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Performance of a tapered pulse tube
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I. Abstract

In a well instrumented pulse tube refrigerator having 1500 W of cooling power at 125 K, we have measured the figure of merit of a tapered pulse tube at several operating points. At operating points near the operating point for which the taper was designed (using the design method described by Olson and Swift, to be published in *Cryogenics* in late 1997), the figure of merit is 0.96. This is close to the theoretical optimum figure of merit 0.97 calculated for this pulse tube considering only two loss mechanisms: heat conduction in the metal pulse tube wall and ordinary thermoacoustic heat transport in the gas within a few thermal penetration depths of the wall. At operating points farther from the design operating point, the measured figure of merit is much lower, as streaming-driven convection adds a third loss mechanism.

II. Introduction

Lee, Kittel, Timmerhaus, and Radebaugh² observed a toroidal steady flow in a pulse tube, realized that it must convect significant heat, explained that it might be a streaming phenomenon driven by the sidewall boundary layer, and proposed that tapering the pulse tube might reduce it. Olson and Swift³ developed this idea further, derived an expression for the taper that suppresses streaming, and presented

measurements showing that this remarkably small taper indeed yields improved pulse-tube-refrigerator performance.

Here, we present further experimental evidence that streaming driven convection is suppressed in a pulse tube tapered according to the formula of Olson and Swift. Our measurements show that, with the correct taper, the heat load on the cold heat exchanger due to the pulse tube can be close to that due to ordinary conduction in the pulse tube wall plus ordinary thermoacoustic heat transport near the pulse tube wall.

III. Apparatus and Instrumentation

The pulse tube refrigerator used for these measurements, shown schematically in Fig. 1, was part of Cryenco's 1997 thermoacoustically driven orifice pulse tube refrigerator, which liquefied natural gas at rates as high as 100 gallons per day.^{4,5} This refrigerator is much more powerful than those usually discussed at the International Cryocooler Conferences; its measured net cooling power was between 1100 W and 1700 W at 125 Kelvin for the data reported here. This high power was achieved primarily through use of large-diameter components; for example, the pulse tube itself had a diameter of 10 cm. The refrigerator used 3.1 MPa helium at 40 Hz; for all measurements reported here, the pressure amplitude was 210 kPa at the pressure sensor near the hot end of the pulse tube (see Fig. 1).

In a pulse tube refrigerator with such a large cooling power, the use of inertance^{6,7} is easier than in smaller systems. A large inertance has relatively little dissipation, so series and parallel orifice valves, as shown in Fig. 1, can be used with a large inertance to allow adjustments of the refrigerator's operating point over a wide range.⁸ In this

paper, we will identify refrigerator operating points by means of the complex acoustic impedance Z at the hot end of the pulse tube, as shown in Fig. 1, just below the middle flow straightener. The magnitude of Z is the ratio of oscillatory pressure amplitude to oscillatory volumetric velocity amplitude; the phase of Z is the phase angle by which pressure leads volumetric velocity.

In the design of our refrigerator, calculations^{9,10} of viscous and thermal losses in the regenerator led us to identify an operating point with the highest expected efficiency at the desired cooling power, which we will call the “design” operating point and which is shown as the open circle in Fig. 2. Analysis using Eq. (13) of Olson and Swift showed that operating points along the line marked “straight” in Fig. 2 would have zero streaming-driven convection if a straight pulse tube were built. We judged that the design operating point was too far from this line, so our pulse tube was built using Eq. (13) for the design operating point, with a taper $(1/A) dA/dx = -0.49 \text{ m}^{-1}$, which is equivalent to a total included angle ϕ of 1.3° with the cold end larger than the hot end. Further analysis using Eq. (13) showed that all operating points on the line marked “as built” would have zero streaming-driven convection with this taper. The five selected experimental operating points, shown as filled circles and labeled “1” through “5,” were chosen to be close to the design operating point and close to the “as-built” line of possible operating points. We predicted that experimental operating points farthest from this line would exhibit the greatest streaming-driven convective heat transfer.

