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PREDICTION OF STRONGLY-HEATED INTERNAL GAS FLOWS

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ABSTRACT

The purposes of the present article are to remind practitioners why the usual textbook approaches may not be appropriate for treating gas flows heated from the surface with "large" heat fluxes and to review the successes of some recent applications of turbulence models to this case. Simulations from various turbulence models have been assessed by comparison to the measurements of internal mean velocity and temperature distributions by Shehata for turbulent, laminarizing and intermediate flows with significant gas property variation. Of about fifteen models considered, five were judged to provide adequate predictions.

1. INTRODUCTION

Gas cooling of heated surfaces offers the advantages of inherent safety, environmental acceptability, chemical inertness, high thermal efficiency and a high temperature working fluid for electrical energy generation and process heating. Consequently, helium and other gas systems are considered as coolants for advanced power reactors, both fission and fusion. Cooling of a gas is important in gas turbine engine and rocket propulsion systems. These applications have in common turbulent flow with significant gas temperature variation along and/or across the cooling channels.

The purposes of the present article are (1) to remind practitioners why the usual textbook approaches may not be appropriate for treating gas flows heated from the surface with "large" heat fluxes and (2) to review the successes (and failures) of some recent applications of turbulence models to this case. Some observations will also be pertinent to cooling of a gas as well. The main message is one of caution to thermal engineers, even for apparently simple cases.

There are enough effects induced by large heat fluxes so that, to concentrate on them and avoid further complications, this paper considers only steady state conditions in a simple geometry: the classical axisymmetric circular tube. The heated region is preceded by a flow development region which yields an approximately fully-developed turbulent velocity profile; heating is then by an approximately uniform wall flux, as from electrical resistance heating. No internal energy generation is treated. The case is further constrained to small tubes, low densities and/or microgravity applications so buoyancy effects are not important; that is, the situation represents *dominant forced convection.* As a first warning, Figure 1 demonstrates the dangers of blind application of recommended "general purpose," commercial computer codes for this limited case (a version of a popular commercial code has been used on a typical engineer's work station).

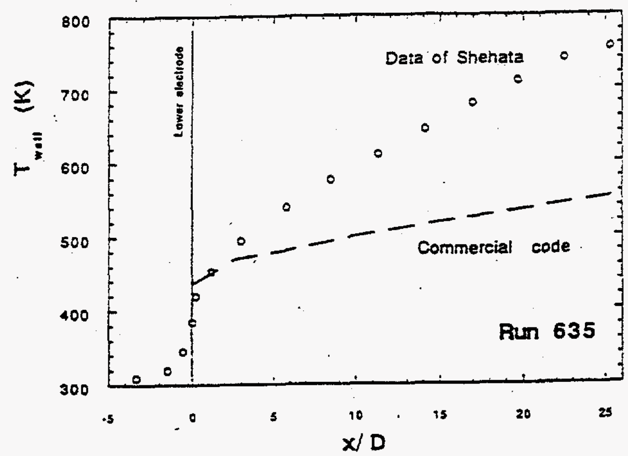

Figure 1 Numerical prediction from commercial general-purpose CTFD code, at conditions of Shehata's Run 635, compared to his measurements.

For a summary of Western studies to about 1982 on the effects of property variation on turbulent and laminar internal gas flows, the reader is referred to an earlier review by McEligot [1986]. While not all-inclusive and concentrating primarily on forced convection, this review can provide a useful introduction to the subject. The present study does not consider particle-laden gas flows; for studies of this promising technique, the reader is referred to the review of Hasegawa, Echigo and Shimizu [1986]. Reviews of mixed convection in vertical tubes are presented by Jackson and coworkers [Jackson, Cotton and Axcell, 1989; Cotton and Kerwin, 1995]. Useful reviews of the status of numerical prediction techniques for turbulent flows have been published by Nagano and Shimada [1995] and Iacovides and Launder [1995]. Nagano and Shimada relate modeling techniques to Direct Numerical Simulations (DNS) and their potential. Iacovides and Launder relate their study to applications for internal cooling passages of gas turbine blades, a complicated problem with some features of the present study; of particular interest is their *conclusion*

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that "the first essential in modeling such flow is to adopt a low-Reynolds-number model for the sublayer region." The reader is referred to these reviews for general background on computational fluid dynamics (CFD), circa 1995.

In our terminology, the viscous layer is operationally defined to include both the so-called "linear" layer, where molecular effects dominate, and the next region where molecular effects are still significant but not dominant. For unheated flows, these regions typically extend to y^+ ($= y[g_c \tau_w/\rho]^{1/2}/\nu$) of about five and thirty, respectively. This usage follows that suggested by Bradshaw [1971]. Emphasis must often be centered on the viscous layer because it tends to provide the greatest uncertainty in predicting the convective thermal resistance [McEligot, 1986].

2. CONSTANT PROPERTY LIMIT

As will be seen later, simulations of some effects of strong heating of a gas involve the low-Reynolds-number turbulent range. In fact, Kawamura [1979] demonstrated nicely that adequate predictions of some phenomena with significant property variation were obtained when their results were also good in the low temperature-difference limit as the gas properties become effectively constant (his Figure 1). Therefore, it is appropriate that codes and their turbulence models be initially examined for fully-developed flow in a circular tube *with the constant properties idealization* to assess their capabilities to simulate low-Reynolds-number flows in the simplest case. In any event, most engineers would want both heat and momentum transfer to be handled adequately for constant properties before treating cases with property variation.

McEligot, Ormand and H. C. Perkins [1966] showed by measurements that for common gases the Dittus-Boelter correlation [1930], with the coefficient taken as 0.021 [McAdams, 1954], is valid to within about five per cent for $Pr \approx 0.7$ and Reynolds numbers greater than about 2500. Thus, this correlation may be employed as a standard of comparison.

Mikielewicz [1994] conducted simulations of the predictive capabilities of a range of turbulence models for fully-developed flow in a circular tube with uniform wall heat flux and the constant properties idealization; eleven models were considered. The Reynolds number range covered was $4000 < Re < 6 \times 10^4$ and the Prandtl number used was 0.7, for air. His tabulated results are plotted in Figure 2. Based on his predictions for Nusselt number, several popular models could be immediately eliminated from further consideration. Some models do not even handle high-Reynolds-number flows well for heat transfer. Several k-ε models designed for use at low Reynolds numbers also gave poor results. At Re = 5000, the Jones and Launder version is over thirty per cent high and predictions by Lam and Bremhorst and by Shih and Hsu are over fifteen per cent above the correlation. The only model considered which gave acceptable results was that of Launder and Sharma; their predictions fall within the estimated experimental uncertainty in the Nusselt number over the full range.

Reasonable predictions have also been provided in the low-Reynolds-number range for this case by McEligot, Ormand and Perkins [1966], McEligot and Bankston [1969], Kawamura [1979], Torii et al. [1991], Ezato et al. [1997] and Nishimura and Fujii [Nishimura et al., 1997] and others. These calculations have covered a range of types of turbulence models, from modified mixing length approaches to a Reynolds stress model coupled to a two-equation model for heat transfer.

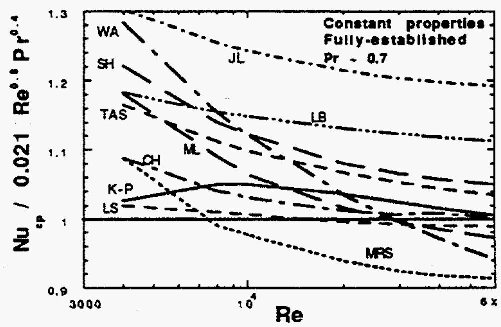

Figure 2. Simulations by Mikielewicz [1994] using various turbulence models (curve labeled KP is the Kurganov-Petukhov correlation).

3. EFFECTS OF LARGE HEAT FLUXES

The effects of temperature on the transport properties of helium are presented in Figure 3. From the perfect gas "law", one recalls that the density also varies significantly -- approximately inversely with the absolute temperature. These trends are typical of most common gases and of binary mixtures of gases; for the former, the Prandtl number is around 2/3 or 0.7, while for the latter in may be as low as 0.2 [McEligot, Taylor and Durst, 1977].

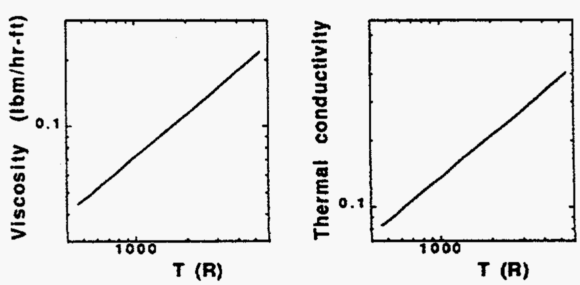

Figure 3. Transport properties of helium.

