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**COMPARISON OF CALCULATIONS OF
FRAGMENT PRODUCTION**

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COMPARISON OF CALCULATIONS OF FRAGMENT PRODUCTION

Gregory H. Canavan

Differences between estimates of fragment production rates in space debris collisions are shown to be due primarily to different choices of the exponent in the fragment production function and the distinction between catastrophic and all collisions. Sensitivity to the fragment production parameter over the range of values consistent with experimental data is discussed.

Introduction. This note compares estimates of fragment production rates in debris collisions through calculations performed with consistent debris distribution functions implicit in integrated collision frequencies provided by Attachment A.¹ It gives a complete set of collision frequencies, defines NASA JSC's prescription for fragment production, and provides references for the parameters used in those comparisons.² That makes it possible to achieve agreement with estimates of average masses per collision and to discuss discrepancies in fragment production rates based on them. The agreement on the average mass involved in all collisions is on the order of 1%, and the agreement on average masses in catastrophic collisions is on the order of 5%, which is within the uncertainties in averaging procedures, once it is recognized that only about 52% of the collisions are catastrophic.

Empirical fragment distribution kernels are reviewed and their proper averaging over debris and collision frequency distributions are derived and applied, leading to a prediction of 102 fragments per collision. That differs from JSC's value of 403 fragments per collision, but the latter includes collisions of all types. When the JSC analysis is restricted to catastrophic collisions and corrected for the fact that only 52% of all collisions are catastrophic, that reduces the discrepancy to a factor of ≈ 2.2 .

The remaining discrepancy is shown to result from the different values used in the two studies for the exponent in the fragment production kernel. The empirical value from the DoD impact experiments is determined and discussed in a companion note. The value that best fits the data is the experimental average. It produces 102 fragments per collision, which also agrees closely with the predictions of the developed by the DoD to model the experiments. The JSC estimates instead use a larger value that predates any serious experiments on fragment production, which is inconsistent with the DoD experiments and model.

This comparison was stimulated by the apparent four-fold contradiction between estimates of the average number of fragments per collision. The differences between the two are resolved by treating only catastrophic collisions, recognizing that only about half of the collisions are catastrophic, and using the experimentally determined fragment production kernel.

When that is done, both analyses agree to within about 5% with the 102 fragments per collision calculated below. Other discrepancies are within averaging differences.

Debris cumulative collision rate estimates for the next century are given in Table I of Attachment A. They are derived from 10 Monte-Carlo simulations of debris growth from current conditions, repeating the launch rate of 1985-1995 in each decade. The growth of cumulative collisions in each case is shown in Fig. 1A of Attachment A, which indicates a variance on the order of 10-20%. That is appropriate for this small number of trials and small enough to make the average quantities computed below meaningful, if noisy.

Table I gives the collision frequency between objects of various sizes, averaged over mass bins with ratios of upper to lower limits of ≈ 2.8 . Summing the entries over projectiles or targets gives an average of 58.4 collisions in 100 years or 0.58 collisions/yr, which is a factor of $\approx 0.58/0.05 \approx 11.6$ -fold greater than the present rate. It is argued that³ that the increase due to cascading can be removed to first order by dividing each element in the matrix by a factor of 11.6, but Table I is used as shown and the results are so normalized where necessary.

Figure 1 shows the average number of collisions of all types per century from Table I of Attachment A as a function of the projectile and target fragment diameters, where the projectile is defined as the colliding partner with the smaller diameter D_p and mass M_p and the target as the partner with the larger diameter D_t and mass M_t , which prevents double counting of collisions. The average collision frequency between projectiles and targets of diameter D_p and D_t is denoted by $F(D_p, D_t)$. The fragment sizes defined in Table I range from a bin containing particles with diameters from 0.1 to 0.16 m, the smallest fragments, to a bin from 6.3 to 10 m, which contains the largest. The coordinates are oriented so that the origin (0.16, 0.16) is at the lower left and the largest fragments (10,10) are at the right.

The value of ≈ 0.2 collisions/century at the origin for fragments from 0.1 to 0.16 m corresponds to the top entry of Table I. There is a detectable number of collisions of these small fragments because of their large number.⁴ For $D_p = 0.16$ m, as D_t increases there are a few local maxima, a sharp increase at $D_t = 1.6$, and a pronounced peak at $D_t = 6.3$ m, after which the number falls, indicating that the most numerous collisions are between the numerous small particles and the few large ones.

