Comparison Between Various Beam Steering Algorithms for the CEBAF Lattice*

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Abstract. In this paper we describe a comparative study performed to evaluate various beam steering algorithms for CEBAF lattice. The first approach that was evaluated used a Singular Value Decomposition (SVD) based algorithm to determine the corrector magnet setting for various regions of the CEBAF lattice. The second studied algorithm is known as PROSAC (Projective RMS Orbit Subtraction And Correction). This algorithm was developed at TJNAF to support the commissioning activity. The third set of algorithms tested are known as COCU (CERN Orbit Correction Utility) which is a production steering package used at CERN. A program simulating a variety of errors such as misalignment, BPM offset, etc. was used to generate test inputs for these three sets of algorithms. Conclusions of this study are presented in this paper.

INTRODUCTION
The CEBAF accelerator consists of a 45 MeV injector, two side-by-side superconducting linacs, and 9 recirculation arcs that recirculate the beam through the linacs up to 5 times for 4 GeV total energy. Beams of different energies are separated at the first spreader and are transported through isochronous arcs to the recombiner at entrance of second Linac. At the exit of second Linac, the beams of different energies are separated again to be sent to either Experimental Halls or through the recirculation arcs. An orbit correction system is required at CEBAF to increase the machine aperture and to steer the beam through any portion of the accelerator for a desired beam delivery objective. A variety of beam steering algorithms of varying characteristics and complexity are available. In this study we have compared three such algorithms.

SVD Based Algorithm
Consider there are M beam position monitors and N corrector magnets available to the beam steering algorithm. Changes in corrector strength $\Delta \theta$ (vector of length N) will reduce the closed orbit error $\Delta x$ (vector of length M). These two vectors are linearly related through a response matrix $R_{ij}$ as indicated by

$$\Delta x_i = \sum_{j=1}^{N} R_{ij} \cdot \Delta \theta_j$$  \hspace{1cm} (1)

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The response matrix can be written in terms of betatron amplitude, phase advance and the tune of the machine as

$$ R_{ij} = \sqrt{\beta_i \beta_j} \sin(\psi_i - \psi_j) $$

(2)

where $\beta_i$, $\beta_j$ are the betatron amplitudes at $i$th BPM and $j$th corrector. The response matrix can be experimentally determined by changing the strength of $j$th corrector by unit excitation and measuring the resulting beam motion at all BPMs while rest of correctors are set to 0. Orbit correction using SVD is inverse of this process of experimental determination of response matrix. Singular Value Decomposition of $R_{ij}$ can be written as

$$ R = U \cdot W \cdot V^T $$

(3)

where $U$ is $M \times M$ unitary matrix, $W$ is $M \times N$ diagonal matrix that contains all the singular values, and $V$ is an $N \times N$ unitary matrix. The inverse of matrix $R_{ij}$ can be obtained from

$$ R^{-1} = V \cdot W^{-1} \cdot U^T $$

(4)

where $W^{-1}$ can be constructed by inverting the singular values and then taking a transpose of the matrix. If any of the singular values are zero, then this singularity can be removed by setting the inverse of that singular value to be zero rather than a large number in the inverse matrix. Including all non-zero singular values to determine the inverse will result in most accurate correction of the closed orbit error. However, if the $R_{ij}$ matrix is nearly singular then, this correction might require unreasonably large corrector settings on a few correctors. This condition can be avoided by eliminating the smallest singular values from the inverse calculation until the corrector settings requirement enters a more reasonable range. The first step in the orbit correction process is obtaining the difference orbit $\Delta x$, which is the difference between the orbit measured by BPMs and the desired reference orbit. Next, the corrector settings are computed using

$$ \Delta \theta = R^{-1} \cdot \Delta x $$

(5)

If the corrector settings turn out to be over the saturation limit of power supplies, then the $R^{-1}$ is recalculated by eliminating the lowest singular value from inverse calculation. Once satisfactory corrector settings are obtained, they could be applied to reduce the closed orbit error. See reference (1) for an implementation of this algorithm at Advanced Photon Source at Argonne National Lab.
PROSAC algorithm

This algorithm (2) utilizes the projection of \( j \) th corrector on the closed orbit as a parameter for selecting the best corrector and iteratively determining its settings to progressively reduce the closed orbit error. That corrector magnet, in \( N \) dimensional space of all available magnets, which has the largest projection on the vector for closed orbit error is considered best. The projection \( V_j \) of \( j \) th corrector effect on orbit is given by

\[
V_j = C_j \cdot \Delta x
\]  

where \( \Delta x \) is the closed orbit error and \( C_j \) is the unit response on \( M \)-BPMs by the \( j \) th corrector. Now the setting for this corrector is calculated using

\[
\Delta \theta_j = W_j \cdot \Delta x
\]  

where \( W_j \) is the \( j \) th column in the response matrix \( R_{ij} \).

