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Computational Model of a Copper Laser

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Abstract

This report describes a computational model of a copper laser amplifier. The model contains rate equations for copper and the buffer gas species (neon and hydrogen), along with equations for the electron temperature, the laser intensity, and the diffusing magnetic field of the discharge. Rates are given for all pertinent atomic reactions. The radial profile of the gas temperature is determined by the time-averaged power deposited in the gas. The presence of septum inserts, which aid gas cooling, is taken into account. Fields are calculated consistently throughout the plasma and the surrounding insulation. Employed in conjunction with a modulator model, the model is used to calculate comprehensive performance predictions for a high-power operational amplifier.

1 Introduction

The copper laser is characterized by laser transitions from an excited pair of states to a low-lying metastable pair [1]. The lasing transitions, of wavelengths 510.6 nm and 578.2 nm, are terminated after some 50-70 ns by accumulation of atoms in the lower states. The laser is important not only because of its visible wavelengths and short pulse length, but also because of its high repetition rate (several kilohertz), its 1-2% electrical efficiency, and the practicality of nearly diffraction-limited operation at an average power of hundreds of watts [2, 3]. During the last two decades, copper lasers have been developed extensively at Lawrence Livermore National Laboratory [2-10], as well as in Russia and the former USSR [11-40], Israel [41-47], Japan [48-56], the UK [57-60], Australia [61-68], and elsewhere.

The subject of this report is a computational model of a copper laser amplifier, as constituted in a computer code. The essence of the model is a set of rate equations for the states of the constituent species (copper, neon, and hydrogen), the electron temperature, the extracted laser intensity, and the magnetic field of the discharge. These equations are coupled to a modulator model [69] by the boundary value of the magnetic field, which gives the plasma current.

In our model, there are seven copper species, three neon species, and five species of atomic and molecular hydrogen. The processes described in the rate equations include electron-impact excitation, deexcitation, and ionization, line radiation with trapping effects, optical pumping, three-body recombination, charge exchange, Penning ionization, and (in the case of hydrogen) molecular dissociation, recombination, and negative-ion attachment/detachment.

The electron temperature equation includes ohmic heating, cooling via inelastic collisions, and equilibration with the gas temperature via elastic collisions, along with effects from other reactions. Detailed rates are given for all reactions.

Another quantity in the model is the gas temperature (the common temperature of atoms and ions), which rises markedly at the center of the discharge and is held to the wall temperature at the boundaries. In view of the large thermal inertia of the system, the gas temperature changes little during a single pulse. It is calculated via a steady state model, in which the radial profile is determined by the time-averaged power deposited in the gas. Recently it has been shown [9] that septa placed vertically along the length of the tube are effective in lowering the gas temperature. This benefits the discharge both by decreasing the prepulse electron temperature and by raising the gas density and thus the discharge impedance. The effects of septa are included in the model.

The model contains a detailed description of field diffusion across the discharge, and it provides the fields within the insulation package. In the version of the model discussed here, the various species are assumed to have a radial profile corresponding to the reciprocal of the gas temperature. This is expected to provide a reasonable approximation to the radial distribution. A version including radial particle and energy diffusion has been developed in large part, but circumstances have impeded its completion.

Several computational models of a copper laser have been developed during the last decades [4, 6, 7, 16, 21, 25, 70, 71, 72], of which the model of Kushner and Warner [7] is closest in spirit to ours. Relative to these, our model has greatly improved treatments of reaction rates, gas heating, and electromagnetics. A recent model by Carman *et al.* [65] has been shown to predict successfully the dynamics of a small-bore laser. Its treatment of the neutral copper atom appears roughly equivalent to ours, although we believe that our rates are more current. It includes two elevated CuII levels, which we neglect. It contains ambipolar and neutral diffusion as well as electron thermal transport, but neglects field diffusion. A two-temperature treatment of the electron energy distribution function is constructed. The geometry is that of a cylindrical tube without septa.

In Section 2, we describe the atomic physics models of copper, neon, and hydrogen. In Section 3, we discuss the discharge dynamics and, in particular, the rate equations for the various species and the electron temperature. Then we turn to the gas heating model (Section 4) and the plasma electromagnetics (Section 5). The computational scheme is outlined in Section 6. In Section 7 we turn specifically to a septum laser and present results describing both a representative operating point and a systematic survey of operating points. Our conclusions are given in Section 8. Four appendices contain details of the atomic rates and the treatment of radiation trapping.

2 Atomic States of the Model

2.1 Copper States

In the copper atom, the lasing transitions begin on J multiplets of the $(4p)^2P$ configuration and end on J multiplets of the $(4s^2)^2D$ configuration. These are also the configurations lying

just above the ground state. The next several higher neutral states play a role in depopulating the upper laser level via electron impact collisions, but that role is expected to diminish for larger excitation energies. The ionized population, however, always is significant, in view of the slow rate of recombination. A reasonable model of the copper atom would therefore contain the ground state, the lasing states (each divided into two J multiplets), an aggregate of the next several states, and the first ionized state. Thus our Cu model has the seven effective states listed in Table 1. The energies are taken from the tabulation by Moore [73]. Here and elsewhere, the approximate electron configurations are abbreviated according to the terminology of Table 2.

Table 1: Cu States of the Model

State	E (eV)
(1) $(4s)^2 S_{1/2}$	0
(2) $(4s^2)^2 D_{5/2}$	1.389
(3) $(4s^2)^2 D_{3/2}$	1.642
(4) $(4p)^2 P_{1/2}^o$	3.786
(5) $(4p)^2 P_{3/2}^o$	3.816
(6) Pseudostate	-
(7) Cu^+	7.724

Table 2: Abbreviations for Electron Configurations

$3d^{10}4s$	$(4s)$
$3d^9 4s^2$	$(4s^2)$
$3d^{10}4p$	$(4p)$
$3d^9 4s 4p$	$(4p')$
$3d^{10}5s$	$(5s)$
$3d^{10}5p$	$(5p)$
$3d^{10}4d$	$(4d)$

In the notation of Table 1, the green lasing transition (510.6 nm) connects states 5 and 2, while the yellow transition (578.2 nm) connects states 4 and 3.

The aggregate state, or pseudostate, represents an average of the nine states immediately above $(4p)^2 P_{3/2}^o$. The configurations and energies of these states are listed in Table 3. We assume that the relative populations of these states are distributed according to local thermodynamic equilibrium at electron temperature T_e . The effective degeneracy is given by

$$g_p(T_e) = \sum_{i=1}^9 g_i \exp [-(E_i - E_1)/T_e], \quad (1)$$

Table 3: Cu States Represented by Pseudostate

	State	E (eV)
(1)	$(4p')^4P^o$	4.922
(2)	$(4p')^4F^o$	5.122
(3)	$(5s)^2S$	5.348
(4)	$(4p')^4D^o$	5.471
(5)	$(4p')^2F^o$	5.508
(6)	$(4p')^2P^o$	5.685
(7)	$(4p')^2D^o$	5.756
(8)	$(5p)^2P^o$	6.122
(9)	$(4d)^2D$	6.191

where the sum runs over the levels of Table 3. At low T_e , this equals the degeneracy of $(4p')^4P^o$, while at temperatures near 10 eV it approaches the sum of the pseudostate degeneracies, $g_p \rightarrow 108$, as shown in Fig. 1. The effective energy of the pseudostate, also shown in Fig. 1, increases from about 4.92 eV [i.e. the energy of $(4p')^4P^o$] at $T_e = 0.1$ eV to about 5.45 eV at $T_e = 10$ eV.

Given these copper states, we require radiation rates, excitation/deexcitation rates, ionization rates, and momentum transfer rates. All of these are discussed in detail in Appendix A.

2.2 Neon States

Since the first excited state of the neon atom has an energy of more than 16 eV, our representation of the Ne atom is quite simple. We use a three-state model consisting of the ground state, a pseudostate representing the 3s and 3p levels, and the first ionized state. The pseudostate is constructed as a statistical average, in the same spirit as the Cu pseudostate, with energy levels taken from Moore [74].

The principal role of neon is to increase the laser impedance. Hence electron-atom momentum transfer is the process of primary importance. The rates for this and the other pertinent processes are given in Appendix B.

