POTENTIAL OF A BERYLLIUM X-RAY LENS*

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ABSTRACT

The use of refractive lenses for focusing x-ray beams has been the subject of publications since the early 1980s. Detailed calculations have been made for different shapes for the refractive lens: cylindrical, spherical, parabolic, and for a Fresnel-type refractive lens. The main drawback to the use of a single refractive lens to focus x-rays is that the index of refraction ($n = 1 - \delta$) is very close to 1, which results in a lens with a very long focal length. Recently Snigerov and others have suggested and experimentally demonstrated, using cylindrical-shaped lenses, that this problem of long focal lengths can be overcome by using many lenses in series. Each lens refracts the photon through a small angle, but the sum of these sequential changes in direction can be moderately large. This increase in effective refraction angle reduces the focal length of the lens to a few meters or less and makes the multi-element lens a much more useful instrument for focusing x-rays. This paper, annualizes the expected performance of a lens consisting of a series of aligned hollow spheres in a beryllium substrate. The use of hollow spheres rather than hollow cylinders produces focusing of the x rays into a small focal spot in contrast to the single-directional focusing of the hollow cylinders, which produces a line focus. The use of beryllium as the substrate results in lower photo cross sections for both scattering and absorption relative to the value of the refractive index as compared to higher-Z materials and results in higher transmission values than for lenses with similar focal lengths made from higher-Z materials. Be is very stiff, which makes it easier to fabricate lenses with thin webs between the lens elements without distorting the surfaces of the neighbor lens element. This plus beryllium's low density, keep the absorption and scattering in the web at a minimum. The calculations suggest that one will be able to make Be lenses with short focal lengths (1 to 2 m) with useable transmissions (10 to 30%). Two multi-element lenses have been constructed: one with 20 1-mm-diameter hollow spheres in an aluminum substrate, and one with 50 hollow spheres, 1 mm in diameter, in a beryllium substrate. Some construction details and calculations of the expected performance, are given for these two multi-element lenses.

Key words: x-ray, beryllium lens, refractive lens, focusing

I. INTRODUCTION

The use of refractive lenses for focusing x-ray beams has been the subject of publications since the early 1980s. Detailed calculations have been made for different shapes for the refractive lens: cylindrical, spherical, parabolic, and for a Fresnel-type refractive lens. Yang(1) gives a good summary of the state of the art as of 1993. Although some of the early publications suggested using high-Z materials, such as gold, platinum, and tungsten (2), by 1993, it was clear that the use of low-Z materials was more favorable for focusing x-rays because the values of the refraction coefficient, δ , relative to the cross sections for scattering and absorption are higher for the low-Z materials. Yang analyzed the use of a Fresnel-type refraction lens in considerable detail, as a solution to the problem of the high absorption of low energy x-rays in refractive lenses. Some attempts to build a Fresnel lens have been published (3).

The main drawback to the use of a single refractive lens to focus x-rays is that the index of refraction ($n = 1 - \delta$) is very close to 1. δ is the order of 10^{-4} to 10^{-7} for x-ray energies in the range of 2 to 100 keV. This means that the focal length of single element lenses will be many meters long and this long focal length will make it difficult to focus the x-ray beam into focal spots that are a few microns in diameter. Thus the minimum size of the focal spot will be limited and with it the enhancement of the flux in the focal spot.

Snigerov and others in a recent paper (4) suggest that refractive lenses can be made with shorter focal lengths by combining a series of aligned single lenses. Each lens refracts the photon by a small amount, but the sum of these sequential changes in direction can be moderately large. This increase in effective refraction angle reduces the focal length to a few meter or less and makes the multi-element lens a much more useful instrument for focusing x-rays. The lens used by Snigerov and others in their experiments consisted of a number of aligned hollow cylinders in an aluminum substrate. The main drawbacks to this type of lens were the strong absorption and scattering in the aluminum that limited the transmission through the lens and the fact that the hollow cylinders gave focusing in only one direction., that is, line focusing. They suggested that two hollow cylinder lenses orientated at right angles to each other could be used to obtain double focusing and much better enhancement of the flux in the focal spot. They also proposed the use of boron for the substrate to reduce the losses due to absorption and scattering.