As D_p increases, similar scaling on D_t is seen, but the prominence of the peaks at large D_t falls due to the reduction in the number of target particles in the distribution. However, there are secondary peaks at 2.5 and 6.3 m, where the large size of the target particles overcomes their lower numbers to produce peaks about 15% as large as that at (0.16, 0.63).

The bottom curve in Fig. 2 shows collisions summed over targets size, $\sum_{D_t} F(D_p, D_t)$, as a function of projectile size. The value is about 16.8 collisions/century at $D_p = 0.16$ m, due largely to the large peak at $F(0.16, 6.3)$. As D_p increases, the sum falls by about an order of

magnitude by 1 m, where it stabilizes at about 3 through $D_p \approx 6$ m, after which it falls by another order of magnitude for the largest 10 m bin, which is consistent with the interpretation of Fig. 1 that collisions primarily involve the large flux of small fragments on the few large targets.

The top curve in Fig. 2 is the cumulative number of collisions, obtained by summing the lower curve up to the indicated D_p , i.e., $\sum_0^{D_p} \sum_{D_t} F(D_p, D_t)$. At $D_p = 0.16$ m the cumulant is equal to the sum. It increases rapidly up to $D_p \approx 1$ m; more slowly thereafter. It reaches about 50% of the total of 58.4/century by $D_p = 0.25$ m and 80% by 2.5 m, again reflecting the dominance of collisions of small projectiles with large targets. Figure 3 shows the sum and cumulative collisions as a function of target size, $\sum_0^{D_t} \sum_{D_p} F(D_p, D_t)$, which increase as $D_t^{3/2}$, again indicating a role for large fragments as targets for the numerous small fragments.

Catastrophic collisions. The dashed lines around the lower left bins in Table I identify collisions that are not catastrophic, i.e., involve ratios of target to projectile masses greater than 1,000, as though to be required for complete fragmentation⁵ on the basis of inspection of the DoD-DNA impact experiment data base.⁶ Smaller impactors cause only partial fragmentation of targets.⁷ Excluding these non-catastrophic collisions produces the catastrophic collisions per century $G(D_p, D_t)$ distribution, where the G is used to differentiate catastrophic collisions from the total collisions denoted by F . G from Table I is shown in Fig. 4, in which the peaks at small D_p and large D_t seen in Fig. 1 are suppressed by the elimination of many numerous collisions involving projectiles too small to cause catastrophic collisions.

Table I indicates that ≤ 0.16 m objects have a maximum mass of 0.73 kg, so they are too small by an order of magnitude to fragment the largest objects with masses up to 8589 kg. Only fragments larger than 0.4 m can fragment them. By eliminating collisions with target to projectile mass ratios over 1,000, Table I reduces the collisions to those in the dashed region. That eliminates 27.8 collisions per century and reduces the number of catastrophic collisions per century to 30.6—a reduction of about a factor of two.

Figure 4 shows that the net result is that the peak seen in Fig. 1 at $D_p = 0.16$ m is eliminated, and the peaks at 0.25 and 0.63 m are attenuated to about the levels of those at 2.5 and 6.3 m, so that the secondary peaks such as the one at (1.6, 1.6) become more visible. Figure 5 shows the sum of G over target size, $\sum_{D_t} G(D_p, D_t)$, which indicates that contributions from all diameter bins are comparable. The cumulant $\sum_0^{D_p} \sum_{D_t} G(D_p, D_t)$ rises rapidly to about 50% of the total of 30.6 by $D_p = 1$ m, increases more slowly thereafter, and reaches about 80% of the total by $D_p = 2.5$ m. Figure 6 shows that the sum and cumulative values as functions of D_t resemble those in Fig. 1, although their slopes fall more for larger targets.