From Figure 3 it can be seen that the temperature dependencies of the viscosity and thermal conductivity can also be approximated by power laws

$$\mu/\mu_{ref} \approx (T/T_{ref})^a \text{ and } k/k_{ref} \approx (T/T_{ref})^b$$

for convenience in numerical predictions.

We consider the idealization of a uniform wall heat flux for convenience in the presentation. With appropriate assumptions [McEligot, 1963], one can approximate the increase in bulk temperature along the tube as

$$(T_b/T_{in}) \approx 1 + 4\,q^+_{in}\,(x/D)$$

and the wall-to-bulk temperature difference as

$$((T_w - T_b)/T_{in}) \approx q^+_{in}\,Re^{1-a}\,Pr^{0.6}/C$$

The non-dimensional heating rate $q^+_{in} = q''_w/Gc_{p,in}T_{in} = q''_w A_{cs}/\dot{m}\,c_{p,in}T_{in}$ evolves naturally from non-dimensionalizing the governing equations and boundary conditions in pipe flow with an imposed wall heat flux distribution [Bankston and McEligot, 1970]. Property effects come in via the (non-dimensional) exponents in the power law representations. With "large" heat fluxes (large q^+), the resulting temperature range causes significant variation of the gas properties, invalidating the use of design relations such as the popular Dittus-Boelter correlation.

If one defines the Reynolds number based on bulk fluid properties as $Re = GD/\mu_b = 4\dot{m}/(\Pi D\mu_b)$, then its value will continuously decrease as the axial distance increases with heating. The perfect gas approximation shows the bulk density likewise to decrease as x increases. From an integral continuity relationship for steady flow, $\dot{m} = \rho_b V_b A_{cs}$, one sees the velocity increases in the streamwise direction. That is, the flow accelerates spatially.

In strongly-heated, internal gas flows the pressure drop can be dominated by the induced acceleration as suggested by McEligot, Smith and Bankston [1970]. Assumptions and approximations involved are steady state, one-dimensional flow, constant cross section and low Mach number. The momentum equation then can be arranged in a non-dimensional form,

$$-\{2\rho_x g_c D_h (dp/dx)/G^2\} \approx 8q^+_x + 4f_{\tau,x} + 2(Gr^*_x/Re_x^2)$$

where the subscript x indicates evaluation of properties at the local bulk temperature. The Grashof number appearing in the body force term is defined as $Gr^*_x = gD_h^3/\nu_x^2$ and g is taken as directed opposite to the flow direction (i.e., upflow).

Thus, general effects of strong heating of a gas are variation of the transport properties, reduction of density causing acceleration of the flow in the central core, and - in some cases - significant buoyancy forces. Growth of the internal thermal boundary layer leads to readjustment of any previously fully-developed turbulent momentum profile, i.e., no truly fully-established conditions are reached because the temperature rises -- leading, in turn, to continuous axial and radial variation of properties such as the gas viscosity. For calculations, the property variation couples the momentum equation to the thermal energy equation so they must be solved "simultaneously."

In an application such as the High Temperature Engineering Test Reactor (HTTR) in Japan, or reduction of flow scenarios in other plants, another complication arises. To obtain high outlet temperatures, design gas flow rates are kept relatively low. For example, at the exit of the HTTR cooling channels, the Reynolds number is about 3500. In this range, the heat transfer parameters may appear to correspond to turbulent flow or to laminar flow or to an intermediate behavior, depending on the heating rate [Bankston, 1970], with consequent differences in their magnitudes (Figure 4). If the designer is to have confidence in a CTFD code, its turbulence model must demonstrate the "proper" predictions in these conditions. The situation where laminar values are measured at Reynolds numbers typifying turbulent flow is called "laminarization" by some authors [Perkins, 1975]. Several authors have developed approximate criteria for laminarization by heating using graphical correlations of the heating rate as $q^+_{in}\{Re_{in}\}$ [McEligot, 1963; Fujii et al., 1991] as in Figure 5; these have been shown by McEligot, Coon and Perkins [1970] to correspond to an acceleration parameter, $K_v = (\nu^2/V_b)/(dV_b/dx) \approx 4q^+_{in}/Re_{in}$, which varies approximately as $1/\dot{m}^2$.

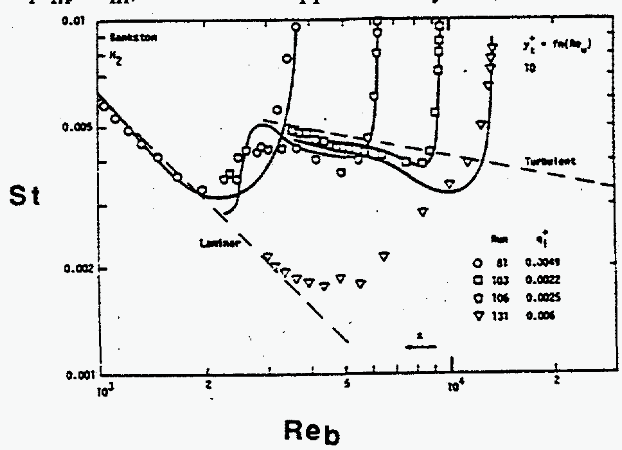

Figure 4. Measurements of Bankston [1965].

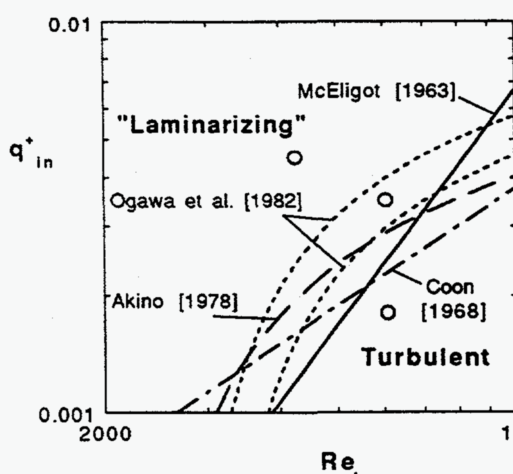

Figure 5. Suggested laminarization criteria [Ezato et al., 1997].

The difficulties in predicting wall temperatures and heat transfer parameters with strong heating and possible laminarization have been demonstrated strikingly by the data of Coon [McEligot, Coon and Perkins, 1970] and of Bankston [McEligot and Bankston, 1969]. Some of Bankston's measurements are shown in Figure 4 along with one attempt to match the data with a simple turbulence model. In this figure, the development is from right to left due to the axial variation of Reynolds number due to the heating; the dashed lines represent accepted empirical correlations for fully-established laminar and turbulent conditions with constant gas properties. One sees the local Stanton numbers decrease below the turbulent correlation for his runs 103 and 106 and then converge towards it downstream (as T_W/T_b approaches unity). For his run 131 the laminar prediction is approached at local Reynolds numbers near 4000.

4. RELATED WORK

Most of the early data for gas heating with significant property variation were obtained with circular tubes of small diameters so forced convection dominated [McEligot, 1986]. With test sections typically less than thirteen mm (D < 1/2 inch) probes could not be inserted for useful profile measurements and, therefore, the experiments could only provide integral wall parameters, such as local heat transfer coefficients and friction coefficients. These data gave initial tests of turbulence models accounting for temperature-dependent transport properties, as demonstrated by McEligot and Bankston [1969], Bankston and McEligot [1970] and Kawamura [1979]. For quasi-developed turbulent flow, experimental results were typified by the empirical correlations of McEligot, Magee and Leppert [1965],

$$Nu_b \approx 0.021\, Re_b^{0.8}\, Pr^{0.4}\, (T_w/T_b)^{-1/2}$$

and

$$(f_b / f_{cp}) \approx (T_w/T_b)^{-0.1}$$

In later studies, approximate agreement with such correlations has been taken as indication of normal turbulent behavior *with gas property variation.*

McEligot [1963] attempted to examine the effects of gas property variation in the laminar-to-turbulent transition region. He found conditions where the local Nusselt number diverged below the turbulent correlation shown above as the axial distance increased and the local Reynolds number thereby decreased. An approximate correlation for this transition was deduced. McEligot estimated flow regimes as turbulent, transitional or laminar based on agreement with the correlations; these studies *might* be considered to be the "discovery" of the process we now call laminarization by heating. Related measurements were obtained by Perkins and Worsoe-Schmidt [1965] and Bankston [1965] at higher temperature ratios during the same time period. Bankston [1966] pointed out that, for the transition or "reverse transition" regime, calculations of the acceleration parameter K_V gave values in the range that Kline et al. [1967] found corresponded to laminarizing conditions in unheated turbulent boundary layers.

