1991 Annual Report
to
The National Science Foundation

December 1, 1991

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I. Summary of Research
Research

The Center research program has a number of different aspects, most of which involve longer term commitments as suggested below.

- **Developent of research software**, both the sponsorship of new programs, and the continued development of existing software. This past year, continuing work was done on the Evolver program of Ken Brakke and the hyperbolic manifolds programs of Jeff Weeks. Under Richard McGehee a major new development was initiated to create four dimensional interactive visualization software focused on “seeing” structures like invariant tori for sympletic maps.

- **Teams of individuals visually exploring mathematical structures** For example, Hsu (a postdoc at MSRI), Kusner (asst. prof. at U. of Mass.), and John Sullivan (Center postdoc) tested various conjectures relating to the Willmore problem by means of the Evolver program. The problem is to find compact surfaces of a given genus in space which minimize the integral of squared mean curvature.

- **Workshops.** The Center will sponsor two major workshops during the current academic year:

  **Computational Crystal Growers Workshop**, organized by Jean Taylor, February 24-29, 1992 (a request for partial support from DOE is pending). Computational models are playing a central role in the analysis of crystal growth, whether the approach is from mathematics, materials science, or physics. Indeed, computational methods have led to theoretical advances and vice versa. The intent of this workshop is to get together many of the people, from each of the three disciplines, that are actively working on various computational models for various types of crystal growth, and thereby to facilitate the exchange of ideas, algorithms, and results.

  It is planned that there be no scheduled talks, apart from an initial introductory round of five-minute talks the first day and a showing of videotapes. Rather, the facilities (space, equipment, and staff) and the layout of the Geometry Center will be used to promote one-on-one and small group interactions. We hope that people will be able to demonstrate their programs in action, either on the workstations or on videotape, and to discuss issues of modeling, programming, etc.

  The people invited have devised a broad range of strategies to model crystal growth. They include direct mappings of manifolds (including graphs of functions over regions in the plane), the crystalline variant of direct mapping, Hamilton-Jacobi formulations, phase-field formulations, the use of characteristics, a new variational formulation, and Monte Carlo simulations on lattices. It has been found in purely theoretical studies that combining various methods leads to proofs inaccessible by a single method alone (e.g., Tom Ilmanen's new regularity proof for motion by mean curvature). We hope there can be similar fruitful interaction among the various computational schemes. At the minimum, we can hope to find which techniques are most promising for which problems.
Visualization of Invariant Sets for Symplectic Maps in Dimension 4, organized by Richard McGehee, March 9-20, 1992. Most questions about the dynamics of symplectic maps in dimension 4 remain unanswered. Even the basic problem of determining the stability of an elliptic fixed point is unresolved, a problem which is rooted in the classic question of the stability of the solar system. This workshop will bring together researchers interested in exploring these questions through computer simulation and visualization. Talks will be limited to two hours per day; most of the time will be devoted to informal discussions centered around computer simulations of symplectic maps. Michael Herman of Ecole Polytech will be in residence at the Center for March and April. He will play a major role in this program which actually extends in a less structured way over March and April. Jurgen Moser of ETH will be associated with the Center during March.

Topics will include: (1) The breakdown of invariant two-dimensional tori. (2) The computation of Arnold's "whiskered tori", which, in this case, are invariant circles. (3) The computation of stable and unstable manifolds for hyperbolic periodic orbits and for hyperbolic invariant circles.

- Communication of research results. Bookwriting teams have come twice this past year to work on "The Geometry and Topology of Three Manifolds" by Codirector W. Thurston. The first volume is nearing completion. "Word Processing in Groups", by D.B.A. Epstein, J.W. Cannon, D.F. Holt, Silvio Levy, M.S. Paterson and W. P. Thurston, was sponsored by the Center and will be published in January. Four of these authors are part of the Center. Both books will have (and already have had) a strong impact in the research community. The Center was instrumental in starting the journal Experimental Mathematics of which the first issue will appear sometime in 1992. It has a very eminent cross-disciplinary board of editors.

Center faculty member Jean Taylor initiated a new kind of conference proceedings for the American Mathematical Society: the proceedings of the special session, Computing Optimal Geometries, that she organized for the 1991 annual meeting, was issued in the form of a video, distributed by AMS.

- Major ongoing research projects. The Center is built on the research programs of its faculty. These programs have flourished and interacted under the Center umbrella. There are many interconnections and commonalities between the fields represented. Most of the education and outreach activities, and the work of the lab, are drawn from them. The following is a representative sample:
Frederick J. Almgren

During the past year I have continued to work in my long term program of work in the geometric calculus of variations. Theoretically this work is in the context of geometric measure theory. The associated development of algorithms lies within the field of computational optimal geometry.

Theoretically I have major progress on theorems guaranteeing the existence of geometric evolutions such as would model the growth of a crystal as it freezes from its melt or its vapor. These are the first such general results and required the development of significant new methods of geometric analysis (the isotropic and completely uniform case, however, was treated slightly earlier by S. Luckhaus). Collaborators in this work are Robert Almgren, Jean Taylor, and Li He Wang.

The computational work I have done has been in association with the Minimal Surface Team of the Center whose members this year were F. Almgren, R. Almgren, K. Brakke, A. Roosen, J. Sullivan, J. Taylor. This past year the team convened the week of July 22 at the Five Colleges Summer Intitute, Mount Holyoke College. The following three weeks were spent at the Center in Minneapolis. R. Almgren and A. Roosen, in particular, made major progress in developing code to model crystal growth. My work and that of J. Taylor made contributions to these schemes.

