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A Study of the Impact of Bias on Accountability (U)

Lockheed Martin Energy Systems, Inc.

April 1996

MASTER

T. R. Harvey
and
S. A. McGuire

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Distribution

T. R. Harvey (3)
S. A. McGuire (5)
Y-12 Plant Records Services (3)
Department Of Energy, Oak Ridge Operations Office, MC&A Branch (1)

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Executive Summary

Introduction

In the area of production the primary goal is to produce a product that meets (or exceeds) customer expectations. This requires that one first obtains statistical control of the process at some desired level of operation. Once this is accomplished, one looks for ways to reduce variations around that level. We take corrective actions only when statistical control charts indicate that some undesirable change has occurred. We do not, for example, concern ourselves with the possibility of relatively small changes that routine control chart tests do not detect. Relatively small biases are of no great concern. However, significant deviations from the desired level of operation are of great concern.

In the area of materials accountability the situation is somewhat different. Here our primary goal is to maintain the best possible records of material inventories taking into account the measuring uncertainties that are associated with such inventories. In addition to the variations of individual values around averages (precision), we must also concern ourselves with those deviations in averages from their true values (biases). We are faced with the problem of knowing true values with exactness, and we are faced with the problem of needing to identify those relatively small biases that behave as random variables while exhibiting little or no statistical significance.

Definitions

The following list of definitions and/or explanations of terms is used throughout the report. An understanding of these will greatly enhance the reader's ability to properly interpret the findings of this report.

1. Precision is defined as two times the standard deviation associated with individual measurements.

2. Precision Factor is precision expressed as a percent of the analytical level of operation.

3. Bias Uncertainty is defined as two times the standard deviation associated with bias fluctuations.

4. Bias Uncertainty Factor is bias uncertainty expressed as a percent of the analytical level of operation.

5. Ratio refers to the ratio of precision uncertainty to bias uncertainty. This is equivalent to saying that Ratio makes reference to the ratio of the standard deviation of individual measurements to the standard deviation associated with bias. The inverse of this ratio is also referenced throughout the report. For example, if we say that the ratio of precision uncertainty to bias uncertainty is 4.0, it follows that the ration of bias uncertainty to precision uncertainty is 0.25.

6. Bias Level is defined as the amount of bias that is exhibited by some number of successive data points.

7. Bias Length is defined as the duration (i.e., the number of successive data points) of any given bias level. Bias length also defines the length of control charts. The terms "Bias Length" and "Control Chart Length" are interchangeable.

8. Bias Group references a group of measurements obtained successively under the same method while that method was operating at the same bias level.

Conclusions

The purpose of this paper is to examine the effects of relatively small bias fluctuations on accountability. In brief, the conclusions arrived at from this study are summarized as follows:

1. When propagation of variance is used to assign limits of uncertainty to an inventory, even relatively small biases are major contributors to that uncertainty (see Table A and Appendix A). To ignore the existence of such biases is to make those limits of uncertainty much too small.

2. Even when it can be assumed that the chart's control limits and center line are based on data generated from a stable and unbiased process, the probability that the usual control chart tests will call attention to relatively small biases is very small.

   Note: When the ratio of precision to bias uncertainty is larger than 4.0, the average number of runs of eight is no more than twice that of the false notices expected when it is known that no bias exists. This would imply that, even though a sizable percentage of such notices might be true, one is likely to conclude that most of them are false because they occur so infrequently.

3. When these small unrecognizable biases exist, one cannot be highly confident that the initial data used to establish the control charts center line and control limits are not also biased by an amount large enough to allow for a ratio of precision to bias uncertainty as large as 4.0. This would imply that in actual practice a proper identification is even more uncertain. It is also a good reason for additional study.

4. Since relatively small biases are difficult to recognize, it is just as difficult to identify the length of such biases when they do exist. Although such bias lengths may vary, it has been estimated by simulation techniques that a bias length of 75 is typical of the average. This, of course, is based on assumptions presented in the attached that seem to be reasonable.

In view of all this, it is recommended that when estimating limits of uncertainty for any inventory, an additional bias uncertainty factor of no less that 25 percent of its corresponding

precision factor be included in the propagation of variance formulas. The purpose of this factor is to cover those biases that we know we cannot even recognize with statistical control charts. This is not to preclude the use of other bias uncertainty factors that we may feel justified in using. A minimum bias length of 75 is recommended when this 25 percent factor is applied. Smaller lengths should be used only when the amount of data available is less than 75 in length.

> Note: Because of number three above, there is good reason to believe that even a percentage of 25 is too small. Although it may be difficult to demonstrate, one larger than 25 percent may not be unreasonable.

It is emphasized that the study was conducted under the assumption that the control chart's center line is unbiased. We know that in the real world this is not true. Now that we are armed with the results of this study, another is planned that takes this fact into account.

The Relative Effects of Precision and Bias Uncertainty on Accountability

Introduction

When dependable standards are available, statistical control charts can be used to detect those biases that are of statistical significance. However, biases can be relatively small and of little or no statistical significance. Unfortunately, even very small biases can have very significant effects on inventory discrepancies. This is very clearly demonstrated by the data shown in Table A (see Appendix A for assistance in interpreting Table A) which were created under the following conditions and/or assumptions (see Diagrams A and B for more detail).

1. Each measuring process is assumed to be stable with its bias behaving over time as a random variable.

2. Randomization is achieved by allowing random variations to occur among bias groups.

3. In addition to a length of twenty-five, bias lengths range from fifty to four hundred in increments of 50.

4. Precision is always assumed to be at least as large as bias uncertainty with their ratios ranging from one to ten in increments of one.

5. Individual measurements carry equal weight.

The relationships shown in this table remain true regardless of:

1. The total number of bias groups, and

2. The total amount of material that is being measured.

As can be seen from Table A, the effects of bias on measuring uncertainty is a very significant amount regardless of the assumptions that are made. In fact, from among the ninety assumptions presented, bias is the major contributor 79 times; is an equal contributor three times; and, is the minor contributor only eight times. Because bias can be such a major contributor, the obvious concern becomes that regarding the probability of being able to detect such biases with the use of statistical control charts. In an attempt to answer this concern, computer simulations were made. The basic approach was to generate, as a base for comparison, large quantities of standard normal data. Also generated was standard normal data with biases introduced that corresponded to conditions presented in Table A.

Items of Explanation

Overview

The purpose of Chart A is to provide the reader with some feel of the biased conditions that are being simulated. The bell shaped curve at the top of the page represents the (standard normal, zero biased) condition. This is the condition against which each of the standard normal, randomly biased conditions are compared. The second line, for example, represents the condition where the standard deviation for precision is 1.0 while the standard deviation for the randomly fluctuating bias is 0.1. A visual comparison of the three standard deviation limits for all distributions are also easily observed by use of this chart.

The Standard Normal, Zero Biased Situations

Table N1 presents a summary of the results obtained when runs of eight were counted for various (standard normal, zero biased) situations. Values presented in this table are used as a base for purposes of comparison against situations of their corresponding (standard normal, randomly-biased) situations. See also the "NO BIAS" lines on Tables R1 through R6 for a listing of the base data. Table N2 presents a similar summary for the result when the number of out-of-control observations are counted. Their corresponding (standard normal, randomly-biased) situations are simulated as shown in Tables S1 through S6. See also the "NO BIAS" lines on these tables.

The Standard Normal, Randomly Biased Situations

There is no way of knowing when, how frequently, or in what behavior pattern randomly fluctuating biases might occur. However, for purposes of this study, the conditions of Table A were used as a starting point for initial simulations. From that point forward, the conditions subsequently simulated were dependent on the results of prior simulations. One might say that many of the simulated conditions were chosen by trial and error. As a matter of record, a total that approaches a billion data points were generated for purposes of this study. Some of this was in the form of repeats for the purpose of confirmation. Some of the data was deemed inappropriate and was discarded. Probably half of this data would have been unnecessary had we known in advance what to expect. About 700 million data points can be accounted for in this report.

TABLE A

The Effects of Precision and Bias Uncertainty on Accountability

Ratio		Number of Measurements per Bias Group	Ratio of Measuring Uncertainty Attributed to Bias to That Attributed to Precision	Limit of Uncertainty Expressed as Percent of Total Uncertainty	
Precision Over Bias Uncertainty	Bias Uncertainty Over Precision			for Precision	for Bias
1	1.000[1]	25	5.00	19.61	98.06
		50	7.07	14.00	99.01
		100	10.00	9.95	99.50
		150	12.20	8.16	99.67
		200	14.10	7.05	99.75
		250	15.80	6.31	99.80
		300	17.20	5.80	99.83
		350	18.60	5.38	99.86
		400	19.80	5.04	99.87
2	0.500	25	2.50	37.14	92.85
		50	3.54	27.22	96.23
		100	5.00	19.61	98.06
		150	6.10	16.17	98.68
		200	7.07	14.00	99.01
		250	7.91	12.55	99.21
		300	8.60	11.55	99.33
		350	9.29	10.71	99.43
		400	9.90	10.05	99.49

[1] Example for interpretation: When the ratio of precision to that of bias uncertainty is 1 to 1 and the bias changes at random with every twenty-five data points, the limit of uncertainty attributed to precision is 19.61 percent of what total uncertainty should be while that attributed to bias uncertainty is 98.06 percent of what total uncertainty should be.

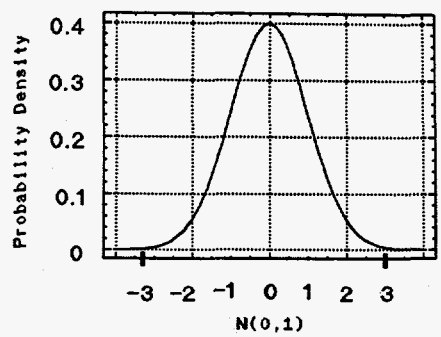
Ratio		Number of Measurements per Bias Group	Ratio of Measuring Uncertainty Attributed to Bias to That Attributed to Precision	Limit of Uncertainty Expressed as Percent of Total Uncertainty	
Precision Over Bias Uncertainty	Bias Uncertainty Over Precision			for Precision	for Bias
3	0.333	25	1.67	51.45	85.75
		50	2.36	39.06	92.06
		100	3.33	28.73	95.78
		150	4.07	23.87	97.11
		200	4.71	20.75	97.82
		250	5.27	18.64	98.25
		300	5.73	17.18	98.51
		350	6.19	15.94	98.72
		400	6.60	14.98	98.87
4	0.250	25	1.25	62.47	78.09
		50	1.77	49.24	87.04
		100	2.50	37.14	92.85
		150	3.05	31.14	95.03
		200	3.54	27.22	96.23
		250	3.95	24.53	96.95
		300	4.30	22.65	97.40
		350	4.64	21.05	97.76
		400	4.95	19.80	98.02
5	0.200	25	1.00	70.71	70.71
		50	1.41	57.74	81.65
		100	2.00	44.72	89.44
		150	2.44	37.90	92.54
		200	2.83	33.33	94.28
		250	3.16	30.15	95.35
		300	3.44	27.91	96.03
		350	3.71	25.99	96.56
		400	3.96	24.49	96.96

Ratio		Number of Measurements per Bias Group	Ratio of Measuring Uncertainty Attributed to Bias to That Attributed to Precision	Limit of Uncertainty Expressed as Percent of Total Uncertainty	
Precision Over Bias Uncertainty	Bias Uncertainty Over Precision			for Precision	for Bias
6	0.166	25	0.83	76.82	64.02
		50	1.18	64.70	76.25
		100	1.67	51.45	85.75
		150	2.03	44.11	89.74
		200	2.36	39.06	92.06
		250	2.64	35.48	93.49
		300	2.87	32.93	94.42
		350	3.10	30.74	95.16
		400	3.30	29.00	95.70
7	0.143	25	0.71	81.37	58.12
		50	1.01	70.35	71.07
		100	1.43	57.35	81.92
		150	1.74	49.75	86.75
		200	2.02	44.36	89.62
		250	2.26	40.48	91.44
		300	2.46	37.69	92.63
		350	2.65	35.27	93.58
		400	2.83	33.33	94.28
8	0.125	25	0.63	84.80	53.00
		50	0.88	74.93	66.23
		100	1.25	62.47	78.09
		150	1.53	54.82	83.64
		200	1.77	49.24	87.04
		250	1.98	45.15	89.23
		300	2.15	42.16	90.68
		350	2.32	39.56	91.84
		400	2.47	37.46	92.72

Ratio		Number of Measurements per Bias Group	Ratio of Measuring Uncertainty Attributed to Bias to That Attributed to Precision	Limit of Uncertainty Expressed as Percent of Total Uncertainty	
Precision Over Bias Uncertainty	Bias Uncertainty Over Precision			for Precision	for Bias
9	0.111	25	0.56	87.42	48.56
		50	0.79	78.63	61.78
		100	1.11	66.90	74.33
		150	1.36	59.34	80.49
		200	1.57	53.69	84.36
		250	1.76	49.47	86.91
		300	1.91	46.35	88.61
		350	2.06	43.61	89.99
		400	2.20	41.38	91.04
10	0.100	25	0.50	89.44	44.72
		50	0.71	81.65	57.74
		100	1.00	70.71	70.71
		150	1.22	63.37	77.36
		200	1.41	57.74	81.65
		250	1.58	53.45	84.52
		300	1.72	50.25	86.46
		350	1.86	47.40	88.05
		400	1.98	45.08	89.26

CHART A

Visual Comparison of Three Standard Deviation Limits



RATIO	STANDARD DEVIATION PRECISION	STANDARD DEVIATION BIAS	
INF	1.0	0.000	
10	1.0	0.100	± 0.300
9	1.0	0.111	± 0.333
8	1.0	0.125	± 0.375
7	1.0	0.143	± 0.429
6	1.0	0.167	± 0.500
5	1.0	0.200	± 0.600
4	1.0	0.250	± 0.750
3	1.0	0.333	± 1.000
2	1.0	0.500	± 1.500
1	1.0	1.000	± 3.000

TABLE N1
RUNS OF EIGHT
Simulations of Statistically-Controlled Situations
Based on 8 Replications of 1,000 Control Charts Each - For Indicated Lengths

Control Chart Length	Number of Data Points		Number of Runs of Eight			
	Each Replication	All Replications	Average	Standard Deviation	3 Std Dev Control Limits	
					Lower	Upper
25	25,000	200,000	74.60	7.95	50.75	98.45
50	50,000	400,000	173.10	13.43	132.81	213.39
100	100,000	800,000	368.16	17.33	316.17	420.15
200	200,000	1,600,000	760.53	24.05	688.38	832.68
300	300,000	2,400,000	1,150.86	32.50	1,053.36	1,248.36
400	400,000	3,200,000	1,543.26	36.85	1,432.71	1,653.81

TABLE N2
THREE SIGMA CONTROL LIMITS
Simulations of Statistically-Controlled Situations
Based on 8 Replications of 1,000 Control Charts Each - For Indicated Lengths

Control Chart Length	Number of Data Points		Number of Values Out of Limits			
	Each Replication	All Replications	Average	Standard Deviation	3 Std Dev Control Limits	
					Lower	Upper
25	25,000	200,000	68.27	8.19	43.70	92.84
50	50,000	400,000	135.75	11.94	99.93	171.57
100	100,000	800,000	271.47	16.13	223.08	319.86
200	200,000	1,600,000	538.56	23.15	469.11	608.01
300	300,000	2,400,000	808.62	30.73	716.43	900.81
400	400,000	3,200,000	1,080.00	36.36	970.92	1,189.08

Analysis

Here at Y-12, either a run of eight successive data points on one side of the centerline or a single value violating the 3-sigma control limits is recognized as a possible out-of-control condition. Because of this, these are the tests that were used when comparing large quantities of data with known biases with large quantities of data with no biases. The results for runs tests are provided in Table R1 through R6 while those for the 3-sigma tests are provided in Tables S1 through S6. The basic approach within each table was to count the number of out-of-control occurrences for various biased situations and to compare these counts with similar counts that were obtained for the (standard normal, zero biased) situation. Contents of the R and S tables are fully described in Appendix B. A study of this appendix is needed for understanding these tables.