Since the 1960s, measurements with larger test sections have been conducted by Hall and Jackson and colleagues at the University of Manchester to examine effects introduced by buoyancy forces aligned with or opposing the flow [Jackson, Cotton and Axcell, 1989]. These studies have led to criteria for significant buoyancy effects in turbulent flow.

By comparison to the thermal entry measurements of Perkins and Worsoe-Schmidt, of McEligot, Magee and Leppert and of Petukhov, Kirillov and Maidanik, Bankston and McEligot [1970] were able to examine the application of eleven algebraic turbulence models for high-Reynolds-number turbulent gas flows with properties varying strongly in both axial and radial directions. Best agreement was found with a van Driest mixing length model with the exponential term evaluated with wall properties,

$$\ell = 0.4\, y\, [\, 1 - \exp\{- y_W^+ / A^+\}\,]$$

where y_W^+ is defined unambiguously as $y(g_c\tau_W/\rho_W)^{1/2}/\nu_W$ and A^+ was taken as 26, as by van Driest. To accommodate low-Reynolds-number turbulent and laminarizing flows, McEligot and Bankston [1969] modified this model (described by Shehata and McEligot [1998]). This modification was developed by comparison to integral quantities, such as the local Stanton number, since internal profile measurements were not then available for guidance.

Most recent work on the topic of laminarization by heating has been conducted in Japan. Measurements of local heat transfer coefficients and friction factors for transitional and laminarizing flows have been obtained by Ogawa et al. [1982] and Ogawa and Kawamura [1986] with circular tubes. Local Nusselt numbers were measured for annuli by Fujii et al. [1991] and Torii et al. [1991]. The first investigator to succeed in applying an advanced turbulence model to laminarization by heating apparently was Kawamura [1979]. He concluded that a modified k-kL model gave good agreement with the experiments. Fujii et al. [1991] employed three types of turbulence models for comparisons to their measurements of strongly-heated turbulent gas flow in an annulus. Torii et al. [1991] and Torii and Yang [1997] applied modified k-ε models for predicting streamwise variation of heat transfer parameters in low-Reynolds-number turbulent and laminarizing flows in circular tubes and annuli. Torii et al. [1993] also attempted to apply the Reynolds-stress model of Launder and Shima to St{Re} data for a circular tube; they concluded that predictions were comparatively poor in the range of turbulent-to-laminar transition.

Since the flows are typically at low Reynolds numbers, it would appear that DNS techniques [Nagano and Shimada, 1995] would be ideal to predict situations such as laminarization induced by strong heating of a gas. While Satake and Kunugi in Japan have been developing a DNS code for circular tubes for this purpose, results do not yet appear to be available with strong property variation. However, for the comparable quasi-developed problem in a two-dimensional channel, Dailey and Pletcher [1998] obtained successful Large Eddy Simulations (LES) employing a Smagorinsky subgrid-scale model with van Driest damping at the walls.

Turbulence models have generally been developed for conditions approximating the constant properties idealization. The few "advanced" turbulence models applied for high heating rates [Kawamura, 1979; Fujii et al., 1991; Torii et al., 1993; Torii and Yang, 1997] were developed without the benefit of velocity and temperature distributions in strongly-heated, dominant forced flow for guidance or testing. Thus, it is not certain whether the agreement with wall temperature data for moderate and strong wall heat fluxes, that was obtained in some cases with such models, was fortuitous or not. Before they can be applied with confidence to gas-cooled systems with high heat fluxes, predictions from turbulence models must be validated by comparison to careful measurements of the internal mean flow and thermal distributions for conditions with significant gas property variation.

5. MEASUREMENTS AVAILABLE

For dominant forced convection *with significant gas property variation* in low Mach number flow of common gases through a circular tube, apparently the only published profile data available to guide (or test) the development of predictive turbulence models have been K. R. Perkins's and Shehata's measurements of mean temperature distributions [Perkins, 1975] and mean velocity distributions [Shehata and McEligot, 1995] for this situation. Their careful measurements are now available to examine this problem and these can serve as the bases for evaluation of predictive techniques. Their experiments concentrated on three characteristic cases with gas property variation: turbulent, laminarizing and intermediate or "subturbulent" (as denoted by Perkins). To the authors's knowledge, the only numerical studies of "advanced" turbulence models for turbulent and laminarizing flows at high heating rates that utilized *internal* data have been those of Mikielewicz at Manchester and Ezato, Nishimura and Fujii in Japan recently.

The experiments of Shehata (and of Perkins) were conducted using an open flow system incorporating a vertical, resistively-heated, circular test section exhausting directly to the atmosphere in the laboratory. The experiment was designed to provide an approximately uniform wall heat flux boundary condition in a tube for ascending air, entering with a fully-developed turbulent velocity profile at a uniform temperature. The heated length was kept relatively short to permit high heating rates with Inconel as the material while possibly approaching quasi-developed conditions. Small single wire probes were introduced through the open exit in order to obtain pointwise temperature and velocity measurements. In addition to the usual difficulties of hot wire anemometry, the temperature range of his experiment introduced additional problems such as radiation corrections; these difficulties, their solutions and related supporting measurements are described by Shehata [1984] and Shehata and McEligot [1995]. Inlet Reynolds numbers ($Re_{in} = 4\dot{m}/(\Pi D \mu_{in})$) of approximately 6000 and 4000 were employed and attention was concentrated on three heating rates chosen to give significant variation of properties for conditions of predominantly forced convection which were considered to be "turbulent," "intermediate," and "laminarizing," respectively ($q^+_{in} \approx$ 0.0018, 0.0035 and 0.0045). The runs are labeled 618, 635 and 445 with the first digit identifying the inlet Reynolds number and the last two indicating the non-dimensional heating rate.

The length of the test section was chosen to permit measurements through and beyond the normal thermal entry region while attaining significant transport property variation, as exemplified by T_w/T_b and T_w/T_i, with common materials and gas. Wall temperatures reached 840 K (1510 R), the maximum wall-to-bulk temperature ratio was about 1.9 and the Mach number was less than 0.013, indicating that compressibility effects would have been negligible. At the entrance the buoyancy parameter Gr_q/Re^2 reached 0.53 for the highest heating rate and lower Reynolds number; it then decreased as x/D increased. The maximum wall-to-inlet temperature ratio was about 2.7, indicating that gas properties such as viscosity varied by a factor of two in the test section. Exit bulk Reynolds numbers were above 3000 in all cases, corresponding to turbulent flow if the test section had been adiabatic.

With the data of Perkins and of Shehata, detailed internal mean profiles for predominantly forced flow in a circular tube are now available for the following approximate conditions:

Re_{in}	q^+_{in}	Temperature	Velocity
8520	0.0010	X	
6020	0.0011	X	
6030	0.0018	X	X
6020	0.0035	X	X
6040	0.0045	X	
4260	0.0045	X	X
3760	0.0055	X	

Tabulations of these data and the boundary and initial conditions are available in the reports of Perkins [1975] and of Shehata and McEligot [1995]. From the graph of laminarization criteria by Fujii et al. [1991] as in Figure 5, one can see these measurements span a range from turbulent behavior at moderate heating rates to rapidly laminarizing flows. From those results further aspects of the development of the mean turbulence structure may also be deduced. For example, once one establishes agreement with measured U{y,x} and T{y,x} fields as in the next section, the distributions of ε/ν, l/r_w, $-\overline{uv}$, $-\overline{vt}$, etc. -- that are required for that matching -- may be considered to be indirectly deduced data [Shehata and McEligot, 1995]. This approach could be thought to be an extension of the earlier technique of deducing eddy diffusivity profiles for simple fully-developed, adiabatic flows by fitting mean velocity profiles. That is, by adjustment of the turbulent models which give $-\overline{uv}$, $-\overline{vt}$, etc., the velocity and temperature distributions have been fit.

