The DHG Sum Rule measured with Medium Energy Photons


1) Brookhaven National Laboratory, 2) INFN-Laboratori Nazionali di Frascati, 3) University Gießen, 4) Norfolk State University and CEBAF, 5) Ohio University, 6) Osaka University, 7) Université de Paris-sud, 8) Università di Roma-II, 9) University of South Carolina, 10) Syracuse University, 11) University of Virginia, 12) Virginia Polytechnic Inst. & State Univ.

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Introduction

The structure of the nucleon has many important features that are yet to be uncovered. Of current interest is the nucleon spin-structure which can be measured by doing double-polarization experiments with photon beams of medium energies (0.1 to 2 GeV). One such experiment uses dispersion relations, applied to the Compton scattering amplitude, to relate measurement of the total reaction cross section integrated over the incident photon energy to the nucleon anomalous magnetic moment. At present, no single facility spans the entire range of photon energies necessary to test this sum rule. The Laser-Electron Gamma Source (LEGS) facility will measure the double-polarization observables at photon energies between 0.15-0.47 MeV. Either the SPring8 facility, the GRAAL facility (France), or Jefferson Laboratory could make similar measurements at higher photon energies.

The Spin Polarizability

The general form of the forward Compton scattering amplitude, as described by Gellmann, Goldberger and Thirring (GGT) [1], is given by

\[
A(\omega) = f(\omega^2) \tilde{e}' \cdot \tilde{e} + i\omega g(\omega^2) \tilde{\sigma} \cdot (\tilde{e}' \times \tilde{e})
\]  

(1)

where \(\tilde{e}\) and \(\tilde{e}'\) are the incident and final photon polarization vectors and \(\tilde{\sigma}\) is the nucleon target spinor. For small values of the photon energy \(\omega\), the functions \(f\) and
$g$ can be expanded in a Taylor series as
\begin{align}
f(W^2) &= f(0) + f'(0)W^2 + \ldots \\
g(W^2) &= g(0) + g'(0)W^2 + \ldots
\end{align}

The leading term $f(0)$ describes Thompson scattering for a point target particle, and the spin-dependent term $g(0)$ is given by the low-energy theorem \cite{2,3} and is proportional to the square of the nucleon anomalous magnetic moment, $\kappa^2$. The derivative terms, when used with the GGT dispersion relations, are related to the spin-dependent cross sections,
\begin{align}
f'(0) &= \frac{1}{4\pi^2} \int_{\omega_0}^{\infty} \frac{\sigma_{1/2} + \sigma_{3/2}}{\omega^2} d\omega \\
g'(0) &= \frac{1}{4\pi^2} \int_{\omega_0}^{\infty} \frac{\sigma_{1/2} - \sigma_{3/2}}{\omega^3} d\omega
\end{align}

where $\omega_0$ is the pion production threshold and $\sigma_J$ is the total cross section for the photon and nucleon spins coupled parallel ($J = 3/2$) or anti-parallel ($J = 1/2$). The top integral is equal to the sum of the electric and magnetic polarizabilities of the target, $\alpha + \beta$, and the bottom integral is referred to as the spin polarizability, $\gamma$. The polarizabilities $\alpha$, $\beta$, and $\gamma$ are fundamental properties of the nucleon and are non-zero only because of the substructure of the nucleon. The polarizabilities are calculable in chiral perturbation theory (ChPT) in terms of fundamental constants such as the axial vector coupling constant $g_A$ and the pion decay constant $F_\pi$ \cite{4}.

While there have been several recent determinations of $\alpha + \beta$ for both the proton and the neutron \cite{5}, which involve the total unpolarized photoabsorption cross section, a direct measurement of $\gamma$ is still lacking, because of the combined requirement of a polarized beam and polarized target. Advances in technology, for both laser-backscattering facilities and polarized targets, provides an opportunity to measure these quantities with unprecedented accuracy. The full-sized (5 cm long, 3 cm diameter) polarized HD targets have been produced recently at Syracuse University, and will be transported to the LEGS experimental area in late 1997. Before going into the experimental details, there is another dispersion integral to describe that can be measured simultaneously with the spin-polarizability.