For the purpose of better understanding, let us take a look at Table R1. It is noted that, when the ratio of precision uncertainty to bias uncertainty is 4, the Type I error for nonadjusted average is 99.99 percent. This would imply that, under these conditions, we are almost certain to detect the presence of bias. However, let us look at this from another perspective. It is noted that the average number of runs of eight observed for the no biased situation is 74.6 while for the 4.0 precision to bias ratio the average number is 143.1. This would imply that while there was an average of 74.6 false notices, there would appear to be only 68.5 true notices. Since one might suspect fewer true notices than false notices, how can one be sure of which are true and which are false? Even though the probability of seeing runs of eight is 99.99 percent, what is the probability that we will be able to see and recognize them as true notices? One approach is to estimate the Type I error with those that appear to be false notices omitted. As can be seen, the 99.99 percent Type I error would appear to be reduced to 0.33 percent.

One might argue that the 0.33% estimate is too low because some of the most deviant runs might be recognized as true biases. However, it is noted that of the total of 1,000 truly biased charts, we would expect no fewer than 68 and no more than 136 of them to contain a run of eight. Of these, no fewer than 37 and no more than 74 would appear to have been false notices while no fewer than 23 and no more than 69 would appear to have been true notices. With these kinds of probabilities and knowing that all 1,000 were biased to some degree, it appears that the chance of identifying and correcting such biases is very small. Although it would likely be larger than 0.33 percent, it is still small.

It should be noted that for all of the R and S charts, the Type I error based on adjusted averages is very low when the ratio of precision uncertainty to bias uncertainty is no less than 4.0. This would imply that the probability of being able to recognize biases of this magnitude is very small. Hence, such biases are very likely to exist even when valid statistical control charts are being used. It would be reasonable therefore to say that the standard deviation associated with bias uncertainty should be no less than 25 percent of that associated with the standard deviation for precision.

It is further noted that the true value for the center line of a control chart is never known. The chart's center line is simply the average of some initial number of statistically-controlled measurement results. As indicated above the estimate of this value could be in error by an amount that is as large as 25 percent of the standard deviation for individual measurements. However, all out-of-control notices, whether runs of eight or 3-sigma limit violations, are based on some deviation from the center line. Because of this, it can be argued that the identification of true notices becomes even more difficult. If, for example, we made some attempt to correct for every run of eight, many of them would be expected to be in the wrong direction. This adds some additional uncertainty to our ability to correctly identify real biases. Although it may be difficult to prove, it could be argued that the standard deviation associated with bias uncertainty should be even larger than the 25 percent associated with the standard deviation for individual measurements.

Determining Bias Length

As can be seen by Table A, the lengths of biases can have much to do with the relative effect of precision uncertainty to that of bias uncertainty. If, for example, we look at the case where the ratio is 4.0, it can be seen that as bias length increases from 25 to 400, ratios of their relative effects increase from 1.2 to 5.0. In view of this, the question then becomes one of how long one might expect a typical bias to endure before it changes to another level. Keep in mind that we are thinking here of biases that are relatively small and unlikely to be detected with the use of statistical control charts. We are not concerning ourselves with those larger biases that control charts are likely to detect. Because of this, we are unlikely to be taking any kind of corrective action. Such biases simply run their course until some random effect changes them.

The problem then becomes one of how to determine the length of these small and difficult-to-identify biases whose individual lengths are unknown. We cannot even be sure of the form they take (sudden shifts in level, cycles, drifts up and down. etc.). We can, however, make assumptions that are reasonable and with computer simulation obtain estimates of their average lengths. In theory, of course, such biases could be very short or they could exist for the entire length of any program: However, we have assumed that such lengths can vary from 10 up to 400 and that they will be in the form of sudden shifts from one level to another. The next question then becomes that of coming up with some estimate of the relative frequencies of these lengths. Do we expect to see more long bias lengths or more short ones? It was reasoned that the routine changes that are made in the system will tend to make the short bias lengths more frequent. With this in mind, the relative frequencies that were chosen are summarized in Table B. Although it is a matter of opinion, Set 1 of Table B was chosen as the most likely representation of reality. However, lest others disagree, additional relationships as represented by Sets 2-6, were simulated. The simulation can be best explained by example.

Note that in Table C, when using the relationships of Set 1, that an average bias length of 67.81 was obtained when the ratio of precision uncertainty to that of bias uncertainty was 4.0 and an inventory of 5,000 items was used. This was arrived at as follows:

1. The standard deviation for individual measurements was assumed to be 1.0 percent while that for the bias uncertainty was assumed to be one forth of that or 0.25 percent.

2. Each of the 5,000 items were assumed to represent 100 grams of material for a total of 500,000 grams.

3. Bias lengths were allowed to occur randomly with probabilities as depicted in Set 1 of Table B.

4. The number of inventory items was kept constant at 5,000 so that exactly 500,000 grams would be the total amount of material. Hence, the last bias length chosen may have been reduced in order that the 5,000 item limit was maintained.

5. A total of 400 iterations of steps 1 through 4 were made.

6. Limits of uncertainty (± 2 standard deviations) were calculated for each of the 400 iterations. These calculations included estimates of total uncertainty as well as estimates that were attributed to precision and bias individually. For each iteration the percent of total uncertainty attributed to both precision and bias uncertainty were estimated.

   Each of these 400 iterations were repeated a total of ten times. Averages for these ten replications were summarized as follows.

Source of uncertainty	Minimum	Maximum	Average
Percent due to bias	86.35	88.65	87.66
Percent due to precision	41.72	44.86	43.26

Hence the ratio of percent of total uncertainty due to bias fluctuations to the percent of total uncertainty due to precision becomes:

$$87.66 / 43.26 = 2.03$$

Using the a portion from Table A we can find the following:

Ratio		Number of Measurements per Bias Group	Ratio of Measuring Uncertainty Attributed to Bias over that Attributed to Precision	Limit of Uncertainty Expressed as Percent of Total Uncertainty	
Precision Over Bias Uncertainty	Bias Uncertainty Over Precision			for Precision	for Bias
	.	.	.	.	.
4	0.250	50	1.77	49.24	87.04
		100	2.50	37.14	92.85
	.	.	.	.	.
	.	.	.	.	.

With interpolation we find that for the ratio of percent of total uncertainty attributed to bias over the percent of the total uncertainty attributed to precision of 2.03, another position in the table can be estimated as follows:

Ratio		Number of Measurements per Bias Group	Ratio of Measuring Uncertainty Attributed to Bias over that Attributed to Precision	Limit of Uncertainty Expressed as Percent of Total Uncertainty	
Precision Over Bias Uncertainty	Bias Uncertainty Over Precision			for Precision	for Bias
	.	.	.	.	.
4	0.250	50	1.77	49.24	87.04
		67.81	2.03	43.26	87.66
		100	2.50	37.14	92.85
	.	.	.	.	.

It is noted that $\sqrt{(87.66)^2 + (43.26)^2} = 97.75$. This is less than the expected value of 100 due to the use of averages across iterations. However, it is reasonably close.

Simulation Results

Now that the procedure for simulation is understood, look at the six sets in Table B to see the actual simulations that were made. Notice that wide variations were made in the relative weight given to individual bias lengths. Notice, in particular, the difference between Sets 1 and 3. Now look at Table C for estimates of their bias lengths. It seems worthy to note that bias lengths for Set 3 average only about 20 higher than those for Set 1. This is not surprising when one considers the fact that Set 3 gives greater weight to the longer bias lengths. Except for Set 6, which practically eliminates the likelihood of longer sets, all other sets fall within the range of Sets 1 and 3.

The following observations are noted:

1. As the ratio of precision uncertainty to that of bias uncertainty increases so does the simulated bias lengths. This is true for all six sets.

2. Increasing the number of items from 5,000 to 50,000 per iteration does not increase the estimates of bias length.

3. Of the sets simulated, Sets 3 and 5 represent the upper limits for bias lengths while Set 6 represents the lower limit. In the opinion of the authors, these sets are also the least likely conditions to actually exist.

4. Sets 1, 2 and 4 do not differ significantly in their estimates of bias lengths. Their average estimated bias length is about 75. In view of this, it would appear that a minimum bias length of 75 is reasonable to assume for all inventories when estimating the length of bias for those biases that too small to see with the use of control charts.

TABLE B

Relative Frequencies of Bias Lengths

Bias Length	Set 1		Set 2		Set 3		Set 4		Set 5		Set 6	
	Relative Frequency	Weight Fraction	Relative Frequency	Weight Fraction	Relative Frequency	Weight Fraction	Relative Frequency	Weight Fraction	Relative Frequency	Weight Fraction	Relative Frequency	Weight Fraction
400	1	0.02222	1	0.04000	1	0.11111	1	0.00855	1	0.25000	0.15	0.04762
350	2	0.04444	1	0.04000	1	0.11111	4	0.03419	0	0.00000	0	0.00000
300	3	0.06667	2	0.08000	1	0.11111	7	0.05983	0	0.00000	0	0.00000
250	4	0.08889	2	0.08000	1	0.11111	10	0.08547	0	0.00000	0	0.00000
200	5	0.11111	3	0.12000	1	0.11111	13	0.11111	0	0.00000	0	0.00000
150	6	0.13333	3	0.12000	1	0.11111	16	0.13675	0	0.00000	0	0.00000
100	7	0.15556	4	0.16000	1	0.11111	19	0.16239	1	0.25000	1	0.31746
50	8	0.17778	4	0.16000	1	0.11111	22	0.18803	1	0.25000	1	0.31746
25	4.5	0.10000	2.5	0.10000	0.5	0.05556	12.5	0.10684	0.5	0.12500	0.5	0.15873
10	4.5	0.10000	2.5	0.10000	0.5	0.05556	12.5	0.10684	0.5	0.12500	0.5	0.15873
	45	1.00000	25	1.00000	9	1.00000	117	1.00000	4	1.00000	3.15	1.00000

TABLE C

Average Bias Length For 10 Replication of 400 Iterations Each Where Each Iteration Consisted of Either 5,000, 25,000 or 50,000 Data Points and Where Bias Length Occur With the Random Frequencies of Table B

RATIO	SET 1			SET 2			SET 3			SET 4			SET 5			SET 6		
	5,000	25,000	50,000	5,000	25,000	50,000	5,000	25,000	50,000	5,000	25,000	50,000	5,000	25,000	50,000	5,000	25,000	50,000
3	63.83	64.78	65.15	61.89	69.08	68.58	88.32	85.44	87.09	60.09	61.85	60.25	74.03	77.63	78.84	40.34	40.04	39.52
4	67.81	68.17	67.46	66.17	70.40	69.05	88.34	86.57	86.41	63.68	65.01	62.81	80.84	78.13	79.85	42.99	40.69	42.50
5	73.55	75.06	71.21	71.13	76.13	74.68	94.69	90.32	90.78	67.22	70.91	67.44	86.62	87.54	85.90	44.51	44.02	45.02
6	72.62	72.48	72.63	74.43	75.61	75.66	94.52	93.51	94.19	69.91	68.39	71.11	85.56	87.08	88.07	47.69	46.87	45.90
7	75.96	74.12	76.20	73.71	79.07	80.02	96.42	94.76	96.59	70.44	70.93	70.96	89.00	89.44	88.92	47.30	46.64	47.19
8	77.94	76.69	78.74	77.41	81.34	79.65	98.79	96.87	97.14	72.01	75.30	73.67	94.58	94.61	91.86	49.05	50.04	49.18
9	80.50	81.94	81.61	78.96	83.66	83.76	104.45	102.06	103.07	78.09	75.14	75.77	97.50	96.35	97.29	48.86	50.06	48.74
10	81.53	82.28	81.44	82.30	84.95	84.03	105.25	104.52	100.24	74.82	77.29	75.45	98.14	99.00	100.45	50.56	51.42	49.20

Ratio: Ratio of the standard deviation of individuals to that of the bias uncertainty.

APPENDIX A

For A Better Understanding of Table A

This study makes the assumption that bias behaves as a random variable. Given that this is true and that such biases go unrecognized, Table A provides a summary of the effects on the accountability of materials under various biased conditions. The real question, of course, becomes one of whether or not statistical control charts will detect such biases. It is rather obvious that the ability to detect such biases is very dependent on their lengths as well as on the size of their fluctuations relative to the variations of individuals. For this reason, the ratio of the standard deviation for individuals (precision) to the standard deviation for bias uncertainty as well as the bias length are used as criteria on which Table A and the subsequent study are based. The specific criteria chosen are presented in the first three columns of the table.

The last two columns provide an illustration, under the indicated conditions, of the contribution made by precision and bias to the total uncertainty. The only remaining column is simply the ratio of these two values.