2.3 Hydrogen States

The addition of about 0.5% hydrogen is known to increase the laser efficiency [75, 76]. In our model, the main effect of hydrogen is to enhance the electron-atom energy exchange rate. That rate varies directly with the momentum transfer rate and inversely with the atomic mass. The momentum transfer cross section of atomic hydrogen is the same order of magnitude as that of neon, and that of molecular hydrogen is nearly an order of magnitude larger. The masses of H and H_2 contribute factors of 20 and 10, respectively, to the energy exchange rate. Thus the quiescent electron temperature is lowered, decreasing the prepulse

metastable copper populations and increasing the optical power. This beneficial effect ceases at a concentration (on the order of a percent) at which the hydrogen overionizes.

Our model includes the hydrogen ground state, an aggregate excited state composed of the $n=2$ and $n=3$ levels, the hydrogen ion, the negative ion, and the ground state of the hydrogen molecule. In practice, only the ground state populations are significant. Between pulses, the split between atomic and molecular hydrogen is determined by the quiescent electron temperature. All the appropriate rates are given in Appendix C.

3 Discharge Dynamics

Exploiting the fact that the tube length greatly exceeds the radius, we divide the plasma into a number of long, thin axial zones. Rate equations for the plasma constituents are set up in each zone, with the various species assumed to have a radial profile corresponding to the reciprocal of the gas temperature in that zone (cf. Sec. 4). The variation of the wall temperature with length, which is regarded as input, determines the copper density within each zone. The vapor pressure of copper was taken from measurements [77, 78] recommended in [79]. As expected, these data conform closely to the expression

$$p = c_1 \exp(-c_2/kT). \quad (2)$$

The coefficients are found to be $c_1 = 3.638 \times 10^8$ torr and $c_2 = 37020$ K.

For the bound states of copper, the rate equations have the form

$$\dot{n}_i = n_e \left[\sum_{j \neq i} (n_j \kappa_{ji} - n_i \kappa_{ij}) - n_i \kappa_{iN} \right] + \sum_{j > i} n_j A_{ji} f_{ji} - \sum_{j < i} n_i A_{ij} f_{ij} + R_i^I, \quad (3)$$

where the index j ranges over other bound states and N denotes the ion. The term proportional to n_e describes electron-impact processes, with the three inner terms corresponding to excitation/deexcitation gains, excitation/deexcitation losses, and ionization losses, respectively. The next two terms describe radiative gains and losses, respectively, with the f factors accounting for radiation trapping. The latter are handled via the Holstein theory, as elaborated in Appendix D.

The term R_i^I , which occurs only in the rates for the lasing levels, describes the effects of optical pumping. We adopt a saturated amplifier model, according to which the upper and lower populations are clamped once the inversion begins. Denoting these densities by n_b and n_a , respectively, we define the pumping terms such that

$$\dot{n}_a = \frac{g_a}{g_a + g_b} [\dot{n}_a^{(0)} + \dot{n}_b^{(0)}], \quad (4)$$

$$\dot{n}_b = \frac{g_b}{g_a + g_b} [\dot{n}_a^{(0)} + \dot{n}_b^{(0)}], \quad (5)$$

where $\dot{n}_a^{(0)}$ and $\dot{n}_b^{(0)}$ denote all the other rates in Eq. (3), and g_a and g_b are the degeneracies. These conditions guarantee that, after the inversion begins ($n_a/g_a = n_b/g_b$), the upper and

lower populations change in synchronization ($\dot{n}_a/g_a = \dot{n}_b/g_b$). The volume rate at which energy is lost by this process is

$$\begin{aligned} Q &= \sum_{i=a,b} E_i [\dot{n}_i^{(0)} - \dot{n}_i] \\ &= (E_b - E_a) \frac{\dot{n}_b^{(0)} - g_b \dot{n}_a^{(0)}/g_a}{1 + g_b/g_a}, \end{aligned}$$

with E_i the energy levels. In our model, this energy is converted to axially directed photons having intensity $I = \ell Q$, with ℓ the length of the axial zone.

The copper ion density changes at the rate

$$\dot{n}_N = n_e \sum_{i < N} n_i \kappa_{iN} - n_e^2 n_N \kappa_{rec}, \quad (6)$$

with the last term giving the three-body recombination rate.

The neon and hydrogen levels obey equations similar to (3) and (6), without the stimulated emission terms. In the case of hydrogen, there are also terms corresponding to molecular dissociation and recombination.

The rate of change of the electron temperature follows from the energy balance equation for the electron energy $U_e = (3/2) n_e k T_e$, which has the form

$$\dot{U}_e = Q_E + Q_{inel} + Q_{el} + Q_{rec} + Q_P. \quad (7)$$

The first term on the right-hand side is ohmic heating,

$$Q_E = E_z^2 / \rho, \quad (8)$$

while the second gives the effects of inelastic losses, summed over atoms,

$$Q_{inel} = n_e \sum_a \sum_{i,j} n_i \kappa_{ij} (E_i - E_j). \quad (9)$$

The third term indicates equilibration with the gas temperature via elastic collisions:

$$Q_{el} = -n_e (T_e - T) \sum_a \frac{3m_e}{M_a} [\kappa_{ea}^{mt}(T_e) n_a^0 + \kappa_{ei}^{mt}(T_e) n_a^+], \quad (10)$$

with n_a^0 and n_a^+ denoting the density of neutrals and ions, respectively. The fourth term, which gives the heating due to recombination, has the form

$$Q_{rec} = [\Delta E_{rec} + b(E_N - E_{N-1} - \Delta E_{rec})] \kappa_{rec} n_e^2 n_N, \quad (11)$$

with ΔE_{rec} indicating the energy difference between the ion ground state and the state to which recombination occurred. We assume that this is a high- n state lying above the pseudostate. As suggested by computer simulations [80], we take $\Delta E_{rec} = (3/2) a k T_e$, with a of order unity. The recombining electron then decays to the pseudostate, with a fraction b of the energy loss attributed to superelastic electron collisions and the remainder to spontaneous radiation. In the absence of detailed information, we take $b = 1/2$. Finally, the term Q_P in Eq. (7) is a small contribution due to Penning ionization.

4 Gas Heating Model

The equation governing the gas energy density is, in principle,

$$\frac{\partial}{\partial t} \left(\frac{3}{2} n T + \frac{1}{2} \rho v^2 \right) + \nabla \cdot \left[\mathbf{v} \left(\frac{5}{2} n T + \frac{1}{2} \rho v^2 \right) - \kappa \nabla T \right] = P, \quad (12)$$

where $\mathbf{v}$ is the gas velocity, κ is the thermal conductivity, and P is the power per unit volume deposited in the gas. The power is given by a sum over momentum transfer collisions between electrons and heavy species as

$$P = n_e (T_e - T) \sum_a \frac{3m_e}{M_a} \left[\kappa_{ea}^{mt}(T_e) n_a^0 + \kappa_{ei}^{mt}(T_e) n_a^+ \right]. \quad (13)$$

If the flow is near sonic, $v \simeq 10^5$ cm/s, disturbances propagate across the tube radius in some 10 μ s. On this time scale the details of the temperature distribution require a treatment of gas kinetics, which is beyond the scope of our model. On a longer time scale, such as the interpulse period, convective disturbances will have damped out, so that one can consider the steady state conduction equation

$$\nabla \cdot (\kappa \nabla T) = -\bar{P}, \quad (14)$$

with $\bar{P}$ the time-averaged power deposited in a particular axial zone. This is the model employed here. In addition, we make the reasonable assumption that the power is deposited uniformly across the discharge cross section. Of course the deposited power density varies from one axial zone to another, since the plasma constituents also vary.

In practice, the code constructs a steady state temperature distribution by following a sequence of several pulses. In the first pulse, the gas temperature is simply taken as the wall temperature. At the end of this pulse, the time-averaged deposition is used to solve for a new temperature distribution, via the last equation. The spatial average of this is then used as the temperature during the second pulse, with a new distribution calculated at the end, and so forth. Convergence within 0.1% is usually obtained after three or four pulses.