The average mass per collision is evaluated by adding M_p and M_t for each element in $F(D_p, D_t)$, forming the product $(M_p + M_t)F(D_p, D_t)$ for each element in the collision matrix, summing over all D_p and D_t , dividing the result by the total of $\sum F(D_p, D_t) = 58.4$ collisions per

century, and multiplying by 0.6 to produce the average mass involved per collision. The factor of 0.6 is because the JSC bins have a ratio of maximum to minimum mass of ≈ 2.8 , so the geometric average mass of each bin is only $\approx 1/\sqrt{2.8} \approx 0.6$ times the mass of the upper limits used to index the bins in Table I. This process is repeated with F replaced by G to determine the average mass per catastrophic collision.

The top curve of Fig. 7 shows the cumulative contributions to average mass per collision of all types summed over D_t as a function of D_p , $\sum_0^{D_p} \sum_{D_t} (M_p + M_t)F(D_p, D_t) / \sum F(D_p, D_t)$, or $\sum MF/\sum F$ for short. The curve rises rapidly due to the numerous small particles. It reaches about half of its asymptotic value of about 2,293 kg by $D_p \approx 0.5$ m, and increases slowly thereafter.

The middle curve shows the cumulative contributions to average mass per catastrophic collision summed over D_t , $\sum_0^{D_p} \sum_{D_t} (M_p + M_t)G(D_p, D_t) / \sum G(D_p, D_t)$, or $\sum MG/\sum G$. The curve starts at a much lower level due to the exclusion of the non-catastrophic small particles but rises rapidly. It reaches half its asymptotic value of 2,103 kg by $D_p \approx 2$ m. That the asymptotes of the top two curves on Fig. 7 are roughly equal reflects the fact that most of the mass in the distribution, and hence most of the mass involved in catastrophic collisions, is in large objects.

The bottom curve shows the cumulative contributions to average mass per catastrophic collision for all collisions, $\sum_0^{D_p} \sum_{D_t} (M_p + M_t)G(D_p, D_t) / \sum F(D_p, D_t)$, or $\sum MG/\sum F$, which gives the mass of fragments produced in an average collision. It starts at a low level due to the exclusion of the non-catastrophic small particles, rises rapidly, and reaches half of its asymptotic value of 1,102 kg by $D_p \approx 2$ m. That its asymptote is only about half the average $\sum MG/\sum G$ for catastrophic collisions reflects the fact that only about half of the collisions are catastrophic. The table below compares the values calculated above and shown in Fig. 1 with those given in Table I of Attachment A.

Average Mass per collision of various types(kg)

	<u>all types</u>	<u>catastrophic</u>	<u>all collisions</u>
Table I	2271	2007	(1052)
Figure 7	2293	2103	1102
difference %	1	5.8	(4.6)

The average masses compare well for the three groupings shown: all types of collisions (both catastrophic and non catastrophic), catastrophic collisions, and all collisions—considering only the mass of collisions that are catastrophic. The 1% difference for all types of collisions is in the noise. However, this type of collision is only an artificial reference, as most of the collisions included in this category do not produce any additional fragments.

The 5.8% discrepancy in catastrophic collisions can be accounted for by slight differences in averaging due to the coarse mass resolution in the Table. The 4.6% discrepancy in all collisions can likewise be accounted for by slight differences in averaging. Table I does not give an average mass for all collisions; however it does give 58.4 collisions for all types and 30.6 for catastrophic, so it is appropriate to multiply the catastrophic 2,007 by $30.6/58.4 = 0.524$ to arrive at the 1,052 used for comparison.

Table I and Fig. 7 agree that the average mass falls by 9-12% for catastrophic collisions as compared to all types of collisions—largely due to the exclusion of the collisions by the smallest fragments with the large ones. They also both indicate that the fragment mass from all collisions falls by about a factor of two, from 2293 to 1102 kg, as shown.

The average mass for catastrophic collisions, $\Sigma MG/\Sigma G \approx 1,102$ kg, is about half the 2293 kg for all collisions. The reason is that only a fraction $\approx 52.4\%$ of the collisions in Table I are catastrophic; in the rest the objects recoil without fragmentation. Thus, in comparing the average mass for catastrophic collisions to the average mass of fragmented objects for all collisions, the mass of fragments is the same in both cases, but the number of collisions in the denominator is twice as large for the latter, so the average mass of fragments produced in all collisions is about half that in catastrophic collisions only.

