Reactions of Halo States: Coulomb Excitations

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ABSTRACT
Coulomb dissociation is a relatively clean probe of the structure of one- and two-nucleon halo nuclei. This is illustrated by the breakup of $^{11}$Be, $^8$B and $^{11}$Li and is discussed in terms of first order perturbation theory. First-order dipole transitions usually dominate the Coulomb dissociation but quadrupole transitions are not insignificant for a proton halo (e.g., $^8$B). Higher-order processes can also distort the observables, such as the momentum distributions of the fragments and the excitation energy spectrum.

1. Introduction

The properties of nuclei far from stability can now be studied at several radioactive beam facilities. Some facilities produce exotic secondary beams from the fragmentation of a primary beam of stable nuclei on a production target. The advantage of this production mechanism, in contrast to ISOL facilities, is that the secondary beam has a relatively high energy per nucleon, close to that of the primary beam (say from 20 MeV/u and up). The properties of the secondary beam particles can then be probed directly in various reactions on a secondary target without the need of a post accelerator. Moreover, the possible β-decay of the secondary beam particles will not play any role during the experiment because of the high velocity.

A large effort has been devoted to study light nuclei that are close to or on the edge of the neutron or proton driplines. In my lectures I will discuss some of them and how their structure can be probed in Coulomb dissociation experiments. A typical situation is a nucleus which has only one bound state, the ground state, and which contains one or two weakly bound valence nucleons and a more tightly bound core. Such nuclei are attractive from a theoretical point of view because the structure properties associated with the valence particle(s), which are probed in the breakup into a core fragment and the released valence particle(s), can possibly be described in terms of simple two- or three-body hamiltonians. This approach is illustrated for $^{11}$Be and $^8$B which contain a single weakly bound neutron and proton, respectively, and for $^{11}$Li which has two loosely bound valence neutrons.

If the valence nucleons are weakly bound and are in an orbit of low angular momentum (preferably an s-wave) one often uses the term a ‘halo’ nucleus to char-
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acterize the system. The valence nucleons will then have a large probability to be outside the core and their density will fall off slowly at large distances from the core. Consequently, the probability that a target nucleus can hit a valence nucleon without distorting the core can be quite high, and cross sections for removing the valence nucleons will be large compared to ordinary nuclei where the valence nucleons are much more tightly bound. Another feature of halo nuclei is a large concentration of dipole strength at very low excitation energies near threshold. As a consequence one has observed very large breakup cross sections on heavy targets where the strong Coulomb dipole field from a target nucleus can excite the relative motion of the core and the valence particles.

Two-neutron halo nuclei, as for example $^{11}$Li, $^{6}$He and $^{14}$Be, are particularly interesting because the adjacent nuclei, $^{10}$Li, $^{5}$He and $^{13}$Be, are all unbound. The pairing, or rather, the two-body attraction between the valence neutrons clearly play an important role in stabilizing these nuclei. The phenomenon is related to pairing in ordinary nuclei but the pair interaction is much stronger in halo nuclei because it is essentially the free $nn$ interaction whereas the pairing force in ordinary nuclei is quenched by nuclear medium effects.

I will discuss how one can simulate the structure of one- and two-nucleon halo nuclei, and how one can probe this structure in Coulomb dissociation experiments on high-Z targets. I will in particular discuss characteristic features of dipole excitations and use the breakup of $^{11}$Be as an illustration. I will also show how quadrupole and also higher-order excitations can distort these features and use the $^{8}$B→$^{7}$Be+p breakup as an example. But let me first mention some of the observables in a breakup reaction that may reflect the structure associated with the valence nucleons in a halo nucleus.

2. Observables in breakup reactions of halo nuclei

The first experiments that were performed with halo nuclei were transmission experiments and total interaction cross sections, $\sigma_I$, were extracted. They are the cross section for all processes that change the identity of the halo nucleus. The breakup reactions that are most sensitive to the halo, however, are those where the core fragment survives intact. They were later measured and will be discussed in the following. A more detailed discussion, including tables and illustrations of measurements that have been performed, can be found in the review by Tanihata.

To be specific, let us consider the variety of measurements that have been performed with beams of the two-neutron halo nucleus $^{11}$Li. Similar measurements have been performed with single-nucleon halo nuclei, such as $^{11}$Be and $^{8}$B, but the $^{11}$Li measurements are more challenging because of the three-body final state. If one valence neutron is removed, then the second neutron will also run away from the core because $^{10}$Li is unstable to neutron decay.

The most obvious observable in the $^{11}$Li→$^{9}$Li+n+n breakup is the two-neutron
removal cross section, $\sigma_{2n}$, which has been measured at various energies (from 20 to 800 MeV/u) and on a variety of targets. The next observable is the momentum distribution of the core fragment and it is commonly projected into a longitudinal and transverse component with respect to the beam axis. The two distributions reflect different aspects of the breakup mechanism and they are particularly interesting for a nuclear induced breakup. In Coulomb dissociation experiments, on the other hand, the longitudinal distribution is probably the most interesting because it is the most direct probe of the breakup mechanism.

A more difficult experiment is to measure the momenta of all three fragments in the final state, namely the $^9\text{Li}$ core and the two neutrons. This has been attempted several times on a high-Z target with the purpose of extracting the low-lying dipole strength of $^{11}\text{Li}$. From the measurements one can construct the momentum distributions for the relative motion of the two neutrons in the final state and for the relative motion of the core fragment and the center of mass of the two neutrons.

From the events one can then determine the cross section as function of the excitation energy of the three-body system. This is often referred to as the decay energy spectrum. Assuming that all events are due to Coulomb dipole excitations, and that first order perturbation theory is a valid approximation, one can then extract, as we shall see, the low-lying dipole strength.

3. The structure of halo nuclei

I will first discuss the structure of $^{11}\text{Be}$ and $^8\text{B}$ associated with a one-neutron and a one-proton halo, respectively, and describe it as a valence nucleon interacting with a core nucleus. To lowest order only the ground state of the core nucleus is considered but in certain cases (as f. ex. $^{11}\text{Be}$) it is necessary also to consider excited states of the core in the halo wave function. For $^8\text{B}$ one may also consider the complications of the non-zero spin of the $^7\text{Be}$ core. This discussion serves as a good introduction to the problems we currently face in the description of $^{11}\text{Li}$.