The average temperature is to be weighted with the gas density, i.e. $\bar{T} = \langle n T \rangle / \langle n \rangle$. From the ideal gas law, this means that the appropriate average temperature is

$$\bar{T} = \langle T^{-1} \rangle^{-1}. \quad (15)$$

The thermal conductivity of neon is given quite accurately by [8]

$$\kappa = \kappa_0 T^m, \quad (16)$$

with $m = 0.639$ and $\kappa_0 = 1.274 \times 10^{-3} \text{ W/(m K}^{m+1})$. In the temperature range 1400–3000 C, the thermal conductivity increases from about 0.14 to 0.22 W/mK.

More generally, it is convenient to write the thermal conductivity in the form $\kappa(T) = \mu'(T)$, so that the heat conduction equation becomes simply

$$\nabla^2 \mu = -\bar{P}. \quad (17)$$

After solving this equation for μ , we obtain the temperature distribution by applying the inverse of μ . We proceed to show how this is done in the cases of interest.

4.1 Cylindrical Laser

Expanding the Laplacian and imposing the boundary conditions $T(a) = T_w$ and $T'(0) = 0$, we find

$$\mu[T(r)] = \mu(T_w) + \frac{1}{4}\bar{P}(a^2 - r^2). \quad (18)$$

The temperature distribution in neon therefore has the form [7]

$$T(r) = T_w [1 + \alpha(1 - x)]^{1/(m+1)}, \quad (19)$$

where $x = (r/a)^2$ and α is a dimensionless parameter proportional to the deposited power per unit length:

$$\alpha = \frac{\bar{P}a^2}{4\mu(T_w)} = \frac{(m+1)\bar{P}a^2}{4\kappa_0 T_w^{m+1}}. \quad (20)$$

Typically one has $1 < \alpha < 2$. The average temperature is then

$$\bar{T} = G_m^{(cyl)}(\alpha) T_w, \quad (21)$$

with

$$G_m^{(cyl)}(\alpha) = \frac{\alpha m}{m+1} [(1 + \alpha)^{m/(m+1)} - 1]^{-1}. \quad (22)$$

The temperature enhancement factor $G_m^{(cyl)}$ is plotted as a function of the deposition in Fig. 2. For typical values of α , it ranges from about 1.2 to 1.5. A sample temperature profile is shown in Fig. 3. Note that the temperature rises from 1490 C at the wall to about 2900 C at the center.

4.2 Septum Laser

We turn to the heat conduction equation, with a 2-dimensional temperature distribution $T(r, \theta)$. The angle is measured with respect to a vertical section from the septum, as shown in Fig. 4. We assume that the radiative coupling is sufficiently strong to force the septum to have the same temperature as the walls. Estimates for a molybdenum septum, with appropriate radiation view factors and emissivities, show that the septum is about 70 C hotter than the wall [9]. Since the difference is only a few percent of the gas temperature, it does not represent a significant effect.

We expand μ in a Fourier series with respect to θ , noting that only cosine terms can contribute because of symmetry:

$$\mu[T(r, \theta)] = \mu(T_w) + \sum_{\ell=0}^{\infty} a_{\ell}(r) \cos(\ell\theta). \quad (23)$$

It is also clear that ℓ will be limited to odd values, in order to enforce the boundary condition that $T = T_w$ along the septum ($\theta = \pm\pi/2$). The solution works out to be

$$T(r, \theta) = \mu^{-1} \left[\mu(T_w) + \frac{4\bar{P}a^2}{\pi} \sum_{\ell \text{ odd}} \frac{(u^2 - u^{\ell})}{\ell(\ell^2 - 4)} (-1)^{(\ell-1)/2} \cos(\ell\theta) \right], \quad (24)$$

with $u = r/a$.

Inserting the specific form of the neon thermal conductivity, we have

$$\bar{T} = T_w G_m^{(s)}(\alpha), \quad (25)$$

in terms of the function

$$G_m^{(s)}(\alpha) = \left[\frac{4}{\pi} \int_0^1 du u \int_0^{\pi/2} d\theta \left(1 + \frac{16\alpha}{\pi} \sum_{\ell \text{ odd}} \frac{(u^2 - u^\ell)}{\ell(\ell^2 - 4)} (-1)^{(\ell-1)/2} \cos(\ell\theta) \right)^{-1/(m+1)} \right]^{-1}. \quad (26)$$

This function is plotted in Fig. 2. A typical temperature profile, for the same deposition as in the cylindrical case, is plotted in Fig. 3. Note that the maximum temperature is now only 2100 C, about 800 C below that of the cylindrical case.

5 Electromagnetics

In this section, we obtain the equations for the driving laser fields, in a realistic one-dimensional approximation. A schematic drawing of the laser assembly is shown in Fig. 5. The salient features are that the plasma is cylindrical, that it is surrounded radially by insulating regions having various permittivities, that the assembly is enclosed in a metal shell except for gaps at which wires leading back to the modulator are attached, and that the entire assembly is rotationally symmetric. Of course the insulator-plasma interface must be straight.

We allow the permittivity of the insulator to be an arbitrary function of r and z . We take the conductivity of the insulator to be zero and that of the metal to be infinite.

We ignore the displacement current in Maxwell's equations, since it gives rise to field variations having time scales shorter than those of interest (nanoseconds to hundreds of microseconds). This is seen as follows. In conductors, the displacement current leads to time scales of order $\tau \simeq \epsilon/\sigma$. For a typical copper plasma, this is in the neighborhood of 10^{-14} s. In insulators the displacement current is relevant for times of order ℓ/c (ℓ a typical dimension), which is generally less than a nanosecond.

In cylindrical geometry, the solutions of Maxwell's equations divide into transverse magnetic ($B_z = 0$) and transverse electric ($E_z = 0$) modes. The former mode is appropriate for our case. Since the fields are independent of θ , the only nonvanishing field components are B_θ , E_r , and E_z . Maxwell's equations then consist of the r -component and z -component of Ampere's law,

$$\frac{\partial B}{\partial z} = -\mu_0 J_r, \quad (27)$$

$$\frac{1}{r} \frac{\partial(rB)}{\partial r} = \mu_0 J_z, \quad (28)$$

the θ -component of Faraday's law,

$$\frac{\partial B}{\partial t} = \frac{\partial E_z}{\partial r} - \frac{\partial E_r}{\partial z}, \quad (29)$$

and Gauss's law,

$$\frac{1}{r} \frac{\partial(r\epsilon E_r)}{\partial r} + \frac{\partial(\epsilon E_z)}{\partial z} = \rho. \quad (30)$$

Here ρ is the free charge and B stands for B_θ . In Ampere's law, the current is given by $\mathbf{J} = \mathbf{E}/\eta$, with η the resistivity. The divergence of the magnetic field vanishes automatically since $B_\theta = B_\theta(r, z)$.

These equations are treated differently, depending on whether we are dealing with the plasma or with insulating regions.

5.1 Fields within Plasma

Within the plasma, we use the fact that the conductivity is nonzero and obtain the electric field from Ampere's law. Inserting it into Faraday's law, we have the following equation for the magnetic field:

$$\frac{\partial B}{\partial t} = \frac{\partial}{\partial r} \frac{D}{r} \frac{\partial(rB)}{\partial r} + \frac{\partial}{\partial z} D \frac{\partial B}{\partial z}, \quad (31)$$

where $D = \eta/\mu_0$. This is a two-dimensional diffusion equation, with diffusion coefficient D . Note that the electric field can be obtained in terms of B via

$$E_r = -D \frac{\partial B}{\partial z}, \quad (32)$$

$$E_z = \frac{D}{r} \frac{\partial(rB)}{\partial r}. \quad (33)$$

Taking into account the simplification that the laser length far exceeds its width, we divide the plasma into a number of long axial zones, as noted in Sec. 3. We expect the magnetic field to vary slowly within each zone and the electric field to be directed primarily in the z direction. At the boundary between zones, however, we expect there to exist a transition layer of width comparable to the plasma radius. Within this layer, the magnetic field should interpolate between its values in the bulk of the zones and the electric field should develop a radial component. If the zones are long compared to the width of the transition layer, we can idealize this situation by ignoring the transition layer. Then in each zone i the magnetic field $B_i(r, t)$ obeys the one-dimensional diffusion equation

$$\frac{\partial B_i}{\partial t} = \frac{\partial}{\partial r} \frac{D_i}{r} \frac{\partial(rB_i)}{\partial r}, \quad (34)$$

with $D_i = \eta_i/\mu_0$ within the zone. At the radial boundary $r = a$, the field must correspond to an enclosed current equal to the plasma current, or

$$B_i(a, t) = \frac{\mu_0}{2\pi a} I_p(t). \quad (35)$$

This holds for each axial zone, since the plasma current is conserved. At the center the magnetic field vanishes.