The fragment production kernel for hypervelocity impacts has been studied in DoD and NASA experiments,⁸ and the few dozen well diagnosed tests have been summarized and interpreted.⁹ The results of those experiments can be fit reasonably well by expressions for the cumulative number of fragments with masses greater than m of the form

$$C = A(M_t/m)^B, \quad (1)$$

where M_t is the target mass, approximately the total fragment mass in those experiments, and A and B are parameters that characterize the number of fragments produced. The value of B can be inferred for each experiment from the slope of the plot of $\ln C$ versus $\ln m$; A is determined by the intercept. The value of B so determined is 0.62 ± 0.07 . The calculations below use this average value of B as a baseline and determine $A = 1/B - 1$ from conservation of mass.¹⁰

The average fragments per collision is determined by averaging Eq. (1) over the collision matrix of Table I to determine the average fragments produced per collision¹¹

$$\langle N \rangle = \Sigma' Z(D_p, D_t, M_f) F(D_p, D_t) / \Sigma F(D_p, D_t), \quad (2)$$

where $Z(D_p, D_t, M_f)$ is the average number of catalog particles of diameter greater than 0.1 m (or fragment mass $M_f \geq 0.26$ kg, according to Table I) produced per collision between objects of mass M_p and M_t . $F(D_p, D_t)$ is the collision rate between them, as represented by the elements of the collision frequency matrix in Table I. The sum is over D_p and D_t , and the prime indicates that the sum is only over catastrophic collisions, in which F reduces to G , as non-catastrophic collisions do not produce additional particles. Z is in general a function of D_p , D_t , M_f , and the

ratio of projectile to target mass for catastrophic interaction. While Z is a function of mass, it is generally not a linear function of mass. Thus, it is not permissible to calculate the average mass per collision

$$\langle M \rangle = \sum (M_p + M_t) G(D_p, D_t) / \sum F(D_p, D_t), \quad (3)$$

and use it in Eq. (1) to estimate the average number of fragments per collision. That would only be correct if Z was linearly proportional to mass, $Z = K(D_p + D_t)$, in which case

$$\langle N \rangle_{\text{linear}} = \sum K(M_p + M_t) G(D_p, D_t) / \sum F(D_p, D_t) = K \langle M \rangle. \quad (4)$$

which is not the case for the kernel of interest for debris. For other dependencies of Z on M_p and M_t , this approximation does not hold. In particular, for the form of Z given by Eq. (1) inferred from the DoD-DNA impact tests

$$\begin{aligned} \langle N \rangle &= \sum' A[(M_p + M_t)/M_s]^B F(D_p, D_t) / \sum F(D_p, D_t) \\ &\neq A \{ \sum' [(M_p + M_t)/M_s] F(D_p, D_t) / \sum F(D_p, D_t) \}^B. \end{aligned} \quad (5)$$

The errors in imposing such an approximation can be quite large for the DoD-DNA $B \approx 0.62$ for which the scaling of Z is far from linear and strongly suppresses the effect of large excursions in mass relative to the linear approximation of Eq. (4). In any case, it is not necessary to use the approximation of linearity, as the full collision frequency matrix and kernel are available for this comparison, and the desired averages can easily be evaluated without approximation.

The average number of fragments per catastrophic collision is determined by averaging the fragmentation kernel of Eq. (1) over the catastrophic collision distribution G of Table I. The kernel, under the assumption that all of the mass of both the projectile and target are converted into fragments is given by

$$C = A[(M_t + M_p) / M_f]^B, \quad (6)$$

where M_f is again the "mass of the smallest debris considered." 0.26 kg, which corresponds to a diameter of 0.1 m, the smallest in the NASA Catalog. Inserting Eq. (6) into Eq. (2) produces the plot of fragment production as a function of M_p and M_t shown in Fig. 8, which resembles the plot of catastrophic collisions in Fig. 4, although the peaks are accentuated by the fragment production in collisions with large particles.

Figure 9 shows the cumulative fragment production summed over D_t as a function of D_p , which is, $\sum_0^{D_p} \sum_{D_t} Z(D_p, D_t, M_f) G(D_p, D_t) / \sum F(D_p, D_t)$, or $\sum ZG / \sum F$. Although the contribution from objects < 0.6 m is reduced, the contribution integrated over target size is roughly the same from 0.6-6 m projectile size bins. The cumulative integral over projectile size reaches about 50% of its full value of 102 fragments by a projectile diameter of about 1.6 m. This asymptotic value is 25-70% larger than the 60-80 fragments per collision estimated in other reports, but is based on a significantly different, cascaded future debris distribution. Figure 10 shows the fragment production rate as a function of target diameter, which increases rapidly with M_t as a result of impacts on large targets by projectiles large enough to be catastrophic.