3.1. One-neutron halo in $^{11}\text{Be}$

The simplest approach to estimate the ground state wave function of a one-neutron halo is to adopt a spherical Woods-Saxon plus spin-orbit well for the interaction with the core, with mass number $A_c$.

$$V_{nc}(r) = V_0 \left(1 - F_{so} (\ell \cdot s) \frac{r_0}{r} \frac{d}{dr} \right) \left(1 + exp((r - R)/a)\right)^{-1}. \quad (3.1)$$

For a realistic choice of the radius $R = r_0 A_c^{1/3}$, the diffuseness $a$, and the spin-orbit strength $F_{so}$, one can then adjust the well depth so that the neutron separation energy $S_n$ is reproduced. The ground state of $^{11}\text{Be}$ is a $1/2^+$ state with $S_n = 0.5$ MeV. The halo wave function must be an s-wave with one radial node, because the lowest s-wave (with no nodes) is already occupied by core neutrons. Some observables are
only sensitive to the asymptotic form of the wave function, which is

\[ \psi(r) \rightarrow \frac{N}{r} e^{-kr}, \quad r \rightarrow \infty, \quad (3.2) \]

where \( k = \sqrt{2\mu_{nc}E_n}/\hbar \), and \( \mu_{nc} \) is the reduced mass.

If we need a more precise wave function, this approach is not good enough. The problem is that the \(^{10}\)Be core (with a 0\(^+\) ground state) has a low-lying 2\(^+\) state (at 3.37 MeV) which easily gets excited. A better approximation, which is commonly referred to as the weak coupling limit, is to include this excitation into the Hamiltonian for the valence neutron. After diagonalizing this Hamiltonian one obtains a wave function that contains both s- and d-waves,

\[ \psi(r) = A\psi_s(r)|0^+ > + B\psi_d(r)|2^+ >. \quad (3.3) \]

where the s-wave is associated with the 0\(^+\) ground state of \(^{10}\)Be, and the d-wave is associated with the 2\(^+\) excited state.

To make this approximation realistic one also needs a spin-orbit interaction that splits d-waves into a \( d_{5/2} \) and a \( d_{3/2} \)-wave. This has been done in Refs. 5 and 6 where the core was treated as a deformed nucleus. The result is a ground state wave function with 80 to 90\% in an \( s_{1/2} \)-wave and the remaining part mostly in a \( d_{5/2} \)-wave. It is also possible to adjust the parameters of the model so that the energy levels of all known positive parity states are reproduced.

If we want to calculate dipole excitations of the valence neutron we also need a model for the final state p-waves. The peculiar thing about \(^{11}\)Be is that the ground state is a 1/2\(^+\) state and not a 1/2\(^-\). The 1/2\(^-\) state is located at an excitation energy of 320 keV. The qualitative explanation is that the coupling to the 2\(^+\) state of the core lowers the 1/2\(^+\) level, whereas the 1/2\(^-\) level is pushed up in energy. A quantitative description of the inversion of the two levels is problematic. The point is that the weak coupling limit is not a good approximation for negative parity states because pair correlations between the p-shell neutrons in the deformed core influence the low-lying negative parity states by Pauli blocking\(^7\).

The empirical approach in Ref. 8 is to apply the weak coupling limit also for negative parity states. The effect of Pauli blocking is simulated by using a different well depth for the negative parity states. It appears that this may be the most practical approach we have at present. The authors state\(^8\) that this approach gives a good description of all known states of \(^{11}\)Be but details are not given. The model was instead applied in a three-body calculation of the ground state of \(^{12}\)Be.

### 3.2. One-proton halo in \(^8\)B

The valence proton in \(^8\)B is bound by only 137 keV. It occupies, to lowest order, a \( p_{3/2} \) orbit. An extra complication is that the \(^7\)Be core has a non-zero spin, \( I_c = 3/2^- \). Thus the ground state of \(^8\)B is a 2\(^+\) state which is formed by coupling the two 3/2\(^-\) spins of the core and the valence proton to the total spin 2\(^+\).
One can again describe the $p_{3/2}$ wave function in terms of a single-particle Hamiltonian which includes the Coulomb interaction and a single-particle potential of the form (3.1). The discussion of the Coulomb dissociation of $^8$B presented in Sect. 7 is based on Ref. 9. The parameters used there to describe the ground state are

$$r_0 = 1.25 \text{ fm}, \quad a = 0.52 \text{ fm}, \quad F_{so} = 0.351 \text{ fm}, \quad V_0 = -44.658 \text{ MeV}. \quad (3.4)$$

States in the continuum consist of single-particle orbits $(k\ell j)$ that are coupled to the core spin $I_c$ to form a total spin $J$. Each spin channel can therefore be characterized by the quantum numbers $|(\ell j; I_c)J\rangle$. A simple way to achieve a good description is to adjust the well depth, $V_0(\ell j, J)$, for each spin channel so that the known features are reproduced. Examples on known channels are the $1^+$ and $3^+$ resonances, associated with $p_{3/2}$ orbits. These resonance can be reproduced with well depths, respectively, of $-42.14$ and $-36.80$ MeV. In the calculations reported in Sect. 7, the well depths for all other spin channels have been set equal to the well depth for the $1^+$ channel. This is probably not the best choice. However, the objective in Sect. 7 is not so much to make accurate predictions but instead to investigate various qualitative features of the Coulomb dissociation of $^8$B.

### 3.3. Two-neutron halo in $^{11}$Li

The ground state of $^{11}$Li has a two-neutron separation energy\(^{10}\) of only $295\pm35$ keV. The simplest model of $^{11}$Li is to describe it as a three-body system, consisting of two valence neutrons interacting with each other and with a spherical and spinless $^9$Li core. This implies a hamiltonian of the form

$$H_{3B} = \frac{p_1^2 + p_2^2}{2m} + V_{nc}(1) + V_{nc}(2) + V_{nn} + \frac{(p_1 + p_2)^2}{2mA_c}. \quad (3.5)$$

In this approach, the only part of the core Hamiltonian that is considered is the last term of (3.5), which is the recoil kinetic energy.