I spent the three weeks following that at the Aspen Center for Physics participating in a session with metalurgists and physicists who were working in geometry and physics of solidification processes. One of the reasons for doing this is to make more likely that when we achieve our long term theoretical and computational goals they will be as useful as possible as tools for scientific research.

During the past year I have been supervising the Ph.D. work of John Steinke and Alice Underwood and the undergraduate research of Ivan Blank (and to a lesser extent David Ben-Zvi). The projects of Underwood, Blank, and Ben-Zvi are directly related to activities of the center.

James W. Cannon

(1) Cannon, colleague Walter Parry of Eastern Michigan University, and Ph.D. student Eric Swenson are working on the algorithmic recognition of hyperbolic groups by means of their recursive patterns at infinity. This program requires the solution, or approximate solution, of a combinatorial version of the Riemann Mapping Theorem. Parry developed a first algorithm which Cannon has implemented on the Sun. The algorithm is interesting but provably runs slowly on certain classes of rather simple examples. Parry, Cannon, and Swenson together have developed a modified algorithm currently being implemented by Cannon and Swenson. The new algorithm is expected, on average, to run many times faster than the old algorithm. In particular, it provably runs well on the simple examples where the other algorithm falters.
The combinatorial Riemann mapping theorem being studied has interesting visual and intuitive content related to, but different from, both the circle packing versions being studied by Rodin, Stephenson, and others, and from the finite electrical network version reported on in Doyle and Snell. It involves a slightly modified version of the classical problem of "squaring rectangles" which can be presented visually by computer graphics and can be presented as a combinatorial puzzle serving as a suitable introduction to the Riemann Mapping Theorem for students with no background in complex variables.

This mapping theorem seems to be a new finite or combinatorial approximation to the classical theorem. Peripheral questions being studied along the way are (a) conditions for convergence to the classical mapping and (b) effectiveness of the algorithms relative to classical algorithms for numerical approximations to the classical Riemann mapping.

(2) Cannon, colleague Walter Parry, colleague Bill Floyd of Virginia Tech, and Ph.D. student Blake Fordham are studying the relationships of amenability to combinatorial group theory with particular concentration on Thompson's group of dyadic homeomorphisms of the unit interval.

Thompson's group interpreted as group of rotations in an infinite binary tree has obvious connections with dynamically managed data structures as studied, for example, by Tarjan, Sleator, and Thurston.

The approach we are taking is to analyze orbits of the generators in the groups, as indicated in the classical paper by Foelner, with respect to the existence or nonexistence of what we call "cooling functions." Again, this notion can be presented to an unsophisticated audience as a physical interpretation of amenability or nonamenability. A nonamenable group is a group that can be cooled without the creation of hot spots.

Normal forms and multiplications of group elements in Thompson's group have been implemented by Fordham on computer. The generator orbits are well-understood. Their relationships to one another are not yet well-understood. Fordham is creating computer programs to visualize their interaction.

(3) The Epstein programs on automatic groups form the first step in several directions:
(a) the algorithmic recognition of negative curvature in a group;
(b) the algorithmic analysis for a negatively curved group of the space at infinity;
(c) the algorithmic analysis of entire infinite sequences of groups: which are finite? which are Euclidean? which are negatively curved? which do not allow easy algorithmic analysis?
(d) in what sense can one say that group presentations vary continuously?
(e) what group presentations are nearby one another?
(f) are most finitely presented groups, as claimed by Gromov, and as indicated by Thurston's results on 3-manifold groups, negatively curved? Ph.D. student Paul Shawcroft is addressing some of these issues. Visualization of the space at infinity has potential here.
Bernard Chazelle

Most of my research this year has concentrated on the derandomization of probabilistic geometric algorithms.

My best result in that direction has been the discovery of an optimal convex hull algorithm in any fixed dimension (which had been an open problem for many years). Although the algorithm is deterministic, it is best understood as a cooling process in statistical thermodynamics. Actually, I have been pursuing the analogy between algorithm design and statistical physics in the hope of shedding light on the design of so-called "potential" functions, which constitutes one of the least understood aspects of amortized algorithms analysis.

Another research accomplishment has been the design of an optimal strategy for cutting hyperplanes to provide for efficient divide-and-conquer strategies. This extends the classical theory of epsilon-nets to geometric range spaces.

Miscellaneous results include new algorithms for geometric optimization based on parametric searching, randomized algorithms for two-dimensional computer graphics (which have the advantage of being extremely simple to implement and analyze).

With my graduate student, L. Palios, we have investigated the decomposition of low-dimensional piecewise-linear manifolds. Our latest result provides an optimal algorithm for partitioning the boundary of a 3-polytope into convex-like pieces.

With another graduate student, Hervé Brönnimann, we have derived new lower bounds for polytope range searching, which almost match the existing upper bounds.

David Dobkin

Dobkin has been working on the organization and development of software for doing computational geometry in 2 and 3 dimensions. A library of procedures is beginning to develop which will be easily usable by others. Difficult issues of building common interfaces, handling degenerate data in robust fashion, developing algorithms which behave efficiently in practice as well as theory continue to be tackled. We are also exploring user interface issues in developing a programming environment in which such algorithms can be implemented in the future.

David B.A. Epstein

At a recent international conference on group theory, a well-known group theorist gave an hour-long purely theoretical lecture proving the existence of certain canonical forms for group elements in a particular group, and deriving the growth function of the group. Subsequent to the lecture, we ran our "automata" program on the same group. After five minutes it produced all the results that had been presented in the lecture, together with proofs of correctness. Both sides were relieved to find that their results agreed with each other.