Evolve "Parts Creation." Attachment A gives an alternate procedure for estimating the number of fragments produced ("parts created") per collision based on a "standard NASA equation to determine the number of objects greater than a given mass," which is

$$CN = 0.4478(M_f/M_e)^{-0.7496}, \quad (7)$$

in which M_f is the "mass of the smallest debris considered and M_e is the "mass of the ejecta (i.e., the total mass for catastrophic collisions)." This expression is of the form of Eq. (6), but involves only aggregate or averaged quantities, so it can be evaluated directly with the quantities calculated above. Substituting the average masses of Table I into Eq. (1) gives

Fragments from average mass of catastrophic collisions

	JSC all types	JSC cat	JSC all coll	Eq. (6)	difference
avg. mass	2271	2007	1052	1102	4.5%
fragments	403	367	226	102	122%

The average JSC mass for all collisions are those given below Table I and repeated above. The calculated value of 403 fragments for all collisions corresponds to using JSC's average mass of 2271 kg from Table I in Eq. (7). However, Fig. 7 shows that only about half of all collisions are catastrophic, so it is inappropriate to use this mass in estimating fragment production. It is done here formally to indicate agreement on the *amount of mass that would be involved* in collisions of all types, not the mass of fragments produced in such collisions, which is much lower, as demonstrated above. Using JSC's mass of 2007 kg for catastrophic collisions from Table I in Eq. (7) gives the 367 fragments in the middle column. However, that number only holds for the fraction of all collisions that are catastrophic.

The third column corresponds to JSC's mass of 1052 kg for all collisions, which gives 226 fragments. The 1102 kg from Eq. (6) for all collisions gives the 102 catalog fragments discussed above. The differences refer to those between the JSC parts creation predictions and those calculated above. The small 4.5% differences between the average masses for all collisions was discussed above.

The 126% difference between JSC parts creation prediction and that of Eq. (6) for all collisions is clearly significant. It comes about in two stages. The first is that, as noted above, it is inappropriate to use the average mass for all types of collisions in estimating the number of fragments, in that only a fraction of them are catastrophic. Reducing to catastrophic collisions reduces the average mass by a factor of 2007/2271 or 12% and the number of fragments by 367/403 or 9%. Averaging to take into account that only 52.4% of the collisions are catastrophic reduces the average mass by an additional factor of 1052/2007 or 48% and the number of fragments by 226/367 or 38%. The product of the two reductions gives the overall reductions of

54% in average mass and 44% in fragments shown. The remaining difference in the number of fragments is due to the JSC procedure's use of a fragment production exponent of 0.7496, which is significantly different than the $B = 0.62$ inferred from experiments, as discussed further below.

Variation of fragments per collision with B can be assessed by changing the value of B in the kernel of Eq. (6) used in the averaging of Eq. (2). Figure 11 shows that this variation is fairly strong. From a value of about 70 fragments per collision at $B = 0.55$, it increases to about 100 at 0.62 and 240 at $B = 0.8$. At $B = 0.75$, the number of fragments is 197, which is close to the JSC value of 193 in Eq. (8). However, value of $B = 0.75$ is not consistent with the DoD impact experiments conducted by DNA test data, as shown in a companion review. The experimental $B = 0.62$ obviously better characterizes the data set best.¹² The use of the average value of $B = 0.62$ from the DNA tests reduces the fragments per collision by an additional factor of $102/197$ or 48%, which reduces the JSC prediction of the number of fragments per collision from above to $226 \times 0.48 = 109$, which is within about 7% of the 102 from Eq. (6).

Figure 11 also shows a curve labeled FASTT, which is from a semi-empirical model developed by DNA to model the experimental results.¹³ It lies slightly below the SAB curve at small B, crosses it at about $B = 0.65$, and lies slightly above it for larger B. It is interesting that the curve developed by DNA to model the DoD data gives a slightly lower value of B than the average value of $B = 0.62$ used in the calculations above, which represents an independent test of the reduction of the DNA data and a confirmation that Eq. (6)'s estimate of the number of fragments produced is conservative.

