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Intensification of Harmonic Spontaneous Radiation with a Novel Undulator

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Abstract. We have calculated the on-axis spectrum of spontaneous radiation emitted by an electron moving along a planar undulator that has a magnetic profile along the axis that approximates a square wave. (This could be obtained in practice by driving a ferromagnetic undulator into saturation by "excessive" current in the windings.) We find considerable enhancement of the harmonic radiation spectrum. We compare the harmonic power emitted by an electron moving through an undulator having a sine-wave field profile with the radiation emitted from an undulator having a "square-wave" profile; the latter is approximated by the first three Fourier components of the undulator magnetic field profile along the axial direction. Examples are computed for 40MeV electrons taking $K < 1$, for spontaneous radiation emitted along the axis of the system. The emission at harmonics $f > 1$ is greatly enhanced for the approximate square-wave magnetic profile: the ratio of the power emitted at f=5 by the "square-wave" undulator to that of the sine-wave undulator is about 15 (whereas the corresponding ratio at f=1 is only 1.5). While this enhancement might be expected because of the appreciable n=1 and n=5 Fourier components of the undulator field, higher odd harmonics are enhanced even more (e.g., x1000 at f= 11). FEL gain at the harmonics should be enhanced by similar factors.

INTRODUCTION

In a typical free electron laser (FEL), the electron beam interacts with a "dipole" undulator that has a sinusoidal magnetic field variation; the electron motion is sinusoidal in the plane transverse to this field and emits odd-numbered harmonic radiations along the axis. However, one need not limit the choice of undulator field profile to the sinusoid, providing other profiles result in significant advantages. In connection with the IFEL accelerator, in the past we have pointed out [1] that the use of an undulator profile that approximates a "square wave" will result in an enhanced acceleration gradient, by as much as a factor of two (equivalent in effect to an increase of laser drive intensity by a factor of four), similar to the helical undulator. This improvement (essentially at the fundamental FEL resonance) results largely from the fact that, for a given peak undulator field amplitude, the rms electron acceleration obtained from the square wave undulator is larger than that from the sinusoid; furthermore, the electron orbit is stable as well. In this paper, we find additional advantages that should result particularly at harmonic numbers $f > 1$ if the undulator field profile is nearly "square wave": namely, a large enhancement of the harmonic spontaneous power radiated, together with enhanced FEL gain. The modification of undulators to enhance FEL gain has been examined in the past[2,3,4,5], usually with a particular design in mind, but with similar conclusions.

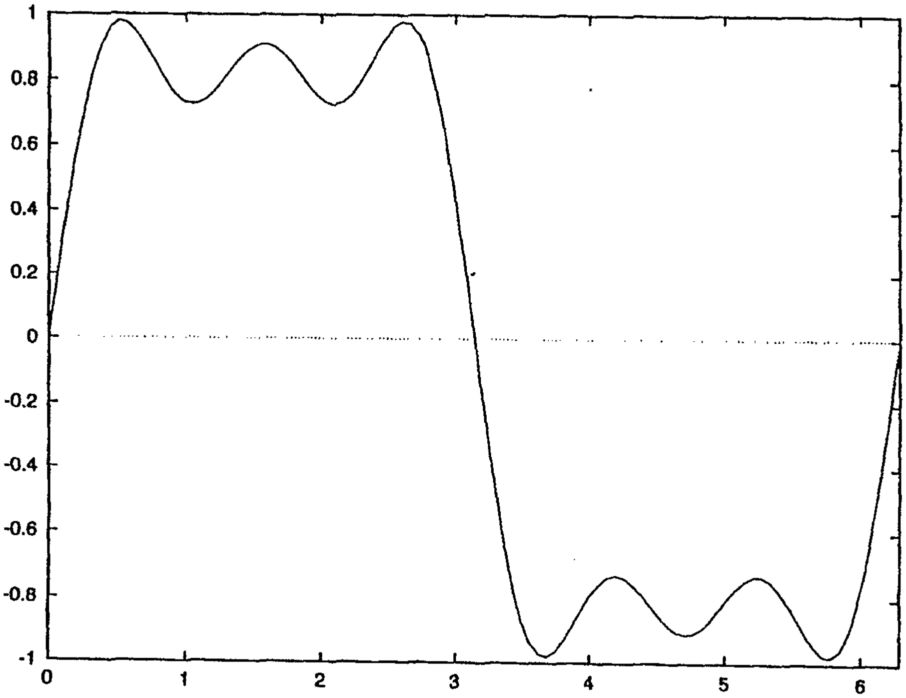

FIGURE 1: Representation of a "square-wave" undulator field by the first three Fourier components. Ordinate, normalized magnetic field; abscissa, axial distance measured in radians over one period (2π).