**The Drell-Hearn-Gerasimov Sum Rule**

Another GGT dispersion relation leads to the sum rule
\begin{equation}
g(0) - g(\infty) = \frac{1}{4\pi} \int_{\omega_0}^{\infty} \frac{\sigma_{1/2} - \sigma_{3/2}}{\omega} d\omega
\end{equation}

where $g(\omega^2)$ is the spin-dependent amplitude given above, with $g(0) = -\alpha\kappa^2/2m^2$. Drell and Hearn \cite{6} and Gerasimov \cite{7} independently made the assumption that $g(\infty) = 0$ because at very high photon energy, Thomson scattering and its spin-dependence are expected to be vanishingly small. With no other assumptions, the DHG sum rule relates the anomalous magnetic moment of the nucleon to the difference
between the total cross section measured with nucleon spin parallel and anti-parallel
to the photons, integrated with weighting factor 1/ω.

Also relevant is that the DHG sum rule is the $Q^2 = 0$ limit of the Bjorkin sum
rule from high-energy lepton scattering. The Bjorkin sum rule is a function of $Q^2$,
and is strictly valid only at $Q^2 = ∞$. However, perturbative QCD can be used to
make corrections that have been shown to agree with experiments at CERN (SMC),
SLAC (E142) and Fermilab (E665). Of course, extrapolation to $Q^2 = 0$ is not reliable
[8], and so this point is determined solely by the DHG measurement. Indeed, it will
provide a significant constraint for models using higher-twist diagrams to extrapolate
the Bjorkin sum rule down to low $Q^2$.

Using a Nd-YLF laser backscattered from 2.8 GeV electrons, the measurement of
the reaction cross section is possible up to a photon energy of 470 MeV at LEGS.
While this does not exhaust the sum rule, the denominator of the integrand suppresses
contributions at higher energies. The LEGS measurements are expected to cover a
significant portion of the sum rule, which could be combined with higher energy
measurements at SPring8 or GRAAL. Both of these other labs have proposals for the
DHG sum rule that depend on the success of the polarized HD target experiment
at LEGS. While the LEGS measurement alone will not determine the validity of the
DHG sum rule, it is an important first step.

One might think that the dominance of the $Δ$ in the LEGS energy range would
allow an unambiguous determination of the DHG integral up to 470 MeV. This is
not the case, as there is a discrepancy between the predictions for the DHG integral
from pion multipole analysis and equation (6) above. Due to the energy weighting in
the denominator, an unknown nucleon resonance with exceptional strength at higher
energy is required to resolve this conflict. If the multipole predictions turn out to be
correct, what would be the consequences? The simplest explanation would be that
$g(∞) ≠ 0$ [11]. The physical origin of this could result from a $J = 1$ fixed pole in the
angular momentum plane [9], although other explanations such as a magnetic moment
for quarks [10] have been proposed. In any case, it would likely stimulate considerable
theoretical dialogue if the DHG sum rule is not satisfied, and perhaps lead to some
new physics. On the other hand, if the multipoles are wrong in the energy region
below 470 MeV, the LEGS measurements will provide the cross sections needed to
correct this problem.

**Estimates from Photoproduction Multipoles**

It is possible to make an estimate of the spin polarizability $γ$ and the DHG sum rule
integrals from the measurements of the different charged meson photoproduction cross
sections. Assuming pion production is the dominant process, the $3/2$ and $1/2$ helicity
cross sections have been estimated [11] using the $N(γ, π)$ multipoles of Workman and
Arndt [12]. The cross sections are parameterized in terms of isovector (VV), isoscalar
(SS) and mixed (VS) channels, where $σ_{p,n} = σ^{VV} + σ^{SS} ± σ^{VS}$.