In this paper, the present authors consider the use of aligned hollow spheres in a light material such as beryllium for refractive lenses. This approach focuses the x-ray beam down to a very small focal spot and is capable of producing much higher concentrations of photons than the hollow cylinder lens mentioned above, without any appreciable aberrations that are generated by the use of crossed hollow cylinders. The use of beryllium as the substrate is important because it is the lowest-Z material that can be fabricated into a lens and results in the least amount of absorption and scattering of the x-ray beam per unit volume. It also gives the best ratio of refractive index to absorption and scattering cross sections. This allows one to make lenses with thick enough webs between the hollow spheres to maintain good spherical shapes, particularly near the central axis of the lens, where most of the focused photons are located, while maintaining minimum absorption and scattering losses in the lens. The use of beryllium substrates is also important because beryllium is a hard, stiff metal that can be polished to a mirror finish and thus allows one to make near perfect lenses.

II. POSSIBLE USES FOR A REFRACTIVE LENS

Refractive lens consisting of aligned hollow spheres produces focusing in both directions with symmetry around the beam axis, and with no change in direction relative to the incident beam. Thus it is different than mirror focusing or bent crystal focusing. It is also different in that it has no high energy or low energy cutoff. The most important will be its use to generate a very small diameter focused beam with a high density of photons in the focus (large number of photons per mm^2) and with a useful number of focused photons. A second application is to produce a well-collimated, parallel beam of photons in a limited energy band. When this type of beam is analyzed with a monochromator using a set of high-order diffraction planes, one can obtain an x-ray beam with a very narrow energy bandwidth with the maximum available flux. It is important to note that this beam is parallel in both the vertical and horizontal direction. This type of collimated beam can also be used to transport the beam to a distant experimental station without the beam expanding with distance. Another application of the refraction lens is to separate out a single diffraction order by focusing the beam through a small aperture. Higher orders (or lower orders) will not be focused at the same distance because of their different energy. In a similar manner, one can use the lens to concentrate a particular energy region of a white beam and/or separate out this energy band before any further analysis or use is made of the beam. This same effect can be used to separate out a single harmonic from an undulator beam. Focusing a 10 keV beam at 1 m from the lens through a small pinhole can produce a beam with an energy band width of 0.30 keV (FWHM). Thus the lens can act as a poor man's monochromator. Because the lens generates an image of the source, it can be used as a diagnostic tool to monitor the source size, shape, and position (4). In this application, the distance from the source to the lens could be chosen short enough to magnify the image of the source for better analysis. More discussion of possible uses is given in the summary.

III. THEORY

The focal length for a hollow sphere lens varies linearly with the radius of the lens and with the square of the energy of the x-ray being focused. The focal length is given by:

$$\text{Focal length} = R / 2N\delta, \quad (1)$$

where R is the radius of the hollow sphere, N is the number of hollow spheres, and δ is the refraction coefficient. The refraction coefficient is very small for this type of lens, $\delta = 3.41 \times 10^{-4} / E^2$ (keV) for a Be lens in the energy range from 6 keV to 60 keV. Below 6 keV, the value of δ increases faster with decreasing energy than $1 / E^2$ (keV), as one approaches the K absorption edge in beryllium. Above 60 keV, the value of $\delta = 3.40 \times 10^{-4} / E^2$ (keV). Thus one must use many aligned hollow spheres to obtain a focal length of a few meters for a lens made with 1-mm-diameter hollow spheres. This type of refraction lens acts like a simple convex lens for visible light and to the first approximation follows the simple lens law:

$$1 / \text{Focal length} = 1 / L_1 + 1 / L_2, \quad (2)$$

where L_1 is the distance from the source to the lens, and L_2 is the distance from the lens to the focus (see figure 1). Thus the closest the lens can be to the source and still focus the x-rays at a minimum distance from the source is twice the focal length.

Figure 1. Schematic of the hollow sphere lens.