This algorithm has a variety of options available for implementing orbit correction scheme. The first option in this algorithm allows for either using the projection \( V_j \) or the normalized projection \( V_j / |C_j| \) for selecting the best corrector. The second set of options allows for three different starting conditions for iterations. First condition is to start with all correctors set as-is and then incremental corrections are applied to reduce the closed orbit error. Second condition requires that all correctors are set to zero and then incremental corrections are applied to reach the desired reference orbit. Third condition starts with performing a simple least squares fit for the current settings of BPMs and correctors and then either one of the above mentioned conditions in the first option could be applied to reach the reference orbit.

The iterative process of reducing the closed orbit error continues with selecting the best corrector and applying the correction until the closed RMS orbit error is reduced to 20% or a user defined fraction of the initial value.

COCU

COCU (Closed Orbit Correction Utilities) is a comprehensive collection of orbit correction algorithms unified under a standard user interface. It has been a major orbit correction tool used at CERN and several other accelerators. The repertoire of algorithms include MICADO, a minimum corrector number routine, MINIMO, an algorithm looking for absolute best corrector combinations, SIMPLEX, a minimization program capable of inequality corrector constraints, and a number of other algorithms. It also performs harmonic analysis in the case of closed orbit in circular machines and conditioning of the input beamline layout to avoid near-degenerate configurations. A detailed description of COCU can be found in reference (3) and
references therein. We have linked the majority of the core COCU program with a graphical user interface to facilitate the data transfer between simulation and orbit correction algorithms.

SIMULATION PROGRAM FOR TEST INPUTS

To compare the performance of the various orbit correction algorithms, simulation program was developed with the CEBAF accelerator as a test bed. Approximately 100 simulation files were generated mimicking all conceivable errors in the machine. These include isolated and distributed optics errors, isolated and distributed misalignment errors in all coordinates, isolated and distributed monitor errors, injection errors in all coordinates, initial corrector kicks and errors, and earth field effects. The use of simulation data helps provide a measure of how each algorithm has performed everywhere against the uncorrected orbit, including areas inaccessible to orbit monitors in the real machine. It also allows creation of special cases where the near-degeneracy of the orbit correction system and corrector magnitude limits are put to test.

RESULTS AND CONCLUSIONS

We have tested over 100 simulation files against the various orbit correction algorithms mentioned above. In most cases all the algorithms produced similar results in terms of the final orbit and overall corrector strengths. We will briefly describe cases where performance of these algorithms differ:

(a). With its inclination to find the minimal set of correctors to control the orbit, in a few cases the MICADO line of algorithms tend to concentrate too much strength into a small number of correctors, as shown in Figure 1.

![Corrector strengths MICADO](image1)

![Orbit correction by MICADO](image2)
Figure 2 shows the orbit before (dotted line) and after (solid line) correction. Corresponding cases using an SVD based algorithm is shown in Figures 3 and 4, which are similar to results from PROSAC.

(b). All algorithms appear to produce large “fighting” correctors due to near-degeneracy of the monitor-corrector response matrix, although PROSAC appears to be the least vulnerable. Figures 5 and 6 show the correction result of an SVD based algorithm where no singular value has been excluded, meaning virtually no constraint on corrector strengths. It can be seen that some correctors conspire to create large local orbit bumps without detection by BPMs (BPM readings are indicated by solid circles). If one proceeds to eliminate singular values such that corrector limits of 1 mrad is imposed, no correction can be successfully accomplished. The MICA-DO line of algorithms produced results more close to those from SVD algorithms.
In this particular case the algorithm PROSAC succeed in reducing the orbit significantly within the corrector limits, without inducing orbit bumps. We are aware of similar problems with PROSAC, but the occurrence is less frequent.

(c). We also compared the behavior of the various algorithms in a region with insufficient monitors such that the information derived from these monitors is inadequate for orbit reconstruction on the order of 5-10 mm. This can be equivalent to evaluating the error handling ability of these algorithms when some monitors are not working. Our conclusion is that all algorithms tested can easily produce undetectable after-correction orbit errors on the same order as the uncertainty in orbit reconstruction by any method. In such cases algorithmic ingenuity apparently can not compensate for fundamental lack of information. Figure 7 shows such a case by SIMPLEX, a COCU algorithm, where a missing BPM towards the end of the line caused undetectable orbit excursion of about 7 mm. Such problems can be rectified only by more BPMs or drastically changed optics.

In summary, we have tested various orbit correction algorithms against simulated orbits. The relative pros and cons are discussed above which may help accelerator controllers in choosing the optimal method to use. Keeping a wide variety of algorithmic options and careful conditioning of the monitor-corrector system to avoid near-degeneracies appear to be the best policy when it comes to orbit correction.

![Figure 7. Uncorrectable orbit due to BPM deficiency](image)

REFERENCES