The results are shown in Fig. 1 for the DHG sum rule [14]. The cross section
differences, without the energy weighting in the DHG integrand, are also shown in
Fig. 1 for comparison (note the relative minus sign). The point to be made here is
that most of the action occurs at the lower energies. At the higher energies, large
differences in the spin-dependent cross sections (from strong nucleon resonances) are
needed to overcome the energy weighting in the denominator. The measurements at
LEGS will cover 90% of the spin polarizability integral and 65% of the DHG integral,
according to these estimates.

A comparison of the estimates with the DHG integral with the values for $g(0)$
(using the proton and neutron magnetic moments) show a large discrepancy [11] in the
VS channel (a subtraction of the proton and neutron spin-dependent cross sections).
The VS channel will have the least systematic errors for the LEGS measurement.
Curiously, the same estimates for the spin polarizability are in quite good agreement
with chiral perturbation theory [11]. The significant discrepancy in the VS channel
suggests that either the multipoles may not be as reliable as some people would like to
believe, or that $g(\infty)$ is in fact not zero. A direct measurement of the spin-dependent
cross sections is needed in order to verify the estimates from multipole analysis. While
there is not sufficient space here to describe the measurement uncertainties, all cross
sections below 470 MeV are planned to have statistical and systematic uncertainties
of a few percent or better.

**The Polarized Target**

Both polarized photon beams and a polarized target are necessary for measurements
of the spin-dependent cross sections. The LEGS facility has a high polarization (70-
99%, depending on the energy) of the photon beam. The addition of a hydrogen and
deuterium target with high polarization makes LEGS uniquely suited to determine
the spin polarizability and DHG integrals up to 470 MeV.

The SPHICE (Strongly Polarized Hydrogen-deuteride ICE) target represents a
new technology based on HD molecules condensed to a solid at very low temperatures
[13]. Once polarized, the coupling between the nucleon spin and the atomic lattice,
which is primarily through molecular rotations, will cause the spin to relax back to
the unpolarized state for molecules with identical particles like $H_2$ or $D_2$. The HD
molecule has no such restriction and can be in the $J = 0$ rotational state, resulting
in very small relaxation rates. The polarization remains 'frozen' for many weeks or
even a few months, depending on the storage temperature [13].

Of course, the same mechanism that slows the decay of the polarization will retard
the growth of polarization. This problem can be overcome by doping the target with a
small amount of Ortho-$H_2$ or Para-$D_2$. The presence of o-$H_2$ and p-$D_2$ are important
for the polarization process since the molecular orbital momentum couples with both
the lattice and the nuclear spin. These states decay in a few days to p-$H_2$ and o-$D_2$
which are 'inert'. By holding the target initially at low temperatures and high fields,
it becomes polarized and stays that way after the decay of the doping agent. The
details are given in reference [13]. In this way, polarizations of 80% for the protons
and 50% (vector) for the deuterons can be obtained.
Figure 1: Isospin decomposition of the DHG integral into vector (VV), scalar (SS) and mixed (VS) components as a function of the upper limit of integration. The dotted curves show the contribution from $N\pi\pi$. 
The SASY detector

The Spin ASYmmetry (SASY) array is a high-efficiency, large solid angle detector that is being constructed at the LEGS facility, see Fig. 2. It consists of several layers to provide complete determination of angle, energy, and particle identity (PID) for all reactions induced by photon beams of up to 470 MeV, on hydrogen and deuterium targets. The polarized HD target, sits in the center of the crystal box detector (XTAL). A solenoid keeps the target in a holding field of 1 Tesla. The XTAL BOX consists of 432 NaI detectors (2.5" by 2.5" cross section), with a lining of 1-inch thick plastic scintillator. The XTAL BOX detector is fully operational and gain matched. It covers a large solid angle (\(\sim 45 - 135^\circ\)) with moderate energy resolution for both charged particles and gamma-rays. A time-projection chamber (TPC) matched to the XTAL BOX dimensions for tracking of charged particles is being designed and will be built during 1998-99.

Figure 2: Schematic of the Spin Asymmetry (SASY) detector at the LEGS facility.