The diameter of the focal spot, d_2 , is given by:

$$d_2 = d_1 \times L_2 / L_1, \quad (3)$$

where d_1 is the diameter of the source. The transmission of the lens is defined as the ratio of the number of photons incident on the focal spot divided by the number of photons incident on the lens area. To obtain the fraction of the incident x-ray beam that passes through the lens and thus the intensity that can be focused, one divides the area of the lens into narrow concentric rings and calculates the transmission of the x-rays through each individual ring. These values for the transmission are then weighted with their relative areas and summed. The transmission of the lens is then the ratio of this sum to the area of the lens. In calculating the transmission through the lens, one must use the total cross section including both the absorption and the scattering cross sections because they both generate losses in the x-ray beam. These transmission calculations can be parameterized in terms of the radius of the hollow sphere, the thickness of the web between the hollow spheres, the total atomic cross section, and the number of hollow spheres. If one doubles the number of hollow spheres and halves the total scattering and absorption cross sections, the transmission remains the same. Thus for any particular lens, the transmission can be parameterized with a single parameter, $N \times \mu/\text{mm}$, where N is the number of hollow spheres, and μ/mm is the total cross section for absorption and scattering for a 1 mm thickness. The material transversed by the beam increases with radius, so

more of the incident flux is absorbed at the larger radii. Thus the size of the beam that exits the lens is a function of the energy of the x rays being focused. Figure 2 is a plot of the relative contribution of the area of concentric rings of equal radial thickness as a function of the radius of the ring. The lens is assumed to consist of 50, aligned, 1 mm diameter, hollow spheres, in a beryllium substrate. For the lowest energy calculated, 4 keV, all of the emerging radiation is concentrated in the

Figure 2. Plot of the relative contribution to the focused flux for concentric rings of equal radial width as a function of their radii. These calculations assume uniform incident flux at the lens face. The radius of the hollow spheres is 0.5 mm.

first 15 percent of the radius. At 6 keV, all of the radiation is in the first 30 percent of the radius. At 10 keV and above, some radiation comes out at all radii. To the extent that the solid lines follow the dashed line, one has the relative importance for the case for no absorption or scattering. For the case where the illumination of the lens is uniform, the dashed line corresponds to the case where the number of photons per unit area is constant. When the solid

Figure 3. A comparison of the transmission values for beams that have smaller radii than the lens. The labels 0.25, 0.5, 0.75 and 1.0 refer to the radii of the beams in terms of the radius of the lens, 1 mm. The lens has N hollow-sphere cut in a beryllium substrate. The diameter of the hollow spheres is 1 mm.

line is one half the value of the dashed line, then the photons per unit area is down to one half, two times the corresponding radii is the FWHM of the emerging beam. These calculation do not take into account the absorption in the web between the hollow spheres, so each curve will be reduced by an amount proportional to this loss. The curves are thus for the case of very thin webs.

In some cases, the beam will be smaller than the lens diameter, and, in this case, the transmission will be higher. Figure 3 compares the relative transmission values for the cases of uniform flux over the whole lens with a radius = 1.0 mm, with the cases for beams with radii of 0.75, 0.50 and 0.25 mm as a function of the parameter, $N \times \mu/\text{mm}$, that was described above.

The enhancement of the flux in the focused beam is defined as the ratio of the number of photons per mm^2 at the focus divided by the number of photons per mm^2 at the lens. The enhancement is given by the equation:

$$\text{Enhancement} = \text{Transmission} \times (2R / d_1)^2 \times (L_1 / L_2)^2, \quad (4)$$

where R and d_1 are constants for any one lens and source, so the enhancement is only a function of $(L_1 / L_2)^2$ for a monochromatic beam. If L_1 is much larger than L_2 , then L_2 does not change much as L_1 varies, and the enhancement is mainly a function of $(L_1)^2$. However, if L_1 is large enough so that the lens is uniformly illuminated, then the number of photons incident on the lens is proportional to $1 / (L_1)^2$, and thus the actual number of photons focused remains relatively constant as L_1 varies. Thus the enhancement is relatively insensitive to the value of L_1 .

