State of Physics at the End of the 20th Century: Massive Neutrinos?

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A brief review of neutrino masses is presented with focus on how masses might appear in unified models. A fall 1996 status report of the LSND (Liquid Scintillator Neutrino Detector) experiment at Los Alamos is given; the statistical evidence for neutrino oscillations is becoming stronger. A summary of a unified model based on SO(18) shows possible complications in understanding neutrino masses.

1 Introduction and General Issues

It is a pleasure to be part of Pete Carruther's 61st birthday celebration. Pete was the Theoretical Division Leader when Murray Gell-Mann, Pierre Ramond and I looked at the "problem" of neutrino masses in the late 70's. We tried to understand neutrino masses in unified models, especially the $G = SO(10)$ model where each "family" of left-handed fermions is assigned the spinor $16$. ($G$ is the gauge group of the theory; representations are in boldface type.) Here it is natural for neutrinos to have Dirac masses of nearly the same size as other masses in the family. In this setting it is hard to understand masslessness of neutrinos; comparatively small masses are easier.

Understanding the empirical patterns of fermion masses, the angles between the mass and current eigenstates of the quarks, and the analogous mixing angles for the leptons for nonzero neutrino masses continues to be challenging. The theory of the strong and electroweak interactions based on $G = SU(3)C \times SU(2)W \times U(1)W$ provides little constraint on these parameters. Quantum loops do not renormalize zero neutrino masses because of the chiral symmetry of zero-mass fermions, and the mixing angles remain zero. However, there is growing evidence that neutrino masses are small but nonzero.

The span of the fermion masses is huge. The ratio of the top-quark mass to the electron mass is about $3.5 \times 10^5$. Evidence for neutrino oscillations suggests nonzero neutrino masses, so there may be larger ratios. Radiative corrections may be responsible for some large mass ratios, but the overall pattern does not appear simple. Masses do not seem to be directly proportional to vector-coupling coefficients of $G$, even after renormalization to low energies.

This talk begins with a brief summary of general issues concerning neutrino masses. This is followed with a status report of the LSND measurement.
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of neutrino oscillations. Finally, we comment on some complications in the fermion mass matrix in a unified model based on $G = SO(18)$, where all the known left-handed fermions are assigned to one irreducible representation.

In the $G = SO(10)$ model, the 45 known left-handed fermionic degrees of freedom are assigned to three $16s$:

$$\mathbf{f}_L = \mathbf{16}_1 + \mathbf{16}_2 + \mathbf{16}_3,$$

(1)

with one $\mathbf{16}$ for each family of quarks and leptons and $\mathbf{f}_L$ is the gauge group representation of the left-handed fermions. For example, $\mathbf{16}_1$ in Eq. (1) contains the left-handed electron, electron neutrino, positron, plus the three colors each of up and down, quarks and antiquarks. Each of these 15 degrees of freedom has a $\Delta \mathcal{I}^W = \frac{1}{2}$ mass except for the neutrino, which is assumed to have zero mass in the usual statement of the standard model. By itself, this one neutral degree of freedom could have a $\Delta \mathcal{I}^W = 1$ Majorana mass that violates lepton number by 2 units. The 16-th degree of freedom of the $\mathbf{16}$ is neutral under the electroweak interactions.

In $SO(10)$ the neutrino mass matrix is that of a four-component fermion with lepton-number violating terms allowed. This is conveniently analyzed in the Weyl basis with Majorana spinors. A Majorana spinor is a Dirac spinor satisfying $\psi = \psi^c$. This constraint equation imposes $\overline{\psi}_R = \psi_R$ and $\overline{\psi}_L = -\overline{\psi}_L$, so it has just two physical degrees of freedom. In terms of a real field $\rho$ the Majorana fields are

$$\psi^T = \left( \rho_R, -\rho_L^\dagger, \rho_L, \rho_R^\dagger \right), \quad \overline{\psi} = -\left( \rho_L^\dagger, \rho_R, \rho_R^\dagger, -\rho_L \right).$$

(2)

Each neutral fermion of the $\mathbf{16}$ contributes a component to $\rho_L$, so $\rho_L^T = (\nu_L, \overline{\nu}_L)$, each with different electroweak quantum numbers.

A Majorana spinor does not necessarily describe a particle that is its own antiparticle. The charge-conjugation operator $C$ is defined to reflect one component of $\rho_L$ to another. Thus, if a spinor $e_L^-$ describes a left-handed electron (and the right handed positron), then a second Majorana spinor $e_L^+$ can be introduced that describes a left-handed positron. These spinors can be put together to form a Dirac electron using $C$. If $e_L^-$ and $e_L^+$ are in the same irreducible representation of $G$, $C$ is associated with $G$. Similarly, it is convenient to describe the two neutral fermions of the $\mathbf{16}$ as two Majorana spinors.