If, for example, we say that the precision for a given method is 4 percent, the uncertainty associated with 100 grams obtained by a single measurement using this method is $\pm$ 4 grams. If we then make 50 such measurements for a total of 5,000 grams, the uncertainty on the 5,000 grams attributed to precision would be

$$\sqrt{50[(100 \text{ grams})(\pm\ 0.04)]^2} = \pm\ 28.28 \text{ grams}$$

Now if it is assumed that the ratio of precision uncertainty to that of bias uncertainty is four, the bias uncertainty becomes 1 percent (one fourth of that of precision). If it is further assumed that all 50 measurements have the same bias level, the bias uncertainty associated with this same 5,000 grams would be

$$(5{,}000 \text{ grams})(\pm\ 0.01) = \pm\ 50 \text{ grams}$$

The combined uncertainty for precision uncertainty and bias uncertainty then becomes

$$\sqrt{50[(100 \text{ grams})(\pm\ 0.04)]^2 + [(5{,}000 \text{ grams})(\pm 0.01)]^2} = \pm\ 57.44$$

Now it can be seen that the $\pm$ 50 grams for bias uncertainty is 87 percent of the total while the $\pm$ 28.8 grams for precision uncertainty is 49 percent of the total. This confirms the values in Table A where the ratio of precision to that of bias uncertainty is four and the bias length is 50. Note also that 87/49 = 1.77 confirms the only remaining value on this line of the table.

APPENDIX B

Description of Values Shown in the R and S Tables

1. **RATIO OF BIAS OVER PRECISION UNCERTAINTY (both R & S Tables)**

Ratio of the standard deviation for bias to the standard deviation of precision. The 'No Bias' notation on the last line makes reference to the (standard normal, zero biased) situation. In theory the ratio here should be a value of infinite size.

2a. **AVERAGE NUMBER OF CHARTS WITH RUNS OF 8 (R Tables only)**

Each line in these tables is based on 20 replications of 1,000 control charts each. A total count of the number of successive runs of eight on one side of the (standard normal, zero biased) center line was obtained for each replication. The average number of these 20 replications is presented in the table.

2b. **AVERAGE NUMBER OF CHARTS WITH 3 S VIOLATIONS (S Tables only)**

Each line in these tables is based on 20 replications of 1,000 control charts each. A total count of the number of values outside the 3 sigma control limits for the (standard normal, zero biased) control chart was obtained for each replication. The average number of these 20 replications is presented in the table.

3a. **STANDARD DEVIATION OBSERVED (R Tables only)**

The standard deviation based on the counts produced from the twenty replications for runs of eight discussed in 2a above.

3b. **STANDARD DEVIATION OBSERVED (S Tables only)**

The standard deviation based on the counts produced from the twenty replications for runs of eight discussed in 2b above.

4. **STANDARD DEVIATION USED (both R and S Tables)**

The standard deviation actually used in subsequent calculations. In some cases the observed standard deviation was used and in others a pooled value is used. Such pooling, when based on individuals that do not differ among themselves by significant amounts, provides for better overall estimates and tends to provide a uniformity to the final results.

5. **AVERAGE NUMBER OF CHARTS: MINIMUM & MAXIMUM (both R & S Tables)**

Of the total of 1,000 charts per replication, we know that the average number with the indicated condition fell within these limits. These are not statistical limits. They are based on counts that were observed and/or on the theoretical counts that could have existed. Better planning in our simulation would have given us averages with statistical limits. However, it is hardly worth the vast amount of simulation that would be required to obtain such information.

6. **AVERAGE PERCENT OF CHARTS: MINIMUM & MAXIMUM (both R & S Tables)**

These are the average number of charts mentioned in 5 above, expressed as a percent of the total number of charts (1,000).

7. **ADJUSTED AVERAGE (both R & S Tables)**

For each of the averages discussed in 2a and 2b, the average number of false notices that one might expect is listed on the 'NO BIAS' line of each table. For each table the corresponding number of false notices is subtracted from each of the averages discussed in 2a and 2b. These differences represent the average number of true notices and are presented in the column for Adjusted Average.

8. **t-CAL (both R & S Tables)**

The 'NO BIAS' line for each table is based on a simulation of the (standard normal, zero biased) situation. This is the base against which all of the biased data is compared. The t calculated value is the result of applying the t-test to contrasting the results of the biased data with the corresponding 3-sigma limits of the bias data.

Example of the t calculation shown in Table R1:

		ADJUSTED	STANDARD DEVIATION		3-SIGMA CONTROL LIMITS	
RATIO	AVERAGE	AVERAGE	OBSERVED	USED	LOWER	UPPER
8.00	88.50	13.90	7.83	9.84	58.98	118.02
No Bias	74.60	0.00	7.95	7.95	50.75	98.45

For nonadjusted average

$$t\text{-cal} = \frac{[74.6 + 3*(7.95)] - 88.5}{9.84} = \frac{98.45-88.5}{9.84} = 1.01$$

For adjusted average

$$t\text{-cal} = \frac{[74.6 + 3*(7.95)] - 13.90}{9.84} = \frac{98.45 - 13.90}{9.84} = 8.59$$

Note: Comparison with the 'NO BIAS' lower limit always yields Type II errors near 100 percent and Type I errors near 0 percent. For this reason, they are ignored. This simply means that the applicable test is a one-sided t-test. Testing the other side is of no value since nothing is there.

9. TYPE I and TYPE II (both R & S Tables)

When the t-test is made, a Type II error is committed when a statistical hypothesis is accepted when it is false, and a Type I error is committed when a statistical hypothesis is rejected when it is true. Under the (standard normal, biased) situation, the Type II column represents the probability of failing to conclude that a bias is present when a bias does exist. Under the (standard normal, zero biased) situation, the Type I column represents the probability of concluding that a bias exists when there is no bias.

DIAGRAM A

PERCENT OF TOTAL UNCERTAINTY ATTRIBUTED TO BIAS UNDER THE INDICATED CONDITIONS (Details in Table A)

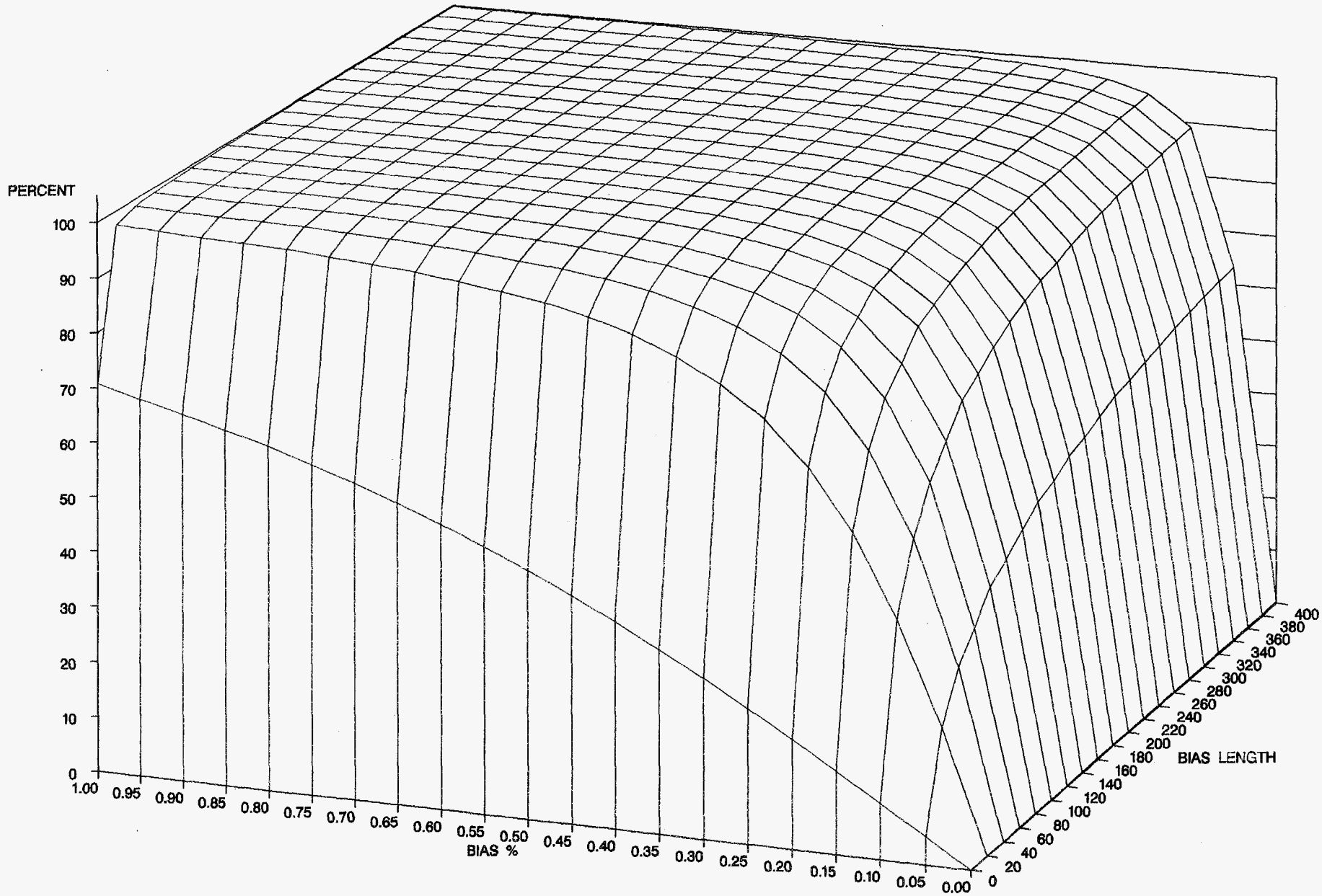


DIAGRAM B

PERCENT OF TOTAL UNCERTAINTY ATTRIBUTED TO PRECISION UNDER THE INDICATED CONDITIONS (Details in Table A)