We obtain an additional piece of information by taking the surface integral of Faraday's law around the r - z cross section of the insulating region. This yields the following relation between the loop voltage and the derivative of the plasma current:

$$V_l - \sum_i \ell_i E_{z,i}(a, t) = L_c \frac{dI_p}{dt}, \quad (36)$$

where V_l is the voltage across the laser assembly (i.e. across the peaking capacitor and its effective series resistor), ℓ_i is the length of a zone, and L_c is the inductance associated with the insulator,

$$L_c = \frac{\mu_0}{2\pi} \int r^{-1} dr dz. \quad (37)$$

Here the integral extends over the insulating cross section in the r - z plane. In this approximation, the various zones can be situated in any order.

5.2 Fields within Insulator

Since the insulator carries no current, the magnetic field is given by the enclosed plasma current. via

$$B(r, z, t) = \frac{\mu_0}{2\pi r} I_p(t), \quad (38)$$

independently of z . Given the magnetic field, we solve for the electric field by combining Gauss's law (with no free charge) and Faraday's law. An efficient way of doing this is to note that, since $\text{div}(\epsilon \mathbf{E}) = 0$, we can represent $\epsilon \mathbf{E}$ in terms of a vector potential as $\epsilon \mathbf{E} = \nabla \times \mathbf{C}$, so that

$$\epsilon E_r = -\frac{\partial C_\theta}{\partial z}, \quad (39)$$

$$\epsilon E_z = \frac{1}{r} \frac{\partial(r C_\theta)}{\partial r}. \quad (40)$$

Then Faraday's law becomes the following static equation for $\varphi \equiv r C_\theta$:

$$r \frac{\partial}{\partial r} \frac{1}{\epsilon r} \frac{\partial \varphi}{\partial r} + \frac{\partial}{\partial z} \frac{1}{\epsilon} \frac{\partial \varphi}{\partial z} = \frac{\mu_0}{2\pi} \frac{dI_p}{dt}. \quad (41)$$

This is an elliptic equation, with boundary conditions determined as follows. First, at the boundary with the plasma, E_z is continuous and so we have a condition on the radial derivative of φ . Second, at a boundary with the metal, the tangential component of the electric field must vanish. This means that

$$\frac{\partial \varphi}{\partial r} = 0 \quad (\text{horizontal boundary}), \quad (42)$$

$$\frac{\partial \varphi}{\partial z} = 0 \quad (\text{vertical boundary}). \quad (43)$$

These are Neumann boundary conditions. Hence Eq. (41) determines φ within an additive constant, and the electric field is determined uniquely.

We can see as follows that charge builds up at the interface between the plasma and the insulator. We construct a narrow container which encloses this interface and which encircles the plasma, and integrate Gauss's law within this container. There is a contribution to the surface integral only within the insulator, since $E_z = 0$ just within the plasma. Hence the surface charge density is

$$\Sigma = \epsilon E_z, \quad (44)$$

so that it is proportional to the normal component of the electric field within the insulator. Actually this charge builds up on the time scale ϵ/σ .

In summary, the insulator fields can be found, at convenient times of interest, by means of a statics calculation. It is important to note that the fields within the plasma can be found without solving for the fields within the insulator.

5.3 Plasma Resistivity

Neglecting convective terms and pressure gradients, electron momentum balance reads

$$en_e(\mathbf{E} + \mathbf{u}_e \times \mathbf{B}) = \mathbf{R}_e, \quad (e > 0) \quad (45)$$

where $\mathbf{R}_e$ is the collisional friction between the electrons and other constituents. This breaks up into an electron-ion contribution and a sum over the electron-atom contributions:

$$\mathbf{R}_e = \mathbf{R}_{ei} + \sum_a \mathbf{R}_{ea}. \quad (46)$$

Each term is the first velocity moment of the appropriate Boltzmann collision integral, e.g. for electron-atom collisions:

$$\mathbf{R}_{ea} = m_e \int d^3v \mathbf{v} C_{ea}. \quad (47)$$

Following Braginskii [81], we assume a Maxwellian atom distribution function. We expand the electron distribution function as a Maxwellian corrected by a drift velocity $\mathbf{u}_e$. The electron-atom friction then reduces to

$$\mathbf{R}_{ea} = -n_a n_e m_e \kappa_{ea} \mathbf{u}_e \quad (48)$$

in terms of the rate coefficient

$$\kappa_{ea} = \frac{m_e}{3T_e} \langle \sigma_{ea}^{mt} v^3 \rangle, \quad (49)$$

with σ^{mt} the momentum transfer cross section. Note that this differs from the usual $\langle \sigma v \rangle$ in that the average contains two additional powers of v . One factor follows from the fact that this is a momentum transfer process, and the other factor follows from the fact that only the part of the electron distribution function proportional to $\mathbf{v} \cdot \mathbf{u}_e$ contributes to the rate. The extra powers of v are normalized by the thermal speed.

The electron-ion friction, projected in the direction perpendicular to the field, has the form [81]

$$\mathbf{R}_{ei} = -m_e n_e \nu_{ei} (1 - \ell^\perp) (\mathbf{u}_e - \mathbf{u}_i)_\perp + m_e n_e \Omega \ell^\wedge \mathbf{n} \times (\mathbf{u}_e - \mathbf{u}_i), \quad (50)$$

with ν_{ei} the electron-ion 90° collision frequency, Ω the electron gyrofrequency, and $\mathbf{n}$ a unit vector in the direction of $\mathbf{B}$. The quantities $\ell^\perp$ and $\ell^\wedge$ are functions of the ratios of the collision frequencies to the gyrofrequency. This component of the friction is all that we require.

Taking the r and z components of the last equation, inserting the expressions for the friction terms, and ignoring the radial electric field, we obtain Ohm's law in the form $J_z = E_z/\eta$, with the resistivity given by

$$\eta = \frac{m_e \nu}{e^2 n_e} \left[1 + \frac{\Omega^2}{\nu \nu'} (1 + \ell^\wedge) \right], \quad (51)$$

with ν the total electron collision frequency (i.e. with both ions and atoms) and ν' the electron-atom collision frequency:

$$\nu = (1 - \ell^\perp) \nu_{ei} + \nu', \quad \nu' = \sum_a n_a \kappa_{ea}^{mt}. \quad (52)$$

The part of η proportional to Ω^2 is the enhancement of the resistivity due to the magnetic field. In typical copper lasers this usually is no more than 5%.

6 Computational Scheme

The variables of the final model consist of the densities of the atomic states, the extracted laser intensity, the magnetic field of the laser, and the modulator variables (voltages, currents, and the fields of the magnetic switches). The modeling of the modulator, a significant problem in itself, is discussed separately [69, 83, 84]. Geometrically, the atomic and laser variables depend on the axial zone, with the dependence entering parametrically via the local wall temperature. The magnetic field depends on both r and the axial zone, via the field diffusion equation (34). We difference this equation in r , obtaining equations for the evolution B at discrete points.

The model thus reduces to a system of ordinary differential equations in time. In a typical problem with five axial zones, driven by the Laser Demonstration Facility modulator [69, 82], there are 90 atomic and laser variables (seven copper states, three neon states, five hydrogen states, the electron temperature, and the two intensity colors, all indexed by the axial zone), 45 magnetic field variables (with 9 radial solution points), and 15 modulator variables, for a total of 150 variables.

We treat all the variables in a unified computational manner, by advancing them in time with an ODE solver. The system is mathematically stiff. We employ the ODE solver LSODKR, a variant of the standard LSODE package [85]. This solver employs Krylov methods in numerical iterations and has root-finding capabilities which are helpful in treating the extracted intensity and the magnetic switches. Details of these methods are discussed in [86] and [87].

