \left(\frac{1 - r_2}{2} \text{sh}(t\sqrt{ka}) + \frac{1 + r_2}{2} \sin(t\sqrt{ka}) \right) r_3 \right] A \sin k\xi \quad . \end{aligned} \quad (3.4.8)$$

In this case the initial perturbation character and further evolution depend on the value of three parameters, r_1, r_2, r_3 . For example, the stability condition of the solution depends on the relation

$$1 - r_1 + (1 - r_2) r_3 = 0 \quad .$$

The parameter r_3 considerably affects the perturbation evolution. Thus if $r_1 = 1$ and $r = 1$ then the solution is stable. But if $r_1 = -1$ and $r_2 = -1$ then the solution will also be stable if $r_3 = -1$.

3.4.5. Perturbations in layer shape and thickness

Consider a more general case where the value of acceleration

$$\mathbf{a} = \frac{\mathbf{p}}{\rho \mathbf{h}_0}$$

is also perturbed, for example by setting a non-constant initial thickness $\mathbf{h}_0$. In so doing, $\mathbf{a}=\mathbf{a}(\xi)$ and the original system becomes non-linear.

Assuming that $\frac{\Delta \mathbf{h}}{\mathbf{h}} = -\mathbf{r}_m \sin \mathbf{k} \xi$, we have $\mathbf{a}=\mathbf{a}_0+\Delta \mathbf{a}$ where $\Delta \mathbf{a}=\mathbf{a}_0 \mathbf{r}_m \sin \mathbf{k} \xi$.

Upon linearization we obtain the system

$$\frac{\partial^2 \mathbf{x}_1}{\partial t^2} = -\mathbf{a}_0 \frac{\partial y}{\partial \xi} \quad (3.4.9)$$

$$\frac{\partial^2 y}{\partial t^2} = \mathbf{a}_0 \frac{\partial \mathbf{x}_1}{\partial \xi} + \Delta \mathbf{a} \quad ,$$

where $\mathbf{x}_1=\mathbf{x}-\xi$, the second part of which contains a small time-independent source $\Delta \mathbf{a}$. Its solution

$$\begin{aligned} \mathbf{x}_1 &= \left[\left(\frac{\mathbf{r}_1 - 1}{2} \mathbf{A} - \frac{\mathbf{r}_m}{2\mathbf{k}} \right) \text{ch}(t\sqrt{\mathbf{k}\mathbf{a}}) + \left(\frac{\mathbf{r}_1 + 1}{2} \mathbf{A} - \frac{\mathbf{r}_m}{2\mathbf{k}} \right) \cos(t\sqrt{\mathbf{k}\mathbf{a}}) + \frac{\mathbf{r}_m}{\mathbf{k}} \right] \cos \mathbf{k} \xi \\ y &= \left[\left(\frac{1 - \mathbf{r}_1}{2} \mathbf{A} + \frac{\mathbf{r}_m}{2\mathbf{k}} \right) \text{ch}(t\sqrt{\mathbf{k}\mathbf{a}}) + \left(\frac{\mathbf{r}_1 + 1}{2} \mathbf{A} - \frac{\mathbf{r}_m}{2\mathbf{k}} \right) \cos(t\sqrt{\mathbf{k}\mathbf{a}}) \right] \sin \mathbf{k} \xi \end{aligned} \quad (3.4.10)$$

corresponds to hypercycloid-like perturbations in shape and sinusoidal perturbations in thickness.

The solution for the case when only sinusoidal perturbations in thickness for a strong thin layer are given is presented in [15].

For a non-equithick layer there can be both exponentially growing and bounded oscillating solutions. The indicators of the solution growth, specifically the conditions of their boundedness, depend on the combined effect of the perturbations in shape and thickness. If the perturbations in thickness are relatively large compared to the perturbations in shape then there are no bounded solutions. Here one can introduce one more dimensionless parameter $\mathbf{r}_4$ determining the relative weight of the perturbations in shape as compared to the perturbations in thickness:

$$r_4 = \frac{Ak}{r_m} \quad . \quad (3.4.11)$$

The condition of boundedness of the perturbation growth in y will take the form

$$\frac{1-r_1}{2}A + \frac{r_m}{2k} = 0 \quad . \quad (3.4.12)$$

For fixed sinusoidal perturbations of the layer thickness one can reduce the rate of perturbation growth by additionally applying compensating hypercycloid perturbations in shape according to (3.4.12). At $r_4=0$, the perturbations in thickness grow exponentially.

3.4.6. Pulsed acceleration

Consider the case of pulsed acceleration (the Richtmyer-Meshkov instability of a thin liquid layer). At $t=0$ let a uniformly thick layer perturbed in the hypercycloid form in the initial state be affected “from below” by a shock pressure pulse that is constant in space. The layer elements promptly attain finite velocities whose value depends on the slope angle of the normal to the element.

Denote $V=J/\rho h_0$, $V>0$, where J is applied shock pulse. One can show that for pulsed acceleration the general solution is

$$x = \xi + (r_1 - k V t) A \cos k\xi \quad (3.4.13)$$

$$y = (1 - V r_1 k t) A \sin k\xi + V t \quad .$$

Then at the perturbation “cusp” with $\sin k\xi=-1$, we obtain

$$\frac{dy}{dt} = V r_1 k A + V \quad . \quad (3.4.14)$$

Hence, if $r_1=-1$ (the cycloid facing downward with its cusps) then the perturbation grows linearly in time according to (3.4.14). If $r_1=0$ (sinusoid), then the perturbation does not grow in y .

3.4.7. Numerical studies for a thin layer

Below we give the results of numerical computations to compare with analytic studies. The computations were performed by the **ROB** technique [42] designed for the problems of non-linear coupling of thin strong shells with continua and by the **KGD**

technique [74] for computing the problems of 2D gas dynamics of compressible fluid in Lagrangian variables.

The first computation series was performed in the thin layer approximation by the **ROB** technique. In the computations the following values were used: density, $\rho=7.8$; layer thickness, $h=0.1$; mass acceleration, $a=g=0.01$. The applied external pressure was estimated from the condition of unperturbed plate equilibrium, $p=\rho hg$. It was assumed that $k=1$, and the amplitude of the perturbation normal to the plate was $A=0.01$. Thirty-six computational points were put on a wavelength. The initial perturbation in the layer shape was set in the hypercycloid form according to (3.4.3). After setting the perturbations in geometry, the thickness h was updated at each computational point from the condition of conservation of initial mass of layer elements.

The computations were made with the parameter values $r_1=-1; 0; 1$. The perturbation amplitude variations with time are presented in Fig. 3.4-1. The numerical solutions do not differ significantly from the analytic results.

In this computation series the perturbed layer element thicknesses were estimated from the condition of conservation of Lagrangian particle mass. Under the same assumption the problem is studied in [61]. However, in the general case the perturbed layer element thickness can be set independently.

Fig. 3.4-1.

The following computations demonstrate considerable variations in the layer dynamics when, instead of considering the behavior of an equimass shell, a close, but somewhat different problem is solved which is physically quite real. Initially, the condition of constant thickness, $h=h_0 \text{ const}$ (at $t=0$), is given for the perturbed layer. Then, at $t>0$, the thickness $h=h(\xi)$ is updated, as usual, at each computational point from the condition of conservation of particle mass. Using these assumptions, computations of the cycloids facing downward and upward with their cusps and the sinusoid ($r_1=-1, 1, 0$) were performed.

Under these conditions the perturbation growth turns out to be practically identical independent of perturbation shape. The growth rate of any perturbation (among those which were considered) equals the growth rate of the sinusoidal perturbation with the same wave number $\mathbf{k}$. In turn, this rate is, as was mentioned above, a factor of two smaller than the growth rate of the cycloidal perturbation of an equimass layer.

The next series of computations was performed by the 2D Lagrangian gas dynamics program [74]. The initial perturbations in velocities were given according to (3.4.5). It was easier to set perturbations in velocities than perturbations in coordinates since in doing so one need not meet the condition of conservation of mass of the Lagrangian cells after setting the perturbations. Here this condition is met automatically.

The parameters for this set of calculations were $\mathbf{B}=0.01$, $\mathbf{a}=1$, $\rho=7.8$, $\mathbf{h}=0.1$, $\mathbf{k}=1$. External pressure $\mathbf{p}=\rho\mathbf{h}\mathbf{a}$ was set on the region boundary. On the other boundary, the pressure was set at $\mathbf{p}=0$. The gas-dynamic equation of state is selected in the Mie-Grueneisen form with the parameters $\rho=7.8$, $\mathbf{c}_0=4.6$, $\mathbf{n}=3$, $\Gamma=2.54$. Five computations were performed where the factor $\mathbf{r}_2$ took the values $\mathbf{r}_2=-1, 0, 1, -2, 2$.

The amplitude of the vertical velocity $\mathbf{V}_y$ vs time is presented in Fig. 3.4-2. By the termination time of the computation, $\mathbf{t}=5$, the initial velocity amplitude had grown by a factor of ~ 70 in the computation with $\mathbf{r}_2=-1$. But in the computation with $\mathbf{r}_2=1$, it remained practically unchanged, neither increasing nor decreasing. Thus, one can state that computational “interferences” in the 2D gas-dynamic computation do not “stir up” stable solutions and at the same time do not significantly reduce their amplitude.

Fig. 3.4-2.

For $\mathbf{r}_2=0$, the growth rate is 2 times lower and for $\mathbf{r}_2=2$, 1.5 times higher than for $\mathbf{r}_2=-1$, as it must be according to the analytic solutions.

Thus, the analytic solutions given here for a thin layer are in good agreement both with the numerical solutions in the shell approximation and with the numerical solutions to the complete system of compressible fluid hydrodynamics equations. Quite a variety of

forms of perturbation evolution are possible depending on values of the initial data which are not severely different from one another. Above, we have considered only the simplest case where $\mathbf{a}=\text{const}$. If, for example, acceleration is time dependent, $\mathbf{a}=\mathbf{a}(t)$, then this can introduce important new features into the layer behavior. Thus, when the acceleration changes sign during motion the “stable” cycloid transforms into the “unstable” and vice versa. This may account for the fact that during plate acceleration and deceleration in experiments [17], the perturbations measured after the plate acceleration stage which arose at this perturbation phase grow very slowly or do not grow at all at small times in the deceleration phase.

The analysis performed here is also important when considering Rayleigh-Taylor instability with respect to a thin layer and continuum semi-space. Below the results of numerical studies are given for a compressible ideal fluid semi-space.

3.4.8. *The evolution of boundary perturbations in a compressible fluid semi-space*

The computations were performed by the 2D Lagrangian gas dynamics program [74]. On the interface the initial velocity perturbations were

$$V_x(0) = r_2 B \cos k\xi e^{-k\eta}, \quad V_y(0) = B \sin k\xi e^{-k\eta}.$$

These perturbations were exponentially decreasing with distance from the interface.

Here r_2 is the basic dimensionless factor determining the character of the interface evolution when perturbations in initial velocities are given. The compressible medium model allows us to consider the evolution of the perturbations in the initial velocities with an arbitrary r_2 , while the incompressible medium model forces us to set $r_2=-1$ to meet the incompressibility condition.

The values $B=0.01$, $a=1$, $\rho=7.8$, $k=1$ were used. Inside the region at the initial time the condition of static equilibrium of a compressible medium under the action of gravitational body force acting parallel to the axis $\mathbf{Y}$ and a static pressure gradient was used. An external constant “counterpressure” consistent with the static equilibrium conditions was imposed on the region interface. The “top” boundary of the computational domain was set quite high so that its presence did not affect the computation results.

The gas-dynamic equation of state is the Mie-Grueneisen form with the parameters $\rho=7.8$, $c_0=4.6$, $n=3$, $\Gamma=2.54$.

Five computations were performed where the factor r_2 took the values $r_2=-1, 0, 1, -2, 2$. Figure 3.4-3 shows interface perturbation amplitude growth vs time for the semi-space problem.

The computed results confirm the sensitivity of the perturbation evolution to r_2 that was detected earlier for a thin layer. The time dependence of “peak” amplitudes for variations in the initial velocity profile is qualitatively the same as for a thin layer. For $r_2=1$,

the perturbation is bounded in the computation up to time $t=5$ in accordance with the analytic results for a thin layer. However, at $t \sim 7$ some “stirring-up” in mass velocities is already noticeable which later on is rapidly amplified. Re-computation with the smoothing program reduced the velocity oscillations by a factor of about two, but the qualitative picture of the oscillations remained the same. The “peak” amplitude grows most severely at $r_2=-2$. It is about ~ 1.5 times as high as at $r_2=-1$. The “peaks” for $r_2=-1$ grow ~ 2 times faster than at $r_2=0$. At $r_2=2$ the perturbation turns over and grows approximately at the same rate as at $r_2=0$.

The growth rate of bubbles is noticeably less than of peaks. At $t \sim 7$, independent of the parameter r_2 , the bubble growth rates reach practically the same level, the difference being no more than 10%.

Fig. 3.4-3. “Peak” amplitude vs time: 1. $r_2=2$, 2. $r_2=1$, 3. $r_2=0$, 4. $r_2=-1$, 5. $r_2=-2$.

The interface shape for various r at times $t=5$ and $t=7$ is presented in Fig. 3.4-4 and Fig. 3.4-5.

$r_2 = -2$

$r_2 = 1$

$r_2 = -1$

$r_2 = 2$

Fig. 3.4-4. Interface shape at $t = 5$.

Fig. 3.4-5. Interface shape at $t = 7$.

In the next computational series the interface of the semi-space was initially a hypercycloid (3.4.3).

Fig. 3.4-6. Perturbation amplitude (peaks) vs time when a perturbation in interface shape is given: 1. $r = 2$, 2. $r = 1$, 3. $r = 0$, 4. $r = -1$.

The time dependence of the perturbation amplitudes (peaks) in the computations with various r_1 are presented in Fig. 3.4-6. It is seen that all considered perturbations exponentially grow at all values of the parameter r_1 . There are no perturbations that change their phase. Growth rates of various perturbations are close, though somewhat different. The jet shape essentially depends on the parameter r_1 . At $r_1 < 0$, the jets are acute and fastest; at $r_1 > 0$, they are blunt and of lower velocity.

As the computed results demonstrate, the case considered here noticeably and qualitatively differs from the earlier case where perturbations in the velocity were initially set. This difference arises from the fact that initial perturbations in the interface shape cause perturbations in the mass distribution along the interface line, thereby profoundly affecting its evolution.

3.5. Instability of a Thin Elastic Layer Under the Action of Perturbations [18]

The first part of this subsection (3.5.1) presents a solution of the problem of thin liquid layer perturbation evolution in 3D. As was mentioned in **Section II**, the terms corresponding to the liquid layer are responsible for the instability effect; strength, generally speaking, leads to stabilization.

The second part of this subsection (3.5.2) gives the solution for an elastic thin layer with 3D perturbations. These solutions are compared with solutions for a liquid layer with 3D perturbations and with solutions for a thin elastic layer with 2D perturbations.

3.5.1. Instability of a thin liquid layer with 3D perturbations

In the space of the 3D observer, we introduce the Cartesian system $\mathbf{R}(\mathbf{X}, \mathbf{Y}, \mathbf{Z})$ (the axis $\mathbf{Z}$ is directed vertically upwards). At the initial time the median surface of the layer of thickness $\mathbf{h}_0$ occupies the position $\mathbf{z}=\mathbf{0}$. The acceleration of gravity g is in opposition to the $\mathbf{Z}$ -axis from the top down, and the applied pressure acts from the bottom upwards.

Take $\mathbf{p}=\rho\mathbf{h}_0\mathbf{g}$, i.e., $\mathbf{a}=\mathbf{p}/\rho\mathbf{h}_0=\mathbf{g}$.

The equation of motion is of the form [66]

$$\rho h_0 \frac{\partial^2 \bar{\mathbf{R}}}{\partial t^2} = -P \left(\frac{\partial \bar{\mathbf{R}}}{\partial \eta} \times \frac{\partial \bar{\mathbf{R}}}{\partial \xi} \right) + \rho h_0 \mathbf{g} . \quad (3.5.1)$$

Here ξ, η , Lagrangian coordinates; $\xi=\mathbf{x}_0$, $\eta=\mathbf{y}_0$, particle position at the initial time. This equation accounts for the law of conservation of mass for the 2D Lagrangian element of the layer $\rho \mathbf{h} \mathbf{S} = \rho h_0 \Delta \xi \Delta \eta = \text{const}$, where $\mathbf{h}$ is its current thickness and $\mathbf{S}$, its area.