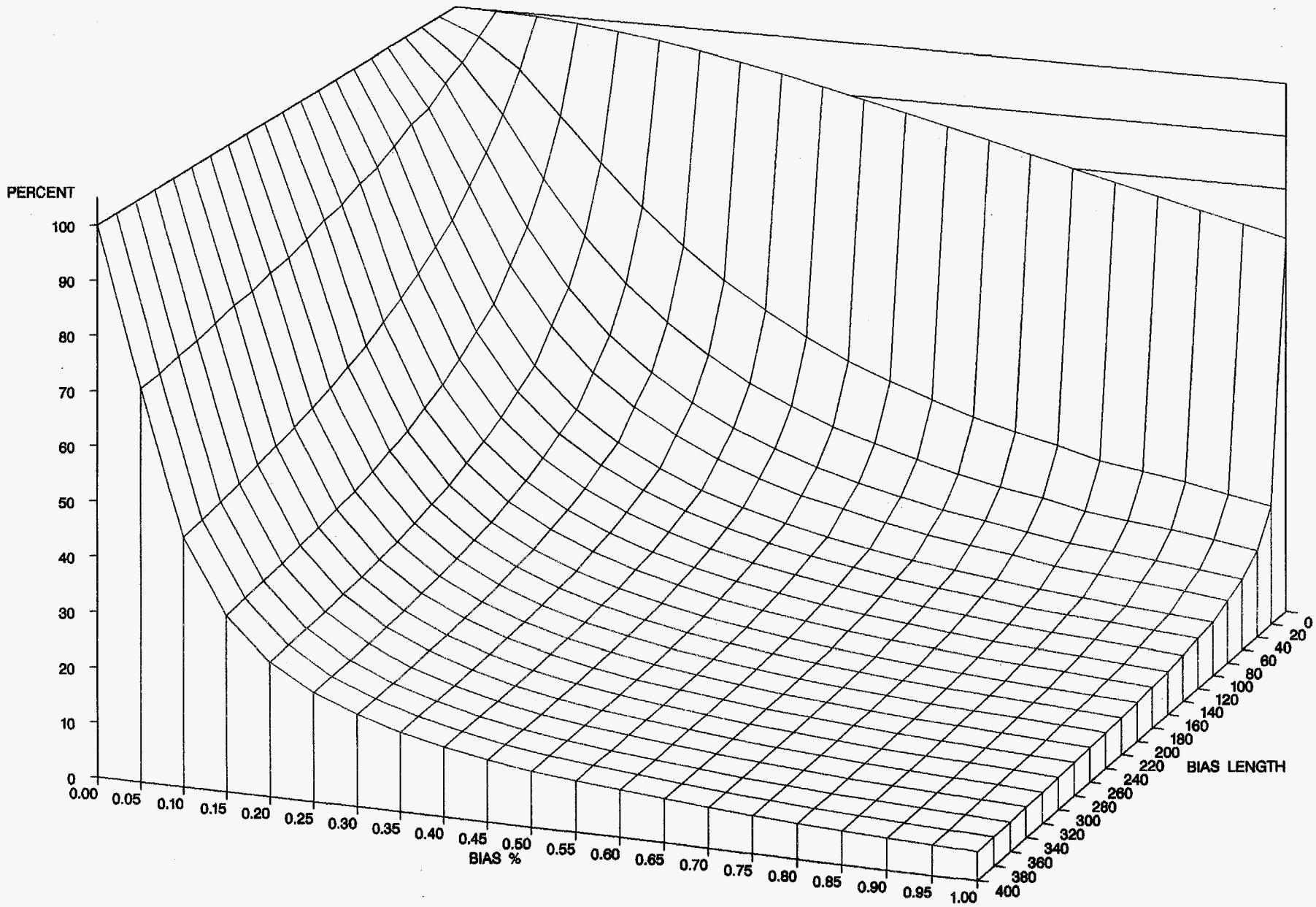


TABLE R1
RUNS OF EIGHT
BASED ON 1,000 BIAS LEVELS OF LENGTH 25 EACH - REPLICATED 20 TIMES

RATIO OF PRECISION OVER BIAS UNCERTAINTY	NUMBER OF RUNS			BASED ON USED STANDARD DEVIATIONS AND AVERAGE NUMBER OF RUNS							ADJUSTED AVERAGE NUMBER OF RUNS	BASED ON USED STANDARD DEVIATIONS AND AVERAGES ADJUSTED FOR NO-BIAS						
				AVG NO. OF CHARTS WITH RUNS OF 8		AVG % OF CHARTS WITH RUNS OF 8		TEST RESULTS FOR NONADJUSTED AVERAGES				AVG NO. OF CHARTS WITH RUNS OF 8		AVG % OF CHARTS WITH RUNS OF 8		TEST RESULTS FOR ADJUSTED AVERAGES		
	AVERAGE	STANDARD DEVIATION																
	# OF RUNS	OBSERVED	USED	MIN	MAX	MIN %	MAX %	t_CAL	TYPE II	TYPE I		ADJ MIN	ADJ MAX	ADJ MIN %	ADJ MAX %	t_CAL	TYPE II	TYPE I
1.00	857.05	28.68	28.68	258	515	25.80%	51.50%	-26.45	0.00%	100.00%	782.45	261	515	26.10%	51.50%	-23.85	0.00%	100.00%
1.50	506.45	26.34	26.34	186	371	18.60%	37.10%	-15.49	0.00%	100.00%	431.85	144	371	14.40%	37.10%	-12.66	0.00%	100.00%
2.00	335.55	17.42	17.42	138	276	13.80%	27.60%	-13.61	0.00%	100.00%	260.95	87	261	8.70%	26.10%	-9.33	0.00%	100.00%
2.50	242.65	14.20	14.20	107	214	10.70%	21.40%	-10.15	0.00%	100.00%	168.05	57	169	5.70%	16.90%	-4.90	0.00%	100.00%
3.00	188.95	10.81	9.84	88	175	8.80%	17.50%	-9.20	0.00%	100.00%	114.35	39	115	3.90%	11.50%	-1.62	6.13%	93.87%
3.50	163.95	11.43	9.84	77	154	7.70%	15.40%	-6.66	0.00%	100.00%	89.35	30	90	3.00%	9.00%	0.93	81.67%	18.33%
4.00	143.10	12.09	9.84	68	136	6.80%	13.60%	-4.54	0.01%	99.99%	68.50	23	69	2.30%	6.90%	3.04	99.67%	0.33%
4.50	125.15	12.65	9.84	60	120	6.00%	12.00%	-2.71	0.69%	99.31%	50.55	17	51	1.70%	5.10%	4.87	99.99%	0.01%
5.20	112.55	11.43	9.84	55	109	5.50%	10.90%	-1.43	8.40%	91.60%	37.95	13	38	1.30%	3.80%	6.15	100.00%	0.00%
5.40	109.20	9.01	9.84	54	107	5.40%	10.70%	-1.09	14.41%	85.59%	34.60	12	35	1.20%	3.50%	6.49	100.00%	0.00%
5.60	105.95	8.43	9.84	52	104	5.20%	10.40%	-0.76	22.76%	77.24%	31.35	11	32	1.10%	3.20%	6.82	100.00%	0.00%
6.20	104.10	9.25	9.84	51	102	5.10%	10.20%	-0.57	28.62%	71.38%	29.50	10	30	1.00%	3.00%	7.01	100.00%	0.00%
6.40	101.70	8.12	9.84	50	100	5.00%	10.00%	-0.33	37.24%	62.76%	27.10	10	28	1.00%	2.80%	7.25	100.00%	0.00%
6.60	99.85	10.00	9.84	49	98	4.90%	9.80%	-0.14	44.42%	55.58%	25.25	9	26	0.90%	2.60%	7.44	100.00%	0.00%
7.50	89.70	9.59	9.84	44	88	4.40%	8.80%	0.89	80.76%	19.24%	15.10	6	16	0.60%	1.60%	8.47	100.00%	0.00%
8.00	88.50	7.83	9.84	44	87	4.40%	8.70%	1.01	83.77%	16.23%	13.90	5	14	0.50%	1.40%	8.59	100.00%	0.00%
8.50	90.25	11.04	9.84	45	89	4.50%	8.90%	0.83	79.26%	20.74%	15.65	6	16	0.60%	1.60%	8.42	100.00%	0.00%
9.00	86.65	8.58	9.84	43	86	4.30%	8.60%	1.20	87.75%	12.25%	12.05	5	13	0.50%	1.30%	8.78	100.00%	0.00%
9.50	83.00	7.66	9.84	41	82	4.10%	8.20%	1.57	93.36%	6.64%	8.40	3	9	0.30%	0.90%	9.15	100.00%	0.00%
10.00	80.80	7.27	9.84	40	80	4.00%	8.00%	1.79	95.56%	4.44%	6.20	3	7	0.30%	0.70%	9.38	100.00%	0.00%
NO BIAS	74.60	7.95	7.95	37	74	3.70%	7.40%	3.00	99.73%	0.27%	0.00	0	0	0.00%	0.00%	12.38	100.00%	0.00%

29,419.26 = POOLED SUMS OF SQUARES ROR RATIOS 3.0 THROUGH 10 INCLUSIVE

96.77386875 = POOLED VARIANCE FOR RATIOS 3.0 THROUGH 10 INCLUSIVE

9.837371028 = POOLED STANDARD DEVIATION FOR RATIOS 3.0 THROUGH 10 INCLUSIVE

98.43 = UPPER CONTROL LIMIT FOR NO BIAS CASE: 74.60+3*8.49

ADJUSTED AVERAGE = OBSERVED AVERAGE - NO BIAS AVERAGE

POOLED DEGREES OF FREEDOM = (20-1)*16 = 304

TYPE II ERROR = PR(FAILING TO REJECT A FALSE HYPOTHESIS)

TYPE I ERROR = PR(REJECTION A TRUE HYPOTHESIS)

TOTAL NUMBER OF DATA POINTS GENERATED PER RATIO = 25*1,000*20 = 500,000

TOTAL NUMBER OF DATA POINTS GENERATED FOR ALL 20 RATIOS = 25*1,000*20*20 = 10,000,000

AVERAGE = AVERAGE NUMBER OF RUNS OF 8

MIN = THE MINIMUM NUMBER OF CHARTS NEEDED TO ACCOUNT FOR THE AVERAGE NUMBER OF RUNS OF 8 (i.e. assuming 3 runs per chart)

MAX = THE MAXIMUM NUMBER OF CHARTS THAT COULD ACCOUNT FOR THE AVERAGE NUMBER OF RUNS OF 8 (i.e. one run per chart)

TABLE R2
RUNS OF EIGHT
BASED ON 1,000 BIAS LEVELS OF LENGTH 50 EACH - REPLICATED 20 TIMES

RATIO OF PRECISION OVER BIAS UNCERTAINTY	NUMBER OF RUNS			BASED ON USED STANDARD DEVIATIONS AND AVERAGE NUMBER OF RUNS							ADJUSTED AVERAGE NUMBER OF RUNS	BASED ON USED STANDARD DEVIATIONS AND AVERAGES ADJUSTED FOR NO-BIAS						
				AVG NO. OF CHARTS WITH RUNS OF 8		AVG % OF CHARTS WITH RUNS OF 8		TEST RESULTS FOR NONADJUSTED AVERAGES				AVG NO. OF CHARTS WITH RUNS OF 8		AVG % OF CHARTS WITH RUNS OF 8		TEST RESULTS FOR ADJUSTED AVERAGES		
	AVERAGE	STANDARD DEVIATION																
	# OF RUNS	OBSERVED	USED	MIN	MAX	MIN %	MAX %	t_CAL	TYPE II	TYPE I		ADJ MIN	ADJ MAX	ADJ MIN %	ADJ MAX %	t_CAL	TYPE II	TYPE I
2.00	733.60	33.11	33.11	227	454	22.70%	45.40%	-15.71	0.00%	100.00%	560.50	95	454	9.50%	45.40%	-10.48	0.00%	100.00%
2.50	537.85	30.48	30.48	191	382	19.10%	38.20%	-10.65	0.00%	100.00%	364.75	61	365	6.10%	36.50%	-4.97	0.00%	100.00%
3.00	437.05	21.26	21.26	168	337	16.80%	33.70%	-10.52	0.00%	100.00%	263.95	44	264	4.40%	26.40%	-2.38	1.40%	98.60%
4.00	320.55	23.32	23.32	138	275	13.80%	27.50%	-4.60	0.01%	99.99%	147.45	25	148	2.50%	14.80%	2.83	99.46%	0.54%
4.50	285.95	16.61	15.37	127	254	12.70%	25.40%	-4.72	0.01%	99.99%	112.85	19	113	1.90%	11.30%	6.54	100.00%	0.00%
5.00	264.95	13.58	15.37	120	240	12.00%	24.00%	-3.36	0.17%	99.83%	91.85	16	92	1.60%	9.20%	7.91	100.00%	0.00%
5.20	253.20	17.85	15.37	115	230	11.50%	23.00%	-2.59	0.90%	99.10%	80.10	14	81	1.40%	8.10%	8.67	100.00%	0.00%
5.40	245.75	14.04	15.37	112	224	11.20%	22.40%	-2.11	2.44%	97.56%	72.65	13	73	1.30%	7.30%	9.16	100.00%	0.00%
5.60	251.25	15.77	15.37	115	231	11.50%	23.10%	-2.46	1.17%	98.83%	78.15	14	79	1.40%	7.90%	8.80	100.00%	0.00%
6.00	234.45	16.81	15.37	108	217	10.80%	21.70%	-1.37	9.33%	90.67%	61.35	11	62	1.10%	6.20%	9.89	100.00%	0.00%
6.20	234.25	19.73	15.37	108	217	10.80%	21.70%	-1.36	9.53%	90.47%	61.15	11	62	1.10%	6.20%	9.91	100.00%	0.00%
6.40	224.05	10.45	15.37	104	208	10.40%	20.80%	-0.69	24.81%	75.19%	50.95	9	51	0.90%	5.10%	10.57	100.00%	0.00%
6.60	222.15	18.18	15.37	104	208	10.40%	20.80%	-0.57	28.77%	71.23%	49.05	9	50	0.90%	5.00%	10.69	100.00%	0.00%
7.00	215.70	18.92	15.37	101	202	10.10%	20.20%	-0.15	44.11%	55.89%	42.60	8	43	0.80%	4.30%	11.11	100.00%	0.00%
7.50	212.65	13.69	15.37	100	200	10.00%	20.00%	0.05	48.10%	51.90%	39.55	7	40	0.70%	4.00%	11.31	100.00%	0.00%
8.00	214.20	10.58	15.37	102	203	10.20%	20.30%	-0.05	47.93%	52.07%	41.10	7	42	0.70%	4.20%	11.21	100.00%	0.00%
9.00	198.60	11.20	15.37	95	189	9.50%	18.90%	0.96	82.60%	17.40%	25.50	5	26	0.50%	2.60%	12.23	100.00%	0.00%
10.00	195.30	13.69	15.37	93	186	9.30%	18.60%	1.18	87.32%	12.68%	22.20	4	23	0.40%	2.30%	12.44	100.00%	0.00%
NO BIAS	173.10	13.43	13.43	84	168	8.40%	16.80%	3.00	99.73%	0.27%	0.00	0	0	0.00%	0.00%	15.89	100.00%	0.00%

62,823.20 = POOLED SUMS OF SQUARES ROR RATIOS 4.5 THROUGH 10 INCLUSIVE

236.1774571 = POOLED VARIANCE FOR RATIOS 4.5 THROUGH 10 INCLUSIVE

15.36806615 = POOLED STANDARD DEVIATION FOR RATIOS 4.5 THROUGH 10 INCLUSIVE

213.39 = UPPER CONTROL LIMIT FOR NO BIAS CASE: 173.10+3*12.312

ADJUSTED AVERAGE = OBSERVED AVERAGE - NO BIAS AVERAGE

POOLED DEGREES OF FREEDOM = (20-1)*14 = 266

TYPE II ERROR = PR(FAILING TO REJECT A FALSE HYPOTHESIS)

TYPE I ERROR = PR(REJECTION A TRUE HYPOTHESIS)

TOTAL NUMBER OF DATA POINTS GENERATED PER RATIO = 50*1,000*20 = 1,000,000

TOTAL NUMBER OF DATA POINTS GENERATED FOR ALL 20 RATIOS = 50*1,000*20*19 = 19,000,000

AVERAGE = AVERAGE NUMBER OF RUNS OF 8

MIN = THE MINIMUM NUMBER OF CHARTS NEEDED TO ACCOUNT FOR THE AVERAGE NUMBER OF RUNS OF 8 (i.e. assuming 6 runs per chart)

MAX = THE MAXIMUM NUMBER OF CHARTS THAT COULD ACCOUNT FOR THE AVERAGE NUMBER OF RUNS OF 8 (i.e. one run per chart)

TABLE R3
RUNS OF EIGHT
BASED ON 1,000 BIAS LEVELS OF LENGTH 100 EACH - REPLICATED 20 TIMES

RATIO OF PRECISION OVER BIAS UNCERTAINTY	NUMBER OF RUNS			BASED ON USED STANDARD DEVIATIONS AND OBSERVED AVERAGES							ADJUSTED AVERAGE NUMBER OF RUNS	BASED ON USED STANDARD DEVIATIONS AND AVERAGES ADJUSTED FOR NO-BIAS						
				AVG NO. OF CHARTS WITH RUNS OF 8		AVG % OF CHARTS WITH RUNS OF 8		TEST RESULTS FOR NONADJUSTED AVERAGES				AVG NO. OF CHARTS WITH RUNS OF 8		AVG % OF CHARTS WITH RUNS OF 8		TEST RESULTS FOR ADJUSTED AVERAGES		
	AVERAGE	STANDARD DEVIATION																
	# OF RUNS	OBSERVED	USED	MIN	MAX	MIN %	MAX %	t_CAL	TYPE II	TYPE I		ADJ MIN	ADJ MAX	ADJ MIN %	ADJ MAX %	t_CAL	TYPE II	TYPE I
1.00	857.05	28.68	28.68	405	809	40.50%	80.90%	-15.23	0.00%	100.00%	488.89	163	809	16.30%	80.90%	-2.40	1.35%	98.65%
1.50	2322.35	78.24	78.24	365	731	36.50%	73.10%	-24.31	0.00%	100.00%	1954.19	163	731	16.30%	73.10%	-19.61	0.00%	100.00%
2.00	1547.40	59.59	59.59	327	654	32.70%	65.40%	-18.92	0.00%	100.00%	1179.24	99	654	9.90%	65.40%	-12.74	0.00%	100.00%
2.50	1154.55	37.65	37.65	303	607	30.30%	60.70%	-19.51	0.00%	100.00%	786.39	66	607	6.60%	60.70%	-9.73	0.00%	100.00%
3.00	919.50	35.00	35.00	278	556	27.80%	55.60%	-14.27	0.00%	100.00%	551.34	46	552	4.60%	55.20%	-3.75	0.07%	99.93%
4.00	679.45	37.65	37.65	244	488	24.40%	48.80%	-6.89	0.00%	100.00%	311.29	26	312	2.60%	31.20%	2.89	99.53%	0.47%
4.50	600.90	26.13	26.13	229	458	22.90%	45.80%	-6.92	0.00%	100.00%	232.74	20	233	2.00%	23.30%	7.17	100.00%	0.00%
5.00	563.90	27.36	23.93	223	446	22.30%	44.60%	-6.01	0.00%	100.00%	195.74	17	196	1.70%	19.60%	9.38	100.00%	0.00%
5.20	549.80	23.45	23.93	219	438	21.90%	43.80%	-5.42	0.00%	100.00%	181.64	16	182	1.60%	18.20%	9.97	100.00%	0.00%
5.40	535.80	22.16	23.93	216	433	21.60%	43.30%	-4.83	0.01%	99.99%	167.64	14	168	1.40%	16.80%	10.55	100.00%	0.00%
5.60	517.90	26.69	23.93	214	427	21.40%	42.70%	-4.09	0.03%	99.97%	149.74	13	150	1.30%	15.00%	11.30	100.00%	0.00%
6.00	503.10	22.50	23.93	210	419	21.00%	41.90%	-3.47	0.13%	99.87%	134.94	12	135	1.20%	13.50%	11.92	100.00%	0.00%
6.20	492.70	22.55	23.93	206	412	20.60%	41.20%	-3.03	0.34%	99.66%	124.54	11	125	1.10%	12.50%	12.35	100.00%	0.00%
6.40	484.20	22.59	23.93	203	406	20.30%	40.60%	-2.68	0.75%	99.25%	116.04	10	117	1.00%	11.70%	12.71	100.00%	0.00%
6.60	487.10	27.73	23.93	206	412	20.60%	41.20%	-2.80	0.57%	99.43%	118.94	10	119	1.00%	11.90%	12.59	100.00%	0.00%
7.00	468.60	29.89	23.93	200	399	20.00%	39.90%	-2.02	2.86%	97.14%	100.44	9	101	0.90%	10.10%	13.36	100.00%	0.00%
7.50	458.50	21.82	23.93	197	393	19.70%	39.30%	-1.60	6.27%	93.73%	90.34	8	91	0.80%	9.10%	13.78	100.00%	0.00%
8.00	447.10	17.23	23.93	195	390	19.50%	39.00%	-1.13	13.70%	86.30%	78.94	7	79	0.70%	7.90%	14.26	100.00%	0.00%
9.00	432.70	19.36	23.93	192	383	19.20%	38.30%	-0.52	30.30%	69.70%	64.54	6	65	0.60%	6.50%	14.86	100.00%	0.00%
10.00	418.40	20.16	23.93	187	374	18.70%	37.40%	0.07	5.75%	94.25%	50.24	5	51	0.50%	5.10%	15.46	100.00%	0.00%
NO BIAS	368.16	17.33	17.33	170	340	17.00%	34.00%	3.00	99.73%	0.27%	0.00	0	0	0.00%	0.00%	24.24	100.00%	0.00%