In the journal *Experimental Mathematics*, many of the philosophical principles underlying the Geometry Center play a central role. The Geometry Center has been a vital element in establishing this journal, and the journal would not have existed without it. Many people associated to the Geometry Center are involved with the journal: Epstein (Chief Editor), Silvio Levy (Editor), Thurston and Marden
Another connection with the Center is that an important paper by Milnor has been accepted for the first issue. An article was written about the journal in Notices of the American Mathematical Society.

Michael H. Freedman

I plan to pursue (jointly with Z.-X. He) the analysis of the energy functional on $C^2$ simple closed curves. We investigate the relation between "modulous" of degree $1$ curve families and linking was made. Initially, we found that there is a universal bound on how fat a solid torus $T$ can be (fat means the degree $1$ curve family is small modulous) if $T$ is to participate in a non-psplit link of $K$ solid tori. We are intrigued by the possibility that Grad $E$ might have only a single stable critical point on the space of unknots. (This is consistent with the function space's topology according to Hatcher's theorem.) Some numerical study will be made by others at The Geometry Center.

Pat Hanrahan

Our work during the last year has concentrated on designing and implementing new 3D computer graphics algorithms and systems. We've concentrated on three areas: tools for making 3D graphics more accessible, volume rendering for scientific visualization, and global illumination calculations for enhancing realism.

In joint work with Jim Lawson, we developed a high-level little language for specifying optical properties of materials and light sources. In related work with Paul Haeberli, we developed a 3D WYSIWYG paint program that allowed the artist to apply paint pigments directly onto 3D shapes. Both these projects are first steps in finding methods for making advanced 3D graphics capabilities easier to use and more accessible.

In the area of scientific visualization we have been developing new volume rendering algorithms. We have developed efficient algorithms for rendering scalar functions defined on 3D Delaunay triangulations. We have also been formulating a theoretical framework for the volume rendering process that has already lead to improved implementations, and finally, with David Laur, we have developed a progressive refinement algorithm suitable for use with conventional high-performance 3D polygon-based graphics hardware.

The central problem in image synthesis today is how to solve the "rendering equation" that governs the transport of light through 3D scenes. We have developed a very efficient new technique that is based on the recent result that the N-body problem can be solved in linear-time.

John H. Hubbard

Take a mapping $f=f_k + f_{k+1} + ... : C^n \rightarrow C^n$, with $f_k$ homogeneous of degree $k$ and non-degenerate. Then there exists a potential of 0 : the limit

$$\lim_{k \to \infty} k^{-m} \log |f^m(x)|$$

exists in a neighborhood of 0. Now the first case of interest is when $f=f_k$ (the homogeneous case). Then $f$ induces a mapping on projective space, and with the help
of the potential we can show that things like the Brohlin measure exist more specifically, there exist unique invariant $(k,k)$-currents, which can be obtained by appropriate pull-backs and averaging. Even in the case of rational functions, this yields a much easier proof of existence of the Brohlin measure. In the non-homogeneous case, we try to understand the support of $dd^c$ (potential function), which turns out to be homeomorphic to the cone over the Julia set of the endomorphism of $\mathbb{P}^{n-1}$, by a homeomorphism which conjugates the mapping to the associated homogeneous mapping. This is the only analog there is of the Bottcher coordinate in higher dimensions. There is a further application of these results to Newton’s method in several variables, which is still filled with open problems.

In preparation with B. West are Volumes Two and Three of Differential Equations, A Dynamical Systems Approach. Also in preparation is a book to be called Three Jewels by Thurston.

**Benoît B. Mandelbrot**

Benoît B. Mandelbrot’s work continues to be largely concerned with certain singular measures called multifractals. The part of this work that is connected with The Geometry Center is the study of harmonic measure for the Laplacian potential around the two dimensional fractal aggregates called DLA. The importance of this measure in physics is that—in combination with an element of randomness—it governs the growth of DLA. Using the Center’s Cray, Drs. Mandelbrot and Evertsz have evaluated this potential, and (thanks to high quality graphics executed at Yale) have made a number of empirical observations that have escaped earlier investigations. This work is done in close connection with Peter W. Jones of Yale and offers a fresh instance of the fruitfulness of graphics in spanning the range from “pure” mathematics to very concrete physics.

**David Mumford**

My work has been on the development of mathematical models for identification of structures in visual signals. The key problem is that these must be robust with respect to the characteristic distortions of visual signals: the combination of smoothly varying deformations with unexpected breaks and discontinuities in which multiple models conflict.

**Charles S. Peskin and David M. McQueen**

**FIBER ARCHITECTURE OF THE AORTIC VALVE:**

We have combined geometry, mechanics, and computation (and also borrowed a little bit of fluid dynamics) to put together a theory which leads to a derivation from first principles of the complicated fiber architecture of the aortic heart valve leaflets. The key hypothesis of the theory is that the uniform pressure load on the closed aortic valve is supported by a one-parameter family of fibers under tension. We have derived an equation of equilibrium for such a structure, a partial differential equation from which all mechanical variables can be eliminated by the proper choice of coordinates. The result is a purely geometric equation, which (with suitable boundary conditions) determines the shape of the leaflets and the arrangement of the fibers.

Surprisingly, this equation takes the form of an evolution equation in which a single fiber moves to sweep out the leaflet surface. In this interpretation, the fiber moves
at a speed proportional to its curvature but in the direction of the binormal vector, not the principal normal. Such an equation of motion has previously been used in fluid dynamics, specifically in the study of liquid helium, to approximate the motion of vortex lines. In that context, the equation is known as the self-induction approximation. (In our case, no approximation has been made, however.) We take the free edge of the aortic valve leaflet as an initial fiber, fix the ends of the fiber, and let the resulting curve evolve according to the computational scheme for the self-induction approximation introduced by T. Buttke.