To detect the presence of this convective heat transfer, we measured the total energy flow $\dot{H}$ up the pulse tube and the acoustic (or pV) power $\dot{W}$ up the pulse tube. In the absence of any heat transfer in the pulse tube (due to conduction,

convection, or radiation), $\dot{H} = \dot{W}$, so the deviation of $\dot{H}/\dot{W}$ from unity is a measure of such undesired heat transfer, which directly reduces the net cooling power of the refrigerator. Hence, this ratio is commonly known as the figure of merit for a pulse tube. In most previous pulse tube refrigerators, the pulse tube figure of merit has typically been in the range from 0.6 to 0.85. For our experiments, all parts of the pulse-tube refrigerator (including valves, inertance, and compliance) were thermally insulated, except for the fluid streams in contact with the heat exchangers. Hence, measurement of the heat carried away from the heat exchanger at the hot end of the pulse tube was a direct measurement of $\dot{H}$. Uncertainty in our measurements of $\dot{H}$ was about 1% [check DVM specs.], due mostly to uncertainty in the measurement of the electric current in a heater in the water stream, which was essentially used to determine the water flow rate. [Greg find ref to this technique]. Pressure sensors in the compliance and near the hot end of the pulse tube allowed measurement of acoustic power $\dot{W}$ up the pulse tube. If the void volumes in the hot heat exchanger, inertance, orifice, valves, and associated plumbing were zero, $\dot{W}$ and Z could be obtained from such pressure measurements in a straightforward way⁸ known to most pulse-tube experimentalists. However, in our hardware these void volumes were not negligible, so we used a complete numerical model⁹ containing the geometry of these components, with the measured pressure amplitudes and phases, to obtain $\dot{W}$ and Z . This more sophisticated method yielded values of $\dot{W}$ and Z 5% to 10% different from those which would have resulted from the assumption of zero void volumes. The uncertainties in these values of $\dot{W}$ and Z were about 2%, due mostly to uncertainty in the compliance volume. [Greg check w Mark.]

IV. Results

Figure 3 shows the experimental values of $\dot{H}/\dot{W}$ for the five experimental operating points, displayed vs how far those operating points are from operating points of predicted zero streaming-driven convection shown as the “as built” line in Fig. 2. To provide a quantitative measure of this distance, the horizontal axis is the difference between the “as-built” value of $(1/A) dA/dx$ and the value that Eq. (13) yields for $(1/A) dA/dx$ at the selected operating point. Operating points 2 and 5, which were closest to the “as built” optimal condition, have the highest value of $\dot{H}/\dot{W}$, approximately 0.96. Based on the lower values of $\dot{H}/\dot{W}$ for the other three operating points, we estimate that operating points 2 and 5 would have yielded an experimental value of $\dot{H}/\dot{W}$ near or below 0.85 if the pulse tube had been straight instead of being tapered according to Eq. (13).

The solid line in Fig. 3 near $\dot{H}/\dot{W} = 0.97$ shows calculated values of $\dot{H}/\dot{W}$ including only thermal conductivity in the stainless-steel pulse tube wall and thermoacoustic boundary-layer heat transport, but not including streaming-driven convection. (As in Fig. 2, the design operating point is indicated by the open circle.) Hence, within our experimental uncertainties of a few percent in $\dot{H}/\dot{W}$, experimental operating points 2 and 5 exhibit no streaming-driven convection.

The dashed lines in Fig. 3 are rough estimates of $\dot{H}/\dot{W}$ taking streaming-driven connection into account, using Eqs. (10) and (11) of Olson and Swift evaluated at the center (axially) of the pulse tube, and calculating the heat load with the assumption that *all* the downward streaming gas is at room temperature and *all* the upward moving gas is at 125 K. These lines are in reasonable agreement with the data. They

must be regarded as rough estimates because many of the assumptions built into Olson and Swift's work are only valid in the zero-streaming limit.

V. Summary

Streaming-driven convection can be eliminated in a pulse tube, either by tapering the pulse tube or by choosing an operating point for which a straight pulse tube generates no streaming. Streaming is eliminated if the pulse tube is tapered according to Eq. (13) of Olson and Swift:

$$\frac{1}{A} \left(\frac{dA}{dx} \right) = -\frac{\omega |p_1|}{\gamma p_m |\langle u_1 \rangle|} \left[\left(1 + \frac{2(\gamma-1)(1-b\sigma^2)}{3\sigma(1+\sigma)} \right) \cos \theta + \left(1 + \frac{2(\gamma-1)(1-b)\sqrt{\sigma}}{3(1+\sigma)} \right) \sin \theta \right] - \frac{(1-b)(1-\sqrt{\sigma})}{3(1+\sigma)(1+\sqrt{\sigma})} \frac{dT_m/dx}{T_m} \quad (1)$$

$$\simeq - (0.75 \cos \theta + 0.64 \sin \theta) \frac{\omega |p_1|}{p_m |\langle u_1 \rangle|} - 0.0058 \frac{dT_m/dx}{T_m}. \quad (2)$$