206,665.15 = POOLED SUMS OF SQUARES ROR RATIOS 5.0 THROUGH 10 INCLUSIVE

572.4796316 = POOLED VARIANCE FOR RATIOS 5.0 THROUGH 10 INCLUSIVE

23.92654659 = POOLED STANDARD DEVIATION FOR RATIOS 5.0 THROUGH 10 INCLUSIVE

420.15 = UPPER CONTROL LIMIT FOR NO BIAS CASE: 368.16+3*19.962

ADJUSTED AVERAGE = OBSERVED AVERAGE - NO BIAS AVERAGE

POOLED DEGREES OF FREEDOM = (20-1)*19 = 361

TYPE II ERROR = PR(FAILING TO REJECT A FALSE HYPOTHESIS)

TYPE I ERROR = PR(REJECTION A TRUE HYPOTHESIS)

TOTAL NUMBER OF DATA POINTS GENERATED PER RATIO = 100*1,000*20 = 2,000,000

TOTAL NUMBER OF DATA POINTS GENERATED FOR ALL 20 RATIOS = 100*1,000*20*28 = 56,000,000

AVERAGE = AVERAGE NUMBER OF RUNS OF 8

MIN = THE MINIMUM NUMBER OF CHARTS NEEDED TO ACCOUNT FOR THE AVERAGE NUMBER OF RUNS OF 8 (i.e. assuming 12 runs per chart)

MAX = THE MAXIMUM NUMBER OF CHARTS THAT COULD ACCOUNT FOR THE AVERAGE NUMBER OF RUNS OF 8 (i.e. one run per chart)

TABLE R4
RUNS OF EIGHT
BASED ON 1,000 BIAS LEVELS OF LENGTH 200 EACH - REPLICATED 20 TIMES

RATIO OF PRECISION OVER BIAS UNCERTAINTY	NUMBER OF RUNS			BASED ON USED STANDARD DEVIATIONS AND OBSERVED AVERAGES							ADJUSTED AVERAGE NUMBER OF RUNS	BASED ON USED STANDARD DEVIATIONS AND AVERAGES ADJUSTED FOR NO-BIAS						
				AVG NO. OF CHARTS WITH RUNS OF 8		AVG % OF CHARTS WITH RUNS OF 8		TEST RESULTS FOR NONADJUSTED AVERAGES				AVG NO. OF CHARTS WITH RUNS OF 8		AVG % OF CHARTS WITH RUNS OF 8		TEST RESULTS FOR ADJUSTED AVERAGES		
	AVERAGE # OF RUNS	STANDARD DEVIATION OBSERVED	STANDARD DEVIATION USED	MIN	MAX	MIN %	MAX %	t_CAL	TYPE II	TYPE I		ADJ MIN	ADJ MAX	ADJ MIN %	ADJ MAX %	t_CAL	TYPE II	TYPE I
1.00	7,952.00	172.85	172.85	463	927	46.30%	92.70%	-41.19	0.00%	100.00%	7,191.47	288	927	28.80%	92.70%	-36.79	0.00%	100.00%
1.50	4,786.55	137.57	137.57	446	892	44.60%	89.20%	-28.74	0.00%	100.00%	4,026.02	162	892	16.20%	89.20%	-23.21	0.00%	100.00%
2.00	3,214.10	113.98	113.98	432	863	43.20%	86.30%	-20.89	0.00%	100.00%	2,453.57	99	863	9.90%	86.30%	-14.22	0.00%	100.00%
2.50	2,384.40	86.53	86.53	419	838	41.90%	83.80%	-17.93	0.00%	100.00%	1,623.87	65	838	6.50%	83.80%	-9.14	0.00%	100.00%
3.00	1,877.55	48.25	48.25	403	806	40.30%	80.60%	-21.66	0.00%	100.00%	1,117.02	45	806	4.50%	80.60%	-5.89	0.00%	100.00%
4.00	1,393.55	35.74	35.74	382	763	38.20%	76.30%	-15.69	0.00%	100.00%	633.02	26	634	2.60%	63.40%	5.59	100.00%	0.00%
5.00	1,160.30	46.26	46.26	367	735	36.70%	73.50%	-7.08	0.00%	100.00%	399.77	16	400	1.60%	40.00%	9.36	100.00%	0.00%
5.20	1,128.60	31.51	31.51	366	731	36.60%	73.10%	-9.39	0.00%	100.00%	368.07	15	369	1.50%	36.90%	14.74	100.00%	0.00%
5.40	1,087.45	39.09	39.09	358	717	35.80%	71.70%	-6.52	0.00%	100.00%	326.92	14	327	1.40%	32.70%	12.94	100.00%	0.00%
5.60	1,087.75	36.97	36.97	364	729	36.40%	72.90%	-6.90	0.00%	100.00%	327.22	14	328	1.40%	32.80%	13.67	100.00%	0.00%
6.00	1,046.95	29.88	28.54	359	718	35.90%	71.80%	-7.51	0.00%	100.00%	286.42	12	287	1.20%	28.70%	19.14	100.00%	0.00%
7.00	955.40	26.87	28.54	350	699	35.00%	69.90%	-4.30	0.02%	99.98%	194.87	8	195	0.80%	19.50%	22.35	100.00%	0.00%
8.00	915.55	32.14	28.54	344	688	34.40%	68.80%	-2.90	0.46%	99.54%	155.02	7	156	0.70%	15.60%	23.75	100.00%	0.00%
8.50	902.85	26.27	28.54	280	559	28.00%	55.90%	-2.46	1.19%	98.81%	142.32	6	143	0.60%	14.30%	24.19	100.00%	0.00%
9.00	882.12	26.57	28.54	333	666	33.30%	66.60%	-1.73	4.97%	95.03%	121.59	5	122	0.50%	12.20%	24.92	100.00%	0.00%
10.00	857.65	31.07	28.54	335	670	33.50%	67.00%	-0.87	80.37%	19.63%	97.12	4	98	0.40%	9.80%	25.77	100.00%	0.00%
NO BIAS	760.53	24.05	24.05	320	640	32.00%	64.00%	3.00	99.73%	0.27%	0.00	0	0	0.00%	0.00%	34.62	100.00%	0.00%

154,740.48 = POOLED SUMS OF SQUARES ROR RATIOS 6.0 THROUGH 10 INCLUSIVE

814.42359 = POOLED VARIANCE FOR RATIOS 6.0 THROUGH 10 INCLUSIVE

28.53810768 = POOLED STANDARD DEVIATION FOR RATIOS 6.0 THROUGH 10 INCLUSIVE

832.68 = UPPER CONTROL LIMIT FOR NO BIAS CASE: 760.53+3*35.262

ADJUSTED AVERAGE = OBSERVED AVERAGE - NO BIAS AVERAGE

POOLED DEGREES OF FREEDOM = (20-1)*10 = 190

TYPE II ERROR = PR(FAILING TO REJECT A FALSE HYPOTHESIS)

TYPE I ERROR = PR(REJECTION A TRUE HYPOTHESIS)

TOTAL NUMBER OF DATA POINTS GENERATED PER RATIO = 200*1,000*20 = 4,000,000

TOTAL NUMBER OF DATA POINTS GENERATED FOR ALL 20 RATIOS = 200*1,000*20*20 = 80,000,000

AVERAGE = AVERAGE NUMBER OF RUNS OF 8

MIN = THE MINIMUM NUMBER OF CHARTS NEEDED TO ACCOUNT FOR THE AVERAGE NUMBER OF RUNS OF 8 (i.e. assuming 25 runs per chart)

MAX = THE MAXIMUM NUMBER OF CHARTS THAT COULD ACCOUNT FOR THE AVERAGE NUMBER OF RUNS OF 8 (i.e. one run per chart)

TABLE R5
RUNS OF EIGHT
BASED ON 1,000 BIAS LEVELS OF LENGTH 300 EACH - REPLICATED 20 TIMES

RATIO OF PRECISION OVER BIAS UNCERTAINTY	NUMBER OF RUNS			BASED ON USED STANDARD DEVIATIONS AND OBSERVED AVERAGES							ADJUSTED AVERAGE NUMBER OF RUNS	BASED ON USED STANDARD DEVIATIONS AND AVERAGES ADJUSTED FOR NO-BIAS						
	AVERAGE	STANDARD DEVIATION		AVG NO. OF CHARTS WITH RUNS OF 8		AVG % OF CHARTS WITH RUNS OF 8		TEST RESULTS FOR NONADJUSTED AVERAGES				AVG NO. OF CHARTS WITH RUNS OF 8		AVG % OF CHARTS WITH RUNS OF 8		TEST RESULTS FOR ADJUSTED AVERAGES		
	# OF RUNS	OBSERVED	USED	MIN	MAX	MIN %	MAX %	t_CAL	TYPE II	TYPE I		ADJ MIN	ADJ MAX	ADJ MIN %	ADJ MAX %	t_CAL	TYPE II	TYPE I
1.00	11,927.00	377.32	377.32	496	993	49.60%	99.30%				10,776.14	292	993	29.20%	99.30%	-25.25	0.00%	100.00%
1.50	7,121.00	245.80	245.8	492	984	49.20%	98.40%	-23.89	0.00%	100.00%	5,970.14	162	984	16.20%	98.40%	-19.21	0.00%	100.00%
2.00	4,830.00	166.00	166.00	488	977	48.80%	97.70%	-21.58	0.00%	100.00%	3,679.14	100	977	10.00%	97.70%	-14.64	0.00%	100.00%
2.50	3,588.00	190.15	190.15	486	971	48.60%	97.10%	-12.30	0.00%	100.00%	2,437.14	66	971	6.60%	97.10%	-6.25	0.00%	100.00%
3.00	2,854.00	104.80	104.80	481	962	48.10%	96.20%	-15.32	0.00%	100.00%	1,703.14	47	962	4.70%	96.20%	-4.34	0.02%	99.98%
3.50	2,429.00	85.03	85.03	482	963	48.20%	96.30%	-13.88	0.00%	100.00%	1,278.14	35	963	3.50%	96.30%	-0.35	63.50%	36.50%
4.00	2,108.00	55.27	55.27	472	943	47.20%	94.30%	-15.55	0.00%	100.00%	957.14	26	943	2.60%	94.30%	5.27	100.00%	0.00%
5.00	1,770.95	64.39	64.39	470	939	47.00%	93.90%	-8.12	0.00%	100.00%	620.09	17	629	1.70%	62.90%	9.76	100.00%	0.00%
5.20	1,716.05	35.63	35.63	467	934	46.70%	93.40%	-13.13	0.00%	100.00%	565.19	16	566	1.60%	56.60%	19.17	100.00%	0.00%
5.40	1,670.95	51.34	51.34	466	932	46.60%	93.20%	-8.23	0.00%	100.00%	520.09	15	521	1.50%	52.10%	14.19	100.00%	0.00%
5.60	1,633.05	41.04	41.04	464	928	46.40%	92.80%	-9.37	0.00%	100.00%	482.19	14	483	1.40%	48.30%	18.67	100.00%	0.00%
6.00	1,580.05	49.76	49.76	464	927	46.40%	92.70%	-6.67	0.00%	100.00%	429.19	12	430	1.20%	43.00%	16.46	100.00%	0.00%
7.00	1,464.60	36.70	35.33	460	919	46.00%	91.90%	-6.12	0.00%	100.00%	313.74	9	314	0.90%	31.40%	26.45	100.00%	0.00%
8.00	1,385.40	37.72	35.33	457	914	45.70%	91.40%	-3.88	0.05%	99.95%	234.54	7	235	0.70%	23.50%	28.69	100.00%	0.00%
9.00	1,336.08	37.62	35.33	454	908	45.40%	90.80%	-2.48	1.13%	98.87%	185.22	6	186	0.60%	18.60%	30.09	100.00%	0.00%
9.30	1,322.90	45.01	35.33	454	908	45.40%	90.80%	-2.11	2.42%	97.58%	172.04	5	173	0.50%	17.30%	30.46	100.00%	0.00%
9.70	1,322.75	25.96	35.33	456	912	45.60%	91.20%	-2.11	2.44%	97.56%	171.89	5	172	0.50%	17.20%	30.47	100.00%	0.00%
10	1,290.83	35.91	35.91	452	903	45.20%	90.30%	-1.18	12.58%	87.42%	139.97	4	140	0.40%	14.00%	30.87	100.00%	0.00%
NO BIAS	1,150.86	32.5	32.5	441	883	44.10%	88.30%	3.00	99.73%	0.27%	0.00	0	0	0.00%	0.00%	38.41	100.00%	0.00%