Because the initial curve has a sharp bend at the center of the free edge, where the three aortic leaflets meet, we anticipate increasing complexity in the computed solution as the bend is progressively better resolved. Indeed the computed solution appears to have a fractal character with increasing complexity at progressively smaller scales. This complexity manifests itself as a branching, braided fiber architecture much like that observed in the real valve leaflet.

John Sullivan
Geometry Computing Group Postdoctoral Fellow

With Steve Altschuler, I have been investigating the behavior of the curve-shortening flow on space curves, and in particular self-similar or soliton solutions. We started by writing software to graph such curves, using The Geometry Center's MinneView viewer. The pictures suggested many conjectures about the behavior (especially asymptotically) of the solutions, which would not have been obvious from the equations. We have now proven most of these conjectures, and are preparing a paper on "Self-similar solutions to the space curve shortening flow" for submission to Experimental Mathematics.

Starting at the Five Colleges Regional Geometry Institute, I have worked with Ken Brakke, Rob Kusner, and Lucas Hsu on Willmore evolution of surfaces. Experimentally, we have found one numerical scheme that works better than other possibilities for estimating the square curvature energy of a smooth surface from a polyhedral approximation. This can be used to drive an evolution towards a Willmore surface, but we still need to prove estimates on the approximation.

I have worked to extend the results of my PhD thesis. Fred Almgren was interested in computing area-decreasing motions, with some cost from volume swept out added to the surface energy to be minimized. It was fairly easy to extend my earlier algorithm for area-minimizing surfaces to handle this case, and this provides one possibility for a numerical crystal-growth scheme. Thanks in part to discussions with Jesus Gonzalez, I was also able to vastly improve the boundary neighborhood mass estimate, the weakest link in the estimates proving the crystalline approximation theorem.

I wrote an expository paper for the Mathmatica Journal [1:3] on "Generating and Rendering Four-Dimensional Polytopes", in which the dodecaplex (or 120-cell) is constructed from its symmetry group. This was illustrated with pictures created with a soap-bubble shader I wrote earlier in the RenderMan language.
Jean Taylor

I supervised work of Andy Roosen on simulations of dendritic crystal growth and Ostwald ripening and wrote a paper with him describing the theoretical framework and the experimental results. The ability to do these computations, and the ability to attract a student as capable as Roosen to work on these problems, is a direct result of having the superb computer equipment that I've received as a result of the Geometry Center.

I completed two long papers, to appear as Overviews in Acta Metallurgica. The figures were generated on equipment provided by The Geometry Center. I finished a short (6 page) Scripta Met paper with Frank Morgan and wrote two short survey articles. I somewhat revised Motion of Curves by Crystalline Curvature in response to referees suggestions. This involved adding several figures, all obtained by the use of equipment obtained through The Geometry Center.

I investigated properties of initially polyhedral 2D surfaces evolving under weighted mean curvature in R3 and expanded some of my computer programs to compute such growth. I began to investigate the effect of edge energies.

I worked on revisions to preprint on Motion of Multiple-Phase Junctions under Prescribed Phase Boundary Velocities.

I also worked on use of crystalline method to compute stationary (but not minimizing) crystalline minimal surfaces.

I worked out the structure of a proposed paper (probably for Scripta Met) on the difference between evolution of soap bubble clusters via diffusion and motion by mean curvature, with application to the shapes of small crystals seen in materials. This paper will be joint with Rob Kusner and Tom Ilmanen.

Major changes in my research directions included putting diffusion in the crystalline motion problems, expanding the motion problems to 3D, and learning more about phase field model.

My future plans include further extending code to 3D, using current code to model different growth laws, and modeling eutectic growth. In addition, I plan on finishing my paper on triple junctions and characteristics; writing a short paper on limiting shapes of motion by mean curvature versus soap froth evolution; and working with Gelfand on use of characteristics for nonconvex mobility functions.
Plans and Increased Pace of Research

The reports above incorporate plans for the coming years.

There is a broad, powerful base of scientific knowledge represented by the Center faculty and associates. As Center programs enlarge and develop, these intellectual resources will be increasingly drawn on.

In mid-December, two to three additional postdocs will be appointed in math and computer science. This will result in a large increase in resident research activity and consequent increase in student and visitor involvement.

The Center's current postdoc, John Sullivan, has accepted an appointment as an assistant professor in the School of Mathematics of Minnesota. He will spread out his one additional postdoc year over two years, alternating full-time service in the School of Mathematics with full-time research activities in the Center.

We are currently working with the Computer Science Department on a possible joint appointment as well. The more this happens, the more postdocs we can appoint and there will be more activity in the Center.

The graphics lab jointly sponsored with MSRI and Berkeley is being readied. Staff member Silvio Levy is available as a consultant in graphics. Center software is used there and new groups of scientists are involved. Through this lab, larger numbers of mathematicians and students will become acquainted with Center software and activities. For example, the noted astrophysicist Andrei Linde, after consulting with Dr. Levy, is using Center tools to render fractal graphics which model the beginning of the universe. We expect this lab will increase the number of participants in the Center.

The remote computing lab jointly sponsored with the School of Mathematics of the University of Minnesota is also nearing readiness for student and faculty use.

In addition, Ken Brakke will spend his sabbatical year in the Center. We plan to appoint at least one other sabbatical visitor.