There have been many calculations of the ground state of $^{11}$Li (see f. ex. Ref. 11) but most of them are probably unrealistic. A critical issue in modeling the ground state as a three-body system is the neutron-core interaction, $V_{nc}$, which is supposed to describe the low-lying structure of the unbound nucleus $^{10}$Li. If $^9$Li were a spherical nucleus, one would expect that elastic scattering of neutrons on $^9$Li would be dominated by a $p_{1/2}$ resonance. Such neutron scattering data are not available but measurements\(^{12}\) of the reaction $^{11}$B($^7$Li,$^8$B)$^{10}$Li do show a resonance structure near $540$ keV. One can easily adjust the potential (3.1) to produce a $p_{1/2}$ resonance at this energy. An example is the parameter set

$$r_0 = 1.27 \text{ fm}, \quad a = 0.67 \text{ fm}, \quad F_{so} = 0.562 \text{ fm}, \quad V_0 = -35.366 \text{ MeV}. \quad (3.6)$$

Using a realistic nn interaction, $V_{nn}$, one can then diagonalize the Hamiltonian (3.5), and one finds a halo ground state that is dominated by p-waves, with 80-90% in $(p_{1/2})^2$ configurations\(^{13}\).
The same measurements\textsuperscript{12} also suggest an unexpected large cross section near threshold. This may reflect a strong neutron-core s-wave scattering which is not reproduced by the parameter set (3.6). Other measurements also suggest a strong s-wave component near threshold. In one of them\textsuperscript{14}, the unbound nucleus $^{10}\text{Li}$ was produced. After its decay, the distribution in the relative velocity of the $^9\text{Li}$ fragment and a single neutron was measured near zero degrees. The distribution showed a strong peak near zero relative velocity, whereas as the forward and backward peaks expected from the decay of a p-wave resonance were absent. A possible explanation is that a mechanism similar to that responsible for the inversion of the $1/2^-$ and $1/2^+$ levels in $^{11}\text{Be}$ is also be present in $^{10}\text{Li}$ and that it should produce a strong neutron-core, s-wave scattering near threshold.

The $\beta$-decay measurement\textsuperscript{15} of $^{11}\text{Li}$ to the first excited $1/2^-$ state of $^{11}\text{Be}$ indicates that the p-wave component of the $^{11}\text{Li}$ ground state is only about 50\%. This fraction is much smaller than the 80-90\% mentioned above, predicted by the three-body calculation that was based on the parameter set (3.6). It remains to be seen if one can develop a model for $^{10}\text{Li}$ and $^{11}\text{Li}$ that can give a consistent description of all observables.

4. Coulomb Excitation Amplitudes

Here we consider the Coulomb excitation of a halo nucleus $(Z,A)$, consisting of a core $(Z_c,A_c)$ and a valence nucleon $(Z_v,A_v)$, in the Coulomb field from a heavy target nucleus $(Z_T,A_T)$. We shall give the general expression for the excitation amplitude which is based on first order perturbation theory and relativistic kinematics. The formalism is quite general and we do not need to make specific assumptions about the structure of the halo nucleus. This will be done in Sect. 5 where the Coulomb dissociation is discussed and the final states in the continuum are specified. In order to appreciate the general expression for the Coulomb excitation amplitude, it is useful first to consider dipole excitations in the non-relativistic limit.

4.1. Non-relativistic dipole excitations

The Coulomb field from a target nucleus can act both on the core and on the valence nucleon of the halo nucleus. Here we are only interested in the part that acts on their relative position $r = r_{xc}$ and causes a breakup. The part that acts on the position $R$ of the target with respect to the halo nucleus leads to an overall scattering of the halo nucleus but we are not interested in that part here and subtract it, i.e.

$$V_{\text{cou}} = Z_T e^2 \left( \frac{Z_v}{|R - A_v r|} + \frac{Z_c}{|R + A_c r|} - \frac{Z_c + Z_v}{R} \right).$$

Expanding this interaction to first order in $r$ we obtain the dipole field,

$$V_{\text{dip}} = Z_T e^2 \frac{Z_v A_v - Z_c A_c}{A} \frac{R \cdot r}{R^3}.$$

\textsuperscript{(4.1a)}
It looks just like the dipole field acting on a particle with an effective charge of
\[ e_1 = \frac{Z_x A_e - Z_c A_x}{A} e. \] (4.1b)

We can now determine the excitation amplitude, from the initial state \( |i > \) to a final state \( |f > \), generated by the dipole field. The excitation energy is expressed as \( \Delta E_{fi} = \hbar \omega \). For a straight line trajectory, \( R = b x + v t z \), we obtain in first order perturbation theory
\[ a_{fi} = \frac{1}{i \hbar} \int_{-\infty}^{\infty} dt e^{i\omega t} \frac{Z T e e_1}{(b^2 + (v t)^2)^{3/2}} \langle f | b x + v t z | i > \]
\[ = \frac{Z T e e_1}{i \hbar v} \frac{2 \omega}{v} [ \langle f | x | i > K_1(\frac{\omega b}{v}) + i \langle f | z | i > K_0(\frac{\omega b}{v})], \] (4.2)
where \( K_0(\xi) \) and \( K_1(\xi) \) are modified Bessel functions. The two terms, due to transverse \( (x) \) and longitudinal \( (z) \) excitations, are discussed in more detail in Sect. 6.

4.2. General Coulomb excitation amplitude

The general expression for the Coulomb excitation amplitude has been derived in Ref. 16. It is based on first order perturbation theory and assumes a straight line trajectory for the relative motion of projectile and target. The result for electric transitions, for a given impact parameter \( b \), can be written in the form of the following multipole expansion,
\[ a_{fi} = \frac{1}{i} \sum_{\lambda \mu} F_{\lambda \mu} \langle f | M_{\lambda \mu} | i >, \] (4.3)
where the F-amplitudes are
\[ F_{\lambda \mu} = \frac{i^{\lambda + \mu} Z T e}{\hbar v \gamma} \sqrt{\frac{16 \pi}{2 \lambda + 1}} \frac{(\omega / v)^{\lambda} G_{\lambda \mu}}{\sqrt{\lambda + \mu)!((\lambda - \mu))!}} K_{\mu}(\frac{\omega b}{\gamma v}), \] (4.4a)
and \( \gamma = 1/\sqrt{1 - (v/c)^2} \).

The \( G_{\lambda \mu} \) factors are all equal to one in the non-relativistic limit. The expressions to be used for dipole and quadrupole excitations are
\[ G_{10} = G_{20} = G_{2 \pm 2} = \frac{1}{\gamma}, \quad G_{1 \pm 1} = 1, \quad G_{2 \pm 1} = \frac{1}{2} \left( 1 + \frac{1}{\gamma^2} \right). \] (4.4b)

The general expressions can be extracted from Ref. 16 which, however, uses a somewhat different notation.