Expanding the outer product, we obtain

$$\begin{aligned} \frac{\partial^2 x}{\partial t^2} &= -\frac{p}{\rho h_0} \left(\frac{\partial y}{\partial \eta} \frac{\partial z}{\partial \xi} - \frac{\partial z}{\partial \eta} \frac{\partial y}{\partial \xi} \right), \\ \frac{\partial^2 y}{\partial t^2} &= \frac{p}{\rho h_0} \left(\frac{\partial x}{\partial \eta} \frac{\partial z}{\partial \xi} - \frac{\partial z}{\partial \eta} \frac{\partial x}{\partial \xi} \right), \\ \frac{\partial^2 z}{\partial t^2} &= -\frac{p}{\rho h_0} \left(\frac{\partial x}{\partial \eta} \frac{\partial y}{\partial \xi} - \frac{\partial x}{\partial \xi} \frac{\partial y}{\partial \eta} \right) - g. \end{aligned} \quad (3.5.2)$$

Thus, in three dimensions, the equations of motion for a thin layer turn out to be non-linear even in the Lagrangian formulation, in contrast to the situation in two dimensions. But the left-hand side of the equations, i.e., the terms with time derivatives, are linear.

To simplify the studies, we linearize the right-hand side of the equations, taking the displacements relative to the equilibrium position $\mathbf{A}$ and their derivatives to be small:

$$A \ll 1, \quad \frac{\partial A}{\partial \xi} \ll 1, \quad \frac{\partial A}{\partial \eta} \ll 1.$$

Set

$$\frac{\partial y}{\partial \eta} \sim 1+A, \quad \frac{\partial z}{\partial \xi} \sim A, \quad \frac{\partial z}{\partial \eta} \sim A, \quad \frac{\partial y}{\partial \xi} \sim A, \quad (3.5.3)$$

$$\frac{\partial x}{\partial \eta} \sim A, \quad \frac{\partial z}{\partial \xi} \sim A, \quad \frac{\partial x}{\partial \xi} \sim 1+A.$$

Denoting $x_1 = x - \xi$, $y_1 = y - \xi$, we obtain

$$\begin{aligned} \frac{\partial^2 x_1}{\partial t^2} &= -a \frac{\partial z}{\partial \xi}, \\ \frac{\partial^2 y_1}{\partial t^2} &= -a \frac{\partial z}{\partial \eta}, \\ \frac{\partial^2 z}{\partial t^2} &= a \left(\frac{\partial x_1}{\partial \xi} + \frac{\partial y_1}{\partial \eta} \right). \end{aligned} \quad (3.5.4)$$

We seek the solution to the governing system (3.5.4) of the form:

$$x_1(\xi, \eta, t) = Ax(t) \cos k\xi \sin n\eta,$$

$$y_1(\xi, \eta, t) = Ay(t) \sin k\xi \cos n\eta,$$

$$z(\xi, \eta, t) = Az(t) \sin k\xi \sin n\eta,$$

where integers $\mathbf{k}$, $\mathbf{n}$ are the numbers of spatial harmonics by two directions.

On substitution, we obtain a system of three second-order ordinary differential equations

$$\begin{aligned} \frac{d^2 Ax}{dt^2} &= -akAz, \\ \frac{d^2 Ay}{dt^2} &= -anAz, \\ \frac{d^2 Az}{dt^2} &= -a(kAx + nAy). \end{aligned} \quad (3.5.5)$$

In the general case, such a system reduces to a single sixth-order equation. However, in this case, the resulting system appears partially degenerate and reduces for a non-trivial solution to one fourth-order equation:

$$\frac{d^4 Az}{dt^4} = a^2(k^2 + n^2)Az \quad (3.5.6)$$

A similar relation is obtained for the incompressible fluid semi-space in [64].

In the 3D formulation, the dispersion equation turns out to be a fourth-order equation in two dimensions and, in general, it is not much different from its 2D analog in structure. This result is indicative of the fact that the peculiarities of the perturbation evolution in two and three dimensions are similar. But, despite this similarity, the variety of various forms of perturbation evolution is noticeably larger in three dimensions. Here one can introduce a new dimensionless parameter

$$m = n/k, \quad (3.5.7)$$

which plays an important role in the spatial perturbation evolution.

As in the 2D formulation [19], we study the evolution of special perturbation forms. Denote

$$\omega = \sqrt[4]{a^2(k^2 + n^2)}.$$

Here, for simplicity, we restrict ourselves to setting only perturbations in layer shape (causing perturbations in thickness that can be estimated from the condition of conservation of mass of layer particles). Consider solutions to system (3.5.4) of the form

$$\begin{aligned} x_1(\xi, \eta, t) &= (A_1 \text{ch}(\omega t) + A_2 \cos(\omega t)) \cos k\xi \sin n\eta, \\ y_1(\xi, \eta, t) &= (B_1 \text{ch}(\omega t) + B_2 \cos(\omega t)) \sin k\xi \cos n\eta, \\ z(\xi, \eta, t) &= (C_1 \text{ch}(\omega t) + C_2 \cos(\omega t)) \sin k\xi \sin n\eta. \end{aligned} \quad (3.5.8)$$

As in two dimensions, introduce the basic dimensionless parameter r . Set

$$r = Ax/Az. \quad (3.5.9)$$

Taking into consideration (3.5.7), we estimate all the coefficients in Eq.(3.5.8) to obtain

$$\begin{aligned}
 Ax(t) &= \left[-\frac{1}{2} \left(\frac{1}{\sqrt{m^2 + 1}} - r \right) \text{ch}(\omega t) + \frac{1}{2} \left(\frac{1}{\sqrt{m^2 + 1}} + r \right) \cos(\omega t) \right] Az \\
 Ay(t) &= \left[-\frac{1}{2} \left(\frac{1}{\sqrt{m^2 + 1}} - rm \right) \text{ch}(\omega t) + \frac{1}{2} \left(\frac{1}{\sqrt{m^2 + 1}} + rm \right) \cos(\omega t) \right] Az \\
 Az(t) &= \left[\frac{1}{2} (1 - r\sqrt{m^2 + 1}) \text{ch}(\omega t) + \frac{1}{2} (1 + r\sqrt{m^2 + 1}) \cos(\omega t) \right] Az
 \end{aligned} \tag{3.5.10}$$

This solution corresponds to the initial data

$$x(0) = \xi + rAz \cos k\xi \sin n\eta,$$

$$y(0) = \eta + rmAz \sin k\xi \cos n\eta,$$

$$z(0) = Az \sin k\xi \sin n\eta,$$

determining the initial perturbed interface given in parametric form in the observer's space, $z = z(x, y)$. At $\mathbf{m}=\mathbf{0}$, we have the relations for two dimensions.

The growth rate of the exponentially growing term depends on the factor C_1 . We find that if $C_1=\mathbf{0}$, i.e.,

$$1 - r\sqrt{m^2 + 1} = 0, \tag{3.5.11}$$

then the perturbation amplitude is limited. Thus, in three dimensions there are limited oscillating perturbations as in two dimensions. If $\mathbf{m}=\mathbf{1}$, this is the perturbation with $\mathbf{r}=1/\sqrt{2} \approx 0.7071$. Note that, in the 2D formulation, the perturbation with $\mathbf{r}=0.7071$ grows without limit while the perturbation with $\mathbf{r}=1$ is limited. Thus, the introduction of perturbations in the second direction can stabilize the total perturbation. But, of course, it can be vice versa.

In the 3D formulation, the growth rate of growing solutions equals

$$\omega^4 = a^2 (k^2 + n^2). \tag{3.5.12}$$

If $\mathbf{n}=\mathbf{k}$, then the growth rate is $\sqrt[4]{2} \approx 1.2$ times larger than in two dimensions where $\mathbf{n}=\mathbf{0}$. The exponent multiplier is considerably dependent on the initial perturbation shape as in two dimensions, and can be of different signs.

We now discuss briefly the case of pulsed acceleration (the Richtmyer-Meshkov instability of a thin liquid layer). Let a shock wave arrive at an initially perturbed layer surface, imparting normal velocities $\mathbf{V}$ to the layer particles. Then we have:

$$\frac{\partial x_1}{\partial t} = -V \frac{\partial z}{\partial \xi}, \quad \frac{\partial y_1}{\partial t} = -V \frac{\partial z}{\partial \eta}, \quad (3.5.13)$$

$$\frac{\partial z}{\partial t} = V \left(1 + \frac{\partial x_1}{\partial \xi} + \frac{\partial y_1}{\partial \eta} \right).$$

The growth rate of the perturbation amplitude $\mathbf{Az}$ is determined by the relation

$$\frac{d\mathbf{Az}}{dt} = -\mathbf{Az} V r(k + nm) \quad (3.5.14)$$

At $\mathbf{r}=\mathbf{0}$, which corresponds to the “sinusoidal” perturbation of an uniformly thick layer, the perturbation amplitude does not grow.

The above analysis, of course, does not exhaust the whole variety of accelerated thin layer perturbation evolution forms in three dimensions. The realm of 3D non-linear dynamics of thin-wall systems is surprisingly diverse and full of unexpected situations.

3.5.2. Study of 3D perturbations in the Kirchhoff-Love approximation

The strength effect is most severely telling for 3D perturbations. The analytic approximations for a thin layer show that as perturbations are introduced in one more direction, the value of the critical acceleration causing loss of stability increases.

In three dimensions, the governing equations for a thin elastic layer, with the median surface being $\mathbf{z}=\mathbf{0}$ at the initial time, are of the form [66]

$$\begin{aligned} \frac{\partial^2 x_1}{\partial t^2} &= c_{ob}^2 \frac{\partial^2 x_1}{\partial \xi^2} + c_{sd}^2 \left(\frac{\partial^2 x_1}{\partial \eta^2} + \frac{\partial^2 y_1}{\partial \xi \partial \eta} \right) - a \frac{\partial z}{\partial \xi}, \\ \frac{\partial^2 y_1}{\partial t^2} &= c_{ob}^2 \frac{\partial^2 y_1}{\partial \eta^2} + c_{sd}^2 \left(\frac{\partial^2 y_1}{\partial \xi^2} + \frac{\partial^2 x_1}{\partial \xi \partial \eta} \right) - a \frac{\partial z}{\partial \eta}, \\ \frac{\partial^2 z}{\partial t^2} &= -\alpha \left(\frac{\partial^4 z}{\partial \xi^4} + 2 \frac{\partial^4 z}{\partial \xi^2 \partial \eta^2} + \frac{\partial^4 z}{\partial \eta^4} \right) + a \left(\frac{\partial x_1}{\partial \xi} + \frac{\partial y_1}{\partial \eta} \right), \end{aligned} \quad (3.5.15)$$

where $x_1 = x - \xi$, $y_1 = y - \eta$,

$$c_{ob}^2 = \frac{2G}{\rho(1-\nu)}, \quad c_{sd}^2 = \frac{G}{\rho}, \quad \alpha = \frac{c_{ob}^2 h^2}{12}, \quad (3.5.16)$$

G is the shear modulus, h is the layer thickness, and ν is the Poisson ratio.

Consider solutions to governing system (3.5.5) of the form

$$x_1 = X_1(t) \cos(k\xi) \sin(n\eta)$$

$$y_1 = Y_1(t) \sin(k\xi) \cos(n\eta)$$

$$z = Z(t) \sin(k\xi) \sin(n\eta).$$

Then, upon substitution, we obtain

$$\frac{d^2 X_1}{dt^2} = -(c_{ob}^2 k^2 + c_{sd}^2 n^2) X_1 - c_{sd}^2 kn Y_1 - akZ$$

$$\frac{d^2 Y_1}{dt^2} = -(c_{ob}^2 n^2 + c_{sd}^2 k^2) Y_1 - c_{sd}^2 kn X_1 - anZ$$

$$\frac{d^2 Z}{dt^2} = -\alpha(k^4 + 2k^2 n^2 + n^4)Z - a(kX_1 + nY_1).$$

Seeking a solution in the form

$$X_1 = C_1 e^{\omega t}, \quad Y_1 = C_2 e^{\omega t}, \quad Z = C_3 e^{\omega t},$$

we obtain

$$C_1(\omega^2 + c_{ob}^2 k^2 + c_{sd}^2 n^2) + C_2 c_{sd}^2 kn + C_3 ak = 0$$

$$C_1 c_{sd}^2 kn + C_2(\omega^2 + c_{ob}^2 n^2 + c_{sd}^2 k^2) + C_3 an = 0$$

$$C_1 ak + C_2 an + C_3(\omega^2 + \alpha(k^4 + 2k^2 n^2 + n^4)) = 0.$$

For this system to have a non-zero solution at non-zero C_1 , C_2 , C_3 , it is necessary that the determinant be equal to zero:

$$\begin{vmatrix} \omega^2 + \mathbf{C}_{ob}^2 k^2 + \mathbf{C}_{sd}^2 n^2 & \mathbf{C}_{sd}^2 kn & ak \\ \mathbf{C}_{sd}^2 kn & \omega^2 + \mathbf{C}_{ob}^2 n^2 + \mathbf{C}_{sd}^2 k^2 & an \\ ak & an & \omega^2 + \alpha(k^4 + 2k^2 n^2 + n^4) \end{vmatrix} = 0 \quad (3.5.17)$$

This is the equation of sixth order in ω or cubic in $\lambda = \omega^2$. As the characteristic matrix is symmetric, all the roots λ appear to be real. Positive λ are associated with indefinitely increasing solutions, negative λ with oscillatory bounded solutions.

Without accounting for strength, for a thin layer $\mathbf{C}_{ob}^2 = \mathbf{C}_{sd}^2 = 0$ and the governing dispersion relation will take the form

$$\omega^4 = a^2 (n^2 + k^2).$$

If the acceleration is $\mathbf{a} = \mathbf{0}$, the characteristic equation takes the form

$$(\lambda + \alpha(k^4 + 2k^2 n^2 + n^4)) ((\lambda + \mathbf{C}_{ob}^2 k^2 + \mathbf{C}_{sd}^2 n^2)(\lambda + \mathbf{C}_{sd}^2 k^2 + \mathbf{C}_{ob}^2 n^2) - k^2 n^2 \mathbf{C}_{sd}^4) = 0$$

and determines stable bounded solutions at all $\mathbf{k}$ and $\mathbf{n}$ and any small thickness.

In this case all roots $\lambda = \omega^2$ of Eq.(3.5.17) will be real and negative. On the other hand, at sufficiently large accelerations exponentially growing solutions will appear.

We determine the critical value of acceleration $\mathbf{a}_{cr}$ as the acceleration corresponding to $\omega = \lambda = \mathbf{0}$. Setting $\omega = \mathbf{0}$ in the characteristic equation, we obtain the relation for critical acceleration $\mathbf{a}_{cr}$:

$$\mathbf{a}_{cr}^2 = \frac{\alpha(k^2 + n^2)^2 \left((\mathbf{C}_{ob}^2 k^2 + \mathbf{C}_{sd}^2 n^2) (\mathbf{C}_{ob}^2 n^2 + \mathbf{C}_{sd}^2 k^2) + \mathbf{C}_{sd}^4 k^2 n^2 \right)}{\mathbf{C}_{sd}^2 (k^2 - n^2)^2 + 2\mathbf{C}_{ob}^2 k^2 n^2}. \quad (3.5.18)$$

Relation (3.5.18) relates the wave number of perturbation $\mathbf{k}$ and $\mathbf{n}$, acceleration $\mathbf{a}$, strength material properties and layer thickness. Recall that $\mathbf{C}_{ob}^2$ and $\mathbf{C}_{sd}^2$ are expressed in terms of the shear modulus $\mathbf{G}$, the Poisson factor ν , and the material density according to formulas (3.5.16).

When the values satisfy relation (3.5.18) there is no perturbation amplitude growth, and the layer is stable. In three dimensions, it is convenient to analyze relation (3.5.18) in terms of the critical acceleration considering all other values as parameters.

Note that when $\mathbf{h} \rightarrow \mathbf{0}$, $\mathbf{a}_{cr} \rightarrow \mathbf{0}$, (that is, for “very thin shells” at quite small accelerations) amplitude growth takes place.