258,237.01 = POOLED SUMS OF SQUARES ROR RATIOS 3.0 THROUGH 10 INCLUSIVE

1359.14216 = POOLED VARIANCE FOR RATIOS 3.0 THROUGH 10 INCLUSIVE

35.33213834 = POOLED STANDARD DEVIATION FOR RATIOS 3.0 THROUGH 10 INCLUSIVE

1248.36 = UPPER CONTROL LIMIT FOR NO BIAS CASE: 1,150.86+3*50.562

ADJUSTED AVERAGE = OBSERVED AVERAGE - NO BIAS AVERAGE

POOLED DEGREES OF FREEDOM = (20-1)*10 = 190

TYPE II ERROR = PR(FAILING TO REJECT A FALSE HYPOTHESIS)

TYPE I ERROR = PR(REJECTION A TRUE HYPOTHESIS)

TOTAL NUMBER OF DATA POINTS GENERATED PER RATIO = 300*1,000*20 = 6,000,000

TOTAL NUMBER OF DATA POINTS GENERATED FOR ALL 13 RATIOS = 300*1,000*20*18 = 108,000,000

AVERAGE = AVERAGE NUMBER OF RUNS OF 8

MIN = THE MINIMUM NUMBER OF CHARTS NEEDED TO ACCOUNT FOR THE AVERAGE NUMBER OF RUNS OF 8 (i.e. assuming 37 runs per chart)

MAX = THE MAXIMUM NUMBER OF CHARTS THAT COULD ACCOUNT FOR THE AVERAGE NUMBER OF RUNS OF 8 (i.e. one run per chart)

TABLES R6
RUNS OF EIGHT
BASED ON 1,000 BIAS LEVELS OF LENGTH 400 EACH - REPLICATED 20 TIMES

RATIO OF PRECISION OVER BIAS UNCERTAINTY	NUMBER OF RUNS			BASED ON USED STANDARD DEVIATIONS AND OBSERVED AVERAGES							ADJUSTED AVERAGE NUMBER OF RUNS	BASED ON USED STANDARD DEVIATIONS AND AVERAGES ADJUSTED FOR NO-BIAS						
				AVG NO. OF CHARTS WITH RUNS OF 8		AVG % OF CHARTS WITH RUNS OF 8		TEST RESULTS FOR NONADJUSTED AVERAGES				AVG NO. OF CHARTS WITH RUNS OF 8		AVG % OF CHARTS WITH RUNS OF 8		TEST RESULTS FOR ADJUSTED AVERAGES		
	AVERAGE # OF RUNS	STANDARD DEVIATION OBSERVED	STANDARD DEVIATION USED	MIN	MAX	MIN %	MAX %	t_CAL	TYPE II	TYPE I		ADJ MIN	ADJ MAX	ADJ MIN %	ADJ MAX %	t_CAL	TYPE II	TYPE I
0.50	28,729.00	555.00	555.00	513	1000	51.30%	100.00%	-48.78	0.00%	100.00%	27,185.74	544	1000	54.40%	100.00%	-46.00	0.00%	100.00%
1.50	9,598.00	321.00	321.00	522	1000	52.20%	100.00%	-24.75	0.00%	100.00%	8,054.74	162	1000	16.20%	100.00%	-19.94	0.00%	100.00%
2.00	6,431.20	194.38	194.38	532	1000	53.20%	100.00%	-24.58	0.00%	100.00%	4,887.94	98	1000	9.80%	100.00%	-16.64	0.00%	100.00%
3.00	3,832.95	99.11	99.11	542	1000	54.20%	100.00%	-21.99	0.00%	100.00%	2,289.69	46	1000	4.60%	100.00%	-6.42	0.00%	100.00%
4.00	2,856.85	64.63	64.63	536	1000	53.60%	100.00%	-18.61	0.00%	100.00%	1,313.59	27	1000	2.70%	100.00%	5.26	100.00%	0.00%
5.00	2,363.15	74.06	74.06	546	1000	54.60%	100.00%	-9.58	0.00%	100.00%	819.89	17	820	1.70%	82.00%	11.26	100.00%	0.00%
6.00	2,100.90	31.55	31.55	544	1000	54.40%	100.00%	-14.17	0.00%	100.00%	557.64	12	558	1.20%	55.80%	34.74	100.00%	0.00%
7.00	1,959.00	42.41	49.23	544	1000	54.40%	100.00%	-6.20	0.00%	100.00%	415.74	9	416	0.90%	41.60%	25.15	100.00%	0.00%
8.00	1,862.65	59.54	49.23	546	1000	54.60%	100.00%	-4.24	0.02%	99.98%	319.39	7	320	0.70%	32.00%	27.11	100.00%	0.00%
9.00	1,791.95	31.70	49.23	548	1000	54.80%	100.00%	-2.81	0.56%	99.44%	248.69	5	249	0.50%	24.90%	28.54	100.00%	0.00%
10.00	1,749.95	57.84	49.23	544	1000	54.40%	100.00%	-1.95	3.29%	96.71%	206.69	5	207	0.50%	20.70%	29.40	100.00%	0.00%
NO BIAS	1,543.26	36.85	36.85	540	1000	54.00%	100.00%	3.00	99.73%	0.27%	0.00	0	0	0.00%	0.00%	44.88	100.00%	0.00%

184,185.53 = POOLED SUMS OF SQUARES ROR RATIOS 7.0 THROUGH 10 INCLUSIVE

2423.493825 = POOLED VARIANCE FOR RATIOS 7.0 THROUGH 10 INCLUSIVE

49.22899374 = POOLED STANDARD DEVIATION FOR RATIOS 7.0 THROUGH 10 INCLUSIVE

1653.81 = UPPER CONTROL LIMIT FOR NO BIAS CASE: 1,543.26+3*65.862

ADJUSTED AVERAGE = OBSERVED AVERAGE - NO BIAS AVERAGE

POOLED DEGREES OF FREEDOM = (20-1)*4 = 76

TYPE II ERROR = PR(FAILING TO REJECT A FALSE HYPOTHESIS)

TYPE I ERROR = PR(REJECTION A TRUE HYPOTHESIS)

TOTAL NUMBER OF DATA POINTS GENERATED PER RATIO = 400*1,000*20 = 8,000,000

TOTAL NUMBER OF DATA POINTS GENERATED FOR ALL 12 RATIOS = 400*1,000*20*13 = 104,000,000

AVERAGE = AVERAGE NUMBER OF RUNS OF 8

MIN = THE MINIMUM NUMBER OF CHARTS NEEDED TO ACCOUNT FOR THE AVERAGE NUMBER OF RUNS OF 8 (i.e. assuming 50 runs per chart)

MAX = THE MAXIMUM NUMBER OF CHARTS THAT COULD ACCOUNT FOR THE AVERAGE NUMBER OF RUNS OF 8 (i.e. one run per chart)

TABLE S1
3 SIGMA CONTROL LIMITS
BASED ON 1,000 BIAS LEVELS OF LENGTH 25 EACH - REPLICATED 20 TIMES

RATIO OF PRECISION OVER BIAS UNCERTAINTY	NUMBER OF 3 SIGMA VIOLATIONS			BASED ON USED STD DEV AND AVG NUMBER OF CHARTS OUT OF 3 SIGMA LIMITS							ADJUSTED AVERAGE NUMBER OUT	BASED ON USED STANDARD DEVIATIONS AND AVERAGES ADJUSTED FOR NO-BIAS						
				AVG NO. OF CHARTS WITH 3S VIOLATIONS		AVG % OF CHARTS WITH 3S VIOLATIONS		TEST RESULTS FOR NONADJUSTED AVERAGES				AVG NO. OF CHARTS WITH 3S VIOLATIONS		AVG % OF CHARTS WITH 3S VIOLATIONS		TEST RESULTS FOR ADJUSTED AVERAGES		
	AVERAGE No. OUT	STD DEVIATION OBSERVED	STD DEVIATION USED	MIN	MAX	MIN %	MAX %	t_CAL	TYPE II	TYPE I		ADJ MIN	ADJ MAX	ADJ MIN %	ADJ MAX %	t_CAL	TYPE II	TYPE I
1.00	841.60	64.92	64.92	176	353	17.60%	35.30%	-11.53	0.00%	100.00%	773.33	31	353	3.10%	35.30%	-10.48	0.00%	100.00%
1.50	314.30	25.79	25.79	110	219	11.00%	21.90%	-8.59	0.00%	100.00%	246.03	10	219	1.00%	21.90%	-5.94	0.00%	100.00%
2.00	188.55	9.85	11.12	80	159	8.00%	15.90%	-8.61	0.00%	100.00%	120.28	5	121	0.50%	12.10%	-2.47	1.16%	98.84%
2.50	135.60	12.84	11.12	62	123	6.20%	12.30%	-3.85	0.05%	99.95%	67.33	3	68	0.30%	6.80%	2.29	98.33%	1.67%
3.00	112.35	11.11	11.12	52	105	5.20%	10.50%	-1.75	4.77%	95.23%	44.08	2	45	0.20%	4.50%	4.39	99.98%	0.02%
3.50	97.75	11.80	11.12	46	93	4.60%	9.30%	-0.44	33.19%	66.81%	29.48	2	30	0.20%	3.00%	5.70	100.00%	0.00%
4.00	90.15	11.69	11.12	43	86	4.30%	8.60%	0.24	59.43%	40.57%	21.88	1	22	0.10%	2.20%	6.38	100.00%	0.00%
4.50	89.20	8.95	11.12	44	87	4.40%	8.70%	0.33	62.65%	37.35%	20.93	1	21	0.10%	2.10%	6.47	100.00%	0.00%
5.20	84.40	11.13	11.12	41	82	4.10%	8.20%	0.76	77.15%	22.85%	16.13	1	17	0.10%	1.70%	6.90	100.00%	0.00%
5.40	81.15	6.95	7.39	40	79	4.00%	7.90%	1.58	93.48%	6.52%	12.88	1	13	0.10%	1.30%	10.81	100.00%	0.00%
5.60	79.95	9.32	7.39	39	78	3.90%	7.80%	1.74	95.13%	4.87%	11.68	1	12	0.10%	1.20%	10.98	100.00%	0.00%
6.20	78.10	5.87	7.39	38	77	3.80%	7.70%	1.99	96.96%	3.04%	9.83	1	10	0.10%	1.00%	11.23	100.00%	0.00%
6.40	77.85	9.34	7.39	38	76	3.80%	7.60%	2.03	97.15%	2.85%	9.58	1	10	0.10%	1.00%	11.26	100.00%	0.00%
6.60	77.05	7.83	7.39	38	76	3.80%	7.60%	2.14	97.70%	2.30%	8.78	1	9	0.10%	0.90%	11.37	100.00%	0.00%
7.50	68.20	7.71	7.39	34	67	3.40%	6.70%	3.33	99.82%	0.18%	-0.07	-1	-1	-0.10%	-0.10%	12.57	100.00%	0.00%
8.00	73.10	8.71	7.39	36	72	3.60%	7.20%	2.67	99.24%	0.76%	4.83	1	5	0.10%	0.50%	11.90	100.00%	0.00%
8.50	69.95	7.37	7.39	35	69	3.50%	6.90%	3.10	99.70%	0.30%	1.68	1	2	0.10%	0.20%	12.33	100.00%	0.00%
9.00	71.90	6.73	7.39	35	70	3.50%	7.00%	2.83	99.47%	0.53%	3.63	1	4	0.10%	0.40%	12.06	100.00%	0.00%
9.50	67.95	6.32	7.39	34	67	3.40%	6.70%	3.37	99.84%	0.16%	-0.32	-1	-1	-0.10%	-0.10%	12.60	100.00%	0.00%
10.00	71.45	7.53	7.39	35	70	3.50%	7.00%	2.89	99.53%	0.47%	3.18	1	4	0.10%	0.40%	12.13	100.00%	0.00%
NO-BAIS	68.27	8.19	8.19	34	67	3.40%	6.70%	3.00	99.73%	0.27%	0.00	0	0	0.00%	0.00%	11.34	100.00%	0.00%

16,438.72	= 1ST POOLED SUMS OF SQUARES ROR RATIOS 2.0 THROUGH 5.2 INCLUSIVE
123.5993857	= 1ST POOLED VARIANCE FOR RATIOS 2.0 THROUGH 5.2 INCLUSIVE
11.11752606	= 1ST POOLED STANDARD DEVIATION FOR RATIOS 2 THROUGH 5.2 INCLUSIVE
133	1st POOLED DEGREES OF FREEDOM = (20-1)*7 = 133
11,427.08	= 2nd POOLED SUMS OF SQUARES ROR RATIOS 5.4 THROUGH 10.0 INCLUSIVE
54.67500909	= 2nd POOLED VARIANCE FOR RATIOS 5.4 THROUGH 10 INCLUSIVE
7.394255141	= 2nd POOLED STANDARD DEVIATION FOR RATIOS 5.4 THROUGH 10 INCLUSIVE
209	2nd POOLED DEGREES OF FREEDOM = (20-1)*11 = 209
92.84	= UPPER CONTROL LIMIT FOR NO BIAS CASE: 68.27+3*8.21
	ADJUSTED AVERAGE = OBSERVED AVERAGE - NO BIAS AVERAGE

TYPE II ERROR = PR(FAILING TO REJECT A FALSE HYPOTHESIS)

TYPE I ERROR = PR(REJECTION A TRUE HYPOTHESIS)