The amplitude (4.3) separates nicely into dynamical factors, the F-amplitudes, and a structure part, which is the multipole matrix elements. A similar expression for magnetic transitions is given in Ref. 16. They do play a role at relativistic energies\(^{17} \), as for example in the Coulomb dissociation of \(^8\)B, but we shall not consider them.
The electric multipole operators for the Coulomb excitation of a halo nucleus are
\[ M_{\lambda \mu} = e_{\lambda} r^\lambda Y_{\lambda \mu}(\hat{r}) = \left( Z_{c} e \left( \frac{-A_{c}}{A} \right)^{\lambda} + Z_{e} e \left( \frac{A_{e}}{A} \right)^{\lambda} \right) r^\lambda Y_{\lambda \mu}(\hat{r}). \] (4.5)

The first factor is the effective multipole charge. It is consistent with dipole charge defined in Eq. (4.1b). One can also easily show that the amplitude (4.3) is consistent with (4.2) in the non-relativistic limit.

The Coulomb dissociation of halo nuclei is usually dominated by dipole transitions. At this point it is useful to point out the difference between the dissociation of a proton and a neutron halo. The effective dipole and quadrupole charges for a proton halo are
\[ e_{1}^{p} = \frac{A_{c} - Z_{c}}{A} e, \quad e_{2}^{p} = \frac{Z_{c} + A_{c}^{2}}{A^{2}} e, \]
and for a neutron halo we obtain
\[ e_{1}^{n} = -\frac{Z_{c}}{A} e, \quad e_{2}^{n} = \frac{Z_{c}}{A^{2}} e. \]

It is noted that the quadrupole charge for a neutron halo can be quite small. Consequently, one can essentially neglect quadrupole transitions in the dissociation process. The quadrupole charge for a proton halo, on the other hand, is large and it is necessary to consider quadrupole transitions as illustrated in Sect. 7 for \(^{8}\text{B}\. \)

5. Coulomb Dissociation of Halo Nuclei

To calculate the multipole matrix element in (4.3) we need to specify the initial and final states. Here we consider a single-nucleon halo nucleus and assume that the initial state is a pure single-particle state of the form,
\[ |i> = \frac{u_{0}(r)}{r} |\ell_{0}l_{0}m_{0}>. \]

The final state is a continuum state. If we ignore the interaction between the emitted nucleon and the core fragment (and also the final state interaction with the target nucleus), the final state is a plane wave with momentum \(\hbar k\), and we can use the expansion
\[ <f| = \frac{\exp(-i\kappa r)}{(2\pi)^{3/2}} = \sqrt{\frac{2}{\pi}} \sum_{\ell m} i^{-\ell} j_{\ell}(kr) Y_{\ell m}(\hat{k}) Y_{\ell m}^{*}(\hat{r}), \] (5.1)
where \(j_{\ell}(kr)\) are spherical Bessel functions which have the asymptotic form,
\[ j_{\ell}(kr) \rightarrow \frac{\sin(kr - \ell\pi/2)}{kr}, \quad r \rightarrow \infty. \] (5.1a)
If the final state interaction between the two fragments is important we can instead use the distorted waves (c. f. chapter 5.B in Ref. 18)

\[ < f | = \sum_{\ell j m} e^{i \delta_{\ell j}(k)} \frac{u_{\ell j m}(r)}{kr} < \hat{k}| \ell j m > < \ell j m | \hat{r} > . \tag{5.2} \]

where \( \delta_{\ell j}(k) = \sigma_{\ell j}^{C} + \delta_{\ell j}^{\mu \nu} - \ell \pi/2 \) is the total phase shift due to the Coulomb, nuclear, and centrifugal potentials. Comparing this expression to the plane wave expansion, Eqs. (5.1-1a), we see that the radial wave function must have the following asymptotic normalization

\[ u_{\ell j m}(r) \to \sqrt{\frac{2}{\pi}} \sin(kr + \delta_{\ell j}(k)), \ r \to \infty. \tag{5.2a} \]

Inserting this final state into the multipole matrix element we obtain

\[ < f | M_{\lambda \mu} | i > = \frac{1}{k} \sum_{\ell j m} e^{i \delta_{\ell j}(k)} < \hat{k}| \ell j m > < j_{0} m_{0} \lambda \mu | j m > \frac{< k \ell j || M_{\lambda} || \ell_{0} j_{0} >}{\sqrt{2 j + 1}}, \tag{5.3} \]

where the reduced matrix element is (c. f. Ch. 3A-2 in Ref. 19)

\[ < k \ell j || M_{\lambda} || \ell_{0} j_{0} >= e_{\lambda}(-1)^{j_{0} \lambda - j} \sqrt{\frac{(2j + 1)(2j_{0} + 1)}{4\pi}} \times < j_{0} \frac{1}{2} \lambda 0 | j_{-} \frac{1}{2} > \int_{0}^{\infty} dr \ u_{\ell j m}(r) \ r^{\lambda} u_{0}(r). \tag{5.3a} \]

We can now calculate the momentum distribution associated with the relative motion of the emitted nucleon and the core fragment in the final state. It is simply the square of the excitation amplitude (4.3) because of the way we have chosen to normalize the final state in Eqs. (5.2-2a), i. e.

\[ \frac{dP(k, b)}{d^{3}k} = |a_{f}i|^{2} = |\sum_{\lambda \mu} F_{\lambda \mu} < f | M_{\lambda \mu} | i > |^{2}. \tag{5.4} \]

The amplitudes for different multipole transitions are here added coherently. This leads to an interference, for example between dipole and quadrupole transitions, and we shall see an example on that in the case of \(^{8}\text{B}\) later on.

5.1. Decay energy spectrum

If we integrate over all orientations of the momentum \( k \), and take the average over the magnetic quantum number \( m_{0} \) of the initial state, we obtain the differential dissociation probability for a given impact parameter,

\[ \frac{dP(k, b)}{k^{2}dk} = \frac{1}{2j_{0} + 1} \sum_{m_{0}} \int d\Omega_{k} \frac{dP}{d^{3}k} = \sum_{\lambda \mu} |F_{\lambda \mu}|^{2} \frac{1}{2\lambda + 1} \frac{dB(E\lambda)}{k^{2}dk}, \tag{5.5} \]
where the multipole strength function is
\[
\frac{dB(E\lambda)}{dk} = \sum_{\ell_j} \frac{|<\ell_j||M_\lambda||\ell_0\rangle|^2}{2\ell_0 + 1}.
\] (5.6)

It is possible to perform the integration over impact parameters analytically. This is usually done from a certain minimum impact parameter \(b_0\) where nuclear absorption starts to take over. Another possibility is that the acceptance of the detector defines a maximum scattering angle of the charged fragments which can then be translated into an equivalent minimum impact parameter. In either case, the expression (5.5) is diagonal in \(\mu\) so the integration over impact parameter involves
\[
\int_{b_0}^{\infty} 2\pi dbb|K_\mu(\omega b/\gamma v)|^2 = \pi b_0^2 \left[|K_{\mu-1}(\xi)|^2 - |K_\mu(\xi)|^2 + \frac{2\mu}{\xi} K_\mu(\xi) K_{\mu-1}(\xi)\right],
\] (5.7)
where \(\xi = \frac{\omega b}{\gamma v}\).