Different choices of the exponent B in the fragmentation kernel are the dominant factor in explaining discrepancies between fragment calculations. The value of B used in Eq. (6) is the average experimental results from the summary DNA report. The JSC kernel is described as the "standard NASA equation to determine the number of objects greater than a given mass." It is not otherwise discussed or explained in the documents referenced in Attachment A. It does not agree with the DNA data or with the FASTT model derived to model it.

Comparisons. These corrections permit a direct comparison between Fig 10 and Fig. 2 of Attachment A. The former is plotted in terms of mass; however, the ordinate scales as $M \sim D^{5/2}$, which means the cumulative fragments, which increase as $\sim D^{5/2}$, scale as $\sim (M^{2/5})^{5/2} \sim M$, as seen in Fig. 2 of Attachment A. As plotted, the slopes do not agree, but when the latter is corrected for the fact that only half of the collisions are catastrophic, which reduces the maximum number of fragments to 102-109, the two curves agree in both slope and value.

They also make it possible to compare with the number of collisions over time shown in Fig. 1 of Attachment A. That curve has a complex curvature because it reflects cascading due to the large fragment production rates assumed in the JSC model. When those rates are corrected, the collision rate remains at roughly the present 0.03 collisions/year throughout, so the expected

number of collisions in the next century is ≈ 0.03 collisions/year $\times 100$ year ≈ 3 collisions. For the ≈ 102 fragments per collision calculated above, that would introduce ≈ 3 collisions $\times 100$ fragments/collision ≈ 300 fragments, which would increase the number of objects at altitudes below 2,000 km by about $300/5000 \approx 6\%$, which would not introduce any cascading. Thus, the actual number of collisions would increase roughly linearly to 3, as shown by the dashed curve added to the bottom of Fig. 1 of Attachment A.

An important point on which only a limited assessment is possible is the treatment of partially catastrophic collisions. The calculations above use the collisions identified in Table I as completely catastrophic, i.e., involving projectiles whose masses are within a factor of 1,000 of that of the target. It is stated that "If this ratio was greater than 1,000, then none of the mass went into fragments,"¹⁴ which is consistent with the limited experimental data for larger ratios.¹⁵ It has been suggested that projectiles with smaller masses could be partially effective, but there is no published data on which to evaluate that suggestion. A more practical issue is that the mass bins in Table I do not differentiate cleanly between collisions that are catastrophic and those that are not, i.e., some contain both types of collisions, with the smaller masses in a bin being too small to cause fragmentation and the larger ones causing full fragmentation.

It is not possible to distinguish the fraction of collisions within each mass bin from the information given, but it is possible to bound the effect of partially catastrophic mixes by assuming that all of them are fully catastrophic. Figure 12 shows the effect of this assumption. The objects larger than 0.25 m in diameter appear to be fully catastrophic, whereas it is only the objects larger than 0.6 m that are catastrophic in the full calculation. Figure 13 shows what that means in terms of the cumulative collision rate of objects up to a given size. The bottom curve is for fully catastrophic collisions; the top curve is for all collisions; and the middle curve is for partially catastrophic collisions, treated as if they were all catastrophic. If the actual fraction of partial collisions that were catastrophic was zero, the collision rate would be given by the bottom curve. If it was 100%, the collision rate would rise to the middle curve. For the expected intermediate values, the curve would lie somewhere between the two curves. Even if all the collisions were catastrophic, the curve would only rise to a value intermediate between the catastrophic and all collision curves, which would quantitatively but not qualitatively alter the conclusions above.

Summary and conclusions. Attachment A contains much of the information needed to complete the comparison of estimates of fragment production. It gives a useful table of collision frequencies, NASA's equation for converting them into fragment production rates, and the appropriate references, which are used as the basis for the comparisons made above. This makes it possible to understand and achieve agreement with JSC's average masses and to discuss the discrepancies in the fragment production rate estimates based on them. The agreement on the

average mass involved in all collisions is on the order of a percent, which is much better than is required for this comparison. The agreement on average masses in catastrophic collisions is on the order of 5%, which is within the uncertainties in averaging procedures, once it is recognized that only about 52% of the collisions in JSC's table are catastrophic and the JSC rates are corrected for that fact.