IV. CALCULATIONS FOR HOLLOW SPHERE BERYLLIUM LENSES

Before calculating the performance of a particular design for this type of lens, it is useful to examine its potential in general terms. To be useful, efficient use of all available flux is necessary. This means that the diameter of the lens should be similar to the diameter of the beam being focused and that the transmission of the lens, focused photons/incident photons, should not be too small. At the Advanced Photon Source, useful part of the beams tend to be about 1-2 mm in vertical height at 30 to 40 m from the source. Most of the analysis this paper uses a 1 mm diameter lens. A lens that is 2 mm in diameter would focus more of the beam but would also increase the distance from the lens to the focus by a factor of 2. This would expand the area of the focus beam by a factor of 4 and reduce the enhancement of the focused flux by a similar amount. It is important to make the web between the hollow spheres as thin as possible to reduce the absorption and scattering of the beam. However, it is important to be careful not to make this web too thin so as to avoid any distortion of the surfaces of the hollow spheres during fabrication. The central region of the lens is the most efficient part of the lens, and thus its shape should be kept as perfect as possible. Experiments performed with aluminum substrates suggested that a web of 0.1 mm was thick enough to maintain the integrity of the hollow sphere shape so this thickness is used in most of our calculations. It may be possible to reduce this thickness, in which case the lenses will be more efficient, that is, have higher transmission. Inspection of the recently fabricated beryllium lens suggests that a 0.05 mm web should be acceptable. This would be particularly helpful for energies below 10 keV.

After selecting a desired transmission for the lens, it is possible to calculate the corresponding number of hollow spheres required. The number of hollow spheres determines the focal length of the lens as a function of x-ray energy. Once the focal length is known, the distance from the lens to the focal spot, L_2 , can be calculated, as well as the diameter of the spot, d_2 , and the enhancement of the flux (increase in the photons per mm^2) in the focal spot over that in the incident beam on the lens. Figure 4a is a plot of the enhancement of the flux (triangles) and the corresponding number of 1-mm-diameter hollow spheres (circles) as a function of the x-ray energy for a lens with a transmission of 10 percent. Figure 4b is a plot of the diameter of the focal spot energy (filled circles, left-hand scale), the focal length of the lens, and the distance from the lens to the focal spot, L_2 , (squares and triangles, respectively, right-hand scale) as a function of x-ray energy. For this calculation, L_1 , the distance from the source to the lens, is assumed to be 40 m, and the diameter of the source is assumed to be 0.20 mm. If the diameter of the source is 0.10 mm then values for the diameter of the focal spot will be reduced by a factor of 2 and the enhancement factor will be increased by a factor of 4. If the height of the source is 0.10 mm

and the width of the source is 0.20 mm then the increase in the enhancement will be a factor of 2. The two plots show that the beryllium lens has considerable potential for making small focal spots ($5\ \mu$ to $10\ \mu$), with high enhancement (1000 to 4000) at reasonable distances from the lens (1 m to 2 m) over a energy range of 5 to 25 keV.

Figure 4a. A plot of the calculated enhancement of the flux (triangles) and the corresponding number of 1-mm-diameter hollow spheres (circles) as a function of the x-ray energy for a lens with a transmission of 10 percent. Figure 4b. A plot of the calculated diameter of the focal spot energy (filled circles, left-hand scale), the focal length of the lens and the distance from the lens to the focal spot, L_2 , (squares and triangles, respectively, right-hand scale) as a function of x-ray energy.

The calculated performance of a similar lens designed for 25 percent transmission is shown in Figure 5a and 5b. Here the enhancement is less because the focal length and thus the distance from the lens to the focus is larger. This makes the diameter of the focal spot larger, which spreads out the flux over a larger area, reducing the enhancement. However, the total flux in the focal spot is 2.5 times larger, which would be favorable if total flux is more important than spot size. The number of photons per mm^2 is very close to the same in both cases because the two effects tend to compensate for each other.

The above calculations assume that the photons per mm^2 are uniform over the lens area. If the x-ray beam FWHM is about the size of the lens or smaller, then this will not be true, and, because the beam will be more concentrated in the center of the lens, the transmission will be higher. In this case, moving the lens from 40 m to 30 m from the source will concentrate the beam in the center of the lens even further and increase the transmission even more. If the diameter of the beam is reduced by one

half, then in a typical case the transmission will double. Similarly, if the diameter of the lens is increased to twice its former value, the transmission will typically double as well. The effect increases as the transmission decreases. For low transmission, the decrease in beam size by a factor of two can increase the transmission by factors of 4 to 5. The increase in transmission with an increase in the diameter of the lens might suggest that one could improve the transmission by using larger lens diameters. However, doubling the diameter increases the focal length by twice as well, and then one needs twice as many hollow spheres to obtain the same focal length. If the web between the hollow spheres is very thin, then the transmission is almost but not quite as large as it was with half as many of the smaller spheres. There are now twice as many webs as well, and the absorption in the extra webs results in a further loss in transmission. When there are a large number of hollow spheres, the absorption in the webs becomes important. If there are 200 spheres, there are 200 webs, and, if they are 0.1 mm thick, then the total web thickness is 20 mm. Even in a light weight material like beryllium, this results in a significant amount of absorption.