We retain the "undulator approximation", namely that not only is the amplitude of the motion K/γ small but also $K< 1$: then the radiation will have sharp lines at the harmonics on the axis since the radiation cone (width ~ $1/\gamma$) overlaps the orbit. In a long undulator, the spectrum becomes sharply peaked at frequencies satisfying

$$\omega_{rf} = \omega_0 f / (1 - \beta_0 \cos\theta) = f\omega_{r1} \quad (1),$$

where $\omega_0 = k_0 c$ (the undulator wavenumber times the speed of light), f is the harmonic number, θ is the angle from the axis of motion along the undulator, ω_{r1} is the FEL resonance frequency for f=1, and ω_{rf} is the resonance frequency for the f^{th} harmonic.

The resonance line widths are all ~ ω_{r1}/N. K is the normalized magnetic vector potential, eB_0/k_0mc^2, and B_0 is the peak undulator field. The undulator is the planar dipole type, and we compute only radiation directed along the axis.

THEORY

We begin by expanding the square-wave undulator field in a Fourier series:

$$B_y(z) = \Sigma F_n \exp(ink_0 z) \qquad (2)$$

where $F_n = 0$ for n= even integer, and = $2B_0 / in\pi$ for n = odd integer. In order to keep the peak amplitude of the "square-wave" undulator comparable with the sinusoid, the coefficient is adjusted to be $3.3B_0/in\pi$, and we use a truncated Fourier series, retaining only the n=1,3,5 terms in the examples which follow, for simplicity. This representation is shown in Fig. 1; however, there are two matters to keep in mind. Going to higher n will "smooth" the top of the undulator field profile, but it also will increase the slope of the square-wave jump. The latter point is where the modelling of the actual saturated field will influence the decision of where to truncate.

As the spectrum requires the velocity, $\beta(t) = \mathbf{v}(t)/c$, we integrate the equations of motion of a relativistic electron, finding:

$$\beta_x = \{3.3\ \omega_B/\pi\omega_0\}\ \{\ \exp(i\omega_0 t) + 1/9\ \exp(3i\omega_0 t) + 1/25\ \exp(5i\omega_0 t)\ \} \qquad (3)$$

and $\beta_z = \beta_0 - \beta_x^2/4\beta_0$, with $\omega_B = eB_0/\gamma_B mc$. From this follows the orbits, z(t) and x(t), e.g.:

$$z = \beta_0 ct - (3.3/\pi)^2(\omega_B^2 c/4i\omega_0^3)\ \{\ (1/2)\exp(2i\omega_0 t) + (1/18)\exp(4i\omega_0 t) + \qquad (4)$$

$$+ (187/12150)\exp(6i\omega_0 t) + (1/900)\exp(8i\omega_0 t) + (1/6250)\exp(10i\omega_0 t)\ \};$$

$$x = (3.3\omega_B c/i\pi\omega_0^2)\ \{\exp(i\omega_0 t) + (1/27)\exp(3i\omega_0 t) + (1/125)\exp(5i\omega_0 t)\ \}. \qquad (5)$$

The terms introduced by the higher order Fourier components of the undulator are easily identified; they will cause frequencies up to $10\omega_0$ to appear in z and β_z. The technique of Fourier decomposition of the "square wave" undulator field profile permits one to retain a representation of the orbit as a sinusoid, thus the term $\exp[i\omega\ r(t)/c]$ in the radiation integral[7] can be replaced by a Bessel series expansion, a useful technique originally used for the sinusoid undulator[6]:

$$\exp(ia\sin\nu) = \sum_{n=-\infty}^{n=\infty} J_n(a)\exp(in\nu)\ , \qquad (6)$$

where the harmonic components of the orbit are represented by $a\sin\nu$.