At $\mathbf{n} = \mathbf{0}$ we obtain the stability criterion for 2D perturbations

$$a_{cr}^2 = \alpha c_{ob}^2 k^4 \quad (3.5.19)$$

or, using notations (3.5.16):

$$a_{cr}^2 = \frac{c_{ob}^4}{12} h^2 k^4 = \frac{1}{3(1-\nu)^2} \frac{G^2 h^2 k^4}{\rho^2}.$$

At $\nu=1/2$ this relation coincides with the criterion obtained for two dimensions in Subsection 3.3.

From relation (3.5.18), it follows that under certain conditions perturbations introduced along the second direction can lead to stabilization of the total perturbation. For example, at $\mathbf{n}=\mathbf{k}$, from formula (3.5.18) we have

$$a_{cr}^2 = 2\alpha k^4 \frac{(c_{ob}^2 + c_{sd}^2)^2 + c_{sd}^4}{c_{ob}^2} \quad (3.5.20)$$

We estimate the ratio of the critical accelerations determined by relations (3.5.19) and (3.5.20). One can show that their ratio q is determined by relation

$$q^2 = 5 - 4\nu + \nu^2, \quad (3.5.21)$$

where again ν is the Poisson factor. At $\nu=1/3$, the critical acceleration for 3D perturbations is approximately 2 times as high as for 2D.

IV. Results of Experiments and Numerical Simulations of Hydrodynamic Instability in Strong Media

This section presents results of experimental studies of hydrodynamic instability in strong media obtained with the experimental methods described in **Section I**.

Numerical simulation results which were obtained using the numerical methods described in **Section II** are also given.

4.1. Experimental Data Obtained Using the Jelly Technique

This section describes experimental studies with the jelly technique. In these experiments, variation in gelatin concentration led to a change in strength properties which, in turn, affected the hydrodynamic instability evolution. The following examples are discussed:

- studies of the local perturbation evolution in plane layer acceleration with different strengths (4.1.1);
- studies of the effect of strength on the evolution of the turbulent mixing zone for cylindrical ring convergence (4.1.2);
- studies of the turbulent mixing effect in the acceleration of layers of different strengths (4.1.3).

The last example is somewhat unusual in its setup and illustrates the effect of strength on the processes which occur inside the turbulent mixing zone.

Using the jelly technique to estimate the critical parameters of instability with strength is discussed in 4.1.4.

4.1.1. The effect of strength on unstable boundary perturbation evolution. Results of local perturbation evolution research

Let us consider the results of the experiments where the source of the initial localized perturbation (**LP**) of a plane boundary of the jelly layer accelerated by gaseous explosive mixture detonation products (**GEM DP**) was a short-term pressure field perturbation relating to interference of the shock wave arising from the reflection of detonation waves from the channel walls.

Reference [33] gives the results of such experiments for the acceleration of plane jelly layers with small gelatin concentration.

The effect of strength on instability evolution was studied in a setup close to [33] (see Fig. 4.1.1-1). The difference was in the use of a cylindric channel and GEM initiation at a single point on the channel axis by electric spark breakdown of the gap. In contrast to the 3D flow in experiments [33], in such a setup an axisymmetric flow with a “point” local perturbation on the symmetry axis arises.

Fig. 4.1.1-1. Scheme of experiments for studying the effect of strength on the evolution of a localized perturbation. (Sizes in mm).

Since the characteristic times of **GEM DP** glow in a closed volume are higher than those of layer movement, we can use the **GEM DP** glow in the process photo recording. The recording was made with a high-speed camera in the frontal projection to meet the moving layer (through the accelerated layer).

Figure 4.1.1-2 gives the photochronogram frames of the tests performed with gelatin concentrations of $C = 4.4\%$ and $C = 35.2\%$.

In the test setup described here, the local perturbation arises at the center of the **GEM DP**-jelly interface.

Under action of **GEM DP** pressure the layer is accelerated and the interface is unstable; at low jelly strength this condition leads to the formation of a turbulent mixing zone. The local perturbation, which at the initial time occupies a small part of the unstable interface, evolves rapidly and occupies almost the whole channel cross section with time.

At a higher gelatin concentration, the evolution of the local perturbation at the same times of photo recording is qualitatively the same, but a change in the turbulent mixing zone (**TMZ**) structure is observed: the small-scale structure on the unstable layer interface at $C=35.2\%$ does not evolve. The local perturbation evolution picture resembles the observations of [33], but the jelly concentration variation leading to increase in strength showed a suppression of the small-scale perturbations.

Fig. 4.1.1-2. Evolution of the local perturbation created on an unstable jelly boundary by shock wave interference for (a) layer made of jelly at $C = 4.4\%$, and (b) layer made of jelly at $C = 35.2\%$. The time in milliseconds counted from the GEM initiation time is given below the photochronogram frames.

Thus, increasing the strength in the plane jelly layer leads to the suppression of small-scale perturbations and turbulent mixing on the unstable interface, while it has little effect on the evolution of the perturbation produced by the local inhomogeneity of the pressure field.

4.1.2. The effect of strength on the evolution of the turbulent mixing zone in the convergence of a cylindrical ring

The number of experimental papers published to date which are devoted to research into the hydrodynamic instability on boundaries of a converging cylindrical shell is relatively small [44–46]. Apparently, this can be accounted for by the technical complexities of making a converging cylindrical detonation (or shock) wave.

The jelly technique considerably simplifies this problem. The general form for setting up such experiments is presented in Fig. 4.1.2-1. Under the pressure of the **GEM** detonation products, the cylindrical jelly layer converges to the center of symmetry.

Fig. 4.1.2-1. Scheme of the experimental facility 1,2—plates made of organic glass; 3—vessel ring; 4—jelly ring; 5—internal cavity of the jelly ring filled with air ($P_0 = 1$ atm); 6—closed volume filled with **GEM**; 7—spark dischargers.

The **GEM** detonation wave front velocity is $D=2450$ m/s and the sound speed in the **GEM** detonation products is $C_s \approx 1040\text{--}1100$ m/s. In the experiment the characteristic velocity of jelly shell movement is ≈ 50 m/s which is lower than the detonation front velocity, or the sound speed in **DP**, jelly or air in the inside cavity. This leads to relatively moderate requirements for symmetry of the converging **GEM** detonation wave front. As the number of the points of (synchronous) **GEM** initiation increases, the amplitude of local perturbations decreases until their disappearance into the general background of evolution of the turbulent mixing zone (**TMZ**).

Figure 4.1.2-1 shows the device on which the experiments were performed[35]. The device consists of upper (1) and lower (2) plates made of organic glass polished to transparency and a vessel ring (3) 19.5 cm in inner diameter and 1 cm in height.

A heated gelatin solution was poured into the device set horizontally without the top cover. When the jelly hardened, the ring (4) was cut out of the whole mass with a knife, the ring being concentric to vessel (3). Upon resetting cover (1), closed volumes (5) and (6) were formed, the former containing air at atmospheric pressure, while the latter was filled with **GEM**, the acetylene-oxygen stoichiometric mixture.

On the internal surface of ring (3) (on its semi-height) 40 spark discharges were positioned. Upon discharger gap break-down with a high-voltage pulse, a converging detonation wave was initiated in **GEM**.

The jelly ring collapses under the pressure of the detonation products in the closed chamber. The picture of the flow which arose was recorded using a high-speed camera.

Some changes introduced in the design [48] allowed us to approach modeling the dynamics of the collapse of a thin shell accelerated by cylindrical **HE** charge detonation products [44,45]. This is achieved through the depressurization of detonation products in the chamber (after a time delay) through channels when a diaphragm is broken.

The variation of the solution concentration allows us to make jellies of various degrees of strength. This method was used to study the effect of strength in the collapse of the jelly shell in the experiments on the device (Fig. 4.1.2-1).

Figure 4.1.2-2 gives the photochronogram frames for the experiment with the jelly shell made of a gelatin solution of 4.4% concentration with jelly strength $\sim 0.1\text{kg/cm}^2$.

The perturbation evolution on an unstable boundary depends on the nature of initial perturbation. On the external boundary there are two sources of initial perturbations:

- shell surface roughness dependent on shell manufacture technology. This is about 40–160 μm ;
- different dynamics of acceleration at different points of the shell surface due to the multi-point character of **GEM** initiation.

From the very beginning of the shell movement, a turbulent mixing zone forms on the external jelly-**DP** boundary. Its thickness increases with time. Bubbles on the internal **TMZ** edge enlarge (in cross-section, the bubble size reaches 4–5 mm). After the change in the acceleration sign ($t \approx 800 \mu\text{s}$) the **TMZ** width reduces and reaches its minimum by time $t=1050 \mu\text{s}$, and the internal **TMZ** edge becomes smooth.

At deceleration, the internal shell boundary becomes unstable and a turbulent mixing layer of jelly and air forms on it. The structure of this zone is small-scale and is not resolved with the optical systems of the recording devices (the resolution is $\sim 0.5 \text{ mm}$). The external edge of the turbulent zone turns out to be perturbed.

Fig. 4.1.2-2. Evolution of the turbulent mixing zone on unstable interfaces of a converging ring made of jelly at $C = 4.4\%$. Time in microseconds is counted from the GEM initiation time.

The photochronogram negatives were measured on a microscope. The boundary radius was estimated through averaging measurements in four directions. The radii measurement error eventually depended on the optical resolution of the recording facility and was about 0.5 mm.

The measurement results are given in Fig. 4.1.2-3. The radius error arises from large-scale boundary distortions. The time error arises from the discrete nature of the recording process.

A one-dimensional computation of the jelly movement was performed for the conditions corresponding to the experiment. In these experiments, jelly can be considered to be an incompressible ideal fluid. The variation in air and detonation product pressure can be computed by the adiabatic law, and the energy release in **GEM** can be considered as prompt and homogeneous. In the computation, the following values are used: air, $P_0=1$ atm, $\gamma=1.4$; jelly, incompressible fluid of density $\rho=1$ g/cc; **GEM DP**, $P_0=13.5$ atm, $\gamma=1.24$. The computation did not account for jelly strength and the turbulent mixing processes.

At the initial stage of the cylindrical shell movement, the acceleration is directed from the periphery to the center of the system. The external ring boundary is thus unstable, while the internal is stable. After the beginning of deceleration and the acceleration sign change, the internal boundary becomes unstable and the external becomes stable. At these times the internal boundary acceleration reaches $\sim 4 \times 10^6$ m/s².

The agreement between the experiment and the computation at the movement stage when the internal boundary is stable is satisfactory [47].

Fig. 4.1.2-3. R-T diagram of layer collapse (experiment, $C = 4.4\%$) EP—explosion products, J—jelly, TMZ₁—turbulent mixing zone of external boundary layer (in conjunction with local perturbations), TMZ₂—turbulent mixing zone of internal layer boundary.

Figure 4.1.2-4 presents photochronogram frames for the test with the jelly shell made of gelatin solution of 13.2% concentration, and jelly strength ~ 0.9 kG/cm². Figure 4.1.2-5 gives the R-T diagram of the flow.

Fig. 4.1.2-4. Evolution of the turbulent mixing zone on unstable interfaces of a converging ring made of jelly at $C = 13.2\%$. The time in microseconds is counted from the GEM initiation time. Notations: RL—reference line; J—jelly; TMZ—turbulent mixing zone; LP—local perturbation; EP—explosion products; A—air.

Fig. 4.1.2-5. R-T diagram of layer collapse (experiment, $C = 13.2\%$); EP—explosion products; J—jelly; TMZ₁—turbulent mixing zone of external boundary, 1—light semi-transparent region, 2—dark opaque region, 3—localized perturbation region, TMZ₂—turbulent mixing zone of internal boundary.

With increasing jelly strength, the character of the **TMZ** evolution on the external shell boundary changes noticeably. At late times (the frame for the time $t=704 \mu s$), one can distinguish three characteristic regions in the **TMZ** image: (1) a light semi-transparent region; (2) a dark opaque region; and (3) the zone of local bubbles. After the acceleration sign change at the shell deceleration stage, these bubbles are not forced to the periphery (as in the previous case), but “stick” in this layer.

On the internal shell boundary, from the very beginning, the small-scale structure **TMZ** arises and evolves. At later times, perturbations of $\lambda \approx 2-3$ mm emerge on the **TMZ** edge.

In the jelly test ($C=35.2\%$) (Figs. 4.1.2.-6, 4.1.2-7), the strength completely suppresses the local bubble zone evolution. Perturbations of $\lambda \approx 3-5$ mm grow on the internal boundary of the shell.

Fig. 4.1.2-6. Evolution of the turbulent mixing zone on interfaces of a converging ring made of jelly at $C = 35.2\%$. The time is counted from the GEM initiation time. Notations: RL - reference line; J - jelly; A - air.

Fig. 4.1.2-7. R-T diagram of layer collapse (experiment, $C = 35.2\%$); EP— explosion products, J—jelly, TMZ_1 —turbulent mixing zone of external boundary, TMZ_2 —turbulent mixing zone of internal boundary.

Figure 4.1.2-8 gives the **TMZ** width vs jelly concentration on the external boundary layer at the time of the layer acceleration sign change. Here L_Σ —total TMZ width, L_1 — semi-transparent region width, L_2 —dark opaque region width, L_3 —local perturbation zone width. L_1 is the width of the semitransparent **TMZ** region, L_2 is the width of the dark opaque region which in the image character can be identified as the small-scale turbulent mixing region, L_3 —the width of the local perturbation zone.

Fig. 4.1.2-8. Dependence on jelly concentration of the TMZ on the external boundary (the data at the time of layer acceleration sign change).

The formation of the semi-transparent region may be related to jelly layer friction on the side walls. The friction and jelly adhesion on the wall result in a thin layer bending in the near-wall region. This layer can fictitiously extend the mixing zone leading one to suggest that the genuine width of the mixing zone is determined by the value L_2-L_1 . The dependence of this value on concentration is given in Fig. 4.1.2-9.

Fig. 4.1.2-9. Dependence of suggested size of the mixing zone at layer boundary on jelly concentration.

Within the concentration range ($C=8.8-17.6\%$) this dependence approaches linear. The extrapolation to the higher concentration region suggests the possibility of complete **TMZ** stabilization on the external boundary for concentrations $\geq 21.6\%$. One should note that the point for $C=4.4\%$ also fits into this dependence. However, the accuracy of its position is low. The character of the dependence on strength of the turbulent mixing zone width should be investigated in further experiments.

To conclude, we note that the increase in jelly strength noticeably affects the one-dimensional movement dynamics. With increasing strength the characteristic collapse rate decreases. This is illustrated by the dependence of the conditional focusing time t_f on concentration (Fig. 4.1.2-10). Here the focusing time is taken as the time when the internal **TMZ** edge reaches the system center.

Fig. 4.1.2-10. Dependence of layer collapse time on jelly concentration.

Thus, the experiments using the jelly technique demonstrate that the presence of strength leads to considerably greater complication of the dynamics of the unstable boundary for converging cylindrical geometry than for planar geometry.

4.1.3. The effect of turbulent mixing on the acceleration of layers of different strength

Comparative experiments with plane layers made of various density materials accelerated by pressure of various temperature gases which are described below allowed us to observe qualitatively the effect of the strength properties of the layer on the acceleration dynamics.

The strength properties of the layer affect not only the features of the surface perturbation evolution, but they also can determine the small-scale structure (the sizes of dispersed particles) of the **TMZ**, and, hence, the rate of the processes which occur in the **TMZ**.

The layers were accelerated either by compressed helium (with pressure about 8 atm and temperature close to room temperature), or by **GEM DP** pressure. According to estimates, the first **GEM DP** are of about 13.5 atm pressure and about 2500°C temperature.

The following was used for the layer material:

- water;
- jelly of gelatin concentration $C=4.4\%$ (with the conditional break-up stress of $\sigma=1.0 \text{ N/cm}^2$);
- jelly of high gelatin concentration ($C=30\%$, $\sigma=20 \text{ N/cm}^2$).

In the last case the unstable boundary was covered with a thin film preventing the jelly from mixing with gas; hereinafter we will call such layers “stabilized”.

Before the acceleration of water and jelly layers by compressed gas, a monolayer of polypropylene particles (size $r \leq 0.5$ mm, density $\rho = 0.92$ g/cm³) was applied on their free surface to set the initial perturbation.

The technique (see Figs. 4.1.3-1a and 4.1.3-2a) using **GEM** to accelerate fluid and jelly layers is described in [33,42,43]; using compressed gas, in [37]. These techniques are also described in detail in review [47]. The tests described differed in the use of piston containers where the fluid (or jelly) layers were located. The container constituted a glass open from the accelerating gas side. During piston acceleration this surface is unstable. There were several tests using each setup. The comparison of results between tests performed in the same setup demonstrates good reproducibility.