Figure 5a. A plot of the enhancement of the flux (triangles) and the corresponding number of 1-mm-diameter hollow spheres (filled circles) as a function of the x-ray energy for a lens with a transmission of 25 percent. Figure 5b. A plot of the diameter of the focal spot energy (filled circles, left-hand scale), the focal length of the lens and the distance from the lens to the focal spot, L_2 , (squares and triangles, respectively, right-hand scale) as a function of x-ray energy for the 25 percent transmission lens.

V. PARALLEL BEAM FOCUSING

Figure 6. A plot of the number of hollow spheres needed to focus the radiation into a parallel beam as a function of energy for the two distances, 30 m and 40 m, of the lens from a point source, shown as circles and diamonds, respectively (scale on the right). The lens transmissions for these two cases are also plotted using the scale on the left.

VI. DESIGN AND CALCULATED PERFORMANCE OF A 50-HOLLOW-SPHERE BE LENS

The first design to be discussed is for a Be lens in which the parameters are chosen to obtain maximum enhancement of the focused beam in the energy region of 8 to 10 keV using a lens with 1-mm-diameter hollow spheres and a 0.1 mm spacing between hollow spheres. This spacing was chosen so that there would be enough material between the hollow spheres to eliminate the possibility that the shape of the hollow sphere would become distorted during manufacture. This optimization led to a lens with 50 hollow spheres.

The calculated enhancement for the lens (the ratio of the number of photons per mm^2 at the focus divided by the number of photons per mm^2 at the lens) as a function of x-ray energy (triangles, left-

It is possible to generate a parallel beam in a limited energy region, by choosing the right number of hollow spheres. The number needed will be much smaller than was required for a short focal length lens and the transmission of the lens can be quite high. This is illustrated in Figure 6, where the number of hollow spheres needed and the corresponding transmission of the lens is plotted versus x-ray energy. In each case, two curves are shown for two values of the distance from the source to the lens, L_1 , 30 m and 40 m. It is possible to use figure 6 to estimate the energy range that can be focused into a parallel beam between these two distances by a lens with a particular number of hollow spheres. For example, a lens with 30 hollow spheres can focus a parallel beam for energies between 35 and 40 keV. The corresponding transmission for these parallel beams will be about 0.68 - 0.69.

hand scale) and the calculated transmission of the lens as a function of energy (filled circles, right-hand scale) are shown in Figure 7a and 7b. L_1 is assumed to be 40 m. The enhancement of 3000 in the 8 to 10 keV region is quite large and demonstrates the potential of this type of lens. The lens should be quite useful (enhancements greater than 900) in the energy range from 6 to 14 keV. The transmission in the 8 to 10 keV energy range varies from 5% to 13%. Above 20 keV, the transmission approaches 50%, but the enhancement falls off dramatically because of the increase in the value of L_2 and thus the increase in size of the focal spot.

Figure 7a . The enhancement of the 50-hollow-sphere beryllium lens as a function of x-ray energy (triangles, left-hand scale) and the transmission of the lens as a function of energy (filled circles, right-hand scale) for the case where $L_1 = 40$ m. The corresponding values of diameter of the focal spot, the focal length, and the value of L_2 are shown in Figure 7b.

If L_1 is assumed to be 30 m rather than 40 m (see Figures 8a and 8b), then the enhancement will be less, 1700 to 2000 in the 8 to 10 keV region, but the photons per mm^2 at the lens will be higher by $(40/30)^2$, which will make up for the decrease in enhancement, and the photons per mm^2 at the focus will be about the same as in the $L_1 = 40$ m case. Again the lens should be quite useful, with enhancements greater than 900, in the energy range from 6 to 14 keV. The transmission in the 8 to 10 keV energy range varies from 7.5% to 15%. Above 20 keV, the transmission approaches 50%, but the enhancement falls off dramatically as L_2 and the focal spot diameter become larger.