Introducing this orbit into the formula for the energy spectrum emitted[6,7] we obtain the following including terms up to n=5, while setting $\theta = 0$:

$$dW/d\Omega d\omega =$$

$$(e^2/4\pi^2 c)[f^2/(1-\beta_0)]^2 [(\omega/\omega_0)(1-\beta_0)-f]^{-2} \sin^2 [N\pi\{(\omega/\omega_0)(1-\beta_0) - f\}]$$

$$\times\ (3.3/\pi)^2 (\omega_B/\omega_0)^2 \{A_1^{\ 2} + (1/81)A_3^{\ 2} + (1/625)A_5^{\ 2} + (2/9)A_1A_3$$

$$+ (2/25)A_1A_5 + (2/225)A_3A_5\} \qquad (7)$$

where:

$$A_\alpha = \sum_{n1,n2,\ldots}^{}\ldots\ldots\sum_{n1,n2,\ldots n8 = -\infty}^{=+\infty} J_{n1}\, J_{n2}\ldots\ldots J_{n8}\, (\delta_{n1+n2+\ldots n8+\alpha,f} + \delta_{n1+n2+\ldots n8-\alpha,f}) \qquad (8)$$

In eq. (8), the Σ's denote summations over all the possible Bessel function of type n_i permitted by the δ's, taking $\alpha = 1,3,5$; the arguments of these Bessel functions are (sequentially):

$$-3.3\nu;\ -(3.3/27)\nu;\ -(3.3/125)\nu;\ (3.3^2/8)\mu;\ (3.3^2/72)\mu;$$
$$[(3.3^2)(187)/48600]\mu;\ (3.3^2/3600)\mu;\ (3.3^2/25000)\mu,$$

{ for example, we would have $J_{n3}(-[3.3/125]\nu)$ },

where $\qquad \nu = (f\omega_B \sin\theta\cos\phi)/\ \pi\omega_0[1-\beta\cos\theta] \qquad (9)$

and $\qquad \mu = (f\omega_B^{\ 2}\cos\theta)\ /\ \pi^2\omega_0^{\ 2}(1-\beta\cos\theta). \qquad (10)$

In these terms, ϕ is usually chosen to be zero, although we display the complete angular dependency in ν and μ. This shows that along the axis, $\nu = 0$, and so the surviving Bessel functions connected with this argument are just the J_0.

In eq. (7), the spectral dependence on ω is the same as it would be for the sinusoidal undulator. In the example of the sinusoidal undulator, only the term A_1 survives for radiation directed along the axis $\theta = 0$; but the new undulator introduces new sources of harmonic radiations.

RESULTS

The spontaneous power was computed numerically, and in Fig. 2 we show a typical result where we have taken K = 0.64 (note that $\omega_B/\omega_0 = K/\gamma \sim \beta_x$) and γ = 80 (40MeV), N=16. Only the peak power emission data point is plotted at each harmonic, all intermediate points taken to be zero, and we compare the sinusoidal undulator with two approximations to the square wave undulator, where we include respectively only the n = 1 and 3 components, or the n= 1,3, and 5 components. The striking feature is the very substantial enhancement of