Another way resident research activity will be increased is by the appointment of additional technical staff who can further amplify the computational and visualization efforts of the researchers and the output of research software.

The two workshops, Computational Crystal Growers Workshop and Visualization of Invariant Sets for Symplectic Maps in Dimension 4, that are currently being organized will take place in February and March/April, 1992. Two half-time math graduate students are heavily involved in the software development activities for that. Graduate students will be well represented in both workshops.

There will be additional workshops in the 1992-93 academic year. Possibilities include dynamical systems in two complex dimensions and development of symbolic mathematics programs with high level graphics features.

Budget removed
III. Updated STC Data Base
Grant Number: 89-20161
Center Director: Albert Marden
Institution Name: University of Minnesota
Title: Computation and Visualization of Geometric Structures
NSF Coordinator: Al Thaler

Fiscal Year: 92

Number of Faculty Participants: 20
Number of Non-Faculty Participants: 3
Number of Visitors: 56
Number of Post Doctorates: 1
Number of Graduate Students: 14
Number of Undergraduate Students: 10
Number of High School Teachers: 12

Ethnicity Summary
- American Indian, Alaskan Native: 0
- Asian or Pacific Islanders: 2
- Black, not Hispanic Origin: 1
- Hispanic: 3
- White, not Hispanic Origin: 102
- Unknown: 8

Nationality Summary
- US Citizens: 101
- Foreign Nationals: 13

Gender Summary
- Number of Males: 99
- Number of Females: 17

Sources and Amounts of Support for the STC (Per 1000s)
- Received Support: $0
- Committed Support: $0
- Pending Support: $653
- Total: $653

Faculty Funding Summary (Other Than STC / Per 1000s)
- Funding from NSF: $835
- Funding from DOD: $0
- Funding from DOE: $0
- Funding from EPA: $0
- Funding from NASA: $0
- Funding from NIH: $0
- Funding from NOAA: $0
- Funding from USDA: $0
- Funding from Other Federal: $0
- Funding from Other: $537
- Total: $1372

Faculty Reported STC Funding (Per 1000s)
- Total: $272
IV. Description of Outreach Activities
Outreach Activities

Education

Last Summer, 1991 we organized the first summer program under Center auspices. There were two parts. The first was a two week intensive course, Geometry and the Imagination, created and taught by John Conway, Peter Doyle, Jane Gilman, and Bill Thurston, two of whom are Center faculty members. Earlier versions had been taught twice before at Princeton. The 60 students were a mix of high school and college students, and high school and college teachers. The course was an extraordinary experiment in providing a course that challenged everyone, from the uninitiated to the mathematically sophisticated. The science reporter of the largest daily in Minnesota, the Star Tribune, wrote, "This was perhaps the most unusual, energetic and mind-expanding math class ever taught in Minnesota." It made heavy use of physical objects (e.g., polyhedra, mirrors) to develop intuition. The course notes have been issued as Center Report 30.

The course has inspired other mathematicians to teach courses at their universities likewise aimed at bringing a conceptual understanding of modern geometry to students without specific technical backgrounds. The methods used in the course to introduce curvature have been particularly influential in this development.

The other part of the summer program was a research and training program for talented senior high school and college students. There were 16 students present for up to 12 weeks. There was a special summer faculty to help each student find a worthwhile project, and to work with them through the summer. The summer faculty are called 'coaches' because of the requirement both of depth and breadth in science, and the ability to motivate. This is a very demanding job and it is difficult to find mathematicians who are willing and able to do it. The coaches for the past summer were Stan Wagon (Macalester), Jeff Ondich (Carleton) and Anthony Iano-Fletcher (Warwick). The first week the students arrived, they had a tutorial by Wagon on the use of Mathematica, and one by Maxwell on the use of the animation package Softimage. Student completed projects included a professional quality program to display knots and links developed in collaboration with staff, a study of sunrises and sunsets on the planet Mercury (up to three a day), a hyperbolic flight simulator developed in collaboration with staff, a classification of four dimensional archimedian solids with John Conway, and the logo for the newly formed European Mathematical Congress with Peter Doyle and Max Karoubi.

At the end of the summer, each student was required to write a report describing his or her activities. These are gathered together in Center Report 33.

Thus, the goals of our summer courses are to a) provide strong, attractive models for teaching aspects of geometry to college students without specific math background, b) provide background for high school math teachers and to assist them in developing materials which can be used in their classrooms, and c) demonstrate the advantages of team teaching that brings together different viewpoints, expertise, and personalities. The purpose of our summer long student program is to provide an intellectually rich, non-competative, non-intimidating experience based a personal exploration in depth into a topic in mathematics or computer science. This too provides an alternative to the normal academic process in mathematics education. The goal is not necessarily to produce more math majors or research mathematicians, rather, to give students a deeper appreciation of mathematics and the role of computing.
In Fall, 1991, the Center began two additional programs. The first is a mentoring program for bright, motivated senior high school girls. The purpose is to get them proficient before college in scientific computing, and to provide an outlet for informal discussions about applying for college, the role of women in science, etc. Tamara Munzner of the Center staff directs the program. With her help, each student has found a long term project which can serve as the vehicle for learning the computing skills.

In addition, the Center has hired as consultant Arnie Cutler, an experienced high school mathematics teacher and a graduate student in curriculum development in the School of Education. Cutler's job is to explore mechanisms that Center activities can be brought into K-12 classrooms, to help the Center develop relationships with the local high schools, and to explore possible areas of joint activities with the University of Minnesota School of Education.