7 Results

We now use the model to calculate performance predictions for the laser amplifier with septum inserts. This device has a radius of 4 cm, a tube length of 273 cm, and a septum insert. It is driven by the Laser Demonstration Facility modulator mentioned above [69, 82]. The simulations presented here do not include the effects of hydrogen. Some results of the model with hydrogen are given in [9].

The laser model requires the plasma tube temperature profile along the length of the tube. This was taken from TOPAZ calculations by A. T. Anderson [88]. A typical profile is shown in Fig. 6. Note that this figure shows only half the length of the tube, since the temperature is symmetrical about the midpoint. The half-tube has been divided into five axial zones. The lengths of these zones are set so that the average copper density at the wall, also shown in Fig. 6, changes in approximately uniform steps.

Figure 7 shows predictions for the lasing intensities, at standard operating conditions ($V_0 = 18$ kV, $p = 40$ torr). Since the ^2D and ^2P densities, as plotted, are divided by their degeneracies, the curves touch during the amplification interval. In the second part of this figure, the calculated extracted intensity is compared with measurements by J. J. Chang [89]. The curves have similar shapes, rising quickly and falling relatively slowly over a period of about 70 ns. The predicted power is 428 W, at a pulse repetition rate of 4.3 kHz, as compared to the measured power of approximately 300 W.

Now we explore the full range of experimental voltages and pressures by collating a large number of runs. Several hundred runs were made to cover the entire space of input parameters (driving voltage, gas pressure, and wall temperature), at the same pulse repetition rate as above. In the runs, the central wall temperature ranged from 1375 C to 1600 C, in steps of 25 C. The selected gas pressure ranged from 10 to 110 torr, in steps of 10 torr; and the voltage varied from 16 to 22 kV, in steps of 0.5 kV. Thus runs were made at $10 \times 11 \times 15 = 1430$ points.

The resulting laser power is plotted as a function of central wall temperature, for selected voltages and pressures, in Fig. 8. These plots show that, at a given voltage, both the optimum wall temperature and the maximum power increase with pressure. At 18 kV, for example, the optimum wall temperature increases from about 1425 C (at 10 torr), to about 1500 C (at 110 torr). At the same time, the maximum power increases from 300 W to 650 W. At a given pressure, on the other hand, neither the optimum wall temperature nor the maximum power is very sensitive to voltage. Thus at a pressure of 30 torr, as the voltage is raised from 16 to 19 kV, the optimum wall temperature ranges from about 1450 C to 1475 C. The corresponding maximum power increases from 330 W to only 430 W.

The main effect of the voltage is to control the power absorbed in the wall, i.e. the ohmic heating in the plasma minus the laser power. Figure 9 shows plots of the absorbed power per unit length, halfway along the length of the tube, versus wall temperature for the same choices of pressure and voltage as above. At given pressure and voltage, the absorbed power is practically independent of the wall temperature. At fixed pressure, however, the absorbed power increases by about 30% as the voltage is raised from 16 kV to 19 kV.

To visualize the detailed dependencies of these results, it is convenient to employ con-

tour plots. This is done via a postprocessor code, which collates and analyzes results from a collection of runs. The first plot in Fig. 10 shows contours of the laser power on the temperature-pressure plane, for a fixed voltage of 17 kV. One sees that the contours form a “mountain”, with a peak of about 600 W for moderate temperatures (about 1500 C) and pressures of 100 torr or greater. Similarly, the second plot in Fig. 10 shows power contours on the temperature-voltage plane, for a fixed pressure of 70 torr. Here again a mountain structure is evident for moderate temperatures and high voltages.

Not all points on these contour plots are accessible in experiment, since the wall temperature is not controlled independently. Instead, it is determined by the absorbed power and the characteristics of the particular thermal package. In particular, the central wall temperature is a function of the central absorbed power per unit length, say $T = F(P_{abs})$. This function can be estimated both from calculations and from calorimetry. Although substantial uncertainties exist, a TOPAZ calculation by A. T. Anderson [88] appears to be realistic and is employed here. It is shown in Fig. 11.

The absorbed power, as calculated by the model, is a function of wall temperature, pressure, and voltage, say

$$P_{abs} = P_{abs}(T, p, V). \quad (53)$$

At a given pressure and voltage, then, the operational wall temperature is determined as the root of

$$T = F[P_{abs}(T, p, V)]. \quad (54)$$

To handle this equation numerically, we fit the absorbed power to a polynomial of the following form in T , p , and V :

$$P_{abs}(T, p, V) = \sum_{i_1=1}^{N_1} \sum_{i_2=1}^{N_2} \sum_{i_3=1}^{N_3} A_{i_1, i_2, i_3} T^{i_1-1} p^{i_2-1} V^{i_3-1}. \quad (55)$$

It was found sufficient to take a quadratic in T and quartics in p and V (i.e. $N_1 = 3$, $N_2 = N_3 = 5$). The wall temperature was then found via a root-solving routine.

Using this form for the wall temperature, and returning to Fig. 10, we see that the system passes along a trajectory on the temperature-voltage plane, as the absorbed power increases. The trajectory is indicated by the dotted line. Note that it ascends the power “mountain” only partially before sliding around and descending the opposite side. A similar trajectory occurs on the temperature-pressure plane. The laser power encountered along these trajectories, plotted in Fig. 12, peaks between 450 and 500 W. To increase the power, one should employ a thermal package which absorbs increased power at a given wall temperature.

Putting all this information together, we show in Fig. 13 the predicted wall temperature and laser power as a function of pressure and voltage, for the particular thermal package. To maintain a fixed wall temperature, one would decrease the pressure and increase the voltage (or vice versa) along a contour. For the laser power, there is a ridge extending from low pressures and high voltages to high pressures and low voltages. The high power regime, which occurs at the latter end of the ridge, is in the neighborhood of 16 kV and 100 torr. The power obtained there, about 500 W, is more than twice that found at the low end of the ridge.

Experimentally, the maximum power occurs at about this point, but it varies less strongly with position on the plane. The maximum experimental power is about 300–350 W. Thus the model prediction is about 45% – 65% high.

Generally speaking, one would expect the model to overestimate the extracted power, since it does not include factors such as misalignment and dirty windows which inevitably occur in experiment. In addition to these, two plasma physics factors could be playing a role in this discrepancy. First, the voltage fall at the cathode should lead to an increased power absorption and thus a distortion of the mountain structure. A primitive model of this effect has been implemented in the code, but it was not employed in the present runs because of uncertainties regarding the parameters of the model and, indeed, the validity of the model itself. A second factor could be radial transport. As discussed earlier, the magnetic field is allowed to diffuse radially, but the plasma constituents are treated as having a radial profile determined by the gas temperature. Since the discharge is heated from the outside, inclusion of radial transport would be expected to decrease the extracted power.

8 Conclusions

We have described a detailed computational model of a copper laser amplifier. We have presented rate equations for the constituent species, along with equations for the electron temperature, the extracted laser intensity, and the diffusing magnetic field. These equations are coupled to the model of the driving modulator by the boundary value of the magnetic field, which gives the plasma current. Rates have been constructed for all appropriate atomic reactions. The radial profile of the gas temperature is determined by the time-averaged power deposited in the gas. We have taken into account the role of septum inserts in cooling the gas.

We have used the model, in conjunction with a modulator model, to calculate comprehensive performance predictions for a high-power septum laser amplifier. This analysis included both a representative operating point and a systematic survey of operating points. In general, the calculated laser power appears to lie about 50% above experiment. Some reasons for this discrepancy were outlined.

To refine the model, one would begin by comparing its predictions with more detailed experimental information, such as the line-averaged densities of the ground [8] and excited [62, 63] states. This would also involve the addition of radial transport. A great deal of this work has actually been done, but attention has subsequently shifted to other projects.

In related work, the model has been used in conjunction with a model of pulse extraction from an amplifier [90]. Here the saturation assumption is removed, and the pulse height and lasing densities are calculated as functions of axial position and time. For a laser without a septum, the saturated power agrees with experiment and the results at lower powers show the expected trend.

In summary, we believe that our model contains the most important physics underlying the copper laser. It has proved to be a very useful tool for understanding copper lasers in the laboratory.

