SANDIA REPORT

SAND98–0220 • UC–705
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Printed January 1998

Tolerance Analysis and Variational Solid Geometry