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AN ANALYTIC SOLUTION TO A DRIVEN INTERFACE PROBLEM

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The frictional properties of sliding metal interfaces at high velocities are not well known from either an experimental or theoretical point of view. The constitutive properties and macroscopic laws of frictional dynamics at high velocities necessary for materials continuum codes have only a qualitative validity and it is of interest to have analytic problems for sliding interfaces to enable separation of model from numerical effects. We present an exact solution for the space and time dependence of the plastic strain near a sliding interface in a planar semi-finite geometry. This solution is based on a particular form for the strain rate dependence of the flow stress and results in a hyperbolic telegrapher equation for the plastic strain. The form of the solutions and wave structure will be discussed

INTRODUCTION

The numerical treatment of dynamic thermo-plastic flow requires sophisticated numerical algorithms and sophisticated modeling of the physics of dynamic material flow. For general problems and general geometries, it is at times difficult to separate algorithmic and numerical effects from effects due to physics models. Test problems play an important role in separating numerical from model dependent effects since they provide closed form solutions with which to compare numerical integrations for the relevant continuum equations. For pure fluids a large number of such test problems exist (1). For bulk dynamic plastic flow there is a small number of exactly soluble test problems (2). However, for sliding material interfaces in situations where plastic dissipation is the major irreversible effect, there are very few test problems. We present here one such test problem which is relevant to a driven material interface and rate dependent plastic flow.

A PLANAR SEMI-INFINITE MODEL PROBLEM

The model problem we consider consists of a material whose shear properties are described by a constant shear modulus G_0 . We further assume that the constitutive relation for the flow stress, τ , is given by a linear rate dependent hardening law of the form

$$\tau = G_0(\hat{\tau}_0 + \beta\dot{\Psi}). \quad (1)$$

Here G_0 , $\hat{\tau}_0$ is the yield strength of the material, which we take to be a constant, and β is another constant having units of time describing a linear rate dependent hardening, $\dot{\Psi}$ is the equivalent plastic strain rate. This form is a gross simplification of the plastic properties of a real material but does capture, in an average way, some of the rate dependent properties of real materials where $\tau \propto \dot{\Psi}^\delta$ at very high strain rates with $0 < \delta < 1$ typically.

We consider a situation in which this material occupies the half space $z \geq 0$ and consider a tangential force $F_x(t)$ which is uniform in the $z = 0$ plane which forms the interface. We further assume that the density, ρ , is a constant, ρ_0 . The latter assumption limits us to shear excitations only. For this symmetric situation, all field quantities depend on (z,t) only. The equations of mass, momentum and energy conservation are

$$\begin{aligned}
\frac{D\rho}{Dt} + \rho \nabla \bullet \vec{v} &= 0, \\
\rho \frac{Dv_i}{Dt} &= \frac{\partial \sigma_{ij}}{\partial x_j}, \\
\rho \frac{De}{Dt} &= \sigma_{ij} \frac{\partial v_i}{\partial x_j},
\end{aligned} \tag{2}$$

Where $\vec{v}$ is the material velocity, e the internal energy per unit mass, and σ_{ij} is the Cauchy stress tensor.

$\frac{D}{Dt}$ is the material derivative, $\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{v} \bullet \nabla$.

We consider the case of purely transverse displacement consistent with constant density. Then

$\frac{\partial v_z}{\partial z} = \frac{\partial v_x}{\partial x} = 0$. Furthermore, steady motion in the x direction and spatial homogeneity in the x and y

directions implies $\frac{\partial \sigma_{xx}}{\partial x} = \frac{\partial \sigma_{xz}}{\partial x} = \frac{\partial v_x}{\partial x} = 0$. We look for a solution with $v_z=0$ and, consistent with this,

$\frac{\partial \sigma_{zz}}{\partial z} = 0$. Then,

$$\rho_0 \frac{\partial v_x}{\partial t} = \frac{\partial \sigma_{xz}}{\partial z}, \tag{3}$$

$$\rho_0 \frac{\partial e}{\partial t} = \sigma_{xz} \frac{\partial v_x}{\partial z}. \tag{4}$$

Because of the homogeneity and transversality, the components of the strain rate tensor satisfy