One can now derive a closed expression for the decay energy spectrum, which is the Coulomb dissociation cross section, \(d\sigma/dE\), as a differential in the excitation energy, \(E_x = S_x + (\hbar k)^2/(2\mu_{xx})\), where \(S_x\) is the separation energy and \(\mu_{xx}\) is the reduced mass. Here we shall only give the expression one obtains from dipole transitions. Thus by inserting Eqs. (4.4a) into (5.5), and making use of (5.7) for \(\lambda=1\) one obtains
\[
\frac{d\sigma}{dE_x}(\lambda=1) = \left(\frac{4\pi Z T e}{3\hbar v}\right)^2 \left(2\xi K_0(\xi) K_1(\xi) - \left(\frac{v}{c}\right)^2 \xi^2 (K_1^2(\xi) - K_0^2(\xi))\right) \frac{dB(E1)}{dE}.
\] (5.8)

An even simpler form can be derived in the limit of low excitation energies and high velocities where \(\xi = \frac{\omega b}{\gamma v} \ll 1\) and
\[
\xi K_1(\xi) \to 1, \quad K_0(\xi) \to \log\left(\frac{1.123}{\xi}\right).
\]
Inserting this into (5.8) we obtain
\[
\frac{d\sigma}{dE_x}(\lambda=1) = \left(\frac{4\pi Z T e}{3\hbar v}\right)^2 \left(2\log\left(\frac{1.123\gamma v}{\omega b}\right) - \left(\frac{v}{c}\right)^2\right) \frac{dB(E1)}{dE}.
\] (5.8a)

If the dissociation is dominated by dipole transitions then it is seen that one can extract the dipole strength directly from the measured decay energy spectrum. In practice one should be concerned about possible contributions from other multipole transitions, from higher-order processes, and from nuclear induced breakup.

5.2. Sum rules

Let me finally mention that the multipole response obeys certain sum rules. The first one is the total multipole strength, obtained by summing over all final states,
\[
B(E\lambda) = \sum \frac{|<f||M_\lambda||i>|^2}{4\pi} = e_\lambda^2 \frac{2\lambda + 1}{\ell_0^2} |<i|e^{2\lambda}|i>|.
\] (5.9)
Thus the total dipole strength is proportional to the mean square radius of the halo. This explains why the Coulomb dissociation of a halo nucleus is so important for a high-Z target. It is noted that this sum rule requires a sum over all final states, including those that are blocked by core nucleons. The total strength extracted from a dissociation measurement may therefore be slightly smaller.

The second sum rule is the energy weighted sum over all states, including allowed and also Pauli blocked states,

\[ S(E\lambda) = \sum_{f,\mu} (E_f - E_i) |< f|M_{\lambda\mu}|i >|^2 = \frac{\lambda(2\lambda + 1)}{4\pi} \frac{e^2}{2m} \frac{\hbar^2}{\gamma^2} < i |r^{2\lambda+2}|i >. \quad (5.10) \]

The energy-weighted dipole strength is seen to be a constant. The average dipole excitation energy, obtained as the ratio of (5.10) and (5.9), will therefore become smaller the larger the mean square radius of the halo is.

6. Neutron halos

To illustrate the formalism presented in Sect. 5 let us first consider the Coulomb dissociation of $^\text{11}$Be into a neutron and a $^\text{10}$Be fragment. Let us assume that the initial state is a pure s-state, bound by $S_n=0.5$ MeV, and that we can ignore the spin-orbit splitting in the final state. The multipole matrix elements (5.3) then takes the form

\[ < f|M_{\lambda\mu}|i > = \frac{e^{i\lambda}(k)}{k} Y_{\lambda\mu}(k) \frac{< k||M_{\lambda}\|0 >}{\sqrt{3}}. \quad (6.1) \]

The momentum distribution for the relative motion of the neutron and the core fragment can now be obtained from Eq. (5.4). We only include dipole transitions since they dominate by far the dissociation of a neutron halo. Thus by inserting (6.1) into (5.4), and making use of the properties of the F-amplitudes (4.4), one finds that the longitudinal and transverse components separate nicely. The result is

\[ \frac{dP(k, b)}{d^2k} = \frac{1}{3} \frac{dB(E1)}{k^2dk} \left[ |F_{10}|^2|Y_{10}|^2 + |F_{11}|^2|Y_{11} - Y_{1-1}|^2 \right] \]

\[ = \frac{1}{b^2} \left( \frac{2Ze^2}{\hbar\gamma\nu} \right)^2 \frac{1}{\gamma^2} \left[ (\xi K_0(\xi))^2 \cos^2(\theta_z) + (\xi K_1(\xi))^2 \cos^2(\theta_x) \right] \frac{1}{3} \frac{dB(E1)}{k^2dk}, \quad (6.2) \]

where $\xi = \omega b/(\gamma\nu)$. The angular dependence is determined by the two projections, $\cos(\theta_z) = e_z \cdot k/k$ and $\cos(\theta_x) = e_x \cdot k/k$, associated with longitudinal and transverse excitations, respectively. The relative importance of the two terms in (6.2) is determined by the modified Bessel functions which are illustrated in Fig. 1. The dissociation is seen to be dominated by transverse excitations when $\xi = \omega b/(\gamma\nu) \ll 1$. Moreover, the dependence on the impact parameter is then $1/b^2$, consistent with the fact that the cross section (5.8a) has a logarithmic dependence on the minimum impact parameter. Longitudinal excitations, on the other hand, are important only
in the (broad) vicinity of $\xi \approx 1$, i.e. when the impact parameter approaches the adiabatic distance $\gamma v/\omega$. Consequently, the Coulomb dissociation of a weakly bound halo nucleus is dominated by transverse excitations at high beam velocities.

![Graph showing the two functions that determine the relative importance of the longitudinal ($n=0$) and transverse ($n=1$) Coulomb dipole excitations.](ANL-P-22,145)

Fig. 1. The two functions that determine the relative importance of the longitudinal ($n=0$) and transverse ($n=1$) Coulomb dipole excitations.