9 Acknowledgments

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A Copper Rates

A.1 Radiative Transition Rates

A critical survey of transition rates for the observed spectral lines of CuI has been presented by Bielski [91]. Matching his cited wavelengths with energy spacings given by Moore [73], one can identify a total of 42 observed transitions pertinent to our model. These begin within or below the pseudostate and terminate on the ground state or the lasing levels. In addition, three ultraviolet transitions probably are significant but are not listed. These are the pair of transitions

$$(5p) {}^2P_{3/2,1/2}^o \rightarrow (4s) {}^2S_{1/2},$$

and also

$$(5p) {}^2P_{3/2}^o \rightarrow (4s^2) {}^2D_{3/2}.$$

The first two transitions are similar to the resonance transitions except that the initial configuration is 5p instead of 4p. For lack of better information, we estimate their oscillator strengths as half that of the corresponding resonance lines. The last transition, which has an initial J of 3/2, has been observed for an initial J of 1/2. We assume that the rates for the two J values have the same proportion as the rates for the two similar lines

$$(4p) {}^2P_{3/2,1/2}^o \rightarrow (4s^2) {}^2D_{3/2}.$$

All 45 lines are listed in Table 4.

Five of the lines shown in the table connect states below the pseudostate. These are the pair of resonance lines (327.39 nm and 324.75 nm), the two lasing lines, and also the weak line connecting the J = 3/2 components of the lasing states (cf. reaction above). Note that the resonance lines are stronger than the lasing lines by nearly two orders of magnitude. There is no radiation from the metastable states.

Since the populations of the individual members of the pseudostate are not followed as functions of time, it is necessary to average the radiation from these levels to the ground state and the lasing states. Averaging the rates over statistical weights appropriate to local thermodynamic equilibrium, we obtain effective T_e -dependent rates to the five lower states. These rates, designated $A_{pi}(T_e)$ ($i = 1, \dots, 5$), are shown in Fig. 14.

A.2 Excitation

The electron-impact excitation cross sections of copper are difficult to measure. Early data for the transitions $(4s) {}^2S \rightarrow (4p) {}^2P$ and $(4s) {}^2S \rightarrow (4s^2) {}^2D$ were obtained by Williams and Trajmar [92] and by Trajmar *et al.* [93], with the second reference superseding the first. A change in the absolute normalization of the results was suggested on theoretical grounds in [94]. The resonance cross section was also measured by Aleksakhin *et al.* [95], with results which tend to confirm this renormalization. In addition, a point at 60 eV was recently obtained by Teubner [96]. All of these experiments were in the energy range of 6–100 eV, which only marginally overlaps the regime of interest.

Table 4: Copper Radiative Transitions in the Model

initial state	final state	λ (nm)	A ($10^7 s^{-1}$)
(4p) $^2P_{1/2}^o$	(4s) $^2S_{1/2}$	327.39	13.6
(4p) $^2P_{3/2}^o$	"	324.75	13.7
(4p') $^4P_{3/2}^o$	"	249.21	.306
(4p') $^4P_{1/2}^o$	"	244.16	.185
(4p') $^4D_{3/2}^o$	"	224.43	.23
(4p') $^4D_{1/2}^o$	"	222.57	4.8
(4p') $^2P_{1/2}^o$	"	218.18	11.18
(4p') $^2P_{3/2}^o$	"	217.90	9.4
(4p') $^2D_{3/2}^o$	"	216.51	5.86
(5p) $^2P_{3/2}^o$	"	202.438	18. (a)
(5p) $^2P_{1/2}^o$	"	202.437	18. (a)
(4p) $^2P_{3/2}^o$	(4s ²) $^2D_{5/2}$	510.54	.195
(4p') $^4P_{5/2}^o$	"	359.40	.0008
(4p') $^4P_{3/2}^o$	"	345.78	.0064
(4p') $^4F_{7/2}^o$	"	333.78	.038
(4p') $^4F_{5/2}^o$	"	329.28	.006
(4p') $^4D_{7/2}^o$	"	309.40	.0296
(4p') $^4D_{5/2}^o$	"	301.08	.13
(4p') $^4D_{3/2}^o$	"	299.84	.0027
(4p') $^2F_{5/2}^o$	"	307.38	.015
(4p') $^2F_{7/2}^o$	"	296.11	.376
(4p') $^2P_{3/2}^o$	"	288.29	.18
(4p') $^2D_{3/2}^o$	"	285.87	.0067
(4p') $^2D_{5/2}^o$	"	282.43	.783
(5p) $^2P_{3/2}^o$	"	261.84	3.07
(4p) $^2P_{1/2}^o$	(4s ²) $^2D_{3/2}$	578.20	.19
(4p) $^2P_{3/2}^o$	"	570.01	.025
(4p') $^4P_{3/2}^o$	"	372.07	.0011
(4p') $^4P_{1/2}^o$	"	360.92	.0036
(4p') $^4F_{5/2}^o$	"	353.03	.0262
(4p') $^4D_{5/2}^o$	"	320.82	.0493
(4p') $^4D_{3/2}^o$	"	319.41	.155
(4p') $^4D_{1/2}^o$	"	315.66	.0176
(4p') $^2F_{5/2}^o$	"	327.98	.07
(4p') $^2P_{1/2}^o$	"	306.89	.0019
(4p') $^2P_{3/2}^o$	"	306.34	.155
(4p') $^2D_{3/2}^o$	"	303.61	.243
(4p') $^2D_{5/2}^o$	"	299.73	.116
(5p) $^2P_{3/2}^o$	"	276.638	.1 (a)
(5p) $^2P_{1/2}^o$	"	276.635	.96
(5s) $^2S_{1/2}$	(4p) $^2P_{1/2}^o$	793.29	2.250
(4d) $^2D_{3/2}$	"	515.31	10.340
(5s) $^2S_{1/2}$	(4p) $^2P_{3/2}^o$	809.24	4.590
(4d) $^2D_{3/2}$	"	522.00	2.180
(4d) $^2D_{5/2}$	"	521.81	12.220

(a) Estimated (see text).

Excitation cross sections in copper were first calculated by Msezane and Henry [97], who employed a three-state basis below 10 eV and a four-state basis from 6 to 100 eV. Later, Scheibner, Hazi, and Henry [98] undertook a calculation of the cross sections in an energy range extending down to 0.1 eV, working with a 4-state basis. Scheibner and Hazi subsequently performed a calculation with a 10-state basis Sche88a (cf. also [100]), resulting in the most comprehensive available set of excitation cross sections. All of these calculations lie below the original data [93] by as much as a factor of five, but are closer (within a factor of two) to the renormalized data.

Very recently, a measurement of the resonance excitation cross section in the low-energy regime (threshold to 8 eV) has been carried out [101]. The results agree within 10% with the 10-state calculations of Scheibner and Hazi.

Fortified by this agreement, we have taken the bulk of our excitation cross sections from the Scheibner-Hazi calculation. The ten basis states employed by them are $(4s)^2S$, $(4s^2)^2D$, $(4p)^2P^o$, and all the pseudostate members listed in Table 3 except for the alkaline-like states $(5s)^2S$ and $(5p)^2P^o$. Fine structure was not taken into account in their calculation. Note the rich structure in the energy range below 10 eV or so. In using these results, we shifted the energy scales so that the thresholds agreed with the energies of Tables 1 and 3. Cross sections involving individual J multiplets were inferred by detailed balancing considerations. In addition, for excitation from the ground state to the two $(4s^2)^2D$ levels and to the two $(4p)^2P$ levels, the cross sections were multiplied by the energy-dependent correction factor

$$f(E) = E/(E + E_0^{exp} - E_0^{cal}), \quad (56)$$

where E_0^{exp} and E_0^{cal} are the experimental and calculated thresholds, respectively.

Reaction rates $\kappa = \langle \sigma v \rangle$ were constructed by averaging with respect to a Maxwellian distribution, via standard Gauss-Laguerre integration.