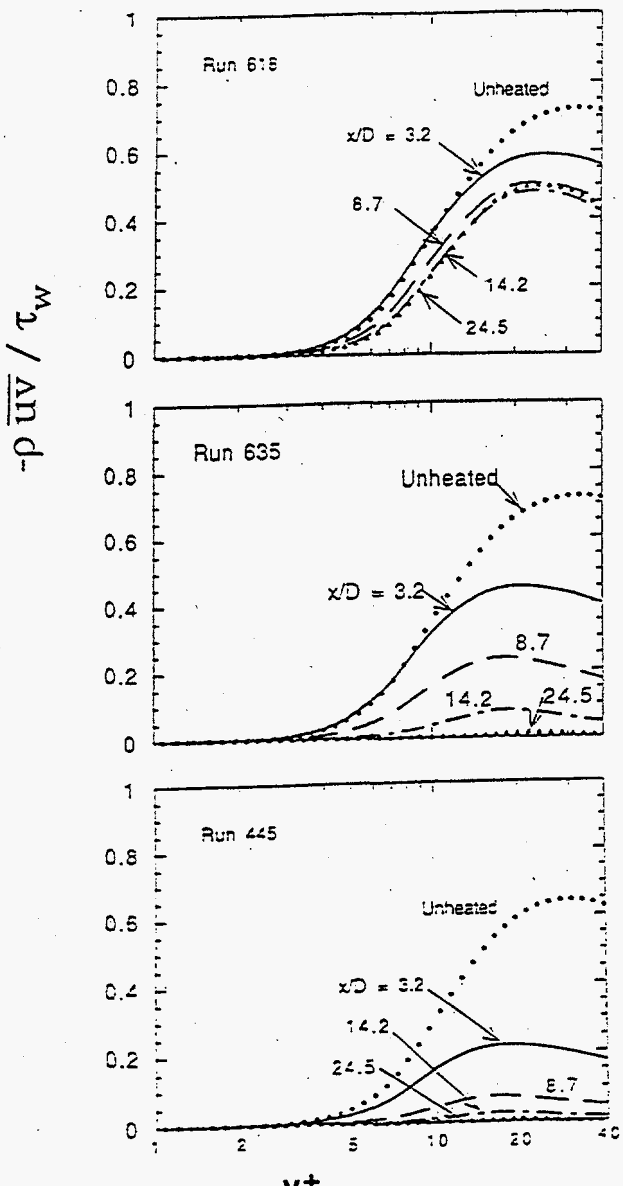

Figure 6 Axial behavior of Reynolds shear stress profiles as deduced from velocity and temperature measurements via numerical diagnostic technique [Shehata and McEligot, 1995].

The Reynolds shear stress or turbulent shear stress distributions, $-\rho\,\overline{uv}$, which are required in order to obtain agreement between predictions and measurements, are shown normalized by the local wall shear stress in Figure 6. For fully-developed, adiabatic turbulent flow the total shear stress, $\mu(\partial U/\partial y) - \rho\,\overline{uv}$, decreases linearly from the wall to the centerline. Therefore, across the viscous layer the normalized Reynolds stress increases from zero to approach the value for the total shear stress, as molecular effects become less important, and then follows this decreasing ramp-like function to zero at the centerline. For each of Shehata's runs, one can discern the beginning of this behavior in the adiabatic entry profile of $-\rho\,\overline{uv}/\tau_w$. With the significant heating of Runs 618 and 635, the deduced levels of these Reynolds stress profiles decrease from the entry to $x/D \approx 3.2$, the first heated profile. For the "turbulent" run (618), the normalized Reynolds stress settles to an approximately constant profile from $x/D \approx 9$ to 25 with slight variation. For the other two runs at higher heating rates, these profiles decrease to negligible values as the axial distance increases. For Run 445 the deduced reduction in the first few diameters is more severe than for Run 635; in a sense, Run 445 laminarizes more quickly (as might be expected for lower Re_i and higher q^+_i).

6. SIMULATIONS WITH GAS PROPERTY VARIATION

Mikielewicz [1994] examined eleven turbulence models developed for forced turbulent flow with the constant property idealization as discussed earlier. For fully-developed flow with constant properties at Reynolds numbers in the range $4000 < Re < 60{,}000$, only the model by Launder and Sharma agreed with the Dittus-Boelter correlation for common gases (coefficient = 0.021) within its estimated experimental uncertainty over the full range; several k-ε models designed for low-Reynolds-number conditions gave poor agreement.

A number of turbulence models, developed for turbulent flows under conditions of uniform fluid properties, were applied by Mikielewicz for the purposes of simulating experiments with *strongly-heated*, *variable property* gas flows at low Reynolds numbers in a vertical circular tube [Shehata, 1984]. The selection of models included a mixing length model, eddy diffusivity models, a one-equation k model and two-equation models of k-ε type with low-Reynolds-number treatments; this selection is representative of models which have been widely used but obviously is not all-inclusive. Thermal energy transport was modeled using a turbulent Prandtl number with its value held constant (0.9).

To illustrate the predictions of *integral heat transfer*, Figure 7 presents the resulting wall temperature distributions for eight models for the three runs. For the thermal design engineer, these predictions are of key importance. In this figure the distance z/D is labeled from the start of the calculation, so the nominal start of heating (usually called x = 0) is at z/D = 14.8 and the near-uniform wall heat flux range is in the region $20 < z/D < 41$. The expected sharp rise of wall temperature after the abrupt increase in wall heat flux (near z/D = 15) is demonstrated by all the models.

In Figure 7 the identification of the various models is via letter labels; the square symbols are the measurements from Shehata [1984]. For the lowest heating rate, "turbulent" Run 618, the trends from model to model are approximately the same as for the results for fully-established heat transfer with constant properties that were presented in Figure 2; models which did not reproduce that condition well predict the behavior with property variation poorly as well. For the purpose of making quantitative comparisons, we examine the predictions at $z/D \approx 40$ where the wall temperature is highest.

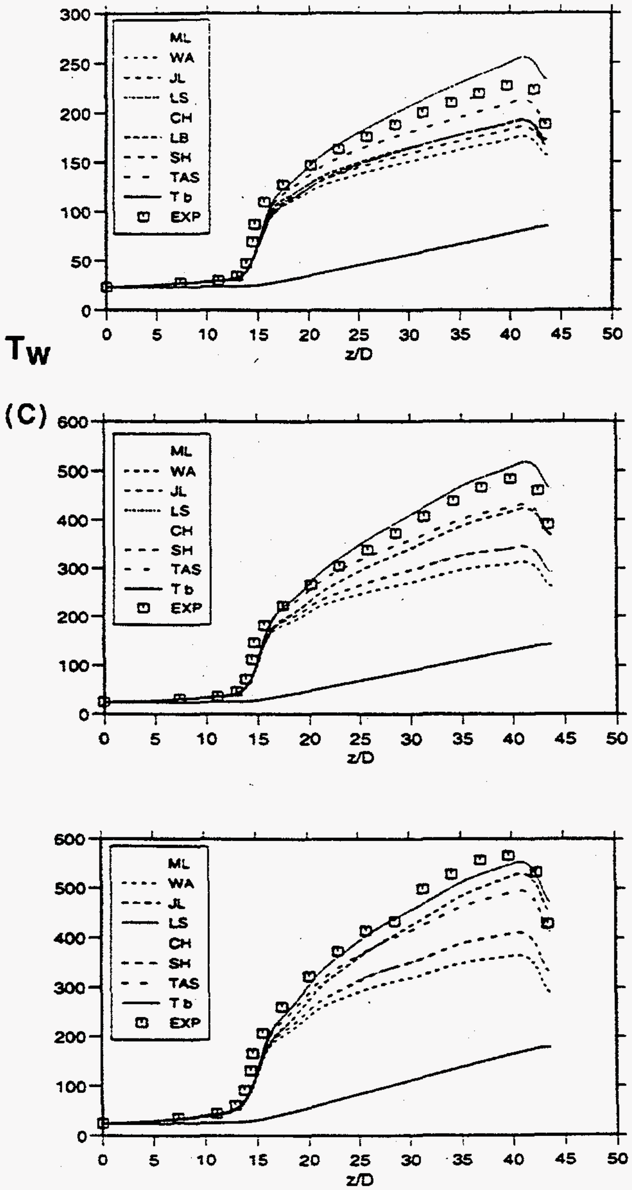

Figure 7 Simulations by Mikielewicz [1994].

The wall-to-bulk temperature difference is underpredicted by over forty per cent in the worst case. The discrepancy is thirty per cent in the case of one "low-Reynolds-number" k-ε model (SH), actually slightly worse than the "original" van Driest mixing length model. The low-Reynolds-number k-ε models of Jones and Launder and of Lam and Bremhorst gave approximately the same results as the mixing length model. Forecasts within twenty per cent of the measured values were yielded by Michelassi, Rodi and Scheurer, by Chien and by Thangam, Abid and Speziale. The model of Launder and Sharma (LS) did slightly better with a conservative overprediction of about thirteen percent.