The most general mass term with Majorana spinors is

$$\mathcal{M} = \frac{1}{2} : \overline{\psi}(S + iP\gamma_5)\psi : = \begin{pmatrix} \rho_R \end{pmatrix} \begin{pmatrix} \rho_L^\dagger & 0 & -M \end{pmatrix} \begin{pmatrix} M^\dagger & 0 \end{pmatrix} \begin{pmatrix} \rho_R \rho_L \end{pmatrix},$$

(3)
where the explicitly hermitian follows from Eq. (2). The colons around the mass operator signifies normal ordering, including a sign change when exchanging a pair of fermion operators. \( M \) is the symmetric matrix,

\[
M = M^T = -\frac{1}{2}(S - iP) - \frac{1}{2}(S - iP)^T.
\] (4)

The diagonalization of the mass matrix would appear to require an analysis of the matrix in Eq. (3). However, by a theorem of Shur it is only necessary to compute the eigenvalues of the matrix \( M^*M \); the matrix \( M^*M \) is Hermitian and positive definite for symmetric \( M \). The eigenvalues of the full mass matrix are plus/minus the square roots of the eigenvalues of \( M^*M \). In the case that \( CP \) is conserved, \( M \) is also a real matrix, and so the eigenvalues of \( M^*M \) are simply the squares of the eigenvalues of \( M \). Thus, a study of the mass operator is a study of the symmetric matrix \( M \).

The mass matrix of a four-component neutrino for the \( \rho_H^1 M \rho_L \) term of Eqs. (3) and (4) is

\[
\rho_H^1 M \rho_L = \left( \begin{array}{c} \nu_R^1 \\ \nu_R^2 \\ \nu_R^3 \\ \nu_R^4 \end{array} \right) \left( \begin{array}{cc} 0 & m \\ m & M_0 \end{array} \right) \left( \begin{array}{c} \nu_L \\ \nu_L \end{array} \right).
\] (5)

The parameter “0” is the \( \Delta I_W = 1 \) mass of the left-handed neutrino (it is not required to be zero, but it certainly expected to be quite small), \( m \) is the Dirac mass for the neutrino, and \( M_0 \) is the \( \Delta I_W = 0 \) right-handed neutrino Majorana mass that could be huge, perhaps \( 10^{10} \) GeV. The two neutrino-like degrees of freedom are mixed by the Dirac \( \Delta I_W = \frac{1}{2} \) mass, \( m \). If the \( \Delta I_W = 1 \) mass matrix element is zero, then the observed neutrino mass can be very small. Solving the mass eigenvalue equation yields approximately \( m^2/M_0 \) for the smaller-mass solution. The neutrino has a mass much than typical Dirac masses in the 16 by the ratio \( m/M_0 \).

Suppose the left-handed fermions are assigned to the representation \( f_L \) as in Eq. (1). By Eq. (4) the mass matrix couples to \( (f_L \times f_L)_S \), where the subscript \( S \) indicates the symmetric part of the product. Both the symmetric and antisymmetric parts of the product

\[
16 \times 16 = 126_S + 10_S + 120_A
\] (6)
can contribute several times each to \( (f_L \times f_L)_S \) in a three-family model, depending on the complexity of the mass operator. \( SO(10) \) has little guidance.

There have been several approaches to finding a more constrained mass matrix. There may be a family symmetry that ties together the three families and further constrains the mass matrix. If such a symmetry is continuous,
one might ask how it is related to the standard model or some unification of it. A simple possibility for providing such constraints is for $f_L$ to be a single, irreducible representation of $G$.

Several unification schemes have been suggested where $f_L$ is irreducible. The $1728$ of $G = E_6$ could contain the known $45$ degrees of freedom. It contains many unobserved states with large values of weak isospin, color and electric charge. There is no obvious reason for all of these masses to be larger than the known quark masses. Another example is the $256$-dimensional spinor representation of $G = SO(18)$. In this case there are still over $200$ states that have not been observed, but their weak and strong assignments do not have large weak isospin, color or electric charge as appear in the $1728$.

The additional degrees of freedom must be sufficiently hidden. The decomposition of the $256$ into $SO(10)$ representations yields eight $16$s and eight $\bar{16}s$. We suppose there is a "color-prime" force that hides all the $\bar{16}s$ and five of the $16$s. The experimentally observed three families are then color-prime singlets. The properties of the mass matrix under $C$ will be described in Sec. 3.

2 Neutrino Oscillations and LSND

The experimental effort to look for small masses by searching for oscillations depends on masses and interaction currents having different origins so they can be out of alignment. Interactions are defined by the gauge symmetries and the coupling of vector bosons to the symmetry currents. The origin of mass is in the symmetry breaking mechanism. The CKM matrix measures this misalignment for the quarks.