The angular dependence of the momentum distribution (6.1) is also quite simple. The part due to longitudinal excitations goes as $\cos^2(\theta_z)$ so the preferred emission of the two fragments is along the $z$-axis. The transverse part is similar but the symmetry axis is the $x$-axis, i.e. the direction of the impact parameter. The simple features of this angular dependence, combined with the fact that transverse excitations dominate the dissociation, are in qualitative agreement with the angular distributions that have been measured by Kakamura et al.\textsuperscript{20} (see their Fig. 3).

6.1. Dipole strength function

It is useful to have an analytic expression for the dipole strength. This can be obtained if we just use the asymptotic form (3.2) as the initial state and assume that the final state is a plane wave. The dipole matrix element generated for example by the $z$ coordinate is then

$$< f|z|i> \propto \int d^{(3)}r \ e^{-ikr} \ z \ \frac{e^{-\kappa r}}{r} = i \frac{d}{dk_z} \int d^{(3)}r \ e^{-ikr} \frac{e^{-\kappa r}}{r}$$

where we have inserted the Fourier transform of the Yukawa function. The dipole matrix elements generated by the $x$ and $y$ coordinates are similar, and summing the
squares we obtain the dipole strength distribution,
\[
\frac{dB(E1)}{k^2 dk} \propto \frac{k^2}{(k^2 + \kappa^2)^4}.
\] (6.4)

This can be converted into a differential in the excitation energy, and with the proper normalization (consistent with the sum rule (5.9)) one obtains\(^2\)
\[
\frac{dB(E1)}{dE_x} = e_1^2 \frac{12}{\pi^2} < r^2 > \frac{(S_n(E_x - S_n))^{3/2}}{E_x^4},
\] (6.5)

where \(< r^2 >\) is the mean square radius of halo ground. From Eq. (5.8) one can now analyze the measured decay energy spectrum\(^2\). A minimum impact parameter of \(b_0 = 12.3\) fm is consistent with the acceptance of the detector. One can then adjust the strength \(< r^2 >\); the best fit to the data is achieved\(^2\) for \(< r^2 > = 41 \pm 9\) fm\(^2\).

The mean square radius of the adopted initial state (3.2) is
\[
< r^2 > = \frac{A}{A_e} \frac{\hbar^2}{4mS_n} = 22.7\ fm^2,
\]
which is much smaller than the best fit value. Thus it appears that the model gives an excellent description of the shape of the dipole strength distribution but the magnitude is off by a factor of two. The reason is that the dipole matrix elements are determined by the far tail of the initial state, which does have the asymptotic form (3.2), but we did not determine the correct normalization.

![Graphs](ANL-P-21.864.867)

**Fig. 2.** The decay energy spectrum for \(^{11}\)Be measured\(^2\) at 72 MeV/u on Pb (left panel), and the longitudinal momentum distribution of \(^{10}\)Be fragments obtained\(^2\) at 63 MeV/u on U (right panel) are compared to the predictions which are based on the Woods-Saxon well discussed in the text.
One can get a more realistic value of \( < r^2 > \) by including the effect of the finite size of the core. An example is shown in the left panel of Fig. 2. The initial and final states were calculated for the Woods-Saxon well (3.1), with \( R=2.7 \text{ fm}, \alpha=0.52 \text{ fm}, F_{so}=0 \) and \( V_0=-61.17 \text{ MeV} \). The initial state is the second s-wave state, which is bound by 0.5 MeV and has a mean square radius of 47 fm\(^2\). The shape of the decay energy spectrum is similar to that predicted by (6.5) but the peak-height is now slightly larger than the data. A possible explanation is that the initial state is not a pure s-wave as discussed in Sect. 3. One should also include a spin-orbit force and consider the possibility that the effective well for the final state p-waves may be different. Such effects were considered in Ref. 6; the resulting decay energy spectrum is below the data. Clearly, one needs a better quantitative understanding of the data.

Another probe of the dipole response is the longitudinal momentum distribution of \(^{10}\text{Be}\) fragments. This distribution can be obtained from Eq. (6.2) by integrating over an appropriate range of impact parameters, and projecting the result onto the beam axis. The prediction of the model, based on the Woods-Saxon well described above, is illustrated in the right panel of Fig. 2 together with the data for a uranium target\(^{22}\). It appears that the two different measurements shown in Fig. 2 are consistent with respect to the shape of the dipole strength distribution.

6.2. Coulomb dissociation of \(^{11}\text{Li}\)

The uncertainties in the structure of the ground state of \(^{11}\text{Li}\) has already been mentioned in Sect. 3.3. In addition, there appears also to be technical difficulties in calculating the dipole response to the three-body continuum, and it is therefore not possible at present to show realistic predictions for the Coulomb dissociation of \(^{11}\text{Li}\).

It is anyway useful to illustrate some of the qualitative features that have been predicted. I shall here make use of a model that we developed at an early stage\(^{23}\). The first assumption made was that we can ignore the recoil energy of the core, i.e. the last term in Eq. (3.5). The second assumption was that we can approximate the interaction between the two valence neutrons by a density-dependent \( \delta \)-function interaction. These two assumptions simplified the calculation of the ground state\(^{23}\) and also the dipole response\(^{24}\), by essentially reducing it to a two-body problem. I shall not repeat the calculational technique here but only summarize the results.

The neutron-core interaction was adjusted to produce a \( p_{1/2} \) resonance at 800 keV, and the free space \( nn \) interaction was determined from an infinite scattering length. Both assumptions are somewhat unrealistic (the neutron-core interaction is too weak and the \( nn \) interaction is too strong). They produced a ground state with a binding energy of 200 keV, somewhat weaker the presently accepted value\(^{10}\) of 295 keV. About 77% of the halo ground state was occupied by \( (p_{1/2})^2 \) configurations.

The calculated dipole response is illustrated in Fig. 3. The calculation included the final state interactions, both between the two neutron and with the core, to all orders. A characteristic feature is the ridge structure of the response that appears when the kinetic energy of one of the neutrons is large. It reflects the 800 keV \( p_{1/2} \)
resonance of the adopted neutron-core Hamiltonian. The two ridges disappear if one ignores the final state interaction with the core.

Fig. 3. Contour plot of the calculated dipole strength of the valence neutrons in $^{11}\text{Li}$, as function of the final state kinetic energies of the two neutrons$^{24}$.