Another major program is planned for Summer, 1992. The two-week intensive summer course will be more focused on high school teachers and providing materials to use in their classrooms. It will be taught to an audience of 30 high school teachers and 20 students who will be enrolled in our 10 week long summer program. In the afternoons, the teachers will work with the instructors to develop materials for their classrooms. Special effort will be made to insure a strong representation of female teachers. The title of the course will be "Chaos and Fractals." It will be taught by a very experienced team of four consisting of Bodil Branner, John Hubbard, Bjorn Felsager, and Mette Vedelsby. The last two have been prominent in math and science curriculum development in the Danish School System and are themselves teachers in Danish gymnasiens. The Head Coach for the summer long program will be Tony Phillips. The program will be enlarged to twenty students. The first week upon arrival, there will again be a Mathematica tutorial by Stan Wagon, and in addition a number of project generating seminars by John Hubbard, Tony Phillips, Stan Wagon, Jeff Weeks and Blaise Morton, a senior engineer at Honeywell. The most demanding part of administering the program is insuring that each student has an interesting project that can be reasonably completed by summer's end.

For Summer 1993, the intensive course will be "Computer Graphics and Art" taught by D. Dobkin, P. Hanrahan, and V. Sorensen. The first two are Center faculty. This will be based on an NSF sponsored undergraduate course in the computer science department at Princeton. The summer program will continue to be refined and its reach extended according to experience and resources.

Videos

During the past year the video "Not Knot" was completed. It was directed by staff member Charlie Gunn and animation consultant Delle Maxwell. A supplement by Epstein, a Center faculty member, and Gunn was written and arrangement was made for professional publication and distribution with the video. The conception and completion of this project spanned several years, and many lessons were learned that have been of considerable value in planning further productions. The video was made to explain what it means for a manifold, specifically the Borromean ring complement, to have hyperbolic structure by seeing what it would be like to live in the space. Both scientifically and educationally this opens many possibilities; the video is the first of what is likely to become a new genre.
Scientifically, it is the first example, at least in mathematics, of how the powerful technology of computer graphics can be used to visualize, optically accurately, spaces with noneuclidean metrics. It also reveals how very difficult it is to communicate mathematics, specifically to write a script to be accessible to a general audience, even a general mathematical audience. Writing the script was one of the most difficult parts of the video. Another difficult part is the artistic input which is animating the scenes to be attractive and inviting for the viewer. Of course the computer graphics itself was completely original. "Not Knot" won professional recognition in the graphics world: it was chosen for the Siggraph 91 Electronic Theater, and it won the Nicograph 1991 award in Japan for the category of scientific computer graphics.

Initially, it was hoped that the video would be accessible and have something to communicate to everyone, from high schoolers to research mathematicians. Our experience is that that can happen, but not without extra help from a lecturer. The scenes of a video go by quickly, and even with every word of the script carefully chosen there is a lot to absorb. This was one of the reasons a 50 page supplement was written. The supplement adds commentary on representative frames from the movie, answering the questions that were asked over many public showings to many different groups, and providing capsule summaries of relevant topics, like knot theory and hyperbolic geometry. We believe the supplement greatly increases the value of the animation. The topics in the summer course were related to the video, which was shown at the beginning and at the end of the course, so the students could compare their understanding.

"Not Knot," as well as Taylor's video, has opened the door to using video as a major output device for communication to research mathematicians, and to a larger scientific, educational and public audience. The production of a number of other Center sponsored videos is underway.

Conferences hosted at The Geometry Center


Helaman Ferguson, speaker
65 registered teachers
postponed due to snow

November 10-11, 1991. Mathematicians and Education Reform (MER) Network. A meeting of three Regional Geometric Initiative and The Geometry Center (DIMACS wanted to but was unable to attend). Attended by NSF program director and AMS representative (Allyn Jackson). There was scientific/education discussion of programs and vertically integrated education model.
V. Management and Planning
Executive Committee
Almgren, F.
Epstein, D.
Hanrahan, P.
Keynes, H., Education Director
Marden, A., Director
McGehee, R., Science Advisor
Thurston, W., Codirector

External Advisory Board
Hyman Bass (Prof. of Math, Columbia Univ.)
James Blinn
Frederick Gehring (Prof. of Math, Univ. of Michigan), Chair
Daniel Gorenstein (Director DIMACS & Prof. of Math, Rutgers)
John Guckenheimer (Dir. of Research Programs, Cornell Theory Center and Professor of Math, Cornell)
Deb Hughes Hallett (Prof. of the Practice in the Teaching of Math, Harvard)
Maria Klawe (Head, Dept. of Computer Science, Univ. of British Columbia)
Jill Mesirov (Dir., Mathematical Sciences Research, Thinking Machines Corp.)
Alan H. Schoenfeld (Chairman, Educ. in Math, Science & Technology, Graduate School of Education, UC-Berkeley)

Director
A. Marden

Codirector
W. Thurston

Education Director
H. Keynes

Science Advisor
R. McGehee

Geometry Computing Group Faculty
Almgren, F.
Cannon, J.
Chazelle, B.
Conway, J.
Dobkin, D.
Douady, A.
Epstein, D.
Freedman, M.
Hanrahan, P.
Hubbard, J.
Keynes, H.
Mandelbrot, B.
Marden, A.
Milnor, J.
Mumford, D.
Peskin, C.
Taylor, J.
Thurston, W.
Wilks, A.

Staff
Administrative
Vail, A.