Also in the case of neutrinos the family identified by the interaction producing the neutrino does not have to line up with the neutrino mass eigenstates. From Eq. (3) the CKM matrix for three neutrinos and three antineutrinos is six-by-six. A neutrino produced in a weak interaction may change its family due to oscillation, for example, a $\bar{\nu}_\mu$ from $\mu^+$ decay, may oscillate to a $\bar{\nu}_e$ if the masses of the $\bar{\nu}_\mu$ and $\bar{\nu}_e$ differ and the mass matrix does not line up with the weak interactions. The crucial parameter describing the oscillation length is $\Delta m^2 = m_{\nu_\mu}^2 - m_{\nu_e}^2$ and the amplitude of the oscillation is $\sin \theta$ where $\theta$ measures the misalignment of the weak interactions and the mass matrix.

At the time of this celebration of Carruther's birthday, the LSND experiment at Los Alamos was half way through its second run. The data here includes the first run (1992-1995) and data from the second run. Within statistics the data from the two runs are consistent with one another, and the overall statistics of the experiment are improved by combining the data. In the second run the tank was placed slightly closer to the beam stop, the length of the
decay channel was changed, and other systematics of the experiment changed slightly.

Most of the neutrinos produced by the secondary interactions and decays resulting from the collision of 800 MeV protons of the LAMPF beam with the beam stop are due to $\pi^+$ and $\mu^+$ decay. Only 11% of the pions produced are $\pi^-$, and 95% of these are absorbed by the beam stop before they can decay into $\mu$s and $\nu$s. Thus, the production by usual weak processes of $\nu_s$ is greatly suppressed. The LSND collaboration finds that\(^4\)

$$\frac{\bar{\nu}_e}{\nu_\mu} = 7.5 \times 10^{-4}. \quad (7)$$

It is not sufficient simply to look for an excess of $\bar{\nu}_s$s; there must be a positive signal of $\bar{\nu}_s$ interactions with the right energy distribution. Particle identification and energy are measured by using both the scintillation and Cerenkov signals in the LSND tank. (The tank is full of baby oil with some scintillator added to it and observed by an array of phototubes.)

The neutrino-oscillation-event signature described here is from the reactions where the $\bar{\nu}_\mu$ has oscillated to $\bar{\nu}_e$, and the $\bar{\nu}_e$ is detected by the reaction,

$$\bar{\nu}_e + p \rightarrow e^+ + n, \quad (8)$$

where the positron is directly observed. There is a positron background at lower energies, but for positrons above 36 MeV, there significant signal. Before other cuts on the data, there is an excess of 63.3 $\pm$ 20.1 events.

The additional signal is the identification of the neutron in Eq. (8) by observing a $\gamma$ with a time delay of 186 $\mu$s from the reaction

$$n + p \rightarrow d + \gamma(2.2 \text{ MeV}). \quad (9)$$

With a fairly tight restriction on the $\gamma$ signal, the excess of $\bar{\nu}_e$ events becomes\(^4\)

$$\text{Excess} = 17.4 \pm 4.7 \text{ events}. \quad (10)$$

The probability of this being a statistical fluctuation is $4.2 \times 10^{-8}$.

It is customary in neutrino oscillation experiments to show solutions (or excluded regions) on a plot of $\Delta m^2$ versus $\sin^22\theta$. This oscillation analysis assumes that only two neutrino types participate in the oscillation, but the analysis does give a good indication where the parameters are likely to lie after additional data is acquired and all three neutrino types are included.

The strongest experimental result is the result given in Eq. (10), which identifies that there is an anomalous source of $\nu_s$s. The solution region for the parameters $\Delta m^2$ and $\sin^22\theta$ in LSND includes a positive signal that lies
outside the exclusion regions of Brookhaven's E776 experiment, the Karmen data and the reactor experiment at Bugey. (All three of these experiments place restrictions on the $\Delta m^2$ and $\sin^2 2\theta$ parameters, with no evidence of a positive signal.) LSND has a positive solution to its signal that lies on a line between $\Delta m^2 = 2 \, eV^2$ and $\sin^2 2\theta = 0.002$ and $\Delta m^2 = 0.3 \, eV^2$ and $\sin^2 2\theta = 0.04$. Other solutions lie in excluded regions by other experiments.

3 The Spinor of $SO(18)$

The Lie algebra of $SO(18)$ is rank 9 and has 153 basis elements. In the Weyl basis, there are 9 diagonal operators, 72 lowering operators (which lower the quantum numbers of a state) and the corresponding 72 raising operators. Although this is a large algebra, the analogous results for $SO(10)$ are well-known and add intuition about the rather large representations of $SO(18)$.