$\dot{\epsilon}_{xx} = \dot{\epsilon}_{zz} = \dot{\epsilon}_{yy} = \dot{\epsilon}_{xy} = \dot{\epsilon}_{yz} = 0$ and $\dot{\epsilon}_{xz} = \frac{1}{2} \frac{\partial v_x}{\partial z}$. We write $\dot{\epsilon}_{ij}$ in terms of its elastic and plastic

components, $\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^{(el)} + \dot{\epsilon}_{ij}^{(pl)}$, and assume that plastic flow is described by the Prandtl-Reuss relation,

$$\dot{\epsilon}_{ij}^{(pl)} = \frac{3}{2} \frac{S_{ij}}{\tau} \dot{\Psi}, \tag{5}$$

where S_{ij} is the deviatoric stress tensor, $S_{ij} = \sigma_{ij} - \frac{1}{3} \delta_{ij} \sigma_{kk}$.

Upon differentiating equation (3) we have,

$$\frac{\partial^2}{\partial t^2} \epsilon_{xz} = \frac{1}{2\rho_0} \frac{\partial^2 \sigma_{xz}}{\partial z^2}, \tag{6}$$

and assuming isotropic elasticity, $\sigma_{ik} = \beta \varepsilon_{\ell\ell}^{(el)} \delta_{ik} + 2G \left(\varepsilon_{ik}^{(el)} - \frac{1}{3} \delta_{ik} \varepsilon_{\ell\ell}^{(el)} \right)$, we have

$$\frac{\partial}{\partial t} \left[\frac{1}{2G} \frac{\partial S_{xz}}{\partial t} + \frac{3}{2} \frac{S_{xz}}{\tau} \frac{\partial \Psi}{\partial t} \right] = \frac{1}{2\rho_0} \frac{\partial^2 S_{xz}}{\partial x^2}. \quad (7)$$

We consider an isotropic von-Mises flow surface, $\frac{3}{2} S_{ij} S_{ij} = \tau^2$ and assume $\varepsilon_{xy}^{(el)} = \varepsilon_{yz}^{(el)} = 0$, again consistent with the above equations. Then $S_{xy} = S_{yx} = 0$. For an initially unstrained sample (assuming ρ constant), we may take $\varepsilon_{zz}^{(el)} = 0$ and then we have $S_{xx} = S_{yy} = -\frac{1}{2} S_{zz} = -\frac{2}{3} \varepsilon_{zz}^{(el)} = 0$. Hence $s_{xz}^2 = \frac{1}{3} \tau^2$ and S_{xz} satisfies

$$\frac{\partial}{\partial t} \left[\frac{1}{G} \frac{\partial S_{xz}}{\partial t} + \sqrt{3} \frac{\partial \Psi}{\partial t} \operatorname{sgn}(S_{xz}) \right] = \frac{1}{\rho_0} \frac{\partial^2 S_{xz}}{\partial z^2}, \quad (8)$$

or in terms of the flow stress,

$$\frac{\partial}{\partial t} \left[\frac{1}{G} \frac{\partial \tau}{\partial t} + 3 \frac{\partial \Psi}{\partial t} \right] = \frac{1}{\rho_0} \frac{\partial^2 \tau}{\partial z^2}. \quad (9)$$

In deriving equation (9) we have made no assumption about G . If we now make the ansatz of constant G and assume the constitutive relationship given in equation (1), then equation (9) reduces to the telegrapher's equation for $\Psi(z, t)$:

$$\frac{\partial^2 \Psi}{\partial t^2} + \frac{3}{\beta} \frac{\partial \Psi}{\partial t} + c^2 \frac{\partial^2 \Psi}{\partial x^2}, \quad (10)$$

with $c^2 \equiv G/\rho_0$.

SOLUTIONS TO THE MODEL PROBLEM

The solutions to the boundary value problem for equation (10) above may be obtained by standard methods (3). If we write Ψ as

$$\Psi(z, t) = w(\zeta, \xi) \exp(-\lambda \xi), \quad (11)$$

where $\lambda = 3/2$ and ζ, ξ are scaled variables $\zeta = z/(\beta c)$, $\xi = t/\beta$, the general solution is

$$w(\zeta, \xi) = w(0, \xi - \zeta) + \lambda \zeta \int_{\xi - \zeta}^{\xi} d\xi' w(0, \xi') \frac{I_1 \left[\lambda \sqrt{(\xi - \xi')^2 - \zeta^2} \right]}{\sqrt{(\xi - \xi')^2 - \zeta^2}}, \quad (12)$$

where I_1 is the Bessel function defined by

$$I_1(x) = \frac{1}{\pi} \int_0^\pi e^{x \cos \theta} \cos \theta d\theta. \quad (13)$$