If streaming is thus eliminated, the heat flow $\dot{Q} = \dot{H} - \dot{W}$ down the pulse tube can be estimated accurately using

$$\begin{aligned} \dot{Q} = & K_{\text{wall}} A_{\text{wall}} \frac{dT_m}{dx} + K A \frac{dT_m}{dx} \\ & + \frac{\Pi \delta_\kappa}{4} \left[\left(\frac{1-\sqrt{\sigma}}{1+\sigma} \cos \theta - \frac{1+\sqrt{\sigma}}{1+\sigma} \sin \theta \right) |p_1| |\langle u_1 \rangle| + \frac{1-\sigma^{3/2}}{1-\sigma^2} \frac{dT_m}{dx} \frac{\rho_m c_p |\langle u_1 \rangle|^2}{\omega} \right] \end{aligned} \quad (3)$$

$$\begin{aligned} \simeq & K_{\text{wall}} A_{\text{wall}} \frac{dT_m}{dx} + K A \frac{dT_m}{dx} \\ & + \Pi \delta_\kappa \left[(0.025 \cos \theta - 0.27 \sin \theta) |p_1| |\langle u_1 \rangle| + 0.51 \frac{dT_m/dx}{T_m} \frac{p_m |\langle u_1 \rangle|^2}{\omega} \right]. \end{aligned} \quad (4)$$

Here, A is the internal area of the pulse tube and A_{wall} is the cross-sectional area of its wall, Π is the internal perimeter of the pulse tube, x is the axial distance from the

cold end of the pulse tube, ω is the angular frequency of oscillation, T_m is the mean temperature, p_m is the mean pressure, $|p_1|$ is the amplitude of the oscillatory pressure, $|\langle u_1 \rangle|$ is the amplitude of the lateral spatial average of the oscillatory velocity, θ is the phase angle by which $\langle u_1 \rangle$ leads p_1 , ρ_m is the mean density of the gas, c_p is its isobaric heat capacity per unit mass, γ is its ratio of isobaric to isochoric specific heats, K is its thermal conductivity, σ is its Prandtl number, $b = (T_m/\mu_m)(d\mu_m/dT_m)$ where μ_m is its dynamic viscosity, $\delta_\kappa = \sqrt{2K/\omega\rho c_p}$ is its thermal penetration depth, and K_{wall} is the thermal conductivity of the wall material. The second form of each equation incorporates numerical values for low-temperature helium gas: $\gamma = 5/3$, $\sigma \simeq 0.69$, and $b \simeq 0.68$. In Eqs. (3) and (4), the first two terms account for ordinary heat conduction, and the last two terms are from Rott's second-order ("thermoacoustic") energy flux¹¹ in the limit $R \gg \delta_\kappa$.¹² (Note that pulse tube end effects can cause dT_m/dx to be significantly larger than $\Delta T/\Delta x$, where ΔT is the temperature difference between the two heat exchangers and Δx is the length of the pulse tube.) In a large pulse tube, a figure of merit $\dot{H}/\dot{W} > 0.95$ is possible, as demonstrated here. In smaller pulse tubes, for which boundary-larger thermoacoustic processes carry a larger fraction of the heat, figures of merit > 0.9 should be possible.

Careful attention to detail is required to achieve such performance. The operating point Z of the refrigerator must be planned very accurately, or means to finely adjust it must be provided. Flow straightening at the ends of the pulse tube must eliminate jet-driven convection. The pulse tube wall roughness must be much less than the viscous penetration depth, and of course the pulse tube must operate in the laminar or weakly turbulent regime.

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VIII. Figure captions

Fig. 1 Schematic of the apparatus. “P” indicates locations of pressure sensors. The angle ϕ of the tapered pulse tube has been greatly exaggerated.

Fig. 2 Complex acoustic impedance Z at the hot end of the pulse tube. The open circle is the design operating point, and the line through the point indicates Z 's for which the as-built taper should suppress streaming. The dashed line indicates Z 's which would have had zero streaming if the pulse tube had been built straight. The filled circles indicate the experimental operating points.

Fig. 3 Figure of merit of the pulse tube at various operating points vs how far each operating point is from the “as-built” line on Fig. 2. Filled circles show experimental values. Open circle shows the design operating point; solid line shows

calculated $\dot{H}/\dot{W}$ neglecting streaming; dashed line shows rough calculated estimate of $\dot{H}/\dot{W}$ including streaming.

Fig. One,

Fig 2

Fig 3

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