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Coupled Spin and Charge Collective Excitations in a Spin Polarized Electron Gas

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Abstract

The charge and longitudinal spin responses induced in a spin polarized quantum well by a weak electromagnetic field are investigated within the framework of the linear response theory. We evaluate the excitation frequencies for the intra- and inter-subband transitions of the collective charge and longitudinal spin density oscillations including many-body corrections beyond the random phase approximation through the spin dependent local field factors, $G_{\sigma}^{\pm}(\mathbf{q}, \omega)$. An equation-of-motion method was used to obtain these corrections in the limit of long wavelengths, and the results are given in terms of the equilibrium pair correlation function. The finite degree of spin polarization is shown to introduce coupling between the charge and spin density modes, in contrast with the result for an unpolarized system.

Keywords: local field corrections, quantum well, collective excitations.

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The equilibrium state of a quasi two dimensional electron system - N particles confined in a quantum well of width L and transverse area A - in the presence of a constant magnetic induction parallel to the 2D layer, corresponds to an imbalance in the spin populations, $N_{\uparrow}$ and $N_{\downarrow}$ respectively, characterized by the degree of polarization, $\zeta = (N_{\uparrow} - N_{\downarrow})/N$. We study the linear response of a spin polarized quantum well (SPQW) to a weak electromagnetic perturbation with arbitrary space and time dependence. Previous investigations of the response functions in these systems were either limited to the case of infinitesimal polarization [1] or they used the random phase or local spin-density approximations. [2] In this work, the spin dependent exchange and correlation effects are included in the local field factors, $G_{\sigma}^{\pm}$, extending the approach of Kukkonen and Overhauser [3] to a spin polarized system.

The SPQW can be described in terms of two spin subband ladders (indexed by n), eigenstates of the unperturbed Hamiltonian, with eigenvalues $\epsilon_{n\sigma}(\mathbf{k})$, and eigenfunctions, $|n\sigma\mathbf{k}\rangle$, 2D plane wave spinors modulated along the width of a symmetric quantum well by an envelope function, $\xi_{n\sigma}(z)$. A weak electromagnetic perturbation - an electric potential, $\phi(\mathbf{r}, t)$, and a magnetic field of arbitrary orientation in the plane of the gas, $\mathbf{b}(\mathbf{r}, t)$ - induces charge density and longitudinal and transverse spin density fluctuations. We focus our attention on the charge and longitudinal spin responses.

The Fourier component of the one particle self consistent Hamiltonian, can be written as [3,4]

$$H_{\sigma}(z) = -e\phi(z) + \gamma\sigma \cdot \mathbf{b}(z) + \int_{-L/2}^{L/2} dz' v_{\mathbf{q}} \left[(1 - G_{\sigma}^{+})\Delta n(z') - \sigma \cdot \Delta \mathbf{m}(z') G_{\sigma}^{-} \right], \quad (1)$$

where γ is the effective Bohr magneton. The 2D wave vector $\mathbf{q}$ is parallel to the QW. (The frequency and wave vector dependence of all the quantities involved is understood.)

Eq. (1) can be used to define the many body local field corrections. $G_{\sigma}^{+}(\mathbf{q}, \omega, z)$, which couples the spin symmetric density fluctuations, $\Delta n(\mathbf{q}, \omega, z)$, through the bare Coulomb interaction, $v_{\mathbf{q}} = 2\pi e^2/q$, is the sum of the parallel and antiparallel spin effects, whereas $G_{\sigma}^{-}(\mathbf{q}, \omega, z)$ describes the difference between same and opposite spin interactions as the coupling constant of the spin antisymmetric magnetic fluctuations, $\Delta \mathbf{m}(\mathbf{q}, \omega, z)$. If $G_{\sigma\sigma}^{xc}$

denotes the same spin exchange and correlation field correction and $G_{\sigma\bar{\sigma}}$ the opposite spin correlation correction, then: [4]

$$G_{\sigma}^{\pm} = G_{\sigma\sigma}^{(xc)} \pm G_{\sigma\bar{\sigma}}^{(c)}. \quad (2)$$

In the first order perturbation theory, the induced fluctuations depend linearly on the effective fields,

$$\Delta n_{\sigma}(\mathbf{q}, \omega, z) = \sum_{n,m} \Pi_{m\sigma n\sigma} \langle nk\sigma | H(\mathbf{q}, \omega, z) | mk\sigma \rangle \zeta_n(z) \zeta_m(z). \quad (3)$$

$\Pi_{m\sigma n\sigma}$ is the non-interacting electron response function:

$$\Pi_{m\sigma n\sigma}(\mathbf{q}, \omega, z) = \frac{1}{A} \sum_{\mathbf{k}} \frac{f_{n\mathbf{k}-\mathbf{q}/2} - f_{m\mathbf{k}+\mathbf{q}/2}}{\hbar\omega - (\epsilon_{m\mathbf{k}+\mathbf{q}/2} - \epsilon_{n\mathbf{k}-\mathbf{q}/2})}, \quad (4)$$

with $\epsilon_{k\sigma} = \hbar^2 k^2 / 2m - \gamma \text{sgn}(\sigma) B$, the energy in the static magnetic field and $f_{nk\sigma}$, the equilibrium distribution of electrons in the subband (n, σ) . The signum function is defined by $\text{sgn}(\sigma) = 1(-1)$ when the spin is parallel (antiparallel) to the field.