For a given $w(0, \xi)$, equation (13) is a closed form solution. One particularly interesting case is that of a constant external force. In that case since $F_x = \frac{A}{\sqrt{3}} \tau$ where, A is the surface area,

$$\dot{\Psi}(0, \xi) = \hat{\tau}_0 \beta (\alpha - 1) = \text{constant with } \alpha = \left(\frac{F_x}{A} \frac{1}{G} \right) \hat{\tau}_0$$

and since we are assuming perfectly plastic behavior $\alpha > 1$. Figure 1 shows the solutions for ψ when $\alpha = 10$ and $\hat{\tau}_0 = 0.000212834$, $\beta = 0.0946 \mu s$, $\rho_0 = 8.84 \text{ g cm}^{-3}$, $G = 0.9397 \text{ Mbar}$ (parameters corresponding roughly to copper) at times $t=2,4,6,8 \mu s$. There is a broad damped excitation formed at the interface whose decrease is roughly exponential in character and which because of the causal nature of equation (10) vanishes for distances greater than ct . The width at half-maximum for this excitation varies predominately with the square root of t , a diffusive characteristic.

The above solutions are continuous solutions to the initial value problem for equation (10). The full set of conservation laws, equations (2) may also admit discontinuous solutions which in the present case correspond to a steady plastic wave. If such a solution were to exist, a nontrivial solution to the jump conditions,

$$s\Delta[u] = \Delta[F], \quad (14)$$

would be possible. In equation (14) we have written the conservation laws in standard form (4) $\frac{\partial u}{\partial t} + \frac{\partial F}{\partial z} = 0$,

s is the wave speed and $u = [\epsilon_{xz}, \rho_0 v_x, \rho_0 (v_x^2 + e)]$, $F = [-\frac{1}{2} v_x, -S_{xz}, -S_{xz}]$. It is easy to show that $s = -S_{xz} / \rho_0 \Delta[v_x]$ and in order to see whether such a steady plastic wave exists we look for a point (z^*, t^*) such that the solution given by equation (12) connects continuously to a constant plastic state. Using equation (14) and eliminating s in favor of $\Delta[v_x]$, it is possible to show that this compatibility condition results in

$$\frac{1}{3} \hat{\tau} + \hat{\tau} \psi = \left(\frac{1}{c} \Delta[v_x] \right)^2, \quad (15)$$

where $\hat{\tau} = \tau / G$. The right hand side of the above equation can be expressed in terms of the spatial derivative of ψ and hence we look for the earliest time t^* for which a z^* exists such that equation (15) is satisfied. This is, in effect, a solution of a Riemann problem for the restricted set of conservation laws. We find typically that there is a minimum time, $t^*_{\min}$, below which there is no solution. For $t > t^*_{\min}$, solutions exist and Fig. 2 shows the progression of solutions using the same parameters as for Fig. 1. The magnitude of ψ is very small relative to the main peak and the wave speed is very nearly the transverse wave speed c .

CONCLUSIONS

We have presented a test problem for use in evaluating materials dynamics codes which treat interfacial slip under circumstances where plastic deformation is an important dissipation mechanism. The semi-infinite test problem assumes only plastic deformation with no elastic transient and accommodates rather general time dependent boundary conditions at the interface. The analytic solution for plastic strain has been given for a particular choice of flow rule which although simple incorporates in an approximate way the rate dependence of the flow stress at very large rates of plastic deformation. More general flow rules give rise to a more complicated pde than the telegrapher's equation and it is more difficult to find closed form solutions. The numerical solution of Eqn. 9 for more complicated flow rules may, however, provide helpful benchmarks for computer codes.

We have limited ourselves to purely transverse excitations. When longitudinal effects are important as well, one must return to the full set of conservation laws and analytic results, for interfacial motion are much more difficult to obtain. We have also neglected thermal conductivity. Thermal conduction introduces a new time scale. For large plastic deformation and high plastic strain rates, it is possible to include such effects via perturbation theory (5). This is frequently the case for dynamic experiments. When this is not the case, the equation for entropy production must be treated explicitly (6). Nevertheless, closed form solutions such as we have presented can afford some simple guidance for when a full treatment is necessary. We hope that, these restrictions notwithstanding, the present test problem will be beneficial for code development where interfacial effects are important.

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