Figure 8a . The enhancement of the 50-hollow-sphere beryllium lens as a function of x-ray energy (triangles, left-hand scale) and the transmission of the lens as a function of energy (filled circles, right-hand scale) for the case where $L_1 = 30$ m. The corresponding values of diameter of the focal spot, the focal length, and the value of L_2 are shown in Figure 8b.

VII. FRESNEL LENS

Yang (1) suggested that the use of a Fresnel-type lens would greatly reduce the absorption and scattering in a refractive lens. Figure 9. illustrates two possible cross sections for a Fresnel lens, one with 6 rings and one with 12 rings, and compares them to the cross section of a normal refractive lens. This type of lens would greatly increase the transmission of lenses designed for the 6 to 15 keV energy range

Figure 9. The cross section of two Fresnel lenses compared to the cross section of a normal refractive lens. The shaded areas are the variable thickness part of the lens, Δx , and the dashed line indicates one half of the web thickness.

Figure 10. The relative contribution of concentric rings to the focused flux is shown as a function of the radius.

VIII. CONSTRUCTION OF A 50-HOLLOW-SPHERE BE LENS

Figure 11. Drawing of the beryllium lens fabricated by Phoenix Precision, Inc. The units are in mm.

The relative importance of successive concentric rings to the focused flux is now much different that it was for the normal lens as shown in Figure 2. A similar plot of relative contribution for the Fresnel lens is shown in Figure 10. Again the dashed line corresponds to the contribution if there is no absorption. Above 6 keV a major portion of the focused beam will come from radii that are larger than one half the radius of the sphere. In this region, the spherical surface has a significantly different slope than the ideal parabolic surface and this will generate slope errors that will focus this radiation at shorter distances from the lens than the radiation that passes through the lens close to the central axis. This will significantly increase the diameter of the focal spot and decrease the enhancement.

One constructs an aligned series of hollow spheres in a substrate by first splitting the substrate in half and then forming a series of matching half spheres in each substrate and assembling the two halves to make the aligned series of hollow spheres. The details of this type of construction are shown in Figure 11. Phoenix Precision, Inc., has constructed this Be lens for the Advanced Photon Source. The Be lens was delivered to Argonne in mid-July, 1997.

IX. CONSTRUCTION AND CALCULATED PERFORMANCE OF A 20-HOLLOW-SPHERE ALUMINUM LENS

A 20-hollow-sphere aluminum lens was constructed by EDM-machining 20 aligned half spheres in two aluminum substrates. The hollow spheres were 1.0 mm in diameter, and the web between the spheres was 0.1 mm thick. The process went quite quickly, and the shape looked spherical to first order. The surface was smooth but a long way from a mirror finish. Experiments were carried out to see if the inside surface could be polished. A cotton tip was cemented to a thin shaft and used to polish the inside surface with 1 μ aluminum oxide polishing compound. The work was slow but did seem to improve the surface condition. The expected performance for the 20-hollow-sphere aluminum lens is plotted in figures 12a and 12b. The strong absorption of the aluminum lens severely limits the enhancement of the flux at the focal spot. The absorption in the 0.1 mm webs between the hollow spheres makes a major contribution to this absorption.

Figure 12a. Plot of the enhancement of the 20-hollow-sphere aluminum lens as a function of x-ray energy (triangles, left-hand scale) and the transmission of the lens as a function of energy (filled circles, right-hand scale) for the case where $L_1 = 40$ m. Figure 12b. Plot of the corresponding values of diameter of the focal spot, the focal length, and the value of L_2 .