Fig. 4.1.3-1a. Scheme of experiments on a layer accelerated by compressed gas (sizes in mm).

Fig. 4.1.3-1b. Layer velocities U as a function of the position S of the stable boundary of a layer accelerated compressed helium. Curves 1 refer to water and jelly layers, 2 to jelly layers with a stabilized boundary.

The experimental results show the considerable role of turbulent mixing in reducing the efficiency of energy extraction from the accelerating gas to the layer.

These results reveal an essential feature. It turns out that the energy losses are higher for water accelerated by hot gas than by cold gas; this effect is not observed for jelly layers.

Figure 4.1.3-1b presents layer velocities U vs displacement S for layer acceleration by compressed helium. The degree of reproducibility from test to test is illustrated in the example of a test series with a stabilized layer with several close curves pertaining to different experiments (the spread of the experimental data in $U(S)$ at $S=45\text{mm}$ is $\sim 1\%$; the error of U evaluation is estimated to be 1.5%).

Fig. 4.1.3-2a. Scheme of experiments on a layer accelerated by pressure produced by detonation of the gaseous explosive mixture (sizes in mm).

Fig. 4.1.3-2b. Layer velocities U as a function of the position S of the stable boundary of a layer accelerated by the pressure produced by the detonation of the gaseous explosive mixture. Curves 1, 2 and 3 refer to water, jelly layers and jelly layer with a stabilized boundary, respectively.

The velocities of jelly layers and water layers are very close to one another and are higher than the velocity of the layer with the stabilized boundary (by 6% at displacements $S=45$ mm).

The results obtained for layer acceleration by **GEM DP** pressure are given in Fig. 4.1.3-2b. In this case the situation is different. As in the previous case, for a layer displacement of $S=45$ mm, the jelly layer velocity turns out to be by 6% higher than the velocity of the layer with the stabilized boundary; however, at the same time the water layer velocity is lower by about 10%. These differences are higher than estimates of possible errors.

Apparently, for acceleration by compressed gas (at room temperature) the higher velocity observed for the layer with the turbulent boundary is a result of the penetration of compressed gas in the form of enlarging bubbles inside the layer mass; as a result, gas pressure acts on a layer of thickness that is decreasing with time and, hence, on an effective mass that is decreasing with time. As a result, the kinetic energy of the layer decreases.

We introduce the following parameter to estimate the reduction in the kinetic energy of the accelerated water and jelly layers:

$$\delta = (E_0(S) - E^*(S))/E_0(S)$$

where S - layer displacement; $E_0(S)$ - kinetic energy of the stabilized layer; $E^*(S)$ - kinetic energy of the layer with the stabilized boundary, and $E^*(S) = m(S) v(S)^2/2$; $m(S)$, $v(S)$ - mass and velocity, respectively, of the layer portion not involved with mixing.

For this set of experiments overall, by the time the displacement of $S=45$ mm is reached the mass of fluid mixed with gas is about 20-25% of the initial total mass of the containers with the layer.

For the water layer accelerated by **GEM DP** pressure, $\delta=35\%$. In other cases the reduction in kinetic energy is almost twice as low, at 20%.

The above results can be treated as follows:

For acceleration by hot gas, an essential cause of the losses can be turbulent heat conduction. Layer material mixing with hot gas must lead to an abrupt increase in the rate of heat flow out of hot gas (due to the increase in the contact area) and, hence, to reduction in the pressure acting on the layer. Owing to high sound speeds in hot gas, the pressure of the volume manages to level off faster than the temperature. Therefore the effect of a reduction in effective pressure due to gas cooling in the **TMZ** should be more pronounced for small **TMZ** volumes and at the stage when the **TMZ** volume is comparable to the **GEM** volume (which takes place in the experiments under consideration).

The estimation of energy losses due to heat transfer from an accelerating gas to a dispersed fluid should involve a model of the material fragmentation process that allows one

to assess the dynamics both of the increase in the thermal contact area and the heat flow as a whole with time.

The fragmentation processes are responsible for the reduction in characteristic radius of drops in dispersed systems due to the instability of large drops with respect to surface strain. It is rather hard to describe these processes, especially for jelly, in any detail. We restrict ourselves to a scale and size analysis of the possible role of heat losses for a water layer.

Assuming that the characteristic velocities of the movement of drops relative to one another in the **TMZ** are proportional to the velocity of the accelerated layer, we estimate the characteristic radius of fluid drops from the equality of the kinetic and surface energy.

Estimates show that by the time the layer velocity reaches 50 m/s, the characteristic drop radius can be 0.002 cm.

The increase in the thermal contact surface area is related to the increase in the total area due to layer material mixing and further drop fragmentation. For a given mass of mixed fluid, the surface area turns out to be inversely proportional to the characteristic radius of the drops. Heat losses can be estimated on the basis of the solution of the problem of heat transfer through the interface of two materials by the linear heat conduction mechanism, neglecting possible small particle evaporation and assuming that, as the surface area increases, the heat flow through new surface portions arises from the time of particle formation.

These estimates show a strong time dependence, $\sim t^{9/4}$, of heat losses by gas in the case under consideration, assuming constant acceleration.

The scales of the losses depend on surface tension (and, evidently, on strength for jelly) and are some tens of percents for water.

The thermophysical properties of water and jelly seem to be similar (by virtue of the small gelatin water solution concentration) but, for jelly layers, energy losses do not increase at the transition from accelerating by compressed gas to accelerating by hot **GEM DP**. This can be related to the difference in the rate of heat loss in the **TMZ** due to the different sizes of water and jelly particles. For jelly, these sizes must be higher due to the strength properties.

The estimates show that the observed difference in heat losses for jelly and water will occur only if the characteristic sizes of water drops and jelly particles are a factor of three different.

Thus the results of experiments performed with various density material layers illustrate a particular physical mechanism, the reduction in efficiency of energy transfer from gas to an accelerated layer due to the heat losses accompanying turbulent mixing on a unstable material interface during acceleration. They show, too, the effect of strength on the energy transfer efficiency.

Note that similar processes can take place in inertial thermonuclear fission experiments. For example, for gas compression by a shell, turbulent mixing can lead to an increase in the area of gas-shell material thermal contact which can lead to shell material cooling and gas compression to higher values, irrespective of the reduction in kinetic energy of the remaining shell part as compared to the case where there is no turbulent mixing.

4.1.4. Using the jelly technique to estimate the critical parameters of instability with strength

Currently the critical instability parameters are estimated in experiments with plates made of structural elements (aluminum, etc.) accelerated by condensed **HE** detonation products. The process is recorded using the x-ray technique and, as a rule, one frame per experiment is obtained.

The use of the jelly technique can contribute considerably to the solution of this problem since the jelly technique:

1. allows the creation of perturbations of any shape and amplitude on the unstable surface under study using simple means;
2. enables experiments to be performed in the laboratory;
3. enables the recording of the whole process of perturbation evolution within one experiment;
4. allows the study of perturbation evolution not only in plane but also in cylindrical geometry [35];
5. allows optical recording of perturbation evolution not only in the side but also in the front projection [33];
6. provides for gradual variation in the jelly strength properties via solution concentration variation.

In addition, the range of the properties is such that one can transfer from the mode of turbulent mixing to its complete suppression by loading the model under study with pressure produced by **GEM** detonation products; this flexibility permits the study of the critical parameters of instability evolution.

At the same time, the strength and viscosity of jelly, including at high loading rates, have not been sufficiently studied thus far. It is also unknown to what measure jellies are a model of ordinary structural materials. Nevertheless, with further improvement and development, the jelly technique can complement the traditional methods for studying the critical instability parameters.

4.2. Experimental study of hydrodynamic instability in metals

Subsection 4.2.1 of this chapter gives the experimental data on Rayleigh-Taylor instability obtained by the pulsed x-ray method. Subsections 4.2.2 and 4.2.3 present experimental data on Richtmyer-Meshkov and Kelvin-Helmholtz instability evolution obtained on preserved metal samples.

4.2.1. Experimental data on Rayleigh-Taylor instability evolution obtained with the pulsed x-ray method

This section describes the results of experimental research into the process of Rayleigh-Taylor instability evolution in a real medium possessing strength which were obtained with the pulsed x-ray method. The results of research into the dependence of perturbation stability on the perturbation initial amplitude and wavelength, as well as research into 2D and 3D perturbation evolution, are given below.

4.2.1.1. Research into the effect of the initial amplitude and wavelength on perturbation stability

As was shown earlier, research into the process of Rayleigh-Taylor instability evolution in strong media performed by various investigators led to basically different conclusions depending on the use of one or another medium strength model. The results of the experimental research into the process of instability evolution in real materials discussed in papers [50,51] showed that the perturbation stability criterion is the perturbation initial amplitude [69]. According to [70], the stability region depends on the critical perturbation wavelength. These conclusions are contradictory to the results of theoretic and computational papers [14,67] which state that for real materials perturbation stability during gravitational instability evolution simultaneously depends on perturbation amplitude and wavelength. However, there were no experimental substantiations of this statement.

For this reason one of the issues considered by us was the experimental determination of the effect of the initial amplitude and wavelength of perturbations on perturbation stability within a wider range of combinations of these parameters. This section discusses the results of our studies. The studies were performed with plates made of annealed aluminum alloy AMg-6. Testing and the experimental assemblies used in the research were described earlier in Section 1.2.1. The studies were performed with plates having 2D initial perturbations of 0.05 mm, 0.1 mm and 0.2 mm amplitudes and 2 mm, 4 mm and 8 mm wavelengths. The testing conditions and results of the experiments on perturbation evolution for various wavelengths and initial amplitudes are given in Table 4.2.1-1. Figures 4.2.1-1 and 4.2.1-2 show x-ray images of the plates under study obtained in the experiments in the unstable and stable modes of perturbation evolution.

Table 4.2.1-1

No	λ , mm	A_0 , mm	D	t, μ s	S, mm	A_n , mm	ΔA_n , mm	A_{sv} , mm	ΔA_{sv} , mm
1	2	0.1	2	4.7	20	0.5	0.15	0	0.1
2	2	0.1	2	6.4	33	0.5	0.1	0	0.05
3	2	0.2	2	2.6	7	1.0	0.15	0.1	0.1
4	2	0.2	2	2.8	8.5	1.5	0.2	0.1	0.1
5	2	0.2	2	3.9	14	Plate disintegration			
6	4	0.05	2	4.7	19	0.55	0.25	0.3	0.15
7	4	0.05	2	6.4	32	0.35	0.15	0.15	0.1
8	4	0.1	2	1.9	3	0.35	0.2	0.1	0.15
9	4	0.1	2	2.5	7	0.45	0.1	0.2	0.1
10	4	0.1	2	3.9	13	0.9	0.2	0.35	0.15
11	4	0.1	2	4.8	20	0.7	0.2	0.35	0.1
12	4	0.1	2	5.0	22	0.8	0.2	0.35	0.1
13	4	0.1	2	7.2	38	Plate disintegration			
14	4	0.2	2	2.5	7	1.35	0.2	0.4	0.1
15	4	0.2	2	3.2	10	1.5	0.2	0.45	0.1
16	8	0.05	2	4.7	20	1.15	0.3	0.6	0.1
17	8	0.05	2	8.3	43	1.0	0.3	0.7	0.3
18	8	0.1	2	2.6	7	0.5	0.15	0.3	0.1
19	8	0.1	2	4.05	16	1.05	0.2	0.65	0.2
20	8	0.1	2	4.75	20	1.3	0.15	0.8	0.15
21	8	0.1	2	6.8	34	1.85	0.35	1.24	0.25
22	8	0.1	2	8.5	45	Plate disintegration

Fig. 4.2.1-1. Unstable modes of perturbation evolution.

Fig. 4.2.1-2. Stable modes of perturbation evolution.

In Table 4.2.1-1 the following notations are used: λ —perturbation wavelength; A_0 —initial perturbation amplitude; D —perturbation dimensionality; t —time interval from the beginning of the loading pressure effect to the time of roentgenography; S —the path length which the plate had traveled by the time of roentgenography; A_n and ΔA_n —the perturbation amplitude on the side of the plate from which it was loaded and the error of this value, respectively; A_{sv} and ΔA_{sv} —the perturbation amplitude from the free plate surface and its measurement error.

Before turning to the analysis of the above experimental results, we would like to note that conventionally all these experiments can be broken into two groups, each with characteristic perturbation evolution and accelerated layer features.

The first group includes the experiments where perturbations are unstable (Fig. 4.2.1-1). In these experiments the material strength of the accelerated layer is insufficient to retard or noticeably decelerate perturbation evolution. The perturbations begin to develop without limit at the stage when the main pressure pulse hits and rapidly lead to disintegration of the accelerated layer. Examples of such experiments are those given in Table 4.2.1-1 under Nos. 3, 4, 5 and 14, 15. Here the classic picture of an instability evolution resembling an instability evolution in fluid is observed. As in a fluid, the perturbations evolve according to a law close to exponential. Plastic strains within the material become so large that the material seems to lose any strength properties; this effect accounts for the similarity between the behavior of this group and fluid behavior.

The second group involves experiments for which initial perturbations are stable (Fig. 4.2.1-1). In these experiments perturbations do not transfer to the mode of unlimited growth as in the first group of experiments. Strength properties of the accelerated layer lead to cessation of perturbation growth, to perturbation stabilization. The perturbations evolve only at the initial stage of the process when the accelerating pressure is close to its maximum. Later on, at later stages, perturbation evolution decelerates under the action of the medium strength properties and, finally, perturbation growth ceases completely. Typical representatives of the second group are the experiments given in Table 4.2.1-1 under Nos. 1, 2 and 6, 7.

Intermediate between the above-mentioned clear-cut representatives of the first and second groups are the experiments where perturbations are practically on the stability boundary. Examples include the experiments given in Table 4.2.1-1 under Nos. 8–13, 16–17 and 18–22. As the numerical simulation results show, under the action of the main pressure pulse the perturbation amplitude in tests 8–13 grew from 7% to 50–60% of the accelerated layer thickness. The plastic strain value therewith reached several hundreds of percents. Despite this, under the action of the medium strength properties, the perturbation evolution was very rapid in the beginning of the acceleration when the main pressure pulse acted, then somewhat decelerated and the perturbation evolution rate decreased. However, most probably, at such strain values real materials begin to lose strength and disintegrate. Apparently, just this occurred when, later on, with practically no loading pressure the plate disintegrated. The disintegration was not only in the form of rupture of thin portions of the accelerated layer where strains are maximal, but also in the form of material continuity loss. In such experiments one of the further reasons for plate disintegration at late stages of the process is the effect of tensile stresses due to the side unloading effect.

It should be noted that the perturbations which are on the stability boundary or in its immediate vicinity are highly sensitive to minor changes (variations) in testing conditions. Such conditions, which can change from test to test, include the initial density of the HE, the residual pressure of atmospheric air in the evacuated chamber and, most important, the value of the initial perturbation amplitude within the manufacture tolerance of the plates under study. Variations in these parameters in either direction may result in variations in perturbation evolution rates not simply in proportion to the initial deviation of these values from their nominal value, but considerably more severe. For example, such a variation can change a stable mode into an unstable mode, or vice versa.

To see the result clearly, it is useful to refer to the perturbation stability diagram (see Section 3.3 of this paper). This diagram is reproduced in Fig. 4.2.1-3. In this figure, symbols “ Δ ” and “ ∇ ” denote the combinations of the perturbation initial amplitude and wavelength used in setting up the experiments made to study the effect of these parameters on the evolution of perturbations in gravitationally unstable aluminum. The symbol “ Δ ” denotes the combinations corresponding to the stable perturbations, the symbol “ ∇ ” denotes those corresponding to the unstable perturbations.

Fig. 4.2.1-3. Perturbation instability region boundary.

According to the experimental data, for perturbations of all wavelengths, a decrease in the initial perturbation amplitude value shifts the process of perturbation evolution from the unstable region towards the stable region. For the perturbations of 2 mm and 4 mm wavelength, the transition from the unstable region to the stable region takes place for initial perturbation amplitudes within the range $0.2 \text{ mm} > A_0 > 0.1 \text{ mm}$, while for the 8 mm perturbation wavelength the transition occurs for initial perturbation amplitudes within the range $0.1 \text{ mm} > A_0 > 0.05 \text{ mm}$.