Technical
Bertilson, S.
Gunn, C.
Levy, Si.
Levy, St.
Munzner, T.
Phillips, M.
Membership of All Advisory Committees

External Advisory Committee:

Hyman Bass (Prof. of Math, Columbia Univ.)
James Blinn
Frederick Gehring (Prof. of Math, Univ. of Michigan), Chair
Daniel Gorenstein (Director DIMACS & Prof. of Math, Rutgers)
John Guckenheimer (Dir. of Research Programs, Cornell Theory Center and Professor of Math, Cornell)
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Alan H. Schoenfeld (Chairman, Educ. in Math, Science & Technology, Graduate School of Education, UC-Berkeley)

Executive Committee:

Frederick J. Almgren, Professor of Mathematics, Princeton University
David Dobkin, Professor of Computer Science, Princeton University
Patrick Hanrahan, Professor of Computer Science, Princeton University
Harvey Keynes, Professor of Mathematics, University of Minnesota
Albert Marden, Director and Professor of Mathematics, Univ. of Minnesota
Richard McGehee, Science Advisor and Professor of Mathematics, U. of MN
William P. Thurston, Codirector and Professor of Mathematics, UC-Berkeley

Center Faculty:

Fred Almgren, Princeton University
James Cannon, Brigham Young University
Bernard Chazelle, Princeton University
John Conway, Princeton University
David Dobkin, Princeton University
Adrien Douady, University of Paris X1, Sud
David Epstein, University of Warwick
Michael Freedman, University of California, San Diego
Patrick Hanrahan, Princeton University
John Hubbard, Cornell University
Harvey Keynes, University of Minnesota
Benoît B. Mandelbrot, IBM and Yale University
Albert Marden, University of Minnesota
John Milnor, SUNY, Stony Brook
David Mumford, Harvard University
Charles Peskin, Courant Institute of Mathematical Sciences
Jean Taylor, Rutgers University
William Thurston, University of California, Berkeley
Allan Wilks, AT&T Bell Laboratories
Directors Report

The Geometry Center is up and running well beyond the scale of the Geometry Supercomputer Project that spawned it. The Center mission of research and education supported by advanced computing, graphics and video technology opens many opportunities and possibilities for having an impact in each of these areas.

The Center is driven by science and is supporting some of the finest mathematics and computer science research produced by Center faculty and associates. At the same time, it is part of the Center mission to inspire increasing numbers of geometers to get involved in activities that the Center can support. In effect, the Center is a new kind of mathematics institute. The Center wants to sponsor small teams of one or more mathematicians to carry out significant scientific/computational/visualization projects. The kinds of workshops we want to promote involve a) computational and theoretical issues with associated code development for modelling important phenomena, b) the ‘experimental’ approach to studying structures that have resisted theoretical explanation, and c) the development of large scale research software. The Center wants to promote research that will benefit from the Center lab and staff expertise.

As the Center is forging new paradigms within the mathematics and computer science communities rather than following well paved roads, its management is quite challenging. An initial idea must be carefully analyzed as to its appropriateness and feasibility. Implementation of a novel program can require great effort. The key factor is to discover and then recruit for active involvement those who are exceptionally motivated, skilled and talented. The first 10 months in the life of the Center include some notable successes in terms of expanding, fruitful research and educational programs. Less noticeable in the written record are the explorations of possible programs for the future. For me as Director, it has been an extremely intense, consuming period, and exhilarating as well.

Research. Through the ‘center without walls’ aspect of The Geometry Center, there is a strong ongoing research program. Its fruits are fundamental for Center activities in education and graphics. We need to do more to bring more mathematicians into the computing/graphics world. We need to encourage more mathematicians to take the risk of developing software to explore, to see what has never been seen and from that to draw inspiration for further research advance. This is one reason behind the Center involvement in starting the journal *Experimental Mathematics*. We want to strengthen our visitor program and increase the number of postdocs and long term visitors on sabbaticals or other outside support. A promising development is the possibility of sharing postdocs with the mathematics and computer science departments.

Education. Beyond the current programs, how can the Center connect to K-12 student and teacher education? How can mathematics research and visualization at the cutting edge connect to the schools? We are actively analyzing the situation with the assistance of high school teacher Arnie Cutler. Ideas include curriculum development involving narrated videos with text supplement, highly structured instructional labs with 3D graphics workstations, and “road shows” with 3D graphics equipment temporarily brought into classrooms with a carefully organized teaching module. In particular, we have had meetings at the School of Education, which is initiating a new program for undergraduate math majors leading to certification and a masters in education. There are ways the Center might have an input in that program. Our efforts seem to be seeding new possibilities in math education within the math department as well.
Technical staff and lab. The lab is the physical heart of the Center and the technical staff (currently six) create, develop and maintain the software tools that make the equipment there useful in the research and educational missions. Among the staff, high levels of expertise exist in areas of graphics programming, mathematics programming, programming languages, editing tools, user interfaces, systems and video management, network management.

The staff carry out major projects of their own, and they serve as consultants. The Center currently has need of more inhouse mathematical programming to attach educational modules to research software and to make selected research software available to a larger audience by improving user interfaces and documentation. More needs to be done; for example, we are investigating possible involvement in symbolic mathematics and computational algebraic geometry development. These are not the kind of activities that are normally supported in academic departments. More generally, the staff need to keep proselytizing the mathematics community explaining the tools available for mathematicians, and welcoming and advising those who want to get involved. Software development focused on getting user friendly programs distributed is an important aspect of the Center mission.

Communication. We want to tell the world what we are doing, what is going on today in geometry. An integral part of carrying out research is the timely communication of results to other researchers. We would like to reach greater numbers than those in a narrow specialty. Then we want to explain current research to students, from elementary school students through college students. We are excited about searching for methods in which presentations can be so layered, using appropriate textual materials, videos, and software. And then we want to communicate to the interested public. There are clear signs of great curiosity about what mathematicians do. Yet it is very challenging to explain contemporary activity in mathematics in a way that can be generally understood. The Center is excited by the challenge.