Our model also requires excitation rates to the two omitted pseudostate members noted above. The optically allowed transitions are, in L-S coupling,

$$(4s)^2S \rightarrow (5p)^2P^o, \quad (4s^2)^2D \rightarrow (5p)^2P^o, \quad (4p)^2P^o \rightarrow (5s)^2S.$$

We estimate their rates by employing the Van Regemorter formula [102]:

$$\kappa_{ab}(T_e) = \frac{16(2\pi^3)^{1/2} a_0^2 E_0^2 f_{ab}}{(3m_e T_e)^{1/2} \Delta E} \tilde{g}(\Delta E/T_e) \exp(-\Delta E/T_e), \quad (57)$$

where f_{ab} is the oscillator strength and $\tilde{g}(\Delta E/T_e)$ is a Maxwellian-averaged effective Gaunt factor (ΔE is the threshold energy and $E_0 = 13.6$ eV).

The final excitation rates are shown in Fig. 15.

A.3 Ionization

Like copper excitation cross sections, the ionization cross sections are difficult to measure near threshold. The earliest ground state measurements were reported by Pavlov *et al.* [103] and by Crawford [104]. The former data began at threshold, while the latter began at about

46 eV. These were followed a few years later by measurements of Schroeer *et al.* [105], which began at 40 eV and were systematically higher than the earlier data. The latest and most comprehensive data are due to Freund *et al.* [106], who obtained results from threshold to 200 eV via a crossed-beam apparatus. These are reasonably close to the data of Pavlov *et al.* Although the nominal uncertainty was 10%, the authors estimated their Cu beam to contain about 30% of its population in the 2D level.

A calculation based on a binary encounter model has been given by Rescigno [107]. This tends to agree with Pavlov *et al.* and Freund *et al.* at low energies and with Schroeer *et al.* at higher energies.

Finally, Lotz [108], has developed an empirical formula according to which electrons are treated as though they could be removed independently from the various subshells. For the ground state, this gives a result quite close to the measurements of Freund *et al.* For excited states, of course, no comparison is available. On the basis of these considerations, the Lotz formula was considered appropriate for describing ionization in the present model.

The Lotz cross section has the form

$$\sigma(E) = \sum_i \eta(E - P_i) a_i n_i \frac{\ln(E/P_i)}{EP_i} \{1 - b_i \exp[-c_i(E/P_i - 1)]\}, \quad (58)$$

where i runs over subshells. Each subshell is characterized by n_i electrons of binding energy P_i , and by the empirical constants a_i , b_i , and c_i . The step function $\eta(E - P_i)$ ensures proper behavior at threshold. The general results are expected to be valid within 30 or 40 percent at the maximum of the cross section.

Table 5 displays the reactions pertinent to the states in our model, and Table 6 lists the appropriate parameters. For ionization of the ground state, for example, there are two terms in the sum, one corresponding to removal of a 4s electron and the other corresponding to removal of a 3d¹⁰ electron. In these terms, we use parameters appropriate to the 4s subshell and 3d¹⁰ subshell, respectively. (Deeper subshells are unimportant in the energy range of interest.) Note that the formulas for the six pseudostate elements with core 4p' (fourth level of Table 5) are similar except for differences in the binding energies (ion energy minus initial energy). The Lotz cross sections and the data of Freund *et al.* are shown in Fig. 16.

The ionization rates, also shown in Fig. 16, were calculated in the same fashion as the excitation rates.

A.4 Momentum Transfer

The electron-atom momentum transfer cross section is derived from the elastic differential cross section by insertion of a factor of $1 - \cos \theta$ before the integration over angles. It is needed for calculating the plasma resistivity and the electron-atom energy exchange rate. Scheibner and Hazi [99] give this cross section for the ground state.

The sole measurement of this quantity in Cu appears to have been performed by Trajmar and coworkers [92, 93] (again the second reference supersedes the first). The authors give five energy points for scattering from the ground state and four points for the states $(4s^2)^2D_{5/2}$, $(4s^2)^2D_{3/2}$, and $(4s)^2P$, with the lowest point at 6 eV. The ground state cross section has

Table 5: Ionization Reactions for Lotz Cross Section

State(s)	Reactions		
(4s) ² S	3d ¹⁰ 4s	→	3d ¹⁰ + e
	"	→	3d ⁹ 4s + e
(4s ²) ² D	3d ⁹ 4s ²	→	3d ¹⁰ + e
	"	→	3d ⁸ 4s ² + e
(4p) ² P ^o	3d ¹⁰ 4p	→	3d ¹⁰ + e
	"	→	3d ⁹ 4p + e
(4p') ⁴ P ^o , (4p') ⁴ F ^o , (4p') ⁴ D ^o , (4p') ² F ^o , (4p') ² P ^o , (4p') ² D ^o	3d ⁹ 4s4p	→	3d ¹⁰ + e
	"	→	3d ⁹ 4s + e
	"	→	3d ⁹ 4p + e
	"	→	3d ⁸ 4s4p + e
(5s) ² S	3d ¹⁰ 5s	→	3d ¹⁰ + e
	"	→	3d ⁹ 5s + e
(5p) ² P ^o	3d ¹⁰ 5p	→	3d ¹⁰ + e
	"	→	3d ⁹ 5p + e
(4d) ² D	3d ¹⁰ 4d	→	3d ¹⁰ + e
	"	→	3d ⁹ 4d + e

Table 6: Parameters for Lotz Cross Section

Subshell	n	a	b	c	Ion Configuration	Energy
3d ⁹	9	1.6	0.95	0.14	3d ¹⁰	7.724
3d ¹⁰	10	1.4	0.96	0.13	3d ⁹ 4s	10.4
4s	1	4.0	0.	0.	3d ⁹ 4p	16.0
4s ²	2	4.0	0.	0.	3d ⁸ 4s ²	16.4
4p	1	4.0	0.	0.	3d ⁸ 4s4p	21.1
4d	1	4.0	0.30	0.60	3d ⁹ 5s	21.1
5s	1	4.0	0.	0.	3d ⁹ 4d	21.9
5p	1	4.0	0.	0.	3d ⁹ 5p	22.6

about the same shape as the Scheibner-Hazi calculation (some structure at low energy and an exponential falloff at higher energies), but it is systematically higher by about a factor of about six. This may not be significant, since the overall normalization of the data is subject to dispute. The $(4p)^2P$ cross section is about 1/6 the of the ground state cross section, and the cross sections from the $(4s^2)^2D$ levels are lower by another factor of six. All the curves are plotted in Fig. 17. Note that the ground state cross section starts at about 10^{-14} cm² at low energy, rises to a peak at a fraction of an eV, and thereafter falls off approximately as $1/E$. As expected, it shows some structure between 2 and 10 eV.

In the model, we employ the Scheibner-Hazi result for the ground state cross section, and we assume that the cross sections from other levels scale as observed experimentally. The pseudostate cross section is taken to be the same as that for $(4p)^2P$. The curves are plotted in Fig. 17.

A.5 Recombination

Because of the existence of autoionizing states, recombination is expected to be an extremely complicated process in the copper atom. Lacking experimental information, however, we simply assume that three-body recombination proceeds as in the hydrogen atom, where classical Monte Carlo calculations [80] are in reasonable agreement with experiment. In this treatment, the recombination rate coefficient has the form

$$\kappa_{rec} = \gamma a^5 (2T_e/m_e)^{1/2}, \quad (59)$$

with $a = e^2/4\pi\epsilon_0 T_e$ and $\gamma \simeq 0.54$. Note that this varies at low temperature as $T_e^{-9/2}$. In conditions characteristic of the laser afterglow ($T_e \simeq 0.2$ eV, $n_e \simeq 5 \times 10^{13}$ cm⁻³), the time scale is about 140 μ s. Thus recombination acts as a bottleneck in the kinetics.

B Neon Rates

The excitation cross section from the Ne ground state to the 3s state is taken from a calculation by Theodosiou [109]. The excitation cross section from ground to the 3p state is taken from experiment [110]. The ground state ionization cross section is taken from a measurement by Fletcher and Cowling [111]. The 3s ionization cross section is taken from experiment [112], while the 3p ionization is taken from a calculation by Hyman [113]. Note that the radiation rate from 3s to ground (1.78×10^8 s⁻¹) requires a trapping correction.

The final excitation and ionization rate coefficients are plotted in Fig. 18.