A similar result takes place as the perturbation wavelength is decreased for a fixed initial perturbation amplitude. Thus, the transition from the unstable region to the stable region for perturbations of 0.05 mm initial amplitude takes place when the perturbation wavelength decreases below $\lambda \sim 8 \text{ mm}$; for perturbations of 0.1 mm initial amplitude, when the initial amplitude perturbation wavelength falls somewhere within the range of $8 \text{ mm} > \lambda > 4 \text{ mm}$; and for perturbations of 0.2 mm initial amplitude, when the initial perturbation wavelength is $\lambda < 2 \text{ mm}$.

Thus, the interface position between the stable region and the unstable region depends both on the initial amplitude and the wavelength of the perturbation.

4.2.1.2. Study of perturbation evolution in various dimensions

The next issue which was studied in the research into gravitational instability in strong media was the relationship between perturbation growth rates in various dimensions.

During gravitational instability evolution in a fluid, 3D perturbations are known to evolve faster than 2D. This was found in numerical study of the process, for example in paper [71], and experimentally substantiated in paper [57]. In the course of computational research into the instability in strong media, it was found that under certain conditions in a strong medium, in contrast to a fluid, 3D perturbation can develop slower than 2D [31].

Section 3.5 of this review contains an analytic solution analysis that also shows that 3D perturbations of a thin liquid layer grow faster than 2D, while for an elastic layer the reverse situation is possible.

In order to verify the result found by numerical simulation, a test series was carried out to compare 2D and 3D perturbation growth in gravitational instability evolution in strong media. The research was performed with plates made of annealed aluminum alloy **AMg-6**. The setups of the experimental assemblies were described earlier in Section 1.2.1. 2D and 3D perturbations of initial amplitude 0.1 mm and wavelengths 4 mm and 8 mm were studied.

Table 4.2.1-2. describes the conditions for the tests and gives the results of the experiments on 2D and 3D perturbation evolution. The notations used in Table 4.2.1-2. are the same as in Table 4.2.1-1.

The results given in Table 4.2.1-2. indicate that for wavelengths of 4 mm and 8 mm, 2D perturbations evolve faster than 3D perturbations of the same amplitude. Moreover, in the 2D perturbation experiments at the last stage of the acceleration process, the plates under study disintegrated. In the 3D perturbation experiments at the same (or close) times, the plates under study did not disintegrate, i.e. three dimensions of the perturbations can result in cessation of their growth and stability (tests 15–19 in Table 4.2.1-2.), while the 2D perturbations of the same initial amplitude and wavelength can be unstable (tests 10–14).

Thus, the results presented in this section show that during gravitational instability evolution in an actual strong medium (aluminum alloy **AMg-6**) there is a wavelength range where 3D perturbations evolve slower than 2D and where in the coordinates “wavelength-initial amplitude” the stability region for 3D perturbations is wider than for 2D.

Table 4.2.1-2.

No	λ , mm	A_0 , mm	D	t, μ s	S, mm	A_n , mm	ΔA_n , mm	A_{sv} , mm	ΔA_{sv} , mm
1	4	0.1	2	1.9	3	0.35	0.2	0.1	0.15
2	4	0.1	2	2.5	7	0.45	0.1	0.2	0.1
3	4	0.1	2	3.9	13	0.9	0.2	0.35	0.15
4	4	0.1	2	4.8	20	0.7	0.2	0.35	0.1
5	4	0.1	2	5.0	22	0.8	0.2	0.35	0.1
6	4	0.1	2	7.2	38	Plate disintegration			
7	4	0.1	3	3.9	14	0.3	0.1	0.1	0.1
8	4	0.1	3	4.75	20	0.4	0.1	0.1	0.1
9	4	0.1	3	6.5	32	0.45	0.1	0.14	0.1
10	8	0.1	2	2.6	7	0.5	0.15	0.3	0.1
11	8	0.1	2	4.05	16	1.05	0.2	0.65	0.2
12	8	0.1	2	4.75	20	1.3	0.15	0.8	0.15
13	8	0.1	2	6.8	34	1.85	0.35	1.24	0.25
14	8	0.1	2	8.5	45	plate disintegration			
15	8	0.1	3	3.65	13	0.4	0.1	0.25	0.1
16	8	0.1	3	4.5	18	0.7	0.2	0.55	0.15
17	8	0.1	3	6.1	28	0.8	0.25	0.55	0.15
18	8	0.1	3	6.7	33	0.95	0.2	0.6	0.15
19	8	0.1	3	8.7	47	1.75	0.2	0.7	0.25

4.2.2. Experimental data on Richtmyer-Meshkov instability obtained using preserved samples

The experimental setup is described in Section 1.3. The experimental results in the form of interface micro-ground ends are given in Figs. 4.2.2-1 and 4.2.2-2 and summarized in Table 4.2.2-1 [26, 28]. The photographs demonstrate the interface perturbation evolution and mutual mixing of metals as the shock-wave pressure increases.

In Table 4.2.2-1 the following notations are used:

Ps - pressure of shock-wave sample implosion, **Gpa**;

Pc - minimum critical pressure under which interface stability upset was noted, **Gpa**;

Pi - the shock-wave implosion pressure range within which it was possible to note the instability evolution, **Gpa**;

a_0, λ_0 (mm) - given initial perturbations (amplitude and wavelength) of the interface or shock-wave front bending.

Table 4.2.2-1.

No RN	Samples	Perturbation setting scheme	a_0 , mm	λ_0 , mm	Ps GPa	Pc , GPa	Pi , GPa
1	Sn-Bi	1.2.9a	0.5	1.6	20;25;27.5;30;35	32.5-35	30-35
			0.2	1.6	30;31;32.5;35		32.5-35
			0.2	3.2	32.5;35		32.5-35
2	Pb-Bi	1.2.9a	0.2	1.1	25;30;31;32.5; 34;35;35.5;37; 38;43;46;50	~31	31-35.5
3	Cd-Bi	1.2.9a	0.5	1.6	35;43;50	43-50	43-50
4	Sn-Pb	1.2.9a	0.5	1.6	30;35;38	~38	35-38
5	Sn-Cd	1.2.9a	0.2	1.6	39;43;47;50; 55.5; 58	55.5-58	55.5-58
			0.5	1.6	35;50	~50	
6	Zn-Bi	1.2.9a	0.5	1.6	50;58	>58	—
7	Bi-Zn	1.2.9a	0.5	1.6	50;58	>58	—

In the experiments it was found that there is a critical shock wave amplitude, **Pc**, at which the interface stability is upset. At **Ps>Pc** the sample metals begin to mix in the volume inside the sealing contour.

The value of **Pc** depends on the sample pair, as well as on the initial perturbation amplitude.

With decreasing initial perturbation amplitude, a_0 , pressure **Pc** somewhat increases; this effect can be accounted for by a decrease in local pressure differences at the interface between the convergence and divergence regions of the shock waves which have passed, the reflected and rarefaction shock waves which cause the perturbation evolution.

In the experiments with the pair **Pb-Bi** performed in the geometry of Fig.1.2-9, it was possible to record the gradual increase in the amplitude of perturbations forming first on the plane interface (penetration of **Bi** in **Pb**) as the shock wave amplitude **Ps** increased (see Fig. 4.2.2-2).

At pressures higher than the upper value of the interval **Pi** the metals homogeneously mix over the volume without noticeable differences in the ground end depending on shock wave pressures (see Fig. 4.2.2-1).

The existence of a critical shock-wave amplitude **Pc** and its relatively weak dependence on **a₀**, in our opinion, testify to a qualitative change in the system strength characteristics. One can try to relate this fact to the melting and subsequent drop of the shear strength of the metals from which the samples are composed during the passage of the shock wave.

Table 4.2.2-2 gives computed pressures **Pm** and temperatures **Tm** of shock-wave melting for some metals. The same table also presents the values of **Pc** from Table 4.2.2-1.

Analysis of these tables reveals a correlation between the beginning of interface instability manifestation and the coordinates of the interval of shock-wave melting for the higher-melting metal from the pair.

Table 4.2.2-2.

No RN	metals	Pm, GPa	Tm, °K	Pc, GPa
1	Bi	16	720	—
2	Sn	32	1020	~33 (Sn-Bi)
3	Pb	34	1560	~31 (Pb-Bi)
4	Cd	47	1510	~50 (Cd-Bi) (Sn-Cd)
5	Zn	59	1780	>58 (Zn-Bi)

a

b

c

Fig. 4.2.2-1.

a

b

c

Fig. 4.2.2-2.

4.2.3. Experimental data on Kelvin-Helmholtz instability in metals

Dynamic studies on preserved samples were performed and the following data was obtained.

4.2.3.1. The dependence on Mach number of the perturbation amplitude and the character of the developing flow at a metal pair interface.

On loading a pair of metals with oblique shock waves in the contact zone, intense shear strains develop, the near-boundary metal layers become severely heated, and cumulative jets can arise. The above effects lead to distortion of the interface profile upon collision. Regular waves and asymmetric distorted waves, as well as layers of mixed component melt, appear.

When the velocity of the contact point (e.g., the velocity of sliding of the cast plate fragments on the immovable plate, see scheme in Fig. 1.2-10) is $V_k < 1.5 \text{ mm}/\mu\text{s}$, no perturbations occur at the interface of Al, Cu, and Fe type metals. (The value of the loading pressure, P_k , is low). The material strength impedes shear flow development. At $V_k \geq 1.5 \text{ mm}/\mu\text{s}$, symmetric wavy perturbations arise at the interface (Fig. 4.2.3-1a). At higher values of V_k , the waves lose their symmetry, the crests become vertical, and the perturbation amplitude somewhat increases (Fig. 4.2.3-1b). Then the transformation of the wavy interface shape into a layer of turbulently mixed materials is observed (Fig. 4.2.3-1c). Next, the perturbation amplitude reaches its maximum, after which it successively decreases (Fig. 4.2.3-1d,e).

Fig. 4.2.3-1.

Figure 4.2.3-2 gives the perturbation amplitude at interfaces of various metal pairs vs the Mach number, $M = V_k / C_0$. For the graphic interpretation of the experimental data, the variable M is selected on similarity grounds in order to compare the perturbation amplitudes at the interfaces of various metal pairs under identical loading conditions (of hydrodynamic flow).

The ascending branch of the dependence $a = f(M)$ is determined by the mode of jet generation at the contact point (the subsonic flow region). A sketch of the developing flow near the collision zone in the frame of reference relating to the contact point is shown in Fig. 4.2.3-3a. The interface perturbations are generated by Kelvin-Helmholtz instabilities during the relative sliding of the quasi-liquid metal layers of the cumulative jet and the immovable plate. The growth in V_k leads to increase in the loading pressure and plastic shear strain intensity in the contact zone. The large metal mass is entrained in the jet flow, and the perturbation amplitude increases.

Fig. 4.2.3-2. Metal layer interface perturbation amplitude vs Mach number. Solid lines: I, Al-Al; II, M1-M1; III, steel 3-steel 3;1, Al-Al [6]; 2, Cu-Cu [12]; 3, Mg-Mg. The dashed line is the generalized dependence $a(M)$ ($\nu=12^\circ$, scaled by the dynamic yield strength Y .)

Fig. 4.2.3-3.

In the critical zone $Co \leq V_k \leq V_{crit}$, detached shock waves are formed which transfer the flow from supersonic to subsonic; for $V_k > V_{crit}$, attached shock waves are formed which swing the flows around; at the point corresponding to $V_k \approx V_{crit}$ the perturbation amplitude reaches its maximum. Thus further growth of V_k through the critical zone to $V_k > V_{crit}$ is attended with swinging the colliding plates around. At the contact point there is no jet generation (see scheme in Fig. 4.2.3-3b). In this mode the interface perturbations are generated by a pure Kelvin-Helmholtz instability mechanism. Behind the oblique shock wave front, the plates are swung about and their separation takes place in the time interval Δt (until

the shock wave passes through the penetrator thickness, reflects from the free surface in the form of a rarefaction wave and reaches the interface). During the interval Δt relative sliding of the two plates occurs. Intense shear at the interface leads to intense shear strains, and thin near-interface metal layers melt off and transfer to the plastic quasi-liquid state. The Kelvin-Helmholtz instability evolves. As V_k grows, the rate of relative metal sliding also increases while, simultaneously, the time of contact decreases. The perturbations do not manage to be completely generated, and their amplitude decreases (the descending branches of the dependence $a=f(M)$, see Fig. 4.2.3-2) [21].

4.2.3.2. The effect of the physical and chemical state of the interface on the nature of the developing perturbation

4.2.3.2.1. The effect of initial material strength

From the results given in Section 4.2.3.1, it follows that the perturbation amplitude (at identical M and $\gamma \approx \text{const}$) is larger for metals with lower strength characteristics (see Fig. 4.2.3-2).

In high-velocity oblique collisions the metal behavior in the vicinity of the contact point is adequately described in terms of hydrodynamics. Very thin near-boundary metal layers, up to several microns in width, transform to the liquid phase. The remaining metal mass near the contact point transforms into a somewhat plastic quasi-liquid state whose behavior is largely dependent on shear stresses τ . On the other hand, there is a relation for the dynamic yield strength which characterizes metal capability to resist shear strains, $Y=2\tau$.

The interface perturbation amplitude is shown to be directly proportional to the loading pressure and inversely proportional to the material strength (pressure destabilizes the interface, and conversely, stabilizes the material strength).

$$a \sim \frac{P}{G} \sim \frac{(\rho_1 + \rho_2)V_k^2}{2(G_1 + G_2)}$$

Rewriting this relation for collision of plates made of the same materials and meaning by the strength of metal its dynamic yield strength, introducing the Mach number $M=V_k/C_0$, and taking into consideration that the product $\frac{\rho V_k C_0}{2}$ is a pressure parameter, we obtain the relation [21]:

$$a \sim \frac{P_k \cdot M}{Y}.$$

4.2.3.2.2. *The effect of the initial temperature*

Copper plates were used which in some tests were cooled to liquid nitrogen temperature.

$T_0 \approx 15^\circ\text{C}$

$T_0 \approx 180^\circ\text{C}$

Fig. 4.2.3-4.

The perturbation amplitude and wavelength are shown to be less for preliminarily cooled copper. Correspondingly, the range of intense strain in the near-interface zone also decreases (see Fig. 4.2.3-4).

Preliminary heating reduces the physical-mechanical characteristics of materials. The interface perturbations are easier to realize. Cooling prior to collision reduces material plasticity which causes a decrease in the perturbation amplitude and wavelength [25].

4.2.3.2.3. *The effect of the contact surface finish*

Plates made of aluminum alloy AMG and copper were used. Super-sonic shock-wave loading ($V_k > C_0$) was realized. Two series of identical tests were performed where the plate contact surfaces were worked to a roughness of no more than $20\ \mu\text{m}$ and polished to smoothness.

No difference in the parameters and shape of the developing perturbations was noted.[22]

4.2.3.2.4. *The effect of the loading condition*

It was shown that with decreasing distance d between plates (2) and (3) (see scheme in Fig. 1.2-11) the perturbation amplitude and wavelength successively decrease. Of the greatest interest are the tests at $d=0$ and $0.1\ \text{mm}$. For the tightly packed arrangement, the perturbation amplitude is approximately twice as high as for the variant $d=0.1\ \text{mm}$:

$a \approx 50 \mu\text{m}$ at $d=1 \text{ mm}$, Fig. 4.2.3-5a;
 $a \approx 20 \mu\text{m}$ at $d=0.1 \text{ mm}$, Fig. 4.2.3-5b;
 $a \approx 45 \mu\text{m}$ at $d=0$, Fig. 4.2.3-5c.

a

b

c

Fig. 4.2.3-5.

At $d=0$ the interface perturbations are a consequence of the Kelvin-Helmholtz instability evolution (under action of the oblique shock wave behind its front swinging-around and the relative sliding of the metal layers along the interface).

At $d=0.1$ mm, a micro-cumulative jet that fills the gap is generated at the contact point. There is relative flow between three materials: the material of the cast plate, the cumulative jet, and the material of the immovable plate. In other words, two parallel tangential discontinuities occur. However, they are known to stabilize one another for wavelengths of the perturbations larger than the distance between the discontinuities. The perturbation amplitude decreases.

Thus, in thin gaps the micro-cumulative jet performs the inverse function: instead of generating interface perturbations, it partially stabilizes them [22].