Administrative staff and fundraising. We currently have a Senior Administrative Director, plus a few student receptionists. We are currently searching for a senior accountant and a more experienced receptionist. We feel that the Center will operate efficiently and effectively by limiting the number of administrative staff to these three positions, which will be filled by experienced, mature individuals able to work together as a team. Our quarterly newsletter has, unfortunately, been much delayed because of lack of staff but, as soon the staff situation is fixed, it should appear on a regular schedule.

As a vigorous program with a large mission, already at the end of the first year of operation, we can see the need for additional resources to carry out the opportunities available. Not only is it necessary to find additional sources of federal funds, but we need to tap the large private sector, especially foundations. To assist us in identifying and soliciting potential private sources of funding, we are considering engaging an experienced consultant. Developing a good approach to foundations and corporations is not unrelated to our need and desire to communicate with the interested public with appropriate written and visual materials.
VI. International Contacts/Visits
Center faculty members Douady and Epstein are based in France and England respectively. Epstein runs a major SERC-funded computational activity at the University of Warwick and there are close scientific relationships to that.

Professor Michael Herman of Ecole Normale Paris will be visiting the Center during March and April, 1992. He is one of the top international experts in dynamical systems and he will be helping direct the workshop on 4-D.

The Symposium ‘Geometry and Computers’ will be in Tokyo, January 20-22, 1992. This was organized jointly by The Geometry Center and Professor Sadayoshi Kojima of the Tokyo Institute of Technology and several members of The Geometry Center will be attending. It is funded entirely by Japanese sources. Professor Kojima is trying to establish in Japan a center similar to ours.

Preliminary discussion have been held with Professor B. Dahlberg, who is scientific director of Volvo, concerning a possible training program.

Anthony Iano-Fletcher, University of Warwick, England, visited the Center in April and again in June-July. He assisted undergraduate summer research students with their projects. Students gave Dr. Iano-Fletcher excellent reviews during their exit interviews.

Maria Iano-Fletcher, University of Leicester, England, visited the Center in April and also in June. She worked with high school students on research projects. She was well-liked by the students and was a positive influence on their work. She has been awarded a grant by the UK Higher Education Initiative to conduct a course along similar lines at Leicester University, England, during the coming academic year.

Three students from the University of Leicester, England, (Harris, Kay and Hooker) attended the two-week summer course, “Geometry and the Imagination.”
APPENDICES
Cannon, J.W.


Dobkin, David


Epstein, David B.A.


Freedman, Michael


Hanrahan, Pat


Hubbard, John H.


Quadratic polynomials and Tableaux, to appear in the proceedings of the Milnor conference.

with P. Papadopol, Superattractive points in \( \mathbb{C}^n \), submitted to Annals of Math.


MacMath, a software package, Springer-Verlag.

Mandelbrot, Benoit B.


Mumford, David


Peskin Charles S. and McQueen, David M.


Taylor, Jean


The Geometry Center was also instrumental in the publication of Computing Optimal Geometries (which was edited by Jean Taylor).
Publications: Use of Center Facilities

Brakke, Kenneth
Surface Evolver Manual, Version 1.65, GCG Research Report 31

Cannon, James

Epstein, David B.A.

Word Processing in Groups, book by Epstein, Cannon, Levy, Thurston, Mike Paterson and Derek Holt (all but the last two are Center members), in press (Jones and Bartlett), 335 pages

Gunn, Charles


Levy, Silvio
Automatic Generation of Hyperbolic Tilings, by Silvio Levy, article accepted for publication in Leonardo, summer 1992 (date tentative)


The Mathematica Journal, edited by Silvio Levy, published by Addison-Wesley, four issues so far.

Experimental Mathematics, journal to be published by Jones & Bartlett, to appear in early 1992, David Epstein (Editor-in-Chief), Silvio Levy (Editor), various members of the Center on the editorial board.

Sullivan, John

"Generating and Rendering Four-Dimensional Polytopes", Mathmatica Journal [1:3]

Taylor, Jean


Simulation of Crystal Growth with Facetted Interfaces (with Andrew Roosen), to appear in Interface Dynamics and Growth, Materials Research Society.


Weeks, Jeffrey

Convex Hulls and Isometries of Cusped Hyperbolic 3-Manifolds, GCG Research Report 32
Appendix II. External Advisory Committee Meetings
The Center has not yet held an External Advisory Committee meeting. A March 1992 meeting is currently being planned.

External Advisory Committee members include:

Hyman Bass (Prof. of Math, Columbia Univ.)
James Blinn
Frederick Gehring (Prof. of Math, Univ. of Michigan), Chair
Daniel Gorenstein (Director DIMACS & Prof. of Math, Rutgers)
John Guckenheimer (Dir. of Research Programs, Cornell Theory Center and Professor of Math, Cornell)
Deb Hughes Hallett (Prof. of the Practice in the Teaching of Math, Harvard)
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Jill Mesirov (Dir., Mathematical Sciences Research, Thinking Machines Corp.)
Alan H. Schoenfeld (Chairman, Educ. in Math, Science & Technology, Graduate School of Education, UC-Berkeley)
Appendix III. Inventions, Patent Applications, Patents
The Center has no inventions, patent applications, nor patents to disclose at this time.
Appendix IV. Awards/Prizes
The Center was awarded a Nicograph 1991 award in the category of scientific computer graphics for its videotape "Not Knot."