X. CONCLUSIONS

The above calculations suggest that beryllium will make an excellent substrate for a refractive lens in the 4 to 30 keV energy range with some special applications up to energies of 60 to 100 keV. Enhancements of the flux (photons per mm^2) in the focal spot will be 1000 or higher in this energy region; with spot sizes of 5 to 10 μ , and focal lengths of 1 to 2 m appear possible. In the more limited energy region of 8 to 15 keV, enhancements of 3000 to 4000 are possible. The refractive lens is much less sensitive to surface roughness than a focusing mirror and is quite insensitive to

instabilities in the principal axis of the lens compared to a focusing mirror. Also, it does not change the direction of the x-ray beam. If these projections are realized in future experiments, these lenses will match the performance of recently reported results for zone plates. The above calculations assume that the hollow spheres are perfect in shape, perfectly aligned, and have smooth inside surfaces. Any deviations from these assumptions will tend to increase the diameter of the focal spot and decrease the enhancement of the flux. For most of the above calculations, the focal spots had values of $5\text{ }\mu$ or larger and precision machining to this tolerance should not be too difficult. Only in the case of the very low energies for the 50-hollow-sphere lens did the diameter of the focal spot drop below $5\text{ }\mu$, approaching $1\text{ }\mu$ at 5 keV. Thus the tests with the 50-hollow-sphere beryllium lens at 5 keV will be a good test as to how well the lens was manufactured. The smoothness of the inside surface of the hollow spheres could be an important parameter for lenses with small focal spot diameters. In which case, it will be necessary to polish the inside surfaces. Preliminary experiments with the beryllium lens suggest that it will be the slope errors rather than the smoothness that will limit the minimum focal spot size. No analysis has been made as to the effect of the refractive lens on the polarization of the x-ray beam, but very little change is expected because most of the x-ray beam will pass through each surface at relatively large incident angles.

The main drawback of the multiply refractive lens is the relatively high total cross section for absorption and scattering of the photons at low energies. Even for low-Z materials, such as beryllium, it is the atomic cross section for absorption and scattering that limits the useful energy range of an individual lens. If and when it becomes possible to fabricate a Fresnel-type lens from low-Z materials, both the useable energy range and the lower limits on the focal length will be substantially reduced. This improvement will both increase the transmission of the lens and will generate smaller diameter focal spots. Both of these effects will improve the enhancement of the flux in the focal spot, and the true potential of the refractive lens will be realized.

The other important drawback with the use of hollow spheres as focusing, is that the shape of a hollow sphere deviates from the desired parabolic shape needed for focusing. The radiation that passes through the lens at larger radii will be bent more strongly and focus at a point closer to the lens than the focus of radiation that passes through the lens close to the central axis. This effect will cause a defocusing of the beam and increase in minimum size of the focal spot.

The main features of the refractive lens are:

1. It focuses in both the vertical and the horizontal direction
2. It does not change the direction of the beam.
3. It focuses one energy at a single distance and all energies simultaneously at different distances.
4. It can be efficiently cooled, so it can withstand high heat loads.

Possible uses of the refractive lens are:

1. To generate a very small diameter focused beam with a high density of photons in the focus (large number of photons per mm^2) and with a useful number of focused photons.
2. To produce a well-collimated, parallel beam of photons in a limited energy band. When this type of beam is analyzed with a monochromator using a set of high-order diffraction planes, one can obtain an x-ray beam with a very narrow energy band with the maximum available flux. This approach will greatly reduce the diffraction of other orders because only the wanted energy band will be parallel and thus be focused efficiently by the monochromator using a set of high order diffraction planes.
3. To separate out a single diffraction order by focusing the beam through a small aperture. Higher orders (or lower orders) will not be focused at the same distance. Thus most of the intensity of the other order diffracted beams will not pass through the small aperture.
4. To produce a well-collimated or focused beam that can be used to transport the x-ray beam to a distant experimental station without having the cross section of the beam expand into a large area.
5. To concentrate a particular energy region of a white beam and/or separate out this energy band before any further analysis or use is made of the beam. The FWHM of the energy band that passes through a pinhole that is approximately the diameter of the source size will be in the range of 1 to 2 percent of the

center energy. This energy resolution could be sufficient for experiments that need high fluxes with only moderate energy resolution.

6. To be used as a diagnostic tool to monitor the source size, shape, and position. In this application, it could be used either with a short focal length refractive lens that makes a small focal spot or with a long focal length lens that magnifies the source size for better analysis. In each case, the ratio of the focal spot diameter to the distance of the focal spot from the lens is the same so the angular resolution of the image is the same.

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