For plate batch loading with the penetrator [see the scheme in Fig. 1.2-11 of Section 1.2 with element (3) being replaced with a batch of plates (3')], the plates swing around behind the oblique shock wave front. Their relative sliding on the interface takes place during wave circulation ($t \approx 2 \mu s$). The Kelvin-Helmholtz instability evolves. The interface becomes of wavy shape. The penetrator and plate batch when moving in the gap d ($t \approx 10 \mu s$) inertially swing around. The relative metal sliding continues. Such coupling results in wave generation at the interface of the plates from the batch (see Fig. 4.2.3-6) [23].

Fig. 4.2.3-6.

Now consider the experiments on the wedge-shaped gap closure. The photograph of the characteristic ground end is given in Fig. 4.2.3-7. It is seen that, in addition to the welded joint of disks (3) and (5) (see scheme in Fig. 1.2-12 of Section 1.2), one can observe the evolution of perturbations at the interface coplanar with the shock wave and copper foil (2).

Fig. 4.2.3-7.

The results of the experiments testify to the fact that the instability is shear-like and is caused by mutual sliding of the steel and copper plates upon collapse of the wedge-shaped gap under them. The tangential velocity discontinuity can be evaluated from the experiment geometry ($\Delta U \approx 0.4 \text{ mm}/\mu\text{s}$). The sliding time equals the time of the wedge-shaped gap collapse ($t \approx 1 \mu\text{s}$). The perturbation wavelength growth is accounted for by the increasing sliding time which leads to reduction in the effective metal viscosity [24]. (See Fig. 4.2.3-8.)

Fig. 4.2.3-8. Perturbation wavelength λ vs distance x along the sample diameter from the oblique collision start point for disks 3 and 5 (see Fig. 1.2-12). Marks 1,2 and 3,4 are the results of processing for two pairs of experiments at different sliding times.

4.2.3.3. Feasibility study of perturbation stabilization and elimination

Loading of flat metal layers (copper, aluminum, magnesium, brass and their combination) with an oblique shock wave and the effect of protective gaskets on the interface state upon loading were experimentally studied. The loading scheme of Fig. 1.2-10 of section 1.2 was used with the addition of the following: a protective layer $10 \leq \Delta \leq 50 \mu\text{m}$ thick was preliminarily applied on the contact surface of immovable plate (2). The mode of loading was chosen to allow interface wave generation to evolve (the subsonic oblique collision).

The stabilizing effect of the metal coatings with respect to the interface perturbation evolution was experimentally found (Fig. 4.2.3-9) (a,c - non coating, b,d - with coating).

Fig. 4.2.3-9.

It was noted that in all the experiments where reliable interface stabilization was observed the immovable plate metal structure did not change. Foils made of the same metals and being of the same thickness do not possess equally strong stabilizing properties.

The results of these experiments allow us to state that the stabilization effect depends on the physical properties of the coating layer. The layer density is $\sim 90\%$ of the crystalline

density of the corresponding metal. The bond of the elementary micro-volumes is extremely weak, described by bonds at tight packing. These indications can be used to apply the theory of non-combining component mixtures to the metal coating layers. The theory postulates that, at very small sizes of a finite number of solid micro-particles, the effect of their dissociation appears. The micro-structure of individual micro-bodies begins to determine the features in the dynamic loading of such mixtures. The system possesses an abnormally high dynamic strength.

In a special experiment series, the dynamic yield strength (dynamic strength) of the copper coating was measured. The value $y \geq 2.5$ GPa at the loading pressure $P \approx 5$ GPa was obtained. This value is considerably higher than the dynamic yield strength of continuous copper ($Y \approx 0.5$ GPa at $P \approx 5$ GPa) and coincides with those for high-strength steel. Interface perturbations, with all other factors being equal, are smaller for stronger materials.

The stabilization effect is also influenced by the thermophysical properties of coating material. Low-melting-temperature metal layers can not attain a large value Y at pulsed loading, since upon complete melting of the layer no microcrystalline interactions remain to manifest themselves.

In a series of one-dimensional thermal computations it was found that under the loading conditions used in the experiments during the time $t \leq 5 \mu s$ (the time of loading) layers $\Delta \leq 10 \mu m$ thick made practically all the metals completely melt. As a result, the interface is intensely heated through and the materials mix. At $10 < \Delta \leq 20 \mu m$ the layers made of low-melting-temperature metals (Sn, Pb, Cd, Bi) get completely melted. The coatings made of Ta, Cu, Ni, Ti, Al, Mg, Zn experience partial melting in the external zone. For all the metals studied, layers $20 < \Delta \leq 30 \mu m$ thick only partially melt in the external zone. The internal zone of the coating layer and the near-interface layers of the loaded sample are weakly heated and characterized with the initial dynamic strength interface in each material [26].

4.2.3.4. On the role of the cumulative jet

Under certain oblique collision conditions specific for each metal pair at the contact point a cumulative jet is generated. The necessary conditions for jet generation are $0 < \gamma < 45^\circ$ (oblique collision realization); $P_k > P^*$ (the loading pressure should secure material transition yield strength; $V_k < C_0$ or $V_k > C_0$, but at the contact point no added oblique shock waves should be generated).

The role of jet generation in dynamic processes is essential. This phenomenon is essential in explosive metal welding, shear instability evolution on metal layer interfaces, and material cumulation. However, the issue of the cumulative jet structure and state remains practically unstudied.

The experiments devoted to this issue were made using setups shown in Fig. 1.2-10 (see Section 1.2). In the experiments the following metals were used: steel 3; Cu; Pb; W; Ta; U. The pulsed x-ray technique was used to record the jet generation process.

It is found that the cumulative jet developing (in the mode $V_k < C_o$) at the contact point is initially dispersed (small separate particles uniformly fill the collision angle). With increasing V_k , the size of particles in the jet decreases.

In **Ta** and **U** the process of cumulative jet generation is fundamentally different. At $V_k \leq 2.5 \text{ mm}/\mu\text{s}$, a compact jet is generated at the contact point. For these materials, the jet is washed away at $V_k \approx 3 \text{ mm}/\mu\text{s}$ ($V_k < C_o$) [27].

4.3 Numerical simulation of hydrodynamic instability in metals

The first subsection (4.3.1) presents the results of the numerical study of the governing parameter effect on instability evolution for an elastic-plastic semi-space (4.3.1.1) and a finite thickness layer (4.3.1.2).

The second subsection (4.3.2) gives the numerical simulation results for experiments on Rayleigh-Taylor instability in metals.

4.3.1. Numerical study of the effect of governing parameters on stability evolution

4.3.1.1. Study of perturbation evolution on the boundary of an elastic-plastic semi-space

The effect of elastic-plastic medium properties on surface perturbation evolution was numerically studied by applying the Sigma system programs [40, 41, 57] to the following problem. A band **DCMP** composed of material $\rho_0 = 7.82 \text{ g/cc}$ in density is considered.

On the boundary **PM** a sinusoidal surface perturbation $y = a_0 \cos nx$, $n = 2\pi/\lambda$ where $a_0 = 0.2 \text{ cm}$, $\lambda = 4 \text{ cm}$ is given. The bandwidth (distance **DC**) equals half the perturbation wavelength. The boundary **DC** is placed sufficiently far from the **PM** surface so that its boundary condition does not affect the solution. The surfaces **PD** and **MC** are rigid walls. On the boundary **PM**, a constant pressure P_0 is applied. The problem geometry is plane. The equation of state of the material is the Mie-Grueneisen type equation [3] with the following parameters:

$$\rho_0 = 7.82 \text{ g/cc}, c_0 = 4.9 \text{ km/s}, n = 3, \Gamma = 2.5477.$$

In the 2D region **DCMP**, the following computational grid was constructed: 10 computational points per perturbation half-period were used; the cell size in the wave propagation direction was set equal to $h_y = 0.02$.

The first computation verified that, if the computational grid was selected sufficiently fine to describe the perturbation evolution, the formulated problem was solved in the gas-dynamic approximation (in this problem P_0 was taken equal to 50 kbar). The perturbation

amplitude growth rate obtained in the 2D computation is $\frac{da}{dt} = 0.037 \text{ km/s}$ which is in good

agreement with the magnitude of this value calculated by the formula

$\frac{da}{dt} = nva_0 = 0.0376$ km/s (mass velocity behind the shock wave front found from the appropriate 1D calculation equals 0.123 km/s).

It was shown in Chapter 2 that an elastic medium will exhibit no amplitude growth; instead oscillations will appear whose period will depend on the shear modulus.

The effect of elastic-plastic medium properties on perturbation evolution was studied for the problem formulated above in a series of computations where different problem parameters (shear modulus G , yield strength Y , sound speed c_0 , boundary pressure P_0) were varied.

In the first set of computations, the effect of material yield strength, Y , on the surface perturbation evolution was studied. In these calculations, the shear modulus and pressure were fixed, $G=500$ kbar and $P_0 = 80$ kbar, while the yield strength varied; Y was taken equal to $\{1,3,5\}$ kbar. The computed results are given in Fig. 4.3.1-1 which shows the time dependence of the complete perturbation amplitude ($a(t)=y_{\max}-y_{\min}$) of the boundary PM.

Fig. 4.3.1-1.

Initially, the perturbation amplitude is seen to grow, though slower than in the gas-dynamics computation; when the amplitude reaches some value a_1 which depends on the material yield strength, the boundary begins to oscillate as for an elastic medium. In the computations it was found that the amplitude increment is

$$\frac{\Delta a}{a_0} = \frac{a_1 - a_0}{a_0} \sim \frac{1}{Y}.$$

The dependence of the perturbation amplitude on the shear modulus G in an elastic-plastic medium was studied. A series of problems was calculated where the yield strength

was $Y=3$ kbar, $P_0 = 80$ kbar and the shear modulus was taken equal to $\{200, 500, 800, 1000\}$ kbar.

The computed results are presented in Fig. 4.3.1-2. It was found that in all these computations the perturbation amplitude grows up to one and the same value a_1 , so that the amplitude increment $\Delta a = a_1 - a_0$ does not depend on the material shear modulus; the results from the shear modulus study indicate that when the boundary reaches the oscillatory mode the oscillation period, as in an elastic medium, is $\sim \frac{1}{\sqrt{G}}$.

Fig. 4.3.1-2.

Figure 4.3.1-3 presents complete perturbation amplitude vs time for various values of applied boundary pressure P_0 (curve 1: $P_0=50$ kbar, curve 2: $P_0=80$ kbar, curve 3: $P_0=100$ kbar); in these computations, the yield strength Y was taken equal to 3 kbar and $G=500$ kbar. The computations showed that the amplitude variation is

$$\frac{\Delta a}{a_0} = \frac{a_1 - a_0}{a_0} \sim (P_0)^2.$$

Fig. 4.3.1-3.

Thus, in the computations it was found that $\frac{\Delta a}{a_0} \sim \frac{(P_0)^2}{Y}$. From the problem parameters one can derive one more value that has the dimension of pressure, ρD^2 , where D is the shock wave velocity in the material which, in the first approximation, equals $D=c_0+\eta u$, and c_0 is the sound speed.

Therefore a computation was made where this parameter was changed (c_0 was taken equal to 3 km/s). Figure 4.3.1-3 shows perturbation amplitude vs time for the computations with various values of c_0 (curve 2: $c_0=4.9$ km/s, curve 4: $c_0=3$ km/s).

Generalizing the results of the 2D numerical computations where the problem parameters (G , Y , P_0 , C_0) were varied, one can make the following conclusions: In an elastic-plastic medium under action of applied pressure, the sinusoidal perturbation amplitude first grows up to a certain value a_1 , with

$$\frac{\Delta a}{a_0} = \frac{a_1 - a_0}{a_0} \sim \frac{(P_0)^2}{Y \rho c_0^2};$$

then the boundary begins to oscillate at a period of $\sim \frac{1}{\sqrt{G}}$.

Note that the perturbation amplitude increment can be estimated from consideration of the energy balance in the region **DCMP**. It is possible to show that

$$\frac{\Delta a}{a_0} \sim \frac{(P_0)^2 a_0}{Y \rho c_0^2 \lambda}.$$

Thus, the numerical computations show that in elastic-plastic medium the perturbation amplitude growth depends on material yield strength, wavelength and perturbation amplitude.

This result can be used to estimate material yield strength at dynamic loading through comparison of available experimental data and results of appropriate numerical computations. The laws (analytic and numerical) found can be used to estimate elastic-plastic media perturbation growth.

4.3.1.2. Numerical study of the effect of the governing parameters on Rayleigh-Taylor instability

Of greatest interest is the case where pressure acting on the layer boundary is many times higher than material yield strength and leads to considerable material compression [14].

In the computations performed using the Lagrangian technique [14], the perturbation growth was studied in a material layer $H_0=2\text{mm}$ thick with parameters close to those of steel during its acceleration due to time-dependent pressure applied from one side, Fig. 4.3-5.

Fig. 4.3-5.

Duration of the forward pressure pulse front ($\Delta\tau$) varied within a wide range, $\Delta\tau = 0.4\text{--}7\mu\text{s}$. The least time was selected so that, during loading in the layer, no shock wave would be generated (shock-free loading). The computations used a simplified Mie-Grueneisen form of the equation of state which neglects the thermal component,

$$P = P_x = \frac{\rho_o C_o^2}{n} \left(\left(\rho / \rho_o \right)^n - 1 \right),$$

with the following parameters: initial density $\rho_0=7.85\text{g/cc}$, sound speed $C_0=3.8\text{ km/s}$, adiabatic exponent $n=5$. Shear modulus and yield strength were taken constant: $G=77\text{ GPa}$; $\sigma_T=1.5\text{ GPa}$. On both the layer surfaces, initial sinusoidal perturbations were given; the equation of the surface layer was

$$Y = \pm 0.5H_0 \mp a_0 \cos(2\pi x / \lambda) .$$

As the computations showed, the surface instability of such a layer depends both on the wavelength and the initial amplitude. Figs. 4.3-6, 4.3-7 present the time variation in the perturbation amplitude on the loaded layer surface at various perturbation wavelengths (Fig. 4.3-6) and initial perturbation amplitudes (Fig. 4.3-7). In these computation variants, $\Delta\tau=2.0 \mu\text{s}$.

Fig. 4.3-6. Perturbation evolution in time for various wavelengths λ/H at $a_0=3.3 \mu\text{m}$. (1— $\lambda/H=1$; 2— $\lambda/H=2$; 3— $\lambda/H=4$.)

Fig. 4.3-7. Perturbation evolution in time for various initial perturbation amplitudes a_0 at $\lambda/H=2$. (1— $3.3 \mu\text{m}$; 2— $9.9 \mu\text{m}$; 3— $16.5 \mu\text{m}$; 4— $33.0 \mu\text{m}$.)

The computed results show that, as the perturbation wavelength increases from $\lambda/H=2$ to $\lambda/H=4$ the process transfers to the instability region. A similar result takes place when the initial perturbation amplitude increases from $a_0=9.9 \mu\text{m}$ to $a_0=16.5 \mu\text{m}$.

Thus, neither the initial perturbation amplitude nor the perturbation wavelength taken individually can be a stability criterion. Figure 4.3-8 shows the computed boundary of the stability region.

Fig. 4.3-8. Elastic-plastic layer stability region boundary. a_D - critical perturbation amplitude estimated by the Drucker theory [69]. $a_D=2\sigma_T H_0/P=0.12\text{mm}$ ($\sigma_T=1.5 \text{ GPa}$, $H_0=2 \text{ mm}$, $P=50 \text{ GPa}$).

To investigate the effect of strength, computations were performed for various material yield strengths. The perturbation wavelength was $\lambda/H=2$, the initial perturbation amplitude $a_0=3.3 \mu\text{m}$, and the pressure pulse forward front duration $\Delta\tau=2 \mu\text{s}$. The computed results (Fig. 4.3-9) show the considerable stabilizing effect of strength. With yield strength decreasing down to 0.5 GPa perturbations grow unboundedly.

Fig. 4.3-9. Temporal dependence of perturbation on loaded layer surface at various material yield strengths σ_T . $\lambda/H=2$, $a_0=3.3 \mu\text{m}$, $\Delta\tau=2 \mu\text{s}$. (1 1.5 GPa; 2 - 1.0 GPa; 3 0.5 GPa.)

The computational study revealed the essential dependence of the perturbation evolution character on the pressure pulse forward front duration ($\Delta\tau$). This dependence manifests itself most clearly for short wavelengths, $\lambda < 1 \text{ cm}$. Figure 4.3-10 shows loaded surface perturbation growth vs time at various times of pressure introduction for $\lambda/H=2$, $a_0=3.3 \mu\text{m}$, $\sigma_T=1.5 \text{ GPa}$.