Comparisons in the cases of the two higher heating rates discriminate amongst the models further. In general, the models showing poor agreement at the lower heating rate remained poor. Numerical difficulties were experienced using a couple models when they were applied under conditions of strongly varying properties and no results were produced. The one-equation model (WA) underpredicted the temperature difference by fifty per cent or more for such conditions. There were some differences in relative rankings of the models as the heating rate was increased and Reynolds number decreased. The JL model improved relative to the CH and TAS low-Reynolds-number k-ε models. At the lower Reynolds number, as one might expect, some of the low-Reynolds-number k-ε models performed better in comparison to the "original" van Driest mixing length model (which was developed for high Reynolds number flows). For strong heating of a gas with property variation, Mikielewicz concluded that the model of Launder and Sharma gave the best predictions overall.

Simulations are next assessed in terms of the *mean velocity and temperature profiles.* Figures 8 to 10 provide direct comparisons between predicted profiles and the data. Presenting velocities in the form of profiles of U/V_b avoids the uncertainties which would have been involved in the use of u_τ for non-dimensionalization. Presenting temperatures in the form T/T_{in} or $(T_w-T)/T_{in}$ checks agreement with the energy and mass balances as well as providing a picture of the development of the thermal layer. In general, the estimated uncertainty in local velocity measurement by Shehata was calculated to be in the range of eight to ten per cent, with the larger value being associated with measurements near the wall. The uncertainty in local temperature measurement was typically one or two percent of the absolute temperature. These estimates are believed to be conservative since comparisons between the integrated and measured total mass flow rates for each profile showed better agreement (three per cent or less, except near the exit in the runs with the two highest heating rates). In contrast to conventional wisdom, when there is significant *gas* property variation, a mass balance is primarily a test of the profile of the quotient $U\{y\}/T\{y\}$ and an energy balance checks the profile $U\{y\}$ (Shehata and McEligot [1995]).

The measured points nearest the wall correspond to values of y^+ from about 3 to 5, depending on the heating rate and station. The location $y/r_w = 0.1$ corresponds to $y^+ \approx 20$ for the entry profiles of Runs 618 and 635. At the last station, $y/r_w = 0.5$ is about $y^+ \approx 60$ and 30 for runs 618 and 445, respectively. A significant portion of the tube is therefore occupied by the viscous layer in all three runs.

The quantity T/T_{in} provides an indication of gas property variation both across and along the tube. In Run 618, T/T_{in} varies by about fifty per cent from the wall to $y/r_w = 0.5$ at the last station, so μ and k vary by about forty per cent in that region. In contrast, in run 445 they vary by more than that in the first three diameters of the heated portion of the pipe. At the first station for each run, the velocity data points near the wall appear to be high relative to what would be indicated by extrapolation of the data further out, a common feature in such measurements (and a good reason for not determining u_τ by fitting to $u^+ = y^+$). Measurements at this location required the greatest insertion of the probe so some aspects of the data reduction are necessarily more uncertain there.

Overall agreement between the Launder Sharma predictions of internal distributions by Mikielewicz and the measurements by Shehata was encouraging though close examination revealed some detailed discrepancies. In such cases there were corresponding differences between predictions of integral parameters and data. Generally, however, the predictions of velocity and temperature profiles are considered to provide a satisfactory description of the observed behavior.

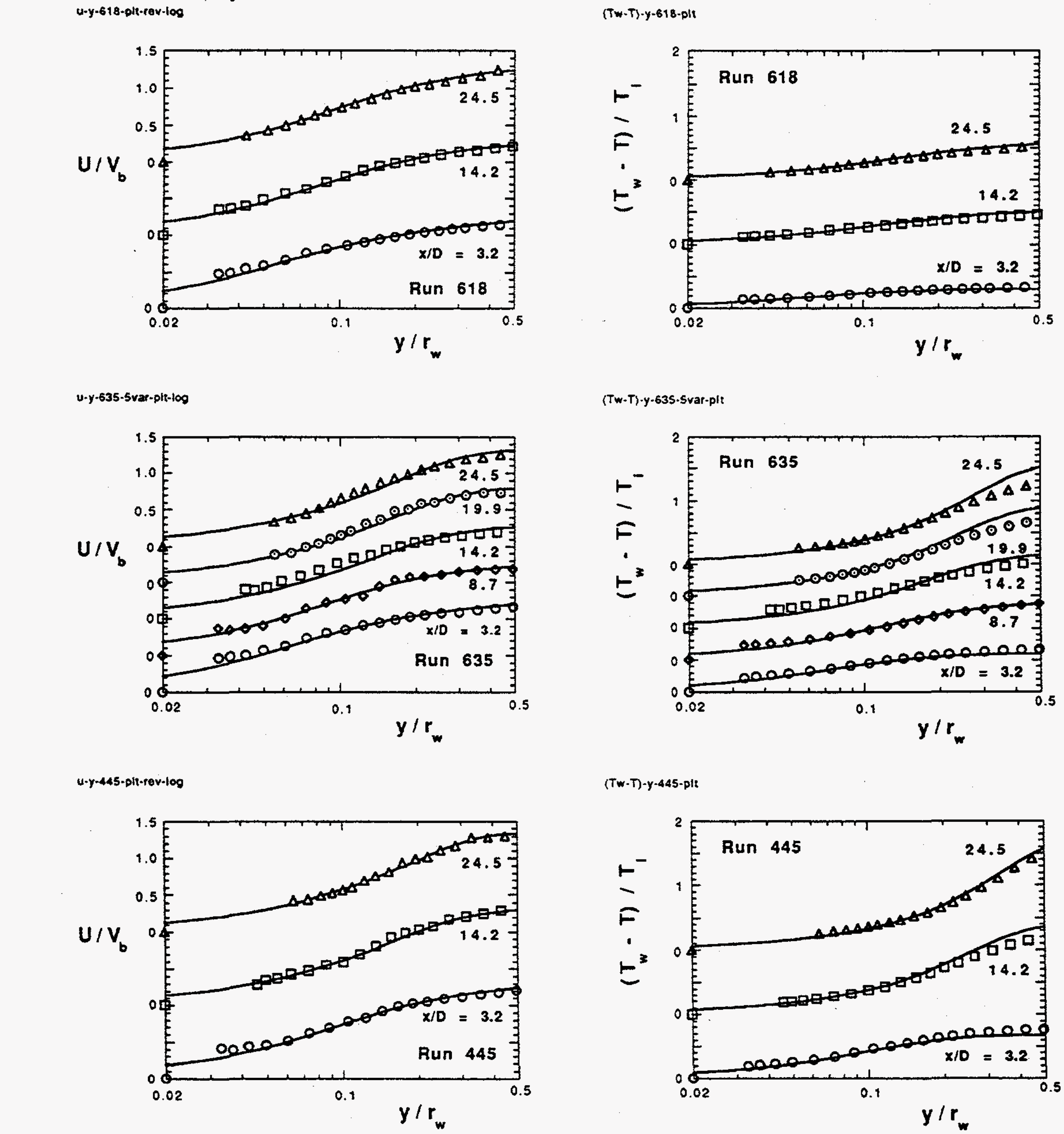

Figure 8 Simulations from modified van Driest model compared to data [Shehata and McEligot, 1998].

In addition to documenting the first mean velocity distributions for these difficult cases, the study by Shehata and McEligot [1998] demonstrated that the simple *modified van Driest model* (developed from wall data alone by McEligot and Bankston [1969]) predicts the mean internal profiles (Figure 8) and non-dimensional pressure drop reasonably well overall. However, Bates et al. [1974] have demonstrated that this model is not appropriate for flows with large buoyancy effects.

Figure 8 presents the comparison between predictions and data for the viscous layer in forms equivalent to wall coordinates while avoiding the uncertainty introduced by the determination of the wall shear stress. In wall coordinates the velocity profile is particularly sensitive to the uncertainty of τ_w since it appears in the numerator for y^+ and the denominator for u^+. By using the semi-log axes the presentation is concentrated on the viscous layer. For the temperature profiles, $t^+_w = (T_w-T)\rho_w u_{\tau,w} c_{p,w}/q''_w$ is represented by $(T_w-T)/T_i$. These comparisons of predictions and measurements are more direct (and more severe) than using wall coordinates based on a wall shear stress which has been fitted in the viscous layer.