TOTAL NUMBER OF DATA POINTS GENERATED PER RATIO = 25*1,000*20 = 500,000

TOTAL NUMBER OF DATA POINTS GENERATED FOR ALL 20 RATIOS = 25*1,000*20*20 = 10,000,000

NUMBER OF DATA POINTS EXPECTED TO VIOLATE 3 SIGMA LIMITS WHEN NO BIAS IS PRESENT = 68

[Pr(|z|>3.0)]*[25*1,000] = {0.002713]*[25,000] = 67.825

TABLE S2
3 SIGMA CONTROL LIMITS
BASED ON 1000 BIAS LEVELS OF LENGTH 50 EACH - REPLICATED 20 TIMES

RATIO OF				BASED ON USED STD DEV AND AVG NUMBER OF CHARTS OUT OF 3 SIGMA LIMITS							ADJUSTED	BASED ON USED STANDARD DEVIATIONS AND AVERAGES ADJUSTED FOR NO-BIAS						
PRECISION	NUMBER OF 3 SIGMA VIOLATIONS			AVG NO. OF CHARTS		AVG % OF CHARTS		TEST RESULTS FOR			AVERAGE	AVG NO. OF CHARTS		AVG % OF CHARTS		TEST RESULTS FOR		
OVER BIAS	AVERAGE	STANDARD DEVIATION		WITH 3S VIOLATIONS		WITH 3S VIOLATIONS		NONADJUSTED AVERAGES			NUMBER	WITH 3S VIOLATIONS		WITH 3S VIOLATIONS		ADJUSTED AVERAGES		
UNCERTAINTY	No. OUT	OBSERVED	USED	MIN	MAX	MIN %	MAX %	t_CAL	TYPE II	TYPE I	OUT	ADJ MIN	ADJ MAX	ADJ MIN %	ADJ MAX %	t_CAL	TYPE II	TYPE I
1.50	625.30	41.01	41.01	175	351	17.50%	35.10%	-11.06	0.00%	100.00%	489.55	10	351	1.00%	35.10%	-7.75	0.00%	100.00%
2.00	366.25	17.42	17.42	136	271	13.60%	27.10%	-11.18	0.00%	100.00%	230.50	5	231	0.50%	23.10%	-3.38	0.16%	99.84%
2.50	268.25	22.50	22.50	111	222	11.10%	22.20%	-4.30	0.02%	99.98%	132.50	3	133	0.30%	13.30%	1.74	95.07%	4.93%
3.00	220.90	13.44	12.71	98	196	9.80%	19.60%	-3.88	0.05%	99.95%	85.15	2	86	0.20%	8.60%	6.80	100.00%	0.00%
4.00	175.20	13.33	12.71	82	163	8.20%	16.30%	-0.29	38.91%	61.09%	39.45	1	40	0.10%	4.00%	10.40	100.00%	0.00%
4.50	170.05	15.09	12.71	79	159	7.90%	15.90%	0.12	45.30%	54.70%	34.30	1	35	0.10%	3.50%	10.80	100.00%	0.00%
5.00	165.30	11.50	12.71	78	157	7.80%	15.70%	0.49	68.63%	31.37%	29.55	1	30	0.10%	3.00%	11.18	100.00%	0.00%
5.20	162.15	9.32	12.71	77	154	7.70%	15.40%	0.74	76.62%	23.38%	26.40	1	27	0.10%	2.70%	11.43	100.00%	0.00%
5.40	156.90	14.75	12.71	75	149	7.50%	14.90%	1.15	86.87%	13.13%	21.15	1	22	0.10%	2.20%	11.84	100.00%	0.00%
5.60	158.35	10.85	12.71	76	152	7.60%	15.20%	1.04	84.44%	15.56%	22.60	1	23	0.10%	2.30%	11.72	100.00%	0.00%
6.00	156.80	13.19	12.71	75	150	7.50%	15.00%	1.16	87.03%	12.97%	21.05	1	22	0.10%	2.20%	11.85	100.00%	0.00%
6.20	150.05	11.67	12.71	71	143	7.10%	14.30%	1.69	94.67%	5.33%	14.30	1	15	0.10%	1.50%	12.38	100.00%	0.00%
6.40	147.95	14.66	12.71	71	142	7.10%	14.20%	1.86	96.07%	3.93%	12.20	1	13	0.10%	1.30%	12.54	100.00%	0.00%
6.60	149.50	13.39	12.71	71	142	7.10%	14.20%	1.74	95.07%	4.93%	13.75	1	14	0.10%	1.40%	12.42	100.00%	0.00%
7.00	148.60	15.69	12.71	71	143	7.10%	14.30%	1.81	95.68%	4.32%	12.85	1	13	0.10%	1.30%	12.49	100.00%	0.00%
7.50	143.70	11.71	12.71	70	139	7.00%	13.90%	2.19	97.95%	2.05%	7.95	1	8	0.10%	0.80%	12.88	100.00%	0.00%
8.00	147.05	10.81	12.71	71	141	7.10%	14.10%	1.93	96.57%	3.43%	11.30	1	12	0.10%	1.20%	12.61	100.00%	0.00%
9.00	138.45	11.47	12.71	67	133	6.70%	13.30%	2.61	99.13%	0.87%	2.70	1	3	0.10%	0.30%	13.29	100.00%	0.00%
10.00	144.45	10.31	12.71	70	140	7.00%	14.00%	2.13	97.70%	2.30%	8.70	1	9	0.10%	0.90%	12.82	100.00%	0.00%
NO-BIAS	135.75	11.94	11.94	66	131	6.60%	13.10%	3.00	99.73%	0.27%	0.00	0	0	0.00%	0.00%	14.37	100.00%	0.00%

49,073.47 = 1ST POOLED SUMS OF SQUARES ROR RATIOS 3.0 THROUGH 10.0 INCLUSIVE

161.425875 = 1ST POOLED VARIANCE FOR RATIOS 3.0 THROUGH 10 INCLUSIVE

12.70534828 = 1ST POOLED STANDARD DEVIATION FOR RATIOS 3 THROUGH 10 INCLUSIVE

1st POOLED DEGREES OF FREEDOM = (20-1)*16 = 304

171.57 = UPPER CONTROL LIMIT FOR NO BIAS CASE: 135.75+3*11.46

ADJUSTED AVERAGE = OBSERVED AVERAGE - NO BIAS AVERAGE

TYPE II ERROR = PR(FAILING TO REJECT A FALSE HYPOTHESIS)

TYPE I ERROR = PR(REJECTION A TRUE HYPOTHESIS)

TOTAL NUMBER OF DATA POINTS GENERATED PER RATIO = 50*1,000*20 = 1,000,000

TOTAL NUMBER OF DATA POINTS GENERATED FOR ALL 20 RATIOS = 50*1,000*20*20 = 20,000,000

NUMBER OF DATA POINTS EXPECTED TO VIOLATE 3 SIGMA LIMITS WHEN NO BIAS IS PRESENT = 136

[Pr(|z|>3.0)]*[50*1,000] = [0.002713]*[50,000] = 135.65

TABLE S3
3 SIGMA CONTROL LIMITS
BASED ON 1000 BIAS LEVELS OF LENGTH 100 EACH - REPLICATED 20 TIMES

RATIO OF PRECISION OVER BIAS UNCERTAINTY	NUMBER OF 3 SIGMA VIOLATIONS			BASED ON USED STD DEV AND AVG NUMBER OF CHARTS OUT OF 3 SIGMA LIMITS							ADJUSTED AVERAGE NUMBER OUT	BASED ON USED STANDARD DEVIATIONS AND AVERAGES ADJUSTED FOR NO-BIAS						
				AVG NO. OF CHARTS WITH 3S VIOLATIONS		AVG % OF CHARTS WITH 3S VIOLATIONS		TEST RESULTS FOR NONADJUSTED AVERAGES				AVG NO. OF CHARTS WITH 3S VIOLATIONS		AVG % OF CHARTS WITH 3S VIOLATIONS		TEST RESULTS FOR ADJUSTED AVERAGES		
	AVERAGE No. OUT	STANDARD DEVIATION OBSERVED	STANDARD DEVIATION USED	MIN	MAX	MIN %	MAX %	t_CAL	TYPE II	TYPE I		ADJ MIN	ADJ MAX	ADJ MIN %	ADJ MAX %	t_CAL	TYPE II	TYPE I
1.00	3,440.20	205.81	205.81	322	645	32.20%	64.50%	-15.16	0.00%	100.00%	3,168.73	32	645	3.20%	64.50%	-13.84	0.00%	100.00%
1.50	1,264.10	69.80	69.80	261	523	26.10%	52.30%	-13.53	0.00%	100.00%	992.63	10	523	1.00%	52.30%	-9.64	0.00%	100.00%
2.00	721.33	47.89	47.89	220	439	22.00%	43.90%	-8.38	0.00%	100.00%	449.86	5	439	0.50%	43.90%	-2.71	0.69%	99.31%
2.50	528.85	29.71	29.71	194	389	19.40%	38.90%	-7.03	0.00%	100.00%	257.38	3	258	0.30%	25.80%	2.10	97.55%	2.45%
3.00	449.10	23.19	23.19	178	356	17.80%	35.60%	-5.57	0.00%	100.00%	177.63	2	178	0.20%	17.80%	6.13	100.00%	0.00%
4.00	366.33	18.80	17.37	156	312	15.60%	31.20%	-2.68	0.75%	99.25%	94.86	1	95	0.10%	9.50%	12.95	100.00%	0.00%
4.50	340.70	17.41	17.37	150	300	15.00%	30.00%	-1.20	12.25%	87.75%	69.23	1	70	0.10%	7.00%	14.43	100.00%	0.00%
5.00	323.45	17.02	17.37	145	290	14.50%	29.00%	-0.21	58.08%	41.92%	51.98	1	52	0.10%	5.20%	15.42	100.00%	0.00%
5.20	324.85	16.68	17.37	146	291	14.60%	29.10%	-0.29	38.85%	61.15%	53.38	1	54	0.10%	5.40%	15.34	100.00%	0.00%
5.40	323.00	19.55	17.37	146	291	14.60%	29.10%	-0.18	57.08%	42.92%	51.53	1	52	0.10%	5.20%	15.45	100.00%	0.00%
5.60	320.68	11.96	17.37	147	294	14.70%	29.40%	-0.05	51.86%	48.14%	49.21	1	50	0.10%	5.00%	15.58	100.00%	0.00%
6.00	308.52	17.38	17.37	138	276	13.80%	27.60%	0.65	73.92%	26.08%	37.05	1	38	0.10%	3.80%	16.28	100.00%	0.00%
6.20	303.20	17.74	17.37	138	276	13.80%	27.60%	0.96	82.52%	17.48%	31.73	1	32	0.10%	3.20%	16.59	100.00%	0.00%
6.40	305.95	15.62	17.37	140	280	14.00%	28.00%	0.80	78.34%	21.66%	34.48	1	35	0.10%	3.50%	16.43	100.00%	0.00%
6.60	301.00	17.33	17.37	137	274	13.70%	27.40%	1.09	85.44%	14.56%	29.53	1	30	0.10%	3.00%	16.71	100.00%	0.00%
7.00	303.75	19.10	17.37	136	271	13.60%	27.10%	0.93	81.73%	18.27%	32.28	1	33	0.10%	3.30%	16.56	100.00%	0.00%
7.50	301.20	18.81	17.37	138	277	13.80%	27.70%	1.07	85.19%	14.81%	29.73	1	30	0.10%	3.00%	16.70	100.00%	0.00%
8.00	289.10	20.08	17.37	136	271	13.60%	27.10%	1.77	95.37%	4.63%	17.63	1	18	0.10%	1.80%	17.40	100.00%	0.00%
9.00	284.85	15.72	17.37	131	263	13.10%	26.30%	2.02	97.09%	2.91%	13.38	1	14	0.10%	1.40%	17.64	100.00%	0.00%
10.00	291.65	12.14	17.37	135	270	13.50%	27.00%	1.62	93.96%	6.04%	20.18	1	21	0.10%	2.10%	17.25	100.00%	0.00%
NOBIAS	271.47	16.13	16.13	127	254	12.70%	25.40%	3.00	99.73%	0.27%	0.00	0	0	0.00%	0.00%	19.83	100.00%	0.00%

131,865.28	= 1ST POOLED SUMS OF SQUARES ROR RATIOS 5.0 THROUGH 10.0 INCLUSIVE
301.7512087	= 1ST POOLED VARIANCE FOR RATIOS 5.0 THROUGH 10 INCLUSIVE
17.37098756	= 1ST POOLED STANDARD DEVIATION FOR RATIOS 5.0 THROUGH 10 INCLUSIVE
361	1st POOLED DEGREES OF FREEDOM = (20-1)*19 = 361
319.86	= UPPER CONTROL LIMIT FOR NO BIAS CASE: 271.47+3*17.98
	ADJUSTED AVERAGE = OBSERVED AVERAGE - NO BIAS AVERAGE

TYPE II ERROR = PR(FAILING TO REJECT A FALSE HYPOTHESIS)

TYPE I ERROR = PR(REJECTION A TRUE HYPOTHESIS)

TOTAL NUMBER OF DATA POINTS GENERATED PER RATIO = 100*1,000*20 = 2,000,000

TOTAL NUMBER OF DATA POINTS GENERATED FOR ALL 20 RATIOS = 100*1,000*20*20 = 40,000,000

NUMBER OF DATA POINTS EXPECTED TO VIOLATE 3 SIGMA LIMITS WHEN NO BIAS IS PRESENT = 272

[Pr(|z|>3.0)]*[100*1,000] = [0.002713]*[100,000] = 271.3

TABLE S4
3 SIGMA CONTROL LIMITS
BASED ON 1,000 BIAS LEVELS OF LENGTH 200 EACH - REPLICATED 20 TIMES

RATIO OF PRECISION OVER BIAS UNCERTAINTY	NUMBER OF 3 SIGMA VIOLATIONS			BASED ON USED STD DEV AND AVG NUMBER OF CHARTS OUT OF 3 SIGMA LIMITS							ADJUSTED AVERAGE NUMBER OUT	BASED ON USED STANDARD DEVIATIONS AND AVERAGES ADJUSTED FOR NO-BIAS						
				AVG NO. OF CHARTS WITH 3S VIOLATIONS		AVG % OF CHARTS WITH 3S VIOLATIONS		TEST RESULTS FOR NONADJUSTED AVERAGES				AVG NO. OF CHARTS WITH 3S VIOLATIONS		AVG % OF CHARTS WITH 3S VIOLATIONS		TEST RESULTS FOR ADJUSTED AVERAGES		
	AVERAGE No. OUT	STANDARD DEVIATION OBSERVED	STANDARD DEVIATION USED	MIN	MAX	MIN %	MAX %	t_CAL	TYPE II	TYPE I		ADJ MIN	ADJ MAX	ADJ MIN %	ADJ MAX %	t_CAL	TYPE II	TYPE I
1.00	6,809.35	250.27	250.27	406	813	40.60%	81.30%	-24.78	0.00%	100.00%	6,270.79	32	813.00	3.20%	81.30%	-22.63	0.00%	100.00%
1.50	2,571.90	143.85	143.85	370	739	37.00%	73.90%	-13.65	0.00%	100.00%	2,033.34	11	739.00	1.10%	73.90%	-9.91	0.00%	100.00%
2.00	1,485.25	50.30	50.30	339	678	33.90%	67.80%	-17.44	0.00%	100.00%	946.69	5	678.00	0.50%	67.80%	-6.73	0.00%	100.00%
2.50	1,075.00	39.63	39.63	312	624	31.20%	62.40%	-11.78	0.00%	100.00%	536.44	3	535.00	0.30%	53.50%	1.81	95.66%	4.34%
3.00	880.25	33.68	33.68	294	589	29.40%	58.90%	-8.08	0.00%	100.00%	341.69	2	342.00	0.20%	34.20%	7.91	100.00%	0.00%
4.00	724.80	22.88	25.05	276	552	27.60%	55.20%	-4.66	0.01%	99.99%	186.24	1	187.00	0.10%	18.70%	16.84	100.00%	0.00%
5.00	647.40	29.76	25.05	261	522	26.10%	52.20%	-1.57	6.62%	93.38%	108.84	1	109.00	0.10%	10.90%	19.93	100.00%	0.00%
5.20	634.35	27.77	25.05	258	517	25.80%	51.70%	-1.05	15.31%	84.69%	95.79	1	96.00	0.10%	9.60%	20.45	100.00%	0.00%
5.40	639.65	22.27	25.05	259	519	25.90%	51.90%	-1.26	11.09%	88.91%	101.09	1	102.00	0.10%	10.20%	20.24	100.00%	0.00%
5.60	623.00	28.98	25.05	255	510	25.50%	51.00%	-0.60	72.17%	27.83%	84.44	1	85.00	0.10%	8.50%	20.90	100.00%	0.00%
6.00	610.50	21.06	25.05	254	509	25.40%	50.90%	-0.10	53.91%	46.09%	71.94	1	72.00	0.10%	7.20%	21.40	100.00%	0.00%
7.00	602.50	20.22	25.05	254	508	25.40%	50.80%	0.22	58.59%	41.41%	63.94	1	64.00	0.10%	6.40%	21.72	100.00%	0.00%
8.00	584.00	25.75	25.05	246	492	24.60%	49.20%	0.96	82.51%	17.49%	45.44	1	46.00	0.10%	4.60%	22.46	100.00%	0.00%
8.50	573.00	22.68	25.05	245	491	24.50%	49.10%	1.40	91.08%	8.92%	34.44	1	35.00	0.10%	3.50%	22.90	100.00%	0.00%
9.00	574.90	25.66	25.05	245	491	24.50%	49.10%	1.32	89.90%	10.10%	36.34	1	37.00	0.10%	3.70%	22.82	100.00%	0.00%
9.50	579.45	23.34	25.05	245	490	24.50%	49.00%	1.14	86.58%	13.42%	40.89	1	41.00	0.10%	4.10%	22.64	100.00%	0.00%
10.00	570.90	28.75	25.05	246	492	24.60%	49.20%	1.48	92.26%	7.74%	32.34	1	33.00	0.10%	3.30%	22.98	100.00%	0.00%
NOBIAS	538.56	23.15	23.15	236	473	23.60%	47.30%	3.00	99.73%	0.27%	0.00	0	0.00	0.00%	0.00%	26.26	100.00%	0.00%