Figure 4.3-10 shows that the system becomes unstable when $\Delta\tau < \Delta\tau^* \approx 0.5 \mu\text{s}$. At $\Delta\tau > \Delta\tau^*$ the layer surface perturbations first grow to a certain amplitude and then make oscillations relative to this amplitude according to the harmonic law. Fig. 4.3-11 shows the maximum perturbation amplitude vs front duration. The minimum boundary perturbation is observed at $\Delta\tau \approx 2 \mu\text{s}$. According to the computations, for perturbation wavelengths $\lambda/H > 4$ ($a_0=3.3 \mu\text{m}$, $\sigma_T=1.5 \text{ GPa}$) the plate is unstable for any pressure introduction times. In the absolute instability region the perturbation growth in a strong layer differs considerably from the perturbation evolution in a fluid. The greatest difference is observed in the short wavelength region (Fig. 4.3-12).

Fig. 4.3-10. Perturbations vs time at various $\Delta\tau$ for $\lambda/H=2$, $a_0=3.3 \mu\text{m}$, $\sigma_T=1.5 \text{ GPa}$. (1— $0.5 \mu\text{s}$; 2— $0.7 \mu\text{s}$; 3— $2 \mu\text{s}$; 4— $4 \mu\text{s}$; 5— $7 \mu\text{s}$.)

Fig. 4.3-11. Maximum perturbation amplitude in layer vs $\Delta\tau$.

Fig. 4.3-12. Perturbation amplitude vs time at various wavelengths for strong material (2,3 $\sigma_T=1.5$ GPa) and for ideal fluid (1,4). $\Delta\tau=2 \mu s$, $a_0=3.3 \mu m$. (1,2 $\lambda/H=4$; 3,4 $\lambda/H=10$.)

The layer shape attained during perturbation evolution in the presence of strength appear to be rather complex and different from the shape of an ideal fluid layer at appropriate loading. The layer shape depends both on wavelength and on the pressure introduction time (Fig. 4.3-13).

Fig. 4.3-13. Characteristic layer shapes.

The analysis of the computed results testifies to the complex character of the instability evolution dependence on the elastic-plastic material properties, on the initial perturbation amplitude and wavelength, and on the character of the accelerating pressure variation in time. The stabilizing strength effect is shown to be retained within a pressure range more than 30 times higher than the material yield strength.

Attention is drawn to the fact that the computed result contradicts Drucker's theory [69]. According to [69], at an initial perturbation amplitude,

$$a_o < a_{cr} \approx \frac{2\sigma_T}{\rho g} \equiv \frac{2\sigma_T H_o}{P},$$

the layer surface must be stable. However, in the considered cases the instability evolution was observed at an initial amplitude considerably less than critical according to Drucker. Moreover, the computations showed strong dependence of stability on perturbation wavelength, whereas, according to [69], the wavelength should play no role. These contradictions result from the fact that the Drucker's theory, based on the ideal plastic analysis, does not account the final velocity of elastic shear wave propagation. Therefore this approximation can be justified only for very short wavelengths where the perturbation propagation time is much less than the characteristic time of perturbation growth.

An interesting result found in the computations is the high sensitivity to the time dependence of the applied pressure, in particular, to the plate loading rate ($\partial P/\partial t$). Within the wave region $\lambda/H \sim 2$ when pressure increases from 0 to $P_{max}=50$ GPa during time $\Delta\tau < \Delta\tau^*$ ($\Delta\tau^* \approx 0.5 \mu s$) the perturbations grow without bound, while at a slower loading ($\Delta\tau > \Delta\tau^*$) the perturbation growth is bounded. The surface perturbations first grow and then become oscillatory; the oscillation frequency and amplitude are practically independent of $\Delta\tau$. The effect of loading rate on the perturbation evolution can be accounted for as follows: At layer loading, a compression wave propagates through the layer. If the pressure is higher than the yield strength, the material in this compression wave is strained in the plastic regime. When the wave has traveled to the layer free surface, a rarefaction wave is generated which travels through the layer in the opposite direction. This process is repeated until the compression and rarefaction waves decay. The faster the pressure introduction, the more intense are the compression and rarefaction waves traveling through the layer. During instability evolution, shear stresses inhibit the perturbation growth. However, as soon as the rarefaction and compression waves of an amplitude higher than yield strength have passed, these stresses are "removed" and within some time interval perturbation growth occurs without experiencing inhibiting stresses. As seen from Fig. 4.3-11, the minimum perturbation growth takes place at $\Delta\tau \approx 2 \mu s$. At more prolonged pressure introduction times, the destabilizing factor seems to be the prolonged increase in pressure which permanently transfers the material to the plastic state. Therefore complete instability suppression is in principle possible only at fairly smooth loading and only in a small wavelength region.

It is characteristic of this problem that the layer shape attained during perturbation evolution also depends on the pressure introduction time. At small $\Delta\tau$, the perturbation shape is close to sinusoidal, while at $\Delta\tau \approx 2 \mu s$ it becomes more complex, Fig. 4.3-13. The

appearance of additional jets is observed. This effect is related to the character of the material elastic-plastic strain.

Since in reality any initial perturbation is of a 3D character, one of the important issues of research is the development of 3D perturbations. Paper [37] mentions an essential difference in development of a single (3D) perturbation in fluid as compared with a prolonged (2D). For a jelly layer, 3D perturbations grow approximately twice as fast as 2D. Such an effect is well known for an ideal fluid. A similar 3D effect is mentioned in [18]. The numerical study of 3D perturbation growth was made in the axisymmetric formulation (the considered configuration constitutes a figure of revolution). The characteristics of the material layer and accelerating pressure are similar to those described above. Figs. 4.3-14 through 4.3-16 present the perturbation amplitude on the loaded layer surface vs wavelength at various times for 2D and 3D perturbations for material of various density and various $\Delta\tau$.

Fig. 4.3-14. Perturbation amplitude vs wavelength at various times for 2D and 3D perturbations. $a_0=3.3 \mu\text{m}$, $\sigma_T=1.5 \text{ GPa}$, $\Delta\tau=2 \mu\text{s}$.

Fig. 4.3-15. Perturbation amplitude vs wavelength at various times for 2D and 3D perturbations. $a_0=3.3 \mu\text{m}$, $\sigma_T=0.5 \text{ GPa}$, $\Delta\tau=2 \mu\text{s}$.

Fig. 4.3-16. Perturbation amplitude vs wavelength at various times for 2D and 3D perturbations. $a_0=3.3 \mu\text{m}$, $\sigma_T=1.5 \text{ GPa}$, $\Delta\tau=0.3 \mu\text{s}$.

Within the short-wavelength region characterized by the most rapid perturbation growth, 3D perturbations are seen to grow slower than 2D. This effect is retained both at decreasing material yield strength and at decreasing pressure rise time $\Delta\tau$ when, as was mentioned above, the intense wave process puts the material in the plastic strain state and the effect of strength is reduced. For increasing wavelengths, the picture of instability evolution becomes similar to fluid instability where 3D perturbations grow faster than 2D. This result is of great practical importance since theoretical and experimental research into 3D perturbation evolution constitutes a more complex problem as compared to studying 2D perturbations.

4.3.2. Numerical simulation of experiments on Rayleigh-Taylor instability evolution

Numerical simulation of Barnes' experiments

In Barnes' experiments [50,52] a thin plate on whose surface sinusoidal initial perturbations were given was subjected to acceleration under the action of gaseous detonation products generated by a plane wave generator. A vacuum gap between **HE** and the plate secured smooth (shock-free) plate loading. The perturbation amplitude growth was recorded radiographically.

In the computations [14] the detonation product (**DP**) effect was simulated by a time-dependent pressure uniformly distributed over the plate surface. Simulating the **DP** action with a pressure boundary condition is valid when the **DP** density is considerably lower than the accelerated plate material density, and while the sound speed is rather high. Comparison with the results of a computation in which **DP** scattering and its effect on the plate was actually calculated showed good agreement with the simplified computation. The numerical study was performed for 1100-0 type aluminum for which most experimental data were obtained.

The time dependence of pressure is shown in Fig. 4.3.2-1.

Fig. 4.3.2-1. Time dependence of pressure on the plate surface.

At this loading level the Mie-Grueneisen form of the equation of state without the thermal component

$$P = P_x = \frac{\rho_0 C_0^2}{n} \left(\left(\frac{\rho}{\rho_0} \right)^n - 1 \right)$$

can be used. The following parameters were set: initial density $\rho_0=2.71$ g/cc; initial sound speed $C_0=5.5$ km/s; adiabatic exponent $n=3.5$. The Poisson factor was taken constant and

equal to 0.34. To model the strength, experimental results were used to estimate the dynamic yield strength at shock-wave loading [72]; according to these estimates, within the pressure range up to 10 **GPa** the yield strength σ_T is described by a linear dependence on pressure of the form:

$$\sigma_T = \sigma_T (1 + 0.04P/\sigma_T), \sigma_T = 0.2 \text{ GPa}. \quad (4.3.2.1)$$

The computed and experimental results for a plate $H_0 = 2.54$ mm thick are presented in Fig. 4.3.2-2. The markers show the experimental points.

Fig. 4.3.2-2. Results of the numerical simulation of the Barnes experiment.

As seen from the figure, the numerical solution describes the experiment fairly well both for the stable mode and for the variant where the unbounded perturbation growth took place. The satisfactory agreement between the computed and experimental data testifies to the utility of performing numerical simulations of instability processes in strong media.

To estimate the strength effect, computations were performed for $\sigma_T = 0.2$ GPa and $\sigma_T = 0.5$ GPa, Fig. 4.3.2-3.

Fig. 4.3.2-3. Perturbation amplitude variation in time on the plate surface at various strengths.

The computed results show the considerable stabilizing effect of strength. Yield strength increase up to 0.5 GPa transfers the process to the stability region.

Numerical simulation of experiments on instability evolution in an accelerated aluminum plate (AMg-6 material)

In these experiments, the process of perturbation evolution on the aluminum plate surface (material AMg-6) was studied with the plate being accelerated under the action of condensed explosive detonation products. A detailed description of the experimental setup results can be found in Section 4.2.1 of this review.

As in the previous case, in computations [31] the detonation product effect was simulated with a pressure whose temporal dependence is presented in Fig. 4.3.2-4. Figure 4.3.2-5 shows the computed plate displacement in time. The experimental points are also given here.

Fig. 4.3.2-4. Pressure vs time.

Fig. 4.3.2-5. Plate displacement vs time.

As shown above, the Barnes experiments can be successfully described using an elastic-plastic model where dynamic yield strength is a pressure function.

During experimental research into the instability of plates made of AMg-6, wavelengths of 2mm, 4mm, and 8mm were considered. As the computations showed, within this wavelength range the strain rate varies within wide limits, up to 10^7sec^{-1} , and it is impossible to describe the whole wavelength range in terms of an elastic-plastic model with pressure-dependent yield strength. A satisfactory agreement between the experimental and numerical results was obtained using the viscous-elastic-plastic material model.

The computations used the Mie-Grueneisen form of the equations of state

$$\mathbf{P} = \mathbf{P}_x + \mathbf{P}_T = \frac{\rho_0 C_0^2}{n} \left(\left(\frac{\rho}{\rho_0} \right)^n - 1 \right) + \Gamma \rho \mathbf{E}_T$$

with the following parameters: initial density $\rho_0=2.64 \text{ g/cc}$; initial sound speed $C_0=5.3 \text{ km/s}$; adiabatic exponent $n=3.5$; Poisson factor $\nu=0.34$; Grueneisen gamma $\Gamma=1.14$.

By analogy with paper [6] dynamic yield strength was taken in the following form:

$$\sigma_T = \sigma_T^0 \left[1 + C_1 \left(1 - \exp(-C_2 \varepsilon_i^p) \right) + C_3 P / \sigma_T^0 \right] \exp \left(-C_4 \left(E_T / E_T^m \right)^2 \right), \quad (4.3.2.2)$$

$$\sigma_T^0 = 0.15 \text{ GPa},$$

where ε_i^p - plastic strain intensity; E_T - current thermal energy; E_T^m - thermal energy required for homogeneous material heating up to melting temperature; C_i - constants. The first term in square brackets describes isothermal hardening, the second square bracket, thermal unhardening. It should be noted that expression (4.3.2.1) is a special case of (4.3.2.2). For the viscosity factor (the Navier-Stokes type viscosity) the dependence on the material state parameters was taken in the same form as for the dynamic yield strength

$$\mu = \mu^0 \left[1 + C_1 \left(1 - \exp(-C_2 \varepsilon_i^p) \right) + C_3 P / \mu^0 \right] \exp \left(-C_4 \left(E_T / E_T^m \right)^2 \right), \quad (4.3.2.3)$$

$$\mu^0 = 260 \text{ Pa} \cdot \text{C}.$$

The constants C_i in expressions (4.3.2.2, 4.3.2.3) are as follows: $C_1=1.4$; $C_2=15$; $C_3=0$; $C_4=5$. With these values of C_i Eq. (4.3.2.2) agrees with the stress dependence on strain for AMg-6 estimated under static conditions.

Figures 4.3.2-6, 4.3.2-7 show the experimental points and computed dependences of the current loaded surface perturbation amplitude on plate displacement for the 2D configuration at various perturbation wavelengths and initial amplitudes. In the computations

for the initial perturbation amplitude, the average value was taken with account of the manufacture error.

Figure 4.3.2-8 shows the instability region boundary for 2D perturbations obtained on the basis of the experimental and numerical results.

From the results presented in Figs. 4.3.2-6 through 4.3.2-8 it follows that increase in the initial perturbation amplitude transfers the process to the instability region. In parallel with this, an increase in the wavelength at the same initial perturbation amplitude leads to a similar result. Thus, perturbation stability depends both on initial amplitude and on perturbation wavelength.

Fig. 4.3.2-6. Effect of initial perturbation amplitude on loaded surface perturbation evolution from plate displacement for 2D configuration.

Fig. 4.3.2-7. Dependence of current loaded surface perturbation amplitude on plate displacement for 2D configuration at various wavelengths.

Figure 4.3.2-8. Boundary of the instability region for 2D perturbations.

The next issue considered during the experiments was that of the effect of perturbation dimensionality on perturbation evolution. Fig. 4.3.2-9 shows the computed and experimental results with 2D and 3D perturbations. These results show that within the considered wavelength range, in contrast to ideal fluid, 3D perturbations evolve slower than 2D.

Fig. 4.3.2-9. Effect of the dimensionality of the perturbation on instability evolution.

Taking into consideration the considerable effect of strength on the perturbation evolution character, the obtained experimental data is a unique source of information for construction of viscous-elastic-plastic models of strong materials within a wide strain rate range (up to 10^7sec^{-1}).

References

1. Taylor G.I. The instability of liquid surfaces when accelerated in a direction perpendicular to their planes// Proceedings of the Roy. Soc., 1950, v.201, No.1065.
2. Richtmyer R.D. Taylor instability in shock acceleration of compressible fluids// Comm. on Pure and Applied Math., 1960, v.13.
3. Zeldovich Ya.B., Raizer Yu.P. Physics of shock waves and high-temperature hydrodynamic phenomena// Moscow, Nauka Publishers, 1966.(trans: Hayes, W. D. and Probstein, R.F., eds, New York, Academic Press, 1966)
4. Popkov, S. Jelly States of Polymers// Moscow, Nauk Publishers, 1974.
5. Berdichevsky V.L. Variational principles of continuum mechanics// Moscow, Nauka Publishers, 1983, 448p.
6. Sedov L.I. Continuum mechanics// Moscow, Nauka Publishers, 1978, 687p.
7. Goldenveizer. Theory of elastic thin shells// Moscow, Nauka Publishers, 1979.(trans: G. Herrmann, ed., New York, Pergamon Press, 1961, *first ed.*).
8. Kachanov L.M. Fundamentals of elasticity theory// Moscow, Nauka Publishers, 1966, 416p.
9. Korotkikh Yu.G., Ugodchilov A.G. Equations of theory of thermal viscosity and plasticity with combined strengthening. Equations of state at small-cycle loading// Moscow, Nauka Publishers, 1981, p.129-167.
10. Computational methods in hydrodynamics. (Ed. by B.Older, S.Fernbakh, M.Rotenberg)// Moscow, Mir Publishers, 1967, 383p.
11. Anuchina N.N., Bakhrah S.M., Zabrodin A.V., Pliner L.A., Sofronov I.D. Studies on hydrodynamic instability of two media interface instability// Computer studies of hydrodynamic instability, Collected scientific works, Ed. by K.I.Babenko, Moscow, IPM, 1981, p.108-162.
12. Bakhrah S.M., Kovalev N.P. Taylor instability in solid bodies// Proc. 5th All-Union Conference on Numerical Methods for Solution of Elasticity and Plasticity Theory Problems, Novosibirsk, 1978, p.15-23.
13. Bakhrah S.M., Kovalev N.P. Taylor instability in inelastic-plastic media// Chislennyye Metody Mekhaniki Sploshnoy Sredy, Novosibirsk, 1980, v.II, No.2, p.5-21.
14. Abakumov A.I., Lebedev A.I., Nizovtseva I.A., Nizovtsev P.N., Rayevsky V.A. Rayleigh-Taylor instability in elastic-plastic medium. Numerical study// VANT. Ser. Teor. i Prikl. Fizika, 1990, No.3, p.14-19.