As $t^+\{y^+\}$ normally shows the same trends as $u^+\{y^+\}$ in fully-established, constant-property flow, the profiles of $(T_w-T)/T_i$ approximate the trends of U/V_b here. Even when the examination concentrates on the near wall region, Runs 618 and 445 (lowest and highest q^+) are predicted well by the modified van Driest model through the viscous layer. (At $x/D \approx 24.5$, $y^+_w \approx 30$ corresponds to $y/r_w \approx 0.26$ for Run 618 and to $y/r_w \approx 0.5$ for 445.)

For Run 635 the velocity predictions agree reasonably well within the viscous layer for all but the profile at $x/D \approx 14$ (at this location, $y^+_w \approx 30$ corresponds to $y/r_w \approx 0.32$ for this run). Non-dimensional temperature profile agreement is approximately the same at this location, underpredicted for $y/r_w < \sim 0.2$ and then overestimated. Further downstream the non-dimensional temperature agrees well for $y/r_w < \sim 0.2$ then diverges for higher values (at $x/D \approx 24.5$, $y/r_w \approx 0.2$ corresponds to $y^+_w \approx 16$, which would be about halfway across the viscous layer in unheated flow). Shehata and McEligot [1995] show that $-\rho\,\overline{uv}/\tau_w$ is predicted to be less than 0.1 at $x/D >$ 14 there in this heated flow (Figure 6) -- so this good agreement near the wall would be a consequence of molecular transport dominating. For temperature distributions these trends continue downstream while agreement of the velocity distribution appears to improve at $x/D \approx 20$ and 24.5. Interpreting this observation in terms of energy transport, one could say that the model predicts a lower rate of turbulent transport than observed from the wall region to the core, starting between $x/D \approx 9$ and 14; *one* explanation could be overprediction of the effective viscous layer thickness at these local conditions. Conceptually, one could adjust the function $A^+\{Re\}$ or other features of a turbulence model to accommodate these details (in fact, Perkins [1975] did improve prediction of $T\{r,x\}$ for "subturbulent" runs by revising $A^+\{Re\}$ at low Reynolds numbers). Figure 8 demonstrates that the *modified* van Driest model does correlate the mean velocity and temperature measurements fairly well overall even for Run 635, the most difficult of the three to predict.

For Run 445, temperature predictions generally look good throughout, despite the fluid property variation being greatest for that case. This result is probably because molecular transport increases in importance once the laminarizing process has begun and so the uncertainties in the turbulence modeling are of less significance. While the mean velocities are slightly high in the central region for $x/D \approx 14$ and 25, they are within the (conservative) estimated experimental uncertainties. Whereas the temperature profiles show close agreement at the thermal entrance and exit, the profile at $x/D \approx 14$ also exhibits the discrepancies seen for Run 618, i.e., overprediction near the wall and underprediction in the core.

The *k-ε turbulence model* of Abe, Kondoh and Nagano [1994], developed for forced turbulent flow between parallel plates with the constant property idealization, has been successfully extended by Ezato [1997] to treat strongly-heated gas flows at low Reynolds numbers in vertical circular tubes. For thermal energy transport, he adopted the turbulent Prandtl number model of Kays and Crawford [1993]. No constants or functions in these models were readjusted.

Under the idealization of constant fluid properties, predictions for fully-established conditions agreed with the Dittus-Boelter correlation for common gases (coefficient = 0.021) within about five per cent for $Re > 3200$. Predicted friction coefficients were about five per cent higher than the Drew, Koo and McAdams [1932] correlation in this range. Below $Re \approx 2000$ both predictions agreed with theoretical laminar values within about one per cent. The local thermal entry behavior, $Nu\{x/D\}$, was further confirmed by comparison to the measurements of H. C. Reynolds [1968] for $Re \approx 4180$ and 6800.

The capability to treat forced turbulent flows with significant gas property variation was again assessed via calculations at the conditions of experiments by Shehata. Predictions forecast the development of turbulent transport quantities, Reynolds stress and turbulent heat flux, as well as turbulent viscosity and turbulent kinetic energy. Results suggest that the run at the lowest heating rate behaves as a typical turbulent flow, but with a reduction in turbulent kinetic energy near the wall. For the highest heating rate all turbulence quantities showed steady declines in the viscous layer as the axial position increased - representative of conditions called laminarizing. For the intermediate run, predictions showed the same trends as the highest heating rate but occurred more gradually with respect to axial distance. These observations are consistent with empirical criteria for these regimes, expressed in terms of $q^+_{in}\{Re_{in}\}$. No direct measurements are available to confirm or refute these turbulence distributions; they must be assessed by examination of the mean velocity and temperature distributions that result from these predictions of the turbulent transport quantities.

Figure 9 provides direct comparisons of the predictions to measured profiles. Run 618 shows a slight overprediction of the velocity profile at $z/D \approx 25$ but otherwise all profiles are good. (In Ezato's calculations, the quantity z represents the distance from the nominal start of heating.) Run 445 predictions look good throughout despite having the largest fluid property variation; again, this result probably occurs because molecular transport increases in importance once the laminarizing process has begun. So if one wishes the most sensitive test of a turbulence model for low-Reynolds-number flows with gas property variation, conditions like those of Run 635 may be key.

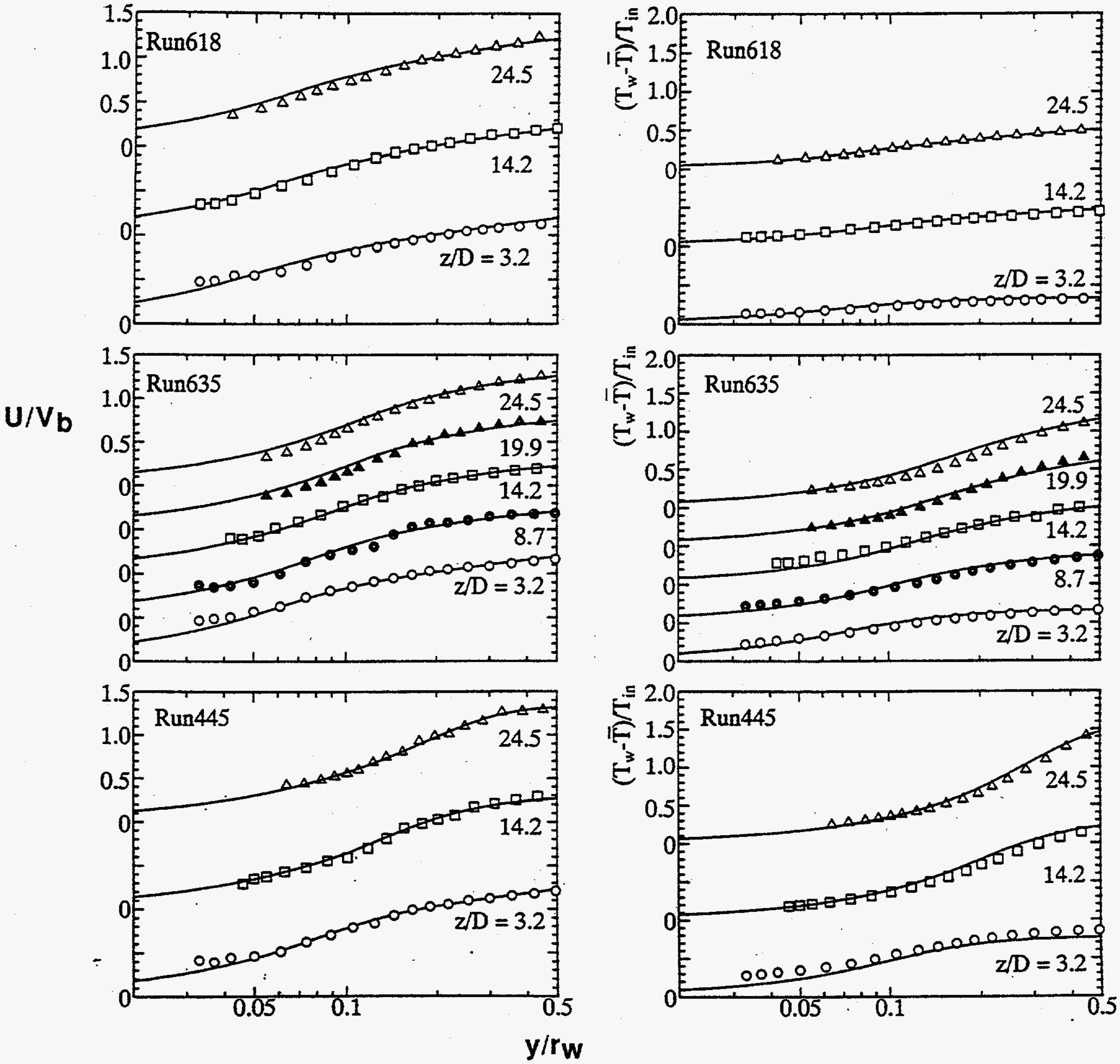

Figure 9 Simulations from k-ε model by Ezato compared to data [Ezato et al., 1997].