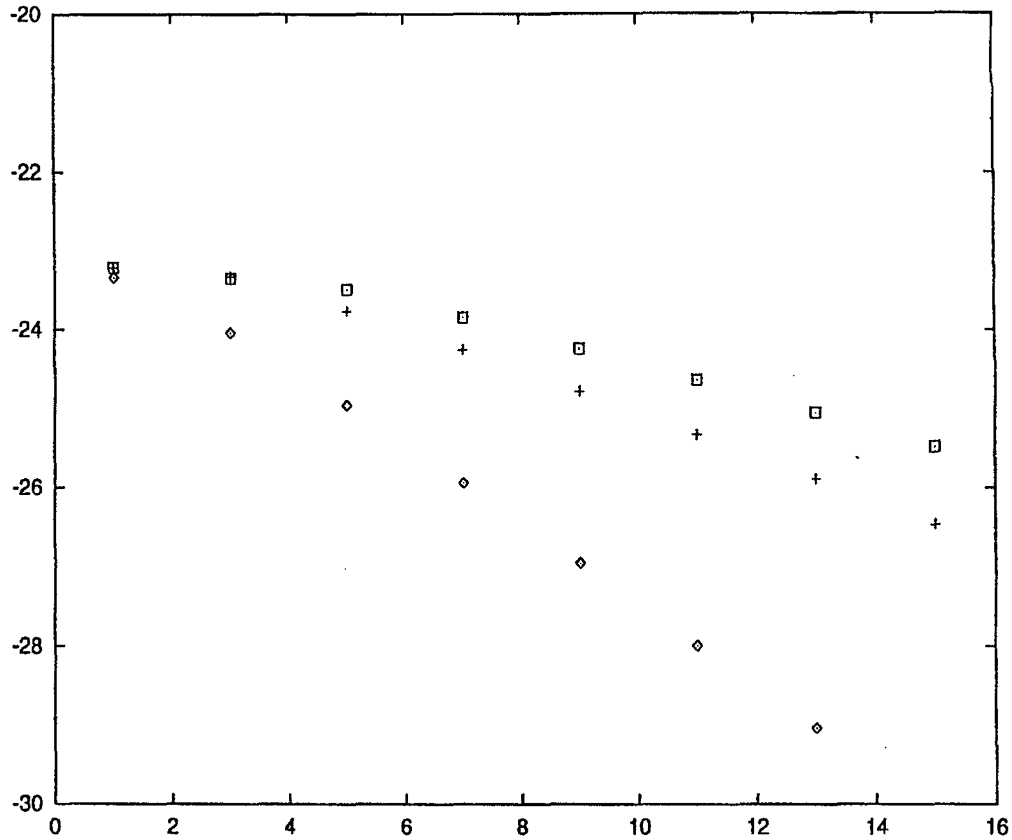

FIGURE 2: Logarithm to base 10 of {dW/dΩdω}, in {watt/ster.-sec^{-1}}, versus f, the harmonic number. The diamonds are the sinusoidal undulator; the crosses and squares are for the square-wave undulator approximated by the terms n =1,3; and by n = 1,3,5 respectively. The undulator parameter is K = 0.64.

spontaneous power emitted at the higher harmonics. That there should be *some* enhancement of radiation is apparant from the fact that electron radiation depends on the square of the electron speed, and the latter is proportional to the integral of the undulator field. The ratio of emission from the "square-wave" undulator to the "sinusoid" is 2.0 for f=1; this is the ratio of the mean square motion of the electron in these two different undulators that have equal peak field amplitudes. For the "approximate" square-wave undulator here, this ratio is about 1.5. However, discounting this factor of 1.5, there is still a remaining factor ~ 10 in enhanced radiation at the fifth harmonic, and larger enhancements at the higher harmonics, to be explained. A truly "square-wave" undulator field axial profile will cause a triangular -wave electron velocity response and a piecewise-quadratic orbit, which is very rich in higher-harmonic content, and this is the major source of this enhancement. This effect would be clearly identifiable in a simple experiment.

The enhancement of harmonic spontaneous emission using the "square-wave" undulator profile has further implications. From Madey's theorem[8], where γ_f and γ_i are the final and initial energy factors of an electron leaving and entering the undulator, and the brackets indicate an ensemble average:

$$<\gamma_f - \gamma_i> = 0.5\ d/d\gamma_i\ [<(\gamma_f - \gamma_i)^2>]. \qquad (11)$$

Since the left-hand side is proportional to the gain, and the quantity within the square brackets is proportional to the spontaneous emission power, it follows that the gain is proportional to the frequency derivative of the spontaneous power spectrum. But, the width of this spectrum ($\sim 1/N$) does not depend on the details of the profile of the undulator field, but rather the number of periods; hence the FEL gain (at small signal and small gain) should be proportional to the peak spontaneous power emitted at the various harmonics. In the illustration described above, the gain of a FEL should be enhanced by about 10 at the fifth harmonic. Notice that enhancements by even larger factors appear at the higher harmonics. Thus the harmonic enhancement effect we have found is not merely an artifact of our representation of the "square-wave" undulator, e.g., that the n=5 Fourier component of the undulator is driving just the n=5 FEL harmonic to high power. It almost goes without saying that for the higher gain indicated here to be realized, the customary conditions on beam quality apply.

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