The momentum transfer cross section has been measured in several experiments. We employ values recommended by Itikawa [114], which are taken from measurements by Robertson [115] and Sol *et al.* [116]. The cross section and the resulting rates are shown in Fig. 19. Note that the cross section is more than an order of magnitude less than that for copper. Electron-Ne collisions dominate, however, since the neon density typically exceeds the copper density by two orders of magnitude.

C Hydrogen Rates

Electron-impact excitation and ionization rates of the hydrogen atom were taken from Vriens and Smeets [117]. The cross section for electron detachment from the negative ion was taken from values recommended by Massey [118]. The attachment rate was calculated via detailed balance, i.e.

$$\kappa(2e + H \rightarrow e + H^-) = \kappa(e + H^- \rightarrow 2e + H) \left(\frac{h^2}{2\pi m_e k T_e} \right)^{3/2} \exp(E_{H^-}/kT_e), \quad (60)$$

with binding energy $E_{H^-} = 0.76$ eV. In comparison, the radiative attachment rate [119] is insignificant. The e-H momentum transfer cross section was taken from a curve recommended by Itakawa [120].

For dissociation of the hydrogen molecule, we used the main channel, $e + H_2 \rightarrow e + H(1s) + H(1s)$, with a rate recommended by Janev *et al.* [121]. The inverse rate was taken from detailed balance,

$$\kappa(e + H + H \rightarrow e + H_2) = \kappa(e + H_2 \rightarrow e + H + H) \left(\frac{h^2}{\pi m_H k T_e} \right)^{3/2} Q_{rot} Q_{vib} \exp(E_{H_2}/kT_e), \quad (61)$$

with $E_{H_2} = 4.48$ eV the molecular binding energy. The rotational and vibrational partition functions have the form

$$Q_{rot}(T) = T/\Theta_r, \quad Q_{vib}(T) = [1 - \exp(-\Theta_v/T)]^{-1}, \quad (62)$$

with $\Theta_r = 2$ K and $\Theta_v = 2000$ K. The e- H_2 momentum transfer rate was taken from Itikawa [114].

These rates are plotted in Figures 20, 21, and 22.

D Radiation Trapping

After an excited atom in a vapor spontaneously decays, the emitted photon may travel only a certain distance before being absorbed by another atom in the lower state. This distance depends not only on the density of lower state atoms but also on the frequency of the photon, which is distributed within the atomic emission spectrum. After a spontaneous lifetime, the absorbing atom will emit another photon, which will travel another frequency-dependent mean free path before again being absorbed. Eventually a photon in this chain will escape from the system. This process of successive emission and absorption, called radiative trapping, increases the effective lifetime of the transition. To describe it in detail, via the radiation transport equation, is far beyond the scope of most discharge models. Instead, following Holstein [122, 123], we adopt a global approach in which the effective decay rate is reduced from the spontaneous rate by an escape factor. In the case of the copper resonance lines, for example, the effective rate is decreased by a factor of approximately 500, depending

on conditions. This correction plays a critical role in the model, since, in its absence, an inversion could not even be attained.

The escape factor depends on the geometry of the system and on the emission lineshape. Although most photons are emitted near line center, where lines are usually Gaussian (due to Doppler broadening), they are quickly absorbed. The relatively fewer photons emitted far from line center, where lines tend to be Lorentzian (due to pressure broadening), are responsible for the bulk of the radiative energy transport. Following Holstein, we calculate the escape factor separately for the two kinds of broadening. We then make the ansatz that the effective escape factor is the greater of these. For the resonance lines, the dominant mechanism is always pressure broadening, while for other lines the dominant mechanism may change during the pulse. Although this treatment obviously represents an idealization, it seems fairly reasonable. In any case, not much more can easily be done.

In calculating the escape factor, we employ the Holstein treatment for cylindrical geometry, except that we integrate over an arbitrary radial density profile rather than solving for the slowest mode.

An important parameter in this treatment is the absorption at line center, given by

$$k = \frac{\lambda^2 n_1 A g_2}{4\pi^2 g_1 \Delta\nu}, \quad (63)$$

where the indices 1 and 2 refer to the lower state and upper state, respectively, and $\Delta\nu$ is the linewidth. For pressure broadening the linewidth can be parametrized as $\Delta\nu = n_0 \sigma v_r / \pi$, with n_0 the density of buffer gas atoms, σ an approximate cross section, and v_r the relative speed. For Doppler broadening, the appropriate linewidth is

$$\Delta\nu = 2 v_{th} / \sqrt{\pi} \lambda, \quad (64)$$

with $v_{th} = (2kT/m)^{1/2}$ the thermal speed.

In the special case of insufficient absorption at line center, trapping can clearly be ignored. We parametrize this situation by the condition $k r_{eff} < 1$, with r_{eff} an effective distance determined by the geometry of the system. In a cylinder, it seems reasonable to set r_{eff} equal to the radius. In the case of a septum, we estimate $r_{eff} = a/2$.

After a calculation, we find that the escape factor is given an expression of the form

$$f = \frac{\alpha c_m b_m}{(k a)^m}, \quad (65)$$

where

$$\alpha = \begin{cases} \pi^{-1/2} & \text{pressure broadening,} \\ [\pi \ln(k r_{eff})]^{-1/2} & \text{Doppler broadening,} \end{cases} \quad (66)$$

and m is $1/2$ and 1 in the respective cases. These last two parameters are such that the transmission function is $T(r) = \alpha (k r)^{-m}$. Also c_m is the constant

$$c_m = \frac{m^2 \Gamma(m)}{(m+1) 2^m [\Gamma(m/2 + 1/2)]^2}, \quad (67)$$

which reduces to 0.139 and 1/4 in the respective cases. Finally, b_m represents an integral over the radial profile:

$$b_m = \int_0^1 du u \bar{n}(u) h_m(u), \quad (u = r/a), \quad (68)$$

where $\bar{n}(u) = n(r)/\langle n \rangle$ is the profile factor, and h_m is the function

$$h_m(u) = \frac{2}{m} \int_0^\pi d\theta \frac{1 - u \cos \theta}{(1 + u^2 - 2u \cos \theta)^{1+m/2}}. \quad (69)$$

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Figure 1: Effective degeneracy of Cu pseudostate; effective energy of the pseudostate.

Figure 2: Temperature correction factor G versus deposition parameter α , for the laser geometries. Note that $G(\alpha) - 1$ unity is plotted.

Figure 3: Typical gas temperature profile for the laser geometries. In the septum case, the profile is in the perpendicular midplane.

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Figure 4: Schematic of septum geometry.

Figure 5: Schematic of laser assembly.

Figure 6: Typical profiles of wall temperature and edge copper density for a septum laser.

Figure 7: Calculated line-averaged lasing densities and amplified optical signal for a septum laser.

Figure 8: Calculated power versus wall temperature at selected voltages and pressures, for a septum laser.

Figure 9: Absorbed power versus wall temperature for a septum laser.

Figure 10: Calculated septum laser power contours. Top: temperature-pressure plane ($V_0 = 17$ kV); bottom: temperature-voltage plane ($p = 70$ torr). The dotted lines show points available to experiment, with the standard thermal package.

Figure 11: Thermal characteristics of septum laser insulation package midway along the tube.

Figure 12: Laser power along the trajectories indicated in Fig. 10. The dashed line indicates a least-squares fit.

Figure 13: Calculated contours of wall temperature and laser power, on the voltage-pressure plane, for a septum laser.

Figure 15: Electron excitation rates from the states of the copper model. The numbers label the final states as in Table 1. The smooth lines superimposed on each curve indicate the least squares fit.

Figure 16: Electron-impact ionization cross sections and rates for the copper model. The numbers label the final states as in Table 1. Also shown are ground state ionization data of Freund *et al.*.

Figure 17: Electron-impact momentum transfer cross sections and rates for the copper model. Also shown are data by Trajmar. The smooth lines line superimposed on the rate indicate the least squares fit.

Figure 18: Neon excitation and ionization rates.

Figure 19: Neon momentum transfer cross section and rate.

Figure 20: Hydrogen excitation and deexcitation rates; hydrogen ionization rates; negative ion detachment cross section and rate. The smooth lines line superimposed on the last curve indicates the least squares fit.

Figure 21: Cross section and rate for momentum transfer in atomic hydrogen and in molecular hydrogen.

Figure 22: Dissociation rate for molecular hydrogen.