For Run 635 agreement between calculations and data is good for the first three stations. Then at $z/D \approx 20$ and 25 the velocity is overpredicted near the wall and at $z/D \approx 25$ the quantity $(T_w - T)/T_{in}$ is overpredicted in the range $0.1 < y/r_W < 0.25$ (corresponding approximately to $8 < y^+ < 20$), i.e., $T\{y\}$ is *under*predicted. In the viscous layer, a higher velocity is caused by a lower value of μ_t, which also leads to a higher thermal resistance in the layer; this higher thermal resistance would interfere with thermal energy transport beyond the region leading, in turn, to lower temperatures further from the heated wall. If one wishes to improve the model further, these observations could provide a starting point. For the present purposes, the level of agreement seen is considered satisfactory.

Overall agreement between the predictions and the measured velocity and temperature profiles is good, establishing confidence in the values of the forecast turbulence quantities -- and the model which produced them. Most importantly, the model also yields predictions which compare well to the measured wall heat transfer parameters and the pressure drop. Thus, thermal design engineers should find calculations based on the turbulence model used by Ezato to be satisfactory for forced low-Reynolds-number turbulent flows with significant gas property variation.

Figure 10 describes the axial flow development after the start of heating in non-dimensional terms, $U/V_b\{x\}$ and linear coordinates. Circles represent Shehata's measurements.

The first set of thermal entry data is for the *"low" heating rate* conditions characterized as *turbulent* by Perkins [1975]. The reference control parameters were $Re_i \approx 6000$ and $q^+ \approx 0.0018$ (i.e., *Run 618*), leading to an acceleration parameter of about 1.2×10^{-6}. The non-dimensional radius of the unheated upstream section is about $r^+ = y^+_c \approx 200$. It is expected that the heat transfer parameters would correspond to turbulent predictions, with allowance for a reduction in local Nusselt number (typically of the order of $(T_w/T_b)^{-1/2}$)) due to the gas property variation along the tube and across the viscous layer. Figure 10a presents the momentum development axially in terms of mean profiles at 3.17, 14.2 and 24.5 diameters. Obtaining the *mean streamwise velocity data* was the main objective of Shehata's work since Perkins [1975] was unable to obtain meaningful velocity measurements in his attempts with an impact tube.

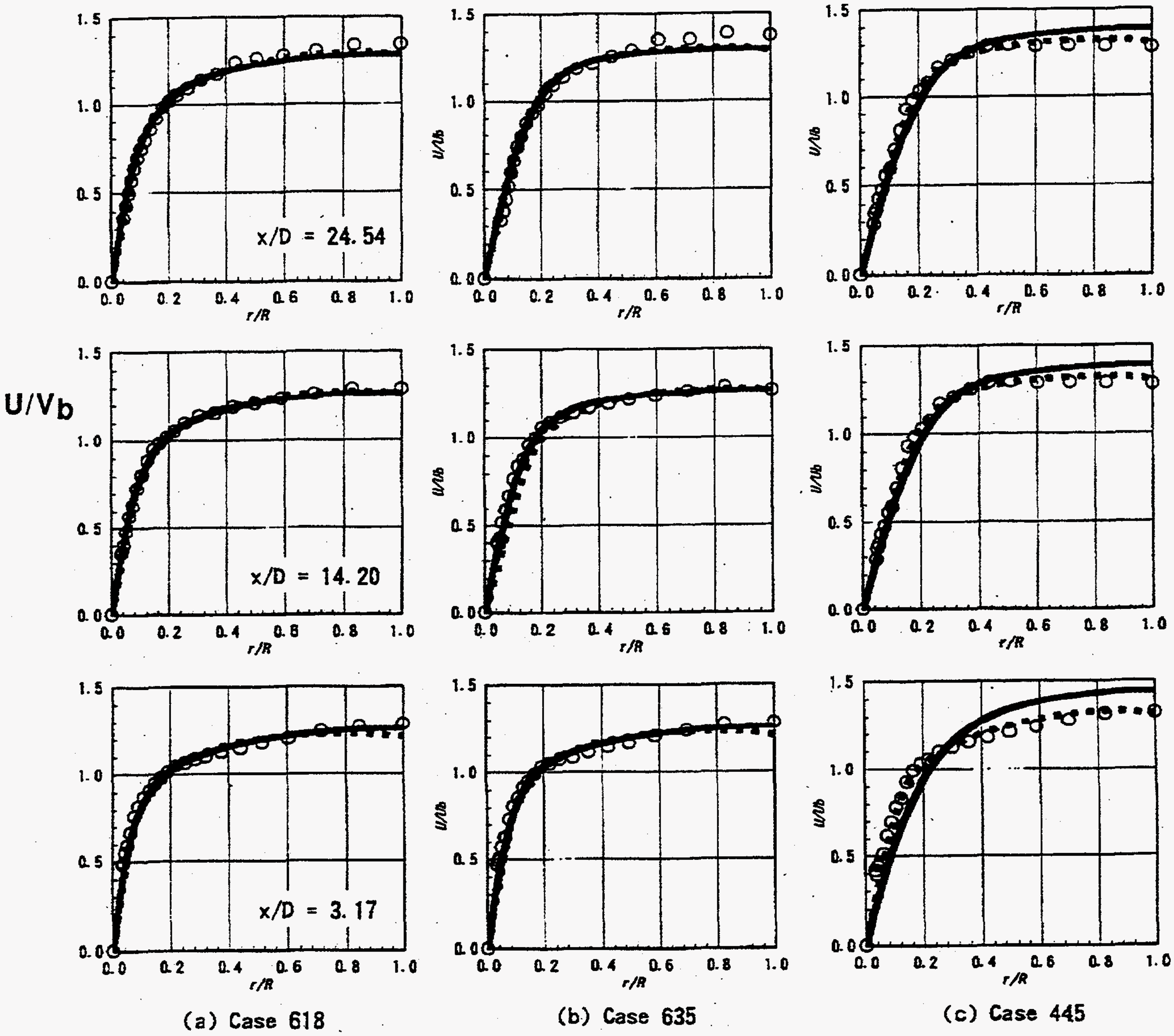

Figure 10 Simulations of Reynolds stress model by Nishimura (solid lines) and k-kL-$\overline{uv}$ model by Fujii (dashed lines) compared to data of Shehata [Nishimura et al., 1997].

Velocity profiles appear representative of normal, developed turbulent flow in a circular tube, as expected. As the flow progresses downstream, the gradient at the wall decreases with the progressive decrease in Reynolds number. No surprises are evident. For these results a thermal boundary layer thickness is operationally defined as the distance from the wall beyond which there is no observable difference from the inlet temperature. At x/D = 3.17, the thermal boundary layer reached about forty per cent of the tube radius. By 14.2 diameters, the thermal boundary layer already extended to the centerline and the profile was typical of a developed turbulent flow, distinguished by a high gradient near the wall (most of the thermal resistance) and a well mixed flow in the interior. The last profile was qualitatively the same, with an increase in level corresponding to the higher bulk temperature. The temperature data of Perkins [1975] behave in much the same way as those of Shehata. Variations between the two are mainly due to minor differences in the heating rates and inlet Reynolds numbers.

The second set of data is at the *intermediate heating rate* (Re_i still 6000 and $q^+ \approx 0.0035$) which was described as *subturbulent* by Perkins [1975]. The acceleration parameter was about 2×10^{-6} along most of the tube. This set has almost twice the heating rate and K_V as the first set, while having the same inlet flow rate. For these conditions, Perkins has shown that the integral heat transfer parameters do not agree with turbulent, variable properties correlations *nor* with laminar, variable properties numerical predictions.

The first three temperature profiles indicated that the thermal boundary layer reached the centerline near x/D = 9 or 10. Near the wall the temperature profile was near linear over a substantial region. This near-linear region grew thicker as the flow progressed downstream, reaching $y/r_W \approx 0.25$ by the last station. Although significant thermal resistance appeared distributed across a large portion of the cross section, a near flat profile was still evident in the central region at the last three locations. This last observation implies that turbulent mixing was not suppressed there (it was not a case of being outside the thermal boundary layer, since the centerline temperature was above $T_W/T_i = 1$).