Self-sustained spin and charge density waves are obtained in the absence of the external field, when the resonance is reached, i.e. the determinant of the system of equations implied by Eq. (3) written for both spin projections is zero. We consider the case in which all of the occupied states are confined to the lowest subband of each spin. In this limit, the condition for collective excitations is:

$$(1 - \chi_{n\sigma 1\sigma} A_{n\sigma}^{n\sigma})(1 - \chi_{n\bar{\sigma} 1\bar{\sigma}} A_{n\bar{\sigma}}^{n\bar{\sigma}}) - \chi_{n\sigma 1\bar{\sigma}} \chi_{n\bar{\sigma} 1\sigma} B_{n\sigma}^{n\bar{\sigma}} B_{n\bar{\sigma}}^{n\sigma} = 0, \quad (5)$$

where $A_{n\sigma}^{n\sigma} = v_{n\sigma 1\sigma}^{n\sigma 1\sigma}(\mathbf{q})(1 - G_{\sigma}^{+} - G_{\sigma}^{-})$, $B_{n\sigma}^{n\sigma'} = v_{n\sigma 1\sigma'}^{n\sigma 1\sigma'}(\mathbf{q})(1 - G_{\sigma}^{+} + G_{\sigma}^{-})$, and $\chi_{n\sigma 1\sigma'} = \Pi_{n\sigma' 1\sigma} + \Pi_{1\sigma n\sigma'}$. $v_{n\sigma n'\sigma'}^{m\sigma' m'\sigma'}(\mathbf{q})$ is the matrix element of the Coulomb interaction, given by

$$v_{n\sigma n'\sigma'}^{m\sigma' m'\sigma'}(\mathbf{q}) = v_{\mathbf{q}} \int_{-L/2}^{L/2} dz \int_{-L/2}^{L/2} dz' \xi_{n\sigma}(z) \xi_{n'\sigma}(z) \xi_{m\sigma'}(z') \xi_{m'\sigma'}(z') e^{-q|z-z'|}. \quad (6)$$

An expression for the local field corrections is needed to solve Equation (5) for $\omega(\mathbf{q})$. Since the collective excitations of a SPQW occur at high frequencies and small wavevectors, an equation-of-motion method is suitable for determining $G_{\sigma}^{\pm}$ as originally proposed by

Niklasson [5] for the 3D unpolarized electron gas, and recently extended to arbitrarily spin polarized systems. [6]

The intra-subband excitations of a SPQW correspond to $n\sigma \rightarrow n\sigma$ or $n\bar{\sigma} \rightarrow n\bar{\sigma}$. ($n = 1$ for the lowest occupied subband.) In these conditions, the results obtained in Ref. [6] become:

$$G_{\bar{\sigma}}^- = \frac{[1 - \zeta \text{sgn}(\tilde{\sigma})]}{N^2(1 - \zeta^2)^2} \sum_{\mathbf{q}', \sigma, \sigma'} [\text{sgn}(\tilde{\sigma}) + \text{sgn}(\sigma)] \{P_{\mathbf{q}\mathbf{q}'}[\text{sgn}(\sigma) - \zeta] - Q_{\mathbf{q}\mathbf{q}'}[\text{sgn}(\sigma') - \zeta]\}, \quad (7)$$

$$G_{\bar{\sigma}}^+ = \frac{[1 - \zeta \text{sgn}(\tilde{\sigma})]}{N^2(1 - \zeta^2)^2} \sum_{\mathbf{q}', \sigma, \sigma'} [1 + \text{sgn}(\tilde{\sigma})\text{sgn}(\sigma)] \{P_{\mathbf{q}\mathbf{q}'}[1 - \zeta \text{sgn}(\sigma)] - Q_{\mathbf{q}\mathbf{q}'}[1 - \zeta \text{sgn}(\sigma')]\}. \quad (8)$$