The basis vectors of the spinor representations of $SO(6 + 4n_f)$ all have unit multiplicity ($n_f$ is number of families); there is just one Hilbert-space vector for each set of $SO(6 + 4n_f)$ quantum numbers, so that eigenvalues of the diagonal operators completely label the basis states.\(^5\)

The three-family model is based on the spinor 256 of $SO(18)$.\(^1\) The $SO(8) \times SO(10)$ branching rule of the 256 is $(8_a, 16) + (8_c, 16)$. The $8_a$ has the $SO^f(3) \times SO^c(5)$ decomposition, $(3, 1) + (1, 5)$; the conjugate spinor $8_c$ has the decomposition $(2, 4)$. Thus, the $SO^f(3) \times SO^c(5) \times SO(10)$ decomposition of the 256 is

$$256 = (3, 1, 16) + (1, 5, 16) + (2, 4, 16).$$

The $SO^f(3)$ is assumed to be the "family symmetry" and the $SO^c(5)$ is a confining "color-prime" symmetry. Without further discussion we simply assume that the $SO^c(5)$ confines so all but the 48 states in $(3, 1, 16)$ are manifested as bound states of color-prime.\(^1\) We also assume that only the color-prime singlets have been observed experimentally. If there is any reality to this model, the composites of non-singlet color-prime states have masses above present experimental observation, even though many masses have $\Delta I^W = \frac{1}{2}$.

The next piece of $SO(18)$ group theory concerns the representations that couple to the mass matrix, $(256 \times 256)_S$, where the subscript $S$ indicates that the mass matrix couples only to the representations in the symmetric part of the tensor product, as derived in Eq. (4),

$$(256 \times 256)_S = 24310 + 8568 + 18.\quad (12)$$

With only three representations there would appear to be strong constraints on the mass matrix. Nevertheless, there are many components in each of these
representations that might have nonzero expectation values after radiative corrections are included.

The part of the analysis discussed here is whether there exists for $SO(18)$ a charge conjugation operator $C$ that that gives a simple classification of the mass-matrix elements. For example, the Dirac masses are all even under $C$. Although $SO(18)$ has the possibility of defining a number of charge conjugations, none of these is particularly attractive.

To define the charge conjugation, divide the Lie algebra into two parts: the symmetric subalgebra $S$, and the remaining operators $A$. A symmetric subalgebra $S$ is defined by the set of commutation relations,

$$[S, S] \subseteq S, \ [S, A] \subseteq A, \ [A, A] \subseteq S. \quad (13)$$

Thus, we can define an automorphism on the full Lie algebra as

$$C(S) = S, \ C(A) = -A. \quad (14)$$

It is obvious that the sign changes of Eq. (14) leave the commutation relations in Eq. (13) unchanged. It is straightforward to derive all the symmetric subalgebras, the action of $C$ of the vectors of a representation, and to count the number of negative and positive eigenvalues of $C$. $^5$

$SO(18)$ has four candidate charge conjugations that flip the signs of the eigenvalues of four or more diagonal operators. These are:

- $S = SO(8) \times SO(10)$, where $C$ flips the signs of 8 operators. In this case $C$ looks exactly like the $C$ for $SO(10)$. In addition it flips all the signs in the diagonal operators in $SO(8)$. ($C$ of $SO(10)$ flips the signs of four of the five diagonal operators, including electric charge and the two diagonal color operators. It cannot flip all five diagonal operators and reflect the 16 to itself.)

- $S = SO(6) \times SO(12)$, where $C$ of $SO(18)$ acts exactly like the $C$ of $SO(10)$ and it also flips just 2 of the signs in the $SO(8)$.

- $S = SO(4) \times SO(14)$, where $C$ flips the signs of 4 diagonal operators. It looks exactly like the $C$ of $SO(10)$ and it flips none the signs in the $SO(8)$.

- The most interesting candidate is associated with the symmetric subgroup $S = SU(9) \times U(1)$, which flips four operators. Moreover, this is the only case that does not pair up half of states in the 256 with the other half. However, it proves impossible to orient the subgroup in $SO(18)$ so that it flips the diagonal color operators and electric charge. The proof
of this claim is based on the fact that $S = SU(5) \times U(1)$ is a symmetric subgroup for $SO(10)$ for which $C$ flips only two of the diagonal operators. The final result can then be built up to the $S = SU(9) \times U(1)$ case for $SO(18)$.) Thus, this case is not a candidate.

In the first three cases no state is transformed outside its 16. The Majorana masses for the neutrinos are transformed into one another, so that $C$ is maximally violated by the mass matrix, as can be seen in Eq. (5).

This analysis leaves us with the conclusion that the mass matrix in $SO(18)$ has mixed behavior under $C$. In every case the $C$ exchanges $\nu_L$ with $\nu_L$ within each $SO(10)$ family. Thus, the behavior of the mass matrix is not simplified. This does not guarantee the failure of the $SO(18)$ model, but does not clarify the origin of the large Majorana mass in Eq. (5).


References