The last set of runs is at the *highest non-dimensional heating rate* and lowest inlet flow rate ($Re_i \approx 4000$ and $q^+ \approx 0.0045$ ---> *Run 445*). These conditions were categorized as being on the borderline between subturbulent and *laminarizing* flow by Perkins (his Figure 8). The acceleration parameter K_V exceeds 3.5×10^{-6}, which is considered by Sreenivasan [1982] and others as near a critical value for laminarization to set in. The measurements at 3.17 diameters showed that the thermal boundary layer had grown to $y/r_W \approx 0.4$ by this distance. Presumably this growth was partially a consequence of upstream axial conduction and axial thermal radiation to the tube wall of the "unheated" entry region, as is evident in earlier Figure 7. By x/D = 14.2 the thermal boundary layer again filled the tube. The temperature profile started to take a parabolic shape, except in the central region where the effects of turbulent mixing still appeared significant. At x/D = 24.5, the measured temperature profile resembled a parabola more closely. One recalls that the analytic profile for fully established laminar flow with constant properties is a fourth-order parabola [Kays and Crawford, 1980] - but, in these data, viscosity and thermal conductivity vary by factors of about two and at this Reynolds number a laminar flow would require a longer distance to become fully established. However, Perkins [1975] did demonstrate that a *laminar, variable properties* calculation - starting from the existing turbulent velocity profile in the entry - *could provide reasonable predictions* of the mean temperature distribution for comparable conditions. The mean velocity profiles in the wall region (Figure 10c) show a near Blasius shape while progressing downstream. In the central region, starting at about half the radius, the profiles are almost flat for the measurements at x/D ≈ 14.2 and 24.5, indicating high turbulent mixing, effects of acceleration, reduced shear stress and/or buoyancy effects there.

Fujii "extended" his *k-kL-*$\overline{uv}$ *model* from the annular case to treat flow in circular tubes. Since his model evolved from the one by Kawamura, it is expected that his results would be comparable to those of Kawamura. *Nishimura* utilized a *Reynolds stress model* (RSM) from Shima *with turbulent heat flux and thermal energy fluctuation equations*; the heat flux equations contain two extra terms to take account of anisotropy of turbulence and velocity turbulence-thermal energy gradient production for the pressure-temperature fluctuation gradient correlation terms [Nishimura, Fujii et al., 1997]. For fully-established, constant-property gas flows, the two turbulence models agreed with accepted correlations of friction and heat transfer in laminar and turbulent regimes and forecast a transition within the Reynolds number range from 2000 to 2400.

To assess their predictive capabilities, simulations from the models of Fujii and Nishimura are compared to the velocity measurements of Shehata in Figure 10. For cases 618 and 635, the results from the two models agree closely with one another and with Shehata's measurements. Only at the last station do the predictions appear to be slightly lower in the central region of the flow, but these deviations are within the estimated experimental uncertainties. For Run 445 the two models do predict observable differences. At the first station the RSM already shows a more laminar appearance for the velocity profile than the k-kL-$\overline{uv}$ model does and at the two later stations the velocity gradient near the wall is slightly less than for the k-kL-$\overline{uv}$ model. For this laminarizing run, the k-kL-$\overline{uv}$ model of Fujii et al. represents the measurements better than the RSM does. However, with the exception of the last station, the differences between the RSM results and the data are still within the estimated experimental uncertainties in general. Agreement of the pressure drop predictions with the experiment is good except in Run 445 where the RSM simulation is about ten per cent low. For the axial wall temperature distributions, the only situation where the results of the two models differ significantly is laminarizing Run 445 where the RSM appears slightly better. In summary, the comparisons to measurements are generally acceptable, giving one confidence in the results predicted for the turbulence quantities which were not measured directly.

7. CONCLUDING REMARKS

The purposes of the present article are (1) to remind practitioners why the usual textbook approaches may not be appropriate for treating gas flows heated from the surface with "large" heat fluxes and (2) to review the successes (and failures) of some recent applications of turbulence models to this case. These objectives have been met to some extent.

Results have been primarily considered for *dominant forced convection* at the conditions of the experiments of Shehata and of Perkins, for cases involving significant variation of density, viscosity and thermal conductivity along and across a circular tube. Examination concentrated on three situations for vertical upflow, labeled "turbulent," "subturbulent" or intermediate and "laminarizing." From the measurements the following observations may be inferred concerning the mean turbulence quantities:

- o the heating disturbed the incoming flow almost immediately in the thermal entry
- o the "turbulent" run recovered to an approximately self-preserving condition
- o in the other two runs, the quantities continued to decrease axially until they were much less than the molecular effects

Simulations of the low-Reynolds-number internal flows were made using about fifteen turbulence models. After preliminary examination treating the gas properties as constant, the ability of the models to handle strongly-heated flows was assessed via calculations at the conditions of experiments by Shehata. Possible effects of buoyancy forces at these conditions were considered and it was concluded that forced convection dominated the heat transfer parameters. Numerical predictions forecast the development of turbulent transport quantities, Reynolds stress and turbulent heat flux, plus mean velocity and temperature distributions, wall heat transfer parameters and pressure drops. In contrast to earlier approaches of other investigators, validation focused on comparisons to the measurements of the developing mean velocity and temperature fields.

One may conclude that existing low-Reynolds-number turbulence models and commercial codes should be used with *caution* for

- o low-Reynolds-number, fully-established, *constant-property* flow and
- o strongly-heated, internal gas flows

For strongly-heated turbulent or laminarizing gas flows, five models were found to give reasonable agreement with the internal profile measurements of Shehata:

- o van Driest model as modified by Bankston and McEligot [1969]
- o k-kL-uv model of Fujii, Akino, Hishida, Kawamura and Sanokawa [1991]
- o k-ε model of Launder and Sharma [1974] as calculated by Mikielewicz [1994]
- o k-ε of Abe, Kondoh and Nagano [1994] by Ezato and Kunugi [1997]
- o Reynolds stress model of Shima by Nishimura [1997]

Since the model of Fujii et al. was derived from the earlier model of Kawamura [1979], it is expected that Kawamura's model would provide good simulations but, to our knowledge, it has not yet been tested directly by comparison to profile measurements. The modified van Driest model is not recommended for conditions with significant buoyancy effects [Bates et al., 1974].

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NOMENCLATURE

( )	function of
A_{cs}	cross-sectional area
c_p	specific heat at constant pressure
D	tube diameter; D_h, hydraulic diameter
g	acceleration of gravity
g_c	units conversion factor, e.g., 1 kg m /(N s^2)
G	mean mass flux, $\dot{m} / A_{cs}$
h	convective heat transfer coefficient, $q''_w/(T_w - T_b)$
k	turbulent kinetic energy, thermal conductivity
l	mixing length
L	turbulent length scale
$\dot{m}$	mass flow rate
p	pressure
q''_w	wall heat flux
r	radial coordinate; r_w, tube wall radius
T	absolute temperature
u_τ	friction velocity, $(g_c \tau_w / \rho_w)^{1/2}$
U	time-mean streamwise velocity component
v	local radial velocity component
V	time-mean radial velocity component
V_b	bulk or mixed-mean streamwise velocity
x	axial coordinate measured from nominal start of heating
y	coordinate perpendicular to the wall
z	axial location

Non-dimensional quantities

f, f_τ	friction factor, $2 \rho_b g_c \tau_w / G^2$
Gr_q	local Grashof number based on heat flux, $g D^4 q''_w /(\nu_b^2 k_b T_b)$
Nu	Nusselt number, e.g., hD/k
Pr	Prandtl number, $c_p \mu/k$
q^+	heat flux parameter, q''_w/Gc_pT; q_i^+, based on inlet conditions, $q''_w/Gc_{p,in}T_{in}$
Re	Reynolds number, $4 \dot{m} / \Pi D \mu$; $Re_{w,m}$, modified wall Reynolds number, $V_b D / \nu_w$
y^+	wall distance coordinate, $y (g_c \tau_w/\rho_w)^{1/2} / \nu_w$

Greek symbols

ε	dissipation of turbulence kinetic energy;
μ	absolute viscosity
ν	kinematic viscosity, μ/ρ
ρ	density
τ	shear stress; τ_w, wall shear stress

Subscripts

b	evaluated at bulk or mixed-mean temperature (or enthalpy)
cp	constant property idealization
i, in	inlet
w	wall, evaluated at wall temperature

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