Fig. 4. Momentum distribution for the relative motion of the two neutrons (a), and for their center of mass motion with respect to the $^9\text{Li}$ fragment (b), obtained in Coulomb dissociation$^{3,25}$ of $^{11}\text{Li}$ at 28 MeV/u on Pb.
The dipole response shows a large concentration of strength at very low excitation energies. So do experiments but the measured peak is located at a somewhat higher excitation energy. The problem is illustrated in more detail in Fig. 4 where the predicted momentum distributions associated with the relative motion of the two neutrons in the final state, and also of the two neutrons with respect to the core fragment, are compared with data. It is seen that the model describes the relative motion of the two neutrons surprisingly well. The center of mass motion of the two neutrons with respect to core, on the other hand, is poorly reproduced. Some of this discrepancy may be due to higher-order dynamical effects as discussed Ref. 26 but most of it is probably due to unrealistic assumptions about the neutron-core interaction.

7. Coulomb dissociation of $^8\text{B}$

The main interest in $^8\text{B}$ concerns the electric dipole strength near threshold. The reason is that it determines the low-energy radiative capture rate of protons on $^7\text{Be}$, which is a determining factor for the abundance of $^8\text{B}$ in the sun. The capture rate has been measured previously but one would like to confirm the result and reduce the uncertainty. The hope is that by measuring the Coulomb dissociation of $^8\text{B}$ one may be able to extract the necessary dipole strength.

The Coulomb dissociation of $^8\text{B}$ has already once been measured on a lead target. A critical issue, which has been debated in the literature, is the influence of E2 transitions. They do not play any role in the radiative capture but they may be important in the Coulomb dissociation. If that is the case, it becomes difficult to extract the dipole strength from the measured decay energy spectrum. One would either have to rely on a model for the E2 strength, or one would need to measure other observables in order to constrain it. We shall see that the longitudinal momentum distribution of $^7\text{Be}$ fragments may provide some information.

Here I discuss some of the basic results that have been obtained in Ref. 9 from the structure model discussed in Sect. 3.2. They include the electric dipole (E1) and quadrupole (E2) strength functions, the decay energy spectrum for the $^8\text{B}\rightarrow^7\text{Be}+p$ Coulomb dissociation, and finally the longitudinal momentum distribution of $^7\text{Be}$ fragments. The relationship to radiative capture is also discussed.

7.1. Electric multipole responses

The calculated electric dipole and quadrupole strengths are shown in Fig. 5. The contribution to the E1 strength from final state s-waves is shown by the dashed curve. It dominates at low excitation energies but d-waves become important at 0.6 Mev and higher excitation energies. The E2 strength has a peak at about 0.6 MeV, which reflects the known $1^+$ resonance associated with $p_{3/2}$ continuum waves; the
dashed curve is the contribution from final state f-waves.

![Graphs showing dipole (E1) and quadrupole (E2) strengths for the \(^{8}\text{B} \rightarrow ^{7}\text{Be} + p\) breakup, as functions of the relative kinetic energy of the two fragments.]

**Fig. 5.** Calculated dipole (E1) and quadrupole (E2) strengths for the \(^{8}\text{B} \rightarrow ^{7}\text{Be} + p\) breakup, as functions of the relative kinetic energy of the two fragments.9

### 7.2. Radiative capture and Coulomb dissociation

The cross section for the radiative capture, \(c + p \rightarrow B + \gamma\), is related to the photoabsorption cross section for the inverse reaction, \(B + \gamma \rightarrow c + p\), by detailed balance.31 For a given electric multipole transition the relation is,

\[
\sigma_{E\lambda}^{(\gamma)} = \left(\frac{E_{\gamma}}{\hbar c k}\right)^2 \frac{2(2I_{B} + 1)}{(2I_c + 1)(2I_p + 1)} \sigma_{E\lambda}^{(\gamma)},
\]

(7.1)

Here \(E_{\gamma}\) is the photon energy, and \(\hbar k\) is the momentum for the relative motion of \(c\) and \(p\). The expression contains the ratio of the phase space factors for the photon and for the relative motion of \(c\) and \(p\), and the statistical weights associated with the spins in the initial and final states. In the case of \(^{8}\text{B}\) we have \(I_{B} = 2\), and \(I_c = 3/2\) and \(I_p = 1/2\) are the ground state spins of \(^{7}\text{Be}\) and the proton. The additional factor of two in (7.1) accounts for the two possible polarizations of the photons emitted in the radiative capture process.

The photoabsorption cross section, on the other hand, can be expressed in terms of the multipole response as follows (c. f. Eq. 3C-16 of Ref. 19)

\[
\sigma_{E\lambda}^{(\gamma)} = \frac{(2\pi)^2(\lambda + 1)}{\lambda((2\lambda + 1)!!)^2} \left(\frac{E_{\gamma}}{\hbar c}\right)^{2\lambda - 1} dB(E\lambda) dE_{\gamma}.
\]

(7.2)
The cross section for the radiative capture of charged particles is usually expressed in terms of the S-factor,

\[ S(E_{\text{rel}}) = E_{\text{rel}} \sigma_{E_\lambda}^r \exp(2\pi\eta(E_{\text{rel}})), \tag{7.3} \]

where \( \eta = \frac{Z_c Z_p e^2}{h v_{\text{rel}}} \). Inserting (7.1) and (7.2) into this expression, one can now calculate the S-factor directly from the multipole response. The S-factor one obtains from the E1 strength shown in Fig. 5 is 17 eV b, at a relative energy of 100 keV. This value is at the lower end of most radiative capture measurements\(^2\). The S-factor that one obtains from the E2 strength shown in Fig. 5 is about three orders of magnitude smaller, so it is insignificant.

Let me mention that one could have used the empirical S-factor to calibrate the s-wave, proton-core interaction. Another possibility is to use the \( n+^7\text{Li} \rightarrow ^8\text{Li}+\gamma \) radiative capture, which has recently been measured\(^3\), to constrain the model.

### 7.3. Decay energy spectrum

The decay energy spectrum for the \(^8\text{B} \rightarrow ^7\text{Be}+p \) breakup has been measured by Motobayashi et al.\(^2\). The data are shown in the left panel Fig. 6. Also shown are contributions predicted from the E1 and E2 strengths shown in Fig. 5, together with their sum (solid curve). The minimum impact parameter was set to 20 fm which corresponds to a \(^8\text{B} \) Coulomb scattering angle of 4.5°. This choice was made because the detection efficiency was small at larger scattering angles.