The correlations between electrons are described by $P_{\mathbf{q}\mathbf{q}'} = \sum_{\mathbf{k}, \mathbf{k}'} f_{\mathbf{k}\sigma; \mathbf{k}'\sigma'}^{(2)}(\mathbf{q}')(\mathbf{q} \cdot \mathbf{q}'/q^2)\tilde{v}_{\mathbf{q}'}/\tilde{v}_{\mathbf{q}}$ and by $Q_{\mathbf{q}\mathbf{q}'} = \sum_{\mathbf{k}, \mathbf{k}'} f_{\mathbf{k}\sigma; \mathbf{k}'\sigma'}^{(2)}(\mathbf{q}')[\mathbf{q} \cdot (\mathbf{q} + \mathbf{q}')/q^2]\tilde{v}_{\mathbf{q}+\mathbf{q}'}/\tilde{v}_{\mathbf{q}}$. $f_{\mathbf{k}\sigma; \mathbf{k}'\sigma'}^{(2)}(\mathbf{q}')$ is the equilibrium two-particle distribution function. Here, $\tilde{v}_{\mathbf{q}} = v_{1\sigma 1\sigma}^{1\sigma 1\sigma}(\mathbf{q})$. For an electron in the first subband $\xi_1(z) = \sqrt{2/L} \cos(\pi/L)z$ and $\tilde{v}_{\mathbf{q}}$ is obtained from Eq. (6), $\tilde{v}_{\mathbf{q}} \sim v_{\mathbf{q}}(1 - qL/2)$. In the long wavelength limit this leads to a $1/q$ divergence for $G_{\sigma}^{\pm}$, a direct consequence of the 2D Coulomb potential.

The dispersion relation of the collective modes is obtained from Eq. (5) where the long wavelength limit for the non-interacting susceptibilities, $\chi_{\sigma\sigma} = n_{\sigma}q^2/m\omega^2$, is used. (n_{σ} is the number of electrons of spin σ in subband n per unit area.) The two solutions describe the density and longitudinal spin waves respectively:

$$\omega_{CD}^2 = (\omega_{P\sigma}^2 + \omega_{P\bar{\sigma}}^2)(1 - \alpha_{CD}q), \quad \omega_{SD}^2 = (\omega_{P\sigma}^2 + \omega_{P\bar{\sigma}}^2) \frac{N_{\sigma}N_{\bar{\sigma}}}{N^2} \alpha_{SD}(\zeta). \quad (9)$$

α_{CD} and α_{SD} are functions of the degree of polarization, ζ , and independent of q in the long wavelength limit:

$$\alpha_{CD} = L \sum_{\sigma} \left[\left(\frac{N_{\sigma}}{N} \right)^2 (1 - 2G_{\sigma\sigma}^{xc}) + \left(\frac{N_{\bar{\sigma}}}{N} \right)^2 (1 - 2G_{\bar{\sigma}\bar{\sigma}}^{xc}) + 2 \frac{N_{\sigma}N_{\bar{\sigma}}}{N^2} (1 - G_{\sigma\bar{\sigma}}^c - G_{\bar{\sigma}\sigma}^c) \right], \quad (10)$$

$$\alpha_{SD} = qL \left[\frac{N_{\sigma}N_{\bar{\sigma}}}{N^2} (1 - G_{\sigma\bar{\sigma}}^c - G_{\bar{\sigma}\sigma}^c) - (1 - 2G_{\sigma\sigma}^{xc}) - (1 - 2G_{\bar{\sigma}\bar{\sigma}}^{xc}) \right]. \quad (11)$$

$\omega_{P\sigma}^2 = (2\pi n_{\sigma} e^2 / \epsilon_s m)q$ is the plasma frequency of a spin- σ 2D electron gas. The q dependence of these coefficients is straightforward to obtain from Eqs. (2), (7), and (8). It is important

to observe that the $1/q$ divergence of the local field corrections is cancelled out among the various terms of α_{CD} . The charge and spin modes are coupled for $\zeta \neq 0$.

The inter-subband excitations correspond to transitions $n\sigma \leftrightarrow m\sigma$ and $n\bar{\sigma} \leftrightarrow m\bar{\sigma}$, with $n \neq m$. In the long wavelength limit, the susceptibilities are approximated by $\chi_{n\sigma m\sigma} \approx 2(m_\sigma - n_\sigma)\omega_{nm\sigma}/\hbar(\omega^2 - \omega_{nm\sigma}^2)$, [7] with $\omega_{nm\sigma}$ the frequency of the subband separation, $\omega_{nm\sigma} = (\epsilon_{n\sigma} - \epsilon_{m\sigma})/\hbar$. For transitions from the lowest subspin band, $m = 1$ and $m_\sigma \gg n_\sigma$. Within these approximations, the values of the local field corrections are given by Eqs. (7) and (8), with $P_{\mathbf{q}\mathbf{q}'} = \sum_{\mathbf{k}, \mathbf{k}'} f_{\mathbf{k}\sigma; \mathbf{k}'\sigma'}^{(2)}(\mathbf{q}')(\hbar\mathbf{q} \cdot \mathbf{q}'/2m\omega_{n1\bar{\sigma}})\tilde{v}_{\mathbf{q}'}/\tilde{v}_{\mathbf{q}}$ and $Q_{\mathbf{q}\mathbf{q}'} = \sum_{\mathbf{k}, \mathbf{k}'} f_{\mathbf{k}\sigma; \mathbf{k}'\sigma'}^{(2)}(\mathbf{q}')[\hbar\mathbf{q} \cdot (\mathbf{q} + \mathbf{q}')/2m\omega_{n1\bar{\sigma}}]\tilde{v}_{\mathbf{q}+\mathbf{q}'}/\tilde{v}_{\mathbf{q}}$. In this case, $\tilde{v}_{\mathbf{q}} = v_{n\sigma 1\sigma}^{n\sigma 1\sigma}$. For a transition to the second subband, $\xi_{2\sigma}(z) = \sqrt{2/L} \sin(2(\pi/L)z)$, and Eq. (6) gives $\tilde{v}_q = -v(q)(10qL^2/9\pi^2)(1 - qL/2)$, a constant in the first order in q . Therefore, the local field corrections are proportional to q . The solutions for the self-sustaining modes are:

$$\omega^2 = \frac{1}{2}[\tilde{\omega}_{n1n1}^2 + \tilde{\omega}_{\bar{n}1\bar{n}1}^2] \pm \frac{1}{2}\{[\tilde{\omega}_{n1n1}^2 + \tilde{\omega}_{\bar{n}1\bar{n}1}^2]^2 + 4\tilde{\omega}_{n1\bar{n}1}^2\tilde{\omega}_{\bar{n}1n1}^2\}, \quad (12)$$

where

$$\tilde{\omega}_{n1n1}^2(q) = \omega_{n1\sigma}^2 \left\{ 1 - \frac{2\pi e^2}{\epsilon_s} \left[\frac{2n_\sigma}{\hbar\omega_{n1}} \left(\frac{10L}{9\pi^2} - \frac{5L^2}{9\pi^2}q \right) (1 - 2G_{\sigma\sigma}^{xc}) \right] \right\}. \quad (13)$$

$$\tilde{\omega}_{n1\bar{n}1}^2(q) = \omega_{n1}^2 \left\{ 1 - \frac{2\pi e^2}{\epsilon_s} \left[\frac{2n_\sigma}{\hbar\omega_{n1}} \left(\frac{10L}{9\pi^2} - \frac{5L^2}{9\pi^2}q \right) (1 - 2G_{\sigma\bar{\sigma}}^{cc}) \right] \right\}. \quad (14)$$

For a spin unpolarized system, Eq. (5) decouples into the well known, inter-subband charge density excitations and spin density excitations.

The utility of this calculation for the collective transitions, is limited by the knowledge of the equilibrium two particle distribution function, $f_{\mathbf{k}\sigma; \mathbf{k}'\sigma'}^{(2)}(\mathbf{q})$, the Fourier transform of the pair correlation function, $g_{\sigma\sigma'}(\mathbf{r})$. For an interacting system, $g_{\sigma\sigma'}(\mathbf{r})$ has to be calculated self consistently, including the many-body effects. In 3D, a direct estimate is obtained by solving the quantum mechanical problem of two electrons moving in a screened Coulomb repulsion, chosen as the potential of a sphere of radius $a = (3n/4\pi)^{1/3}$ uniformly charged with e for $r \leq a$ and zero for $r > a$. [8] In this model, $g_{\sigma\sigma} = [1 + \zeta \text{sgn}(\sigma)]|R(r)|^2/2$ and $g_{\sigma\bar{\sigma}} = [1 - \zeta \text{sgn}(\sigma)]|R(r)|^2/2$, where $R(r)$ is the radial wave function of the pair,

$$R(r) = \frac{\alpha}{a} \left(\frac{r}{2a} - \frac{r^2}{4a^2} \right) + \frac{\alpha}{a}, \text{ for } r \leq a \quad (15)$$

and $R(r) = \sin(kr - z)/r$ for $r > a$. In these expressions, k is the momentum of the relative motion, $a_B = \hbar^2/me^2$, the Bohr radius, and $r_s = a/a_B$. α and z are determined from the condition that $R(r)$ and its derivative are continuous at $r = a$. Thus, $\alpha = 8ka/(8 + 3r_s)$. For $r \ll L$, the pair correlation function obtained above can be a good first approximation in the quasi 2D case.

In this paper we investigate the non spin flip collective excitation spectrum of a SPQW at long wavelengths. In the limit of a linear response approximation, the exchange correlation effects among electrons are included in terms of spin-dependent local field corrections. The spin polarization is seen to introduce new features to the collective excitation spectrum. Our results might offer an interesting test for different many-body approximations by comparison with the observed dispersion in resonant Raman scattering. [9]

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