Empirical fragment distribution kernels are reviewed and proper methods for averaging them over debris and collision frequency distributions are derived, leading to a prediction of 102 fragments per collision for the cascaded distributions of Table I. That contrasts with JSC's value of 403 fragments per collision, but the latter includes collisions of all types. When the JSC analysis is restricted to catastrophic collisions and corrected for the fact that only 52% of all collisions are catastrophic, that reduces the discrepancy to a factor of $226/102 \approx 2.2$.

The remaining discrepancy is shown to result from the different values used in the two studies for the exponent in the fragment production kernel. The empirical value from the DoD impact experiments is inferred and discussed in a companion note. The value that best fits the data is the experimental average value of $B = 0.62$, which produces the 102 fragments per collision cited above. Those calculations also agree quite closely with the predictions of the FASTT model developed by DNA to model the DoD impact experiments. The JSC estimates instead use the larger value of ≈ 0.75 given in Attachment A, which leads to the 226 fragments cited above. JSC's exponent predates any serious experiments on fragment production and which is inconsistent with the results of the DoD experiments and FASTT model.

This comparison was stimulated by the apparent contradiction between the statement that the average debris collision would "produce about 480 fragments larger than 10 cm"¹⁶ and the result that the average number of fragments per collision is 60-80.¹⁷ For the conditions and collision frequencies specified by JSC for this comparison, this apparent discrepancy is resolved by treating only catastrophic collisions, recognizing that only about half of collisions are catastrophic, and using the experimentally determined fragment production kernel. When that is done, both analyses agree to within about 5% with the 102 fragments per collision computed above. The majority of the discrepancy can be traced to a factor of ≈ 2 in fragments per catastrophic collision from JSC's fragmentation formula and a further factor of ≈ 2 two due to the confusion between catastrophic and total collisions in JSC's statement. collision. The remaining discrepancy is within the errors expected for the cascaded distributions used for comparison.

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Parts Creation

The objectives of this exercise were to determine (1) the "average" amount of mass involved in a natural collision and (2) the number of large debris (diameters greater than 10 cm) created in such a collision. This minimum debris size is important since it is generally sufficient to induce satellite breakups in a subsequent collision.

Methodology. Ten 100-year Monte Carlo runs of the EVOLVE code were executed with a base epoch of January, 1996. A traffic model repeating the historic space launches of 1988-1995 (inclusive) every eight years was employed. Consequently, the large number of LEO communications satellites (along with their rocket bodies and operational debris) scheduled for launch during the next several years were not modeled. Explosions were allowed to occur at historic rates. Only statistics on collisions involving objects both with a diameter of 10 cm or greater (≥ 260 gm) were collected. Each random collision was evaluated to determine if conditions existed for complete (catastrophic) fragmentation. Statistics were compiled for both catastrophic and non-catastrophic collisions.

Results. A total of 554 collisions occurred during the ten Monte Carlo runs, representing an average of 55.4 collisions per 100-year simulation. Figure 1 illustrates the cumulative number of collisions for each of the ten Monte Carlo runs. The non-linear growth is indicative of the growing satellite population greater than 10 cm in diameter. Table 1 summarizes the collision types by size (or mass) categories. For example, on average, two collisions occurred each run between objects in the 0.40-0.63 m and in the 3.98-6.31 m size bins (5.89-16.67 kg and 1071.3-3033.4 kg mass bins, respectively). The geometric and arithmetic means of total mass involved in the collisions are indicated beneath Table 1. The average combined mass of objects involved in all collisions was 2271 kg (geometric) or 2585 kg (arithmetic).

Using the standard NASA equation to determine the number of objects greater than a given mass,

$$CN = 0.4476 (M_r / M_o)^{-0.7486}$$

where, CN = cumulative number of particles with mass M_r or larger
 M_r = mass of the smallest debris considered
 M_o = mass of the objects (i.e., total mass for catastrophic collisions),

the number of debris diameter 10 cm or greater is 403 (geometric) or 444 (arithmetic) (Figure 2).