![Figure 6](ANL-P-22144)

Fig. 6. The left panel shows the measured\(^2\) and calculated\(^9\) decay energy spectrum for \(^8\text{B} \), obtained at 46.5 MeV/u on a lead target. The right panel shows the measured and calculated longitudinal momentum distribution of \(^7\text{Be} \) fragments obtained at 41 MeV/u on a gold target\(^3\).
The calculations indicate that E2 transitions do play a significant role in first order perturbation theory. However, higher-order dynamical processes cannot be ignored in the present experiment. This is discussed in Sect. 7.5. It should also be mentioned that M1 transitions, which have not been considered here, make a small contribution around 0.6 MeV. The M1 contribution becomes much more important at higher beam energies.

7.4. Asymmetry in longitudinal momentum distribution

The expression (5.4) for the relative momentum distribution of the two fragments shows that E1 and E2 amplitudes add coherently and they can therefore produce an interference. The simplest way to see the effect is to measure the longitudinal momentum distribution of the $^7$Be fragments. The result of a recent measurement is shown in the right panel of Fig. 6 together with the prediction of the model (dashed curve). The prediction shows a large asymmetry, due to the E1+E2 interference. The data do indicate an asymmetry but the uncertainty in the measurement is large. The solid curve shows the result one obtains when the higher-order dynamical processes are included as discussed below.

7.5. Higher-order dynamical effects

One way to study the influence of higher-order processes in Coulomb dissociation is to solve numerically the time evolution of the halo wave function in the time-dependent Coulomb field from the target nucleus. This method has previously been applied to neutron halos, and it was also recently applied to the Coulomb dissociation of $^8$B. The main results that were obtained are summarized below.

A peculiar feature of the higher-order dynamical effects is that the dissociation probability for $^8$B is smaller than obtained in first order perturbation theory. Moreover, it is not additive: the sum of the dissociation probabilities obtained with pure E1 and E2 fields is larger than obtained by the combined effect of the E1+E2 fields. On the other hand, if one changes the sign of the atomic charge of the target nucleus one gets the opposite result, namely that the sum is smaller than the combined effect of the E1+E2 fields. These features suggest that the most dominant higher-order correction is of third order in the Coulomb field from the target. Such a term arises when a given final state, reached by a first-order process, can also be reached by a second-order process. An example is a first-order E1 transition to a final state which also can be reached by a second-order process consisting for example of an E1 followed by an E2 transition. It is noted that this mechanism does not appear for a pure E1 field.

The result that has been obtained for the decay energy spectrum is shown by the solid curve in Fig. 7. The dynamical calculation includes the monopole, the dipole and the quadrupole Coulomb fields from the target nucleus. It was performed as in Fig. 6, for a 46.5 MeV/u $^8$B beam on a lead target and with a minimum impact parameter of 20 fm. The dashed curves show the results obtained in first order perturbation theory, from E1 and from E1+E2 transitions, respectively. It is
seen that the dynamical calculation agrees with first order perturbation theory only at very large excitation energies. At the maximum, the dynamical result is close to the result obtained from first-order E1 transitions. Thus it appears that one would exaggerate the influence of E2 transitions simply by adding the first-order E1 and E2 contributions, as it was done in Ref. 29 and also in Fig. 6.

![Decay energy spectrum for $^8$B obtained at 46.5 MeV/u on Pb.](image)

Fig. 7. Decay energy spectrum for $^8$B obtained at 46.5 MeV/u on Pb. The solid curve is the result of the dynamical calculation, to all orders in the E1 and E2 Coulomb fields from the target nucleus. The dashed curves are the first-order results from E1 and E1+E2 transitions, respectively.

The longitudinal momentum distribution of $^7$Be fragments is also affected by higher-order processes. The result is shown by the solid curve in the right panel of Fig. 6. The asymmetry is smaller than obtained in first order perturbation theory, represented by the dashed curve. In order to extract the E2 strength from measurements of the asymmetry, it would clearly be an advantage if one could eliminate or reduce the effect of higher-order processes. One way to achieve this is to increase the beam velocity or the impact parameter. However, this would also reduce the asymmetry produced by the first-order E1+E2 interference because the E2 excitation probability falls off faster than the E1 probability as a function of these parameters.

A better way to reduce the effect of higher-order processes, and still keeping the same first-order asymmetry, is to use a lighter target. One concern, though, is a possibly stronger influence of nuclear induced breakup. The longitudinal momentum distribution of $^7$Be fragments has recently been measured again for a variety of targets and at different beam energies, and with much better statistics. Some of the data show a clear asymmetry. By analyzing the different measurements, one may
hopefully achieve a better understanding of the influence of higher-order processes and also be able to extract information about the E2 strength. Thus one may finally be able to make an accurate determination of the low-lying dipole strength and (from detailed balance) of the associated astrophysical S-factor.

8. Final remarks

Coulomb dissociation can be, as we have seen, a simple and useful tool to test and extract information about the structure of one- and two-nucleon halo nuclei. This is the situation when first-order perturbation theory applies and when the dissociation is completely dominated by a single multipole transition, as for example the electric dipole. The situation is more difficult when quadrupole or other multipole transitions play a role, as for example in the dissociation of a proton halo. A way to identify the presence of quadrupole transitions is to measure the asymmetry in the momentum distributions of the fragments which is generated by the interference of dipole and quadrupole transitions. By analysing the asymmetry it may also be possible to quantify the relative strength of dipole and quadrupole transitions.

Another complication in the Coulomb dissociation of weakly bound halo nuclei is that the transitions to the continuum are very strong, and higher-order processes may become significant at low beam energies and on high-Z targets. A simple way to reduce the effect of higher-order processes is to use a higher beam energy, or to use a somewhat lower-Z target. In the latter case one may also have to consider and account for the nuclear induced breakup before one can isolate and extract the electromagnetic transition strength of interest.

The structures of the three halo nuclei discussed in these lectures, namely $^8{\text{B}}$, $^{11}{\text{Be}}$ and $^{11}{\text{Li}}$, are still the subject of some uncertainty. The illustrations that I have made concerning the Coulomb dissociation of these nuclei were based on simplifying assumptions about their structure. This was done in order to emphasize and illustrate the qualitative features that can be observed in Coulomb dissociation experiments. From our current knowledge (also discussed by other lecturers) it should be possible to develop more realistic structure models. It remains to be seen if such models can provide a better description of the available Coulomb dissociation data. It would also be desirable in this connection if the uncertainty in the measurements could be reduced so that the structure models could be tested much more accurately.


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10. References

34. B. Davids (private communications).