178,819.82 = 1ST POOLED SUMS OF SQUARES ROR RATIOS 4.0 THROUGH 10.0 INCLUSIVE

627.4379533 = 1ST POOLED VARIANCE FOR RATIOS 4.0 THROUGH 10 INCLUSIVE

25.04871161 = 1ST POOLED STANDARD DEVIATION FOR RATIOS 4.0 THROUGH 10 INCLUSIVE

285 1st POOLED DEGREES OF FREEDOM = (20-1)*15 = 285

608.01 = UPPER CONTROL LIMIT FOR NO BIAS CASE: 538.56+3*31.013

ADJUSTED AVERAGE = OBSERVED AVERAGE - NO BIAS AVERAGE

TYPE II ERROR = PR(FAILING TO REJECT A FALSE HYPOTHESIS)

TYPE I ERROR = PR(REJECTION A TRUE HYPOTHESIS)

TOTAL NUMBER OF DATA POINTS GENERATED PER RATIO = 200*1,000*20 = 4,000,000

TOTAL NUMBER OF DATA POINTS GENERATED FOR ALL 20 RATIOS = 200*1,000*20*20 = 80,000,000

NUMBER OF DATA POINTS EXPECTED TO VIOLATE 3 SIGMA LIMITS WHEN NO BIAS IS PRESENT = 543

{Pr(|z|>3.0)}*[200*1,000] = [0.002713]*[200,000] = 542.6

TABLE S5
3 SIGMA CONTROL LIMITS
BASED ON 1,000 BIAS LEVELS OF LENGTH 300 EACH - REPLICATED 20 TIMES

RATIO OF PRECISION OVER BIAS UNCERTAINTY	NUMBER OF 3S VIOLATIONS			BASED ON USED STD DEV AND AVG NUMBER OF CHARTS OUT OF 3 SIGMA LIMITS							ADJUSTED AVERAGE NUMBER OUT	BASED ON USED STANDARD DEVIATIONS AND AVERAGES ADJUSTED FOR NO-BIAS						
	AVERAGE	STANDARD DEVIATION		AVG NO. OF CHARTS WITH 3S VIOLATIONS		AVG % OF CHARTS WITH 3S VIOLATIONS		TEST RESULTS FOR NONADJUSTED AVERAGES				AVG NO. OF CHARTS WITH 3S VIOLATIONS		AVG % OF CHARTS WITH 3S VIOLATIONS		TEST RESULTS FOR ADJUSTED AVERAGES		
	No. OUT	OBSERVED	USED	MIN	MAX	MIN %	MAX %	t_CAL	TYPE II	TYPE I		ADJ MIN	ADJ MAX	ADJ MIN %	ADJ MAX %	t_CAL	TYPE II	TYPE I
1.00	10,116.00	533.00	533.00	453	906	45.30%	90.60%	-17.29	0.00%	100.00%	9,307.38	32	906	0.03	0.91	-15.77	0.00%	100.00%
1.50	3,715.00	194.00	194.00	428	857	42.80%	85.70%	-14.51	0.00%	100.00%	2,906.38	10	857	0.01	0.86	-10.34	0.00%	100.00%
2.00	2,186.55	102.00	102.00	413	826	41.30%	82.60%	-12.61	0.00%	100.00%	1,377.93	5	826	0.01	0.83	-4.68	0.01%	99.99%
2.50	1,610.00	55.00	55.00	398	797	39.80%	79.70%	-12.89	0.00%	100.00%	801.38	3	797	0.00	0.80	1.81	95.68%	4.32%
3.00	1,332.45	49.00	49.00	389	778	38.90%	77.80%	-8.81	0.00%	100.00%	523.83	2	524	0.00	0.52	7.69	100.00%	0.00%
3.50	1,182.00	39.00	39.00	378	756	37.80%	75.60%	-7.21	0.00%	100.00%	373.38	2	374	0.00	0.37	13.52	100.00%	0.00%
4.00	1,085.00	39.00	39.00	368	735	36.80%	73.50%	-4.72	0.01%	99.99%	276.38	1	277	0.00	0.28	16.01	100.00%	0.00%
5.00	976.05	30.00	29.13	356	713	35.60%	71.30%	-2.58	0.91%	99.09%	167.43	1	168	0.00	0.17	25.18	100.00%	0.00%
5.20	958.85	25.00	29.13	355	710	35.50%	71.00%	-1.99	3.04%	96.96%	150.23	1	151	0.00	0.15	25.77	100.00%	0.00%
5.40	943.30	31.00	29.13	351	702	35.10%	70.20%	-1.46	8.05%	91.95%	134.68	1	135	0.00	0.14	26.30	100.00%	0.00%
5.60	947.75	31.00	29.13	354	709	35.40%	70.90%	-1.61	6.18%	93.82%	139.13	1	140	0.00	0.14	26.15	100.00%	0.00%
6.00	915.70	33.00	29.13	349	697	34.90%	69.70%	-0.51	69.24%	30.76%	107.08	1	108	0.00	0.11	27.25	100.00%	0.00%
7.00	898.45	18.00	29.13	348	691	34.80%	69.10%	0.08	53.19%	46.81%	89.83	1	90	0.00	0.09	27.84	100.00%	0.00%
8.00	870.15	29.00	29.13	344	687	34.40%	68.70%	1.05	84.71%	15.29%	61.53	1	62	0.00	0.06	28.81	100.00%	0.00%
9.00	854.78	29.00	29.13	340	680	34.00%	68.00%	1.58	93.47%	6.53%	46.16	1	47	0.00	0.05	29.34	100.00%	0.00%
9.30	873.00	36.00	29.13	346	693	34.60%	69.30%	0.95	82.42%	17.58%	64.38	1	65	0.00	0.07	28.72	100.00%	0.00%
9.70	844.45	26.00	29.13	338	677	33.80%	67.70%	1.93	96.60%	3.40%	35.83	1	36	0.00	0.04	29.70	100.00%	0.00%
10.00	852.98	29.00	29.13	341	683	34.10%	68.30%	1.64	94.15%	5.85%	44.36	1	45	0.00	0.05	29.40	100.00%	0.00%
NOBIAS	808.62	30.73	30.73	333	665	33.30%	66.50%	3.00	99.73%	0.27%	0.00	0	0	0.00	0.00	3.00	99.63%	0.37%

225,682.00	= 1ST POOLED SUMS OF SQUARES ROR RATIOS 5.0 THROUGH 10.0 INCLUSIVE
848.4285714	= 1ST POOLED VARIANCE FOR RATIOS 5.0 THROUGH 10 INCLUSIVE
29.12779723	= 1ST POOLED STANDARD DEVIATION FOR RATIOS 5.0 THROUGH 10 INCLUSIVE
285	1st POOLED DEGREES OF FREEDOM = (20-1)*14 = 285
900.81	= UPPER CONTROL LIMIT FOR NO BIAS CASE: 808.62+3*44.047
	ADJUSTED AVERAGE = OBSERVED AVERAGE - NO BIAS AVERAGE

TYPE II ERROR = PR(FAILING TO REJECT A FALSE HYPOTHESIS)

TYPE I ERROR = PR(REJECTION A TRUE HYPOTHESIS)

TOTAL NUMBER OF DATA POINTS GENERATED PER RATIO = 300*1,000*20 = 6,000,000

TOTAL NUMBER OF DATA POINTS GENERATED FOR ALL 20 RATIOS = 300*1,000*20*20 = 120,000,000

NUMBER OF DATA POINTS EXPECTED TO VIOLATE 3 SIGMA LIMITS WHEN NO BIAS IS PRESENT = 814

[Pr(|z|>3.0)]*[300*1,000] = [0.002713]*[300,000] = 813.9

TABLE S6
3 SIGMA CONTROL LIMITS
BASED ON 1,000 BIAS LEVELS OF LENGTH 400 EACH - REPLICATED 20 TIMES

RATIO OF PRECISION OVER BIAS UNCERTAINTY	NUMBER OF 3 SIGMA VIOLATIONS			BASED ON USED STD DEV AND AVG NUMBER OF CHARTS OUT OF 3 SIGMA LIMITS							ADJUSTED AVERAGE NUMBER OUT	BASED ON USED STANDARD DEVIATIONS AND AVERAGES ADJUSTED FOR NO-BIAS						
				AVG NO. OF CHARTS WITH 3S VIOLATIONS		AVG % OF CHARTS WITH 3S VIOLATIONS		TEST RESULTS FOR NONADJUSTED AVERAGES				AVG NO. OF CHARTS WITH 3S VIOLATIONS		AVG % OF CHARTS WITH 3S VIOLATIONS		TEST RESULTS FOR ADJUSTED AVERAGES		
	AVERAGE No. OUT	STANDARD DEVIATION OBSERVED	STANDARD DEVIATION USED	MIN	MAX	MIN %	MAX %	t_CAL	TYPE II	TYPE I		ADJ MIN	ADJ MAX	ADJ MIN %	ADJ MAX %	t_CAL	TYPE II	TYPE I
0.50	72,053.00	3,679.00	3,679.00	493	987	49.30%	98.70%	-19.26	0.00%	100.00%	70,973.00	178	987	17.80%	98.70%	-18.97	0.00%	100.00%
1.50	5,080.00	193.60	193.60	476	952	47.60%	95.20%	-20.10	0.00%	100.00%	4,000.00	10	952	1.00%	95.20%	-14.52	0.00%	100.00%
2.00	2,911.00	124.11	124.11	472	943	47.20%	94.30%	-13.87	0.00%	100.00%	1,831.00	5	943	0.50%	94.30%	-5.17	0.00%	100.00%
3.00	1,783.00	70.15	70.15	454	908	45.40%	90.80%	-8.47	0.00%	100.00%	703.00	2	703	0.20%	70.30%	6.93	100.00%	0.00%
4.00	1,458.00	35.95	34.59	445	890	44.50%	89.00%	-7.77	0.00%	100.00%	378.00	1	378	0.10%	37.80%	23.45	100.00%	0.00%
5.00	1,309.00	42.57	34.59	435	870	43.50%	87.00%	-3.47	0.13%	99.87%	229.00	1	229	0.10%	22.90%	27.76	100.00%	0.00%
6.00	1,220.00	32.38	34.59	426	853	42.60%	85.30%	-0.89	80.87%	19.13%	140.00	1	140	0.10%	14.00%	30.33	100.00%	0.00%
7.00	1,194.00	33.08	34.59	429	859	42.90%	85.90%	-0.14	55.58%	44.42%	114.00	1	114	0.10%	11.40%	31.08	100.00%	0.00%
8.00	1,178.00	29.22	34.59	431	862	43.10%	86.20%	0.32	62.39%	37.61%	98.00	1	98	0.10%	9.80%	31.55	100.00%	0.00%
9.00	1,147.00	31.40	34.59	424	849	42.40%	84.90%	1.22	88.07%	11.93%	67.00	1	67	0.10%	6.70%	32.44	100.00%	0.00%
10.00	1,141.00	30.35	34.59	427	854	42.70%	85.40%	1.39	90.97%	9.03%	61.00	1	61	0.10%	6.10%	32.61	100.00%	0.00%
NOBIAS	1,080	36.36	36.36	417	834	41.70%	83.40%	3.00	99.73%	0.27%	0.00	0	0	0.00%	0.00%	32.70	100.00%	0.00%

227,300.89	= *1ST POOLED SUMS OF SQUARES ROR RATIOS 4.0 THROUGH 10.0 INCLUSIVE*
1196.32048	= 1ST POOLED VARIANCE FOR RATIOS 4.0 THROUGH 10 INCLUSIVE
34.58786608	= 1ST POOLED STANDARD DEVIATION FOR RATIOS 4.0 THROUGH 10 INCLUSIVE
190	1st POOLED DEGREES OF FREEDOM = (20-1)*10 = 190
1189.08	= UPPER CONTROL LIMIT FOR NO BIAS CASE: 1,080+3*57.08
	ADJUSTED AVERAGE = OBSERVED AVERAGE - NO BIAS AVERAGE

TYPE II ERROR = PR(FAILING TO REJECT A FALSE HYPOTHESIS)

TYPE I ERROR = PR(REJECTION A TRUE HYPOTHESIS)

TOTAL NUMBER OF DATA POINTS GENERATED PER RATIO = 400*1,000*20 = 8,000,000

TOTAL NUMBER OF DATA POINTS GENERATED FOR ALL 20 RATIOS = 400*1,000*20*20 = 160,000,000

NUMBER OF DATA POINTS EXPECTED TO VIOLATE 3 SIGMA LIMITS WHEN NO BIAS IS PRESENT = 1,086

[Pr(|z|>3.0)]*[400*1,000] = [0.002713]*[400,000] = 1,085.2