In the forward direction, there will be three conventional wire chambers followed by a Cerenkov detector, the scintillator bars, and a Pb-glass wall. This combina-
tion allows rejection of $e^+e^-$ pairs and high-efficiency detection of both neutral and charged particles, although with less energy resolution than the XTAL BOX. The plastic scintillator bars cover most of the angular range for recoiling nucleons from single-pion production. The Pb-glass wall is for detection of photon energies and angles, largely from $\pi^0$ production. Both the Pb-glass detectors and all of the plastic bars are now gain matched. The Cerenkov detector has been tested in beam and rejects the pair production events, reducing the raw trigger rate by a factor of $\sim 4$.

**Simulation Studies for SASY**

There are three essential components to the simulation of the proposed experiments: 1) the definition of the detectors (geometry, material, etc.), 2) the event generator (based on estimated or measured reactions), and 3) the detector resolution and response (from calibrations). The simulations were carried out using the GEANT software from CERN. Two independent versions of the geometry and detector response were implemented in order to check the consistency of the results. The event generator uses cross section predictions from a multipole analysis of the available data. In the case of a deuterium target, Fermi motion of the nucleons was included. Comparison of real data from the SASY detector at incident photon energies in the range 220-320 MeV are currently being compared with the GEANT simulations in order to improve the detector calibrations.

In order to measure the reaction cross section for polarized photons incident on a polarized target, the detector efficiency must be known to a reasonable accuracy. The simulations for both neutral and charged pions have been analyzed to extract the detection efficiency for the SASY geometry. The $\pi^0$ detection efficiency as a function of the pion polar angle is shown in Fig. 3 for several pion energies. The efficiency is largest (about 60%) at angles covered by the XTAL BOX array (about 45°-135°). At forward angles, the Pb-glass wall provides a reasonable coverage, giving roughly 40% efficiency to detect the $\pi^0$ with a proper reconstructed mass and energy. At angles greater than 135°, the detection efficiency drops to almost zero for $E_{\pi} > 25$ MeV.

Detecting the charged pions gives a slightly different challenge for the analysis software. Both $\pi^+$ and particularly $\pi^-$ can be absorbed on the nuclei in the detector crystals. The result of this, if it happens within the time window when the electronics are recording the detector pulse height, is to give a detected energy different from the pion kinetic energy. As the pion energy increases, the number of events with detected energy not matching the pion kinetic energy increases, giving a reduced efficiency. The detection efficiency is shown in Fig. 4 for $\pi^+$ (top) and $\pi^-$ (bottom) at the kinetic energies shown, with only the XTAL BOX detector geometry. At forward angles, the charged pions are detected with high efficiency in the plastic scintillator bars.

Extracting a reliable reaction cross section separately for the protons and neutrons in the polarized HD target requires a determination of the pion charge ($\pi^+$ or $\pi^-$). This is not possible for the XTAL BOX alone, and so it is essential to track the curvature of the pions in the magnetic holding field of the target. Simulations of the TPC tracking are in progress, and preliminary indications using the current TPC
Figure 3: Efficiency of $\pi^0$ detection in SASY at various $T_\pi$.

Figure 4: Efficiency of $\pi^\pm$ detection in SASY at various $T_\pi$. 
design show that the pion charge is correctly determined over all of the SASY detector solid angle.

**Summary and Conclusions**

A high-precision measurement of the spin-polarizability and the Drell-Hearn-Gerasimov sum rule is now possible with the advent of high-polarization solid HD targets at medium energy polarized photon facilities such as LEGS, GRAAL and SPring8. Other facilities with lower polarization in either the photon beam or target (or both) are also pursuing these measurements because of the high priority associated with this physics. The Spin-asymmetry (SASY) detector that will be used at LEGS has been briefly outlined in this paper. The detector efficiencies have been explored with simulations studies using the GEANT software, with the result that both charged and uncharged pions can be detected with a reasonable efficiency (> 30%) over a large solid angle. Tracking with a TPC, which will be built at LEGS over the next few years, will improve the capabilities of these measurements.

**References**