These numbers actually represent the most conservative (least debris generated) NASA assessment. In reality, the mass of each object will be transformed into two separate, distinct debris clouds with greater total debris than assuming a single fragmentation. Hence, if two objects, one 800 kg and one 1400 kg, collided, two debris clouds of 184 debris and 261 debris, respectively, (total = 465) would be created instead of a single debris cloud of 394 debris, as calculated with the equation above with a total mass of 2200 kg. In addition, if a lower mass equivalence of 145 gm for 10 cm diameter debris or if a combination of low intensity explosion/collision fragment generation (part of the mass is converted using low intensity equations and part of the mass is converted using collision equations) techniques are used, as is sometimes seen in the literature, the total number of debris generated is again greater than the values cited above and in Figure 2.

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COLLISIONAL INTERACTION COMBINATIONS
(EVOLVE 100 YEAR PROJECTION, BUSINESS AS USUAL)
(10 MONTE CARLO ITERATIONS)

DIAM(M)	TOTAL											
0.10	0.20	0.20										
0.16	0.10	0.00	0.10									
0.25	0.50	0.20	0.30	0.00								
0.40	0.40	0.20	0.00	0.20	0.00							
0.63	1.00	0.30	0.40	0.20	0.10	0.00						
1.00	3.60	0.90	0.60	0.50	0.10	0.40	1.10					
1.58	2.80	0.90	0.60	0.30	0.40	0.20	0.10	0.30				
2.51	9.00	2.80	1.90	0.80	0.80	0.40	1.20	0.80	0.30			
3.98	28.90	8.00	5.50	3.50	2.00	1.50	1.20	2.50	1.60	3.10		
6.31	11.90	3.30	2.20	1.60	0.70	0.70	0.70	0.50	0.90	0.70	0.60	
10.00												
TOTAL	58.40	16.80	11.60	7.10	4.10	3.20	4.30	4.10	2.80	3.80	0.60	
DIAM(M)	0.10	0.16	0.25	0.40	0.63	1.00	1.58	2.51	3.98	6.31	10.00	
MASS(KG)	0.26	0.73	2.08	5.09	16.67	47.20	133.6	378.4	1071.3	3033.4	8588.9	

SUM HORIZONTAL: COLLISIONS INVOLVING A GIVEN SIZE AND SMALLER

SUM VERTICAL: COLLISIONS INVOLVING A GIVEN SIZE AND LARGER

TOTAL NUMBERS OF COLLISIONS INSIDE BOXES

DASHED LINE: ALL COLLISIONS ABOVE AND TO THE RIGHT OF LINE ARE CATASTROPHIC

DOTTED LINE: ALL COLLISIONS BELOW AND TO THE LEFT OF LINE ARE NON-CATASTROPHIC

AVERAGE MASS INVOLVED IN COLLISIONS (IN KILOGRAMS):

ALL COLLISIONS INVOLVING OBJECTS LARGER THAN 10 CM:

ONLY CATASTROPHIC COLLISIONS (ABOVE / RIGHT OF DASHED LINE):

MOSTLY CATASTROPHIC COLLISIONS (ABOVE / RIGHT OF DOTTED LINE):

AVERAGE MASS USES GEOMETRIC OR ARITHMETIC MEAN FOR THE SIZE BIN

CATASTROPHIC COLLISION: COMPLETE STRUCTURAL FRAGMENTATION OF BOTH IMPACTING BODIES

GEOMETRIC

ARITHMETIC

2271

2586

2807

2286

2083

2372

Table 1

COLLISIONS BETWEEN OBJECTS GREATER THAN 10 CM DIAMETER (TEN 100-YEAR MONTE CARLO RUNS)

Figure 1A

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Figure 2A

Fig. 1. coll p cent 2d

Fig. 1. Collisions per century

Fig. 2. Collisions versus projectile size

Fig. 3. Collisions versus target size

Fig. 4. Catastrophic collisions per century

Fig. 5. Catastrophic collisions versus projectile size

Fig. 6. Catastrophic collisions versus target size

Fig. 7. Average mass vs projectile size

Fig. 8. Fragment production per century per bin

Fig. 9. Fragment production versus projectile size

Fig. 10. Fragment rate versus target size

Fig. 11 Fragments per total collision vs B

Fig. 12 Collisions for partial fragmentation

Fig. 13. Cumulative collisions vs size for partial fragmentation