15. Nizovtsev P.N., Rayevsky V.A. Approximate analytic solution for the problem of Rayleigh-Taylor instability in strong medium// VANT. Ser. Teor. i Prikl. Fizika, 1991, No.3, p.11-17.
16. Lebedev A.I., Nizovtsev P.N., Rayevsky V.A., Solovyev V.P. Rayleigh-Taylor instability in strong materials// Submitted to DAN Publishing House.
17. Ivanov A.G., Novitsky E.Z., Ogorodnikov V.A., Pinchuk S.Yu. Plate acceleration to hypersonic velocities. Instability at deceleration against air// PMTF, 1982, No.2, p.90-94.(trans: J. Appl. Mech. Tech. Phys. v. 23, p. 242-245.)
18. Bakhrakh S.M., Simonov G.P. Studying Rayleigh-Taylor instability of a thin liquid layer in 3D setting-up// VANT. Ser. Mat. Modelir. Fiz. Protssessov, 1995, No.3, p.33-38.
19. Bakhrakh S.M., Simonov G.P. Analytic and numerical study of Rayleigh-Taylor instability of a thin liquid layer. // VANT. Ser. Mat. Modelir. Fiz. Protssessov, 1995, No.3, p.39-46.
20. Bakhrakh S.M., Grishina G.V., Kovalev N.P., Meshkov E.E., Tolshmyakov A.I., Yanilkin Yu.V. Some issues of experimental and numerical study of Taylor instability// Chisl. Metody Mekhaniki Splashnoy Sredy, Novosibirsk, 1979, v.10, No.1, p.17-30.
21. Drennov O.B. On state of metal layer interface within a wide range of oblique collision velocity variation range// FGV, No.2, V.28, 1991, p.118-123.(trans: Combustion, Explosion, and Shock Physics, v. 27, p. 237-241.)
22. Drennov O.B. On perturbation evolution on aluminum-copper interface in time-dependent mode of oblique collision// PMTF No.3 (175), 1989, p.41-44.
23. Drennov O.B., Mikhailov A.L. Coupling of aluminum samples by gap-free scheme at super-sonic mode of shock-wave loading// FKhom, No.3, 1995, p.84-87.(trans: Phys. Chem. of Mat. Treatment, v.27, 1993, p. 412-414.)
24. Mikhailov A.L. Shear instability of interface in metals// FGV, V.15, 1979, p.158-161.
25. Drennov O.B. Effect of initial cooling of copper samples to cryogenic temperatures on interface state at oblique collision// FKhom, No.4, 1993, p.124-127.
26. Drennov O.B., Mikhailov A.L., Osipov R.S., Rodigina L.D. Perturbation stabilization of metal layer interface at Kelvin-Helmholtz instability evolution// Khimicheskaya Fizika, No.8, v.12, 1993, p.1150-1152.
27. Drennov O.B. On structure of cumulative jet realizing at oblique collision of metal plates// VANT. Ser. Teor. i Prikl. Fizika, No.3, 1991, p.7-10.
28. Mikhailov A.L., Osipov R.S. Interface Instabilities between Shock Compressed Metal Layers// Paper Submitted for the 1995 APS Topical Conference "Shock Compression of Condensed Matter " August 13-18, 1995, Seattle.
29. Drennov O.B., Mikhailov A.L., Osipov R.S., Fedorov A.V. Strength and destruction of plane ampules at shock-wave testing// Problemy Prochnosti, No.10, 1989, p.120.

30. Mikhailov A.L., Drennov O.B., Osipov R.S., Fedorov A.V., Tsyganov V.A. Explosion-proof containers for studying materials at megabar range (up to 100-150 GPa) shock-wave effects// Presentation at Symposium on accident-resistant containers and safety, Albuquerque, USA, 1993.
31. Lebedev A.I., Nizovtsev P.N., Rayevsky V.A. Rayleigh-Taylor instability in solids// 4 - International Workshop on the Physics of Compressible Turbulent Mixing, Cambridge, 1993, p.81-93.
32. Volchenko O.I., Zhidov I.G., Klopov B.A., Meshkov E.E., Popov V.V., Rogachev V.G., Tolshmyakov A.I. A technique for modeling of time-dependent incompressible fluid flows// A.S.1026154, published on 06/30/83, Invention Bulletin No.24, 1983.
33. Volchenko O.I., Zhidov I.G., Meshkov E.E., Rogachev V.G. Localized perturbation evolution on unstable boundary of accelerated liquid layer// Pis'ma v ZhTF, v.15, No.1, 1989, p.47-50.(trans: Sov. Tech. Phys. Lett., v.15, 1989, p. 19-21.)
34. Rogachev V., Zhidov I., Klopov B., Meshkov E., Tolshmyakov A. Non-stationary flows in the proximity of angular points of gas accelerated liquid layer// 3rd Intl. Workshop on the Physics of Compressible Turbulent Mixing, Abbey of Royaumont (France) - June 17-19, 1991, p.1-8.
35. Meshkov E., Nevmerzhitsky N., Rogachev V., Zhidov I. Turbulent mixing development investigation with converging jelly rings// The Proc. of the 4th Intl. Workshop on the Physics of Compressible Turbulent Mixing. 29 March - 1 April 1993, Cambridge, Engl and Ed. by P.Linden, D.Youngs & S.Daniel. Pr. by Cambridge Univ, Press, Cambridge, England (1993), p.578-585.
36. Meshkov E., Nevmerzhitsky N. About turbulent mixing dynamics at unstable boundary of liquid layer, accelerated by compressed gas// 3rd Intl. Workshop on the Physics of Compressible Turbulent Mixing. Abbey of Royaumont (France) - June 17-19, 1991, p.467.
37. Zhidov I.G., Meshkov E.E., Nevmerzhitsky N.V. Experimental study of mixing dynamics on unstable boundaries of accelerated liquid layers// Lebedev FIAN Preprint No.56, 1990, p.63.
38. Bazhenov V.G., Zefirov S.V., Kibets A.I. On numerical implementation of variational-difference moment scheme for solving non-linear problems of non-thin shell dynamics at pulsed effects// Applied Problems of strength and plasticity. Solution methods.All-Union Inter-University Collected Papers/ Gorky University, 1988, p.66-73.
39. Bakhrakh S.M., Kovalev N.P. Employment of the splitting method for elastic-plastic flow computation// Proc. 2nd All-Union Conference on Numerical Methods for Solution of Elasticity and Plasticity Theory Problems, Novosibirsk, Nauka Publishers, 1972, p.68-74.
40. Bakhrakh S.M., Kovalev N.P., Pavlusha I.N., Shanin A.A. Entropy scheme for elastic-plastic flow computation// Numerical Methods for Solution of Elasticity and Plasticity Theory Problems (Proc.4th Conference), Novosibirsk, Nauka Publishers, 1976, Part 1, p.22-35.

41. Bakhrakh S.M., Kovalev N.P., Pavlusha I.N. Method for computing elastic-plastic flows// Proc.3d All-Union Conference on numerical methods for solving elasticity and plasticity theory problems, Novosibirsk, Nauka Publishers, 1974, Part 1, p.22-36.
42. Bakhrakh S.M., Pevnitsky A.V., Simonov G.P., Solovyev B.P. Dynamics of thin-wall chambers surrounded by fluid at pulsed loading// VANT. Ser. Met. i Progr. Chisl. Resh. Zad. Mat. Fiziki, 1984, No.1(15), p.37-40.
43. Bakhrakh S.M., Kovalev N.P. Rayleigh-Taylor instability in strong media// VANT. Ser. Mat. Modelir. Fiz. Protsessov, 1994, No.3, p.33-37.
44. M.Legrand, N.Toque. Interface instabilities occurring during an explosive driven implosion// The Proc. of 3rd Int. Workshop on the Physics of Compressible Turbulent Mixing, Abbey of Royaumont, France, p.9-18.
45. M.Legrand, N.Toque. Richtmyer-Meshkov and Rayleigh-Taylor instabilities inducing a turbulent flow during a cylindrical implosion// The Proc. of the 4th Int. Workshop on the Physics of Compressible Turbulent Mixing, Cambridge, England. Cambridge Univ. Press, Cambridge, England, 1993, p.568-577.
46. V.A.Ogorodnikov, A.G.Ivanov, G.Ya.Karpenko, A.D.Kovtun, A.A.Demidov, L.A.Tolstikova. The importance of shear strength for development of disturbances and instability during acceleration, convergence and retardation of shells// The Proc. of the 4th Int. Workshop on the Physics of Compressible Turbulent Mixing, Cambridge, England. Cambridge Univ. Press, Cambridge, England, 1993, p.586-595.
47. V.A.Andronov, I.G.Zhidov, E.E.Meshkov, N.V.Nevmerzhitsky, V.V.Nikiforov, A.N.Razin, V.G.Rogachev, A.I.Tolshmyakov, Yu.V.Yanilkin. Computational and Experimental Studies of Hydrodynamic Instabilities and Turbulent Mixing// Review of VNIIEF Efforts. LA-12896, Los Alamos National Laboratory, 1995.
48. E.E.Meshkov, N.V.Nevmerzhitsky, V.G.Rogachev, I.G.Zhidov. The use of jelly technique to study hydrodynamic perturbation evolution at unstable plane and cylindrical surfaces// The Proc. of 5rd Int. Workshop on the Physics of Compressible Turbulent Mixing, Stony Brook, 1995, World Scientific, New Jersey, 1996.
49. Dubovik A.S. Photographic recording of high-speed processes// Moscow, Nauka Publishers, 1975.(trans: Aksenov, A. trans., New York, John Wiley, 1981).
50. Barnes J.F., Blewett P.J., McQueen R.G., Meyer K.A., Venable D. Taylor instability in solids// J. Appl. Phys, 1974, Vol.45, p.727-732.
51. Barnes J.F., Janney D.H., London R.k., Meyer K.A., Sharp D.H. Further experimentation on Taylor instability in solids// J. Appl. Phys, 1980, Vol.51, No.9, p.4678-4679.
52. Ivanov A.G., Mineyev V.N., Rozhdestvensky B.F., Svidinsky V.A., Tsytkin V.I. Research into the process of copper plate acceleration by detonation products through a gap// ZhTF, V.XLV, No.3, 1975. (trans: Tsytkin, et al., Sov. Phys. Tech. Phys. v. 20, 1975, p. 387-390.)
53. Zubarev V.N., Telegin G.S., Zhernokletov M.V. Condensed HE detonation product expansion isentropes// PMTF, 1969, No.4, p.127-132.

54. Arinin V.A., Tagirov V.R. Processing of Auroral TV Images// The Proceeding of 22nd European Meeting on Atmospheric Studies by Optical Methods, in press.
55. Seidel A.N. Physical value measurement errors// Translated from English. Leningrad, Nauka Publishers, 1974.
56. Taylor J. Introduction to theory of errors// Translated from English. Moscow, Mir Publishers, 1985.
57. Batalova M.V., Bakhrakh S.M. et al. Sigma complex for computing 2D gas dynamics problems// Proc.All-Union Workshop on numerical methods of viscous fluid mechanics. Novosibirsk, USSR Academy of Sciences Siberian Branch, 1969.
58. Bakhrakh S.M., Glagoleva Yu.P. et al. Concentration method base computation of gas-dynamic flows// DAN SSSR, V.257, No.3, 1981.(trans: Sov. Phys.-Doklady, v. 26, 1981, p.276-278.)
59. Bakhrakh S.M., Spiridonov V.F., Shanin A.A. Method for computing gas-dynamic flows of inhomogeneous medium in Lagrangian-Eulerian coordinates// DAN SSSR, 1984, V.276, No.4, p.829-833.(trans: Sov. Phys.-Doklady, v. 29, p. 443-445.)
60. Yanilkin Yu.V., Shanin A.A., Kovalev N.P. et al. EGAK program complex for computing 2D flows of multi-material medium// VANT. Ser. Mat. Modelir. Fiz. Protsessov, 1993, No.4, p.69-75.
61. Ott E. Nonlinear evolution of Rayleigh-Taylor instability of a thin layer// Phys. Rev. Lett., 1972, Vol.29, p.1429.
62. Gasilov V.A., Goloviznin V.M., Taran M.D. et al. On numerical simulation of Rayleigh-Taylor instability in compressible fluid// Preprint/ USSR Academy of Sciences IPM. - Moscow, 1979, No.70.
63. Manheimer W., Colombant D., Ott E. Three-dimensional, nonlinear evolution of the Rayleigh-Taylor instability of a thin layer// Phys. Fluids, 27(8), Aug, 1984, p. 2164-2175.
64. Volkova R.A., Kruglyakova L.V., Mikhailova N.V., Tishkin V.F., Favorsky A.P. On simulation of Rayleigh-Taylor instability in incompressible fluid in 3D setting up// Preprint/ USSR Academy of Sciences IPM. - Moscow, 1985, No.86.
65. Samarsky A.A., Popov Yu.P. Difference methods for solution of gas dynamics problems// Third Supplemented Edition.- Moscow, Nauka Publishers, 1992, 424p.
66. Volmir A.S. Non-linear dynamics of plates and shells// Moscow, Nauka Publishers, 1972, 432p.
67. Robinson A.C., Swegle J.W. Acceleration instability in elastic-plastic solids. 1.Numerical simulations of plate acceleration, 2.Analytical techniques// J. Appl. Phys, 1989, v.66, No.7, p.2838-2872.

-
68. Landau L.D., Lifshitz E.M. Theoretical physics. In 10 volumes. V.VII. Elasticity theory. Text-book// Fourth Improved and Supplemented Edition.- Moscow, Nauka Publishers, 1987, p.248.(Third English ed., Revised by E.M. Lifshitz, et al, Pergamon Press, NewYork, 1986.)
69. Drucker D.C. "Taylor instability " of surface of an elastic-plastic plate. *Mechanics Today*. Vol.5, 1980, p.37-47.
70. Miles J.W. Taylor instability of a flat plate// General Atomics Rep, GAMD-7335, 1960.
71. Ofer D., Hecht J., Shvarts D., Zinamon Z., Orszag S.A., McCrory R.L. Two and three dimensional analysis of nonlinear Rayleigh-Taylor instability// 4-th International workshop on the physics of compressible turbulent mixing. Cambridge, England, 29 March - 1 April 1993, p.119-128.
72. Bat'kov Yu.V., Glushak B.L., Novikov S.A. Strength of aluminum, copper and steel behind shock wave front// FGV, 1989, V.25, No.5, p.126-132.
73. Bat'kov Yu.V., Glushak B.L., Novikov S.A. Governing equation of aluminum and magnesium for high-velocity straining in shock waves// FGV, 1992, No.1, p.84-89. (trans: Combustion, Explosion, and Shock Waves, v. 28, 1992, p. 79-83.)
74. Bakhrah S.M., Spiridonov V.F., Ladagin V.K. Construction of a conservative scheme for 2D time-dependent gas dynamics computation// Proc. 4-th All-Union Workshop on numerical methods of viscous fluid mechanics. Novosibirsk, USSR Academy of Sciences Siberian Branch, 1978, p.51-63.
75. Bellman R., Pennington P. Effects of surface tension and viscosity on Taylor instability// *Quart. J. Appl. Math.*, 1954, p.151-162.
76. Izmailova V. Surface phenomena in protein compounds// Moscow, Chemia Publ., 1988, p. 239.

Technical Information Department • Lawrence Livermore National Laboratory
University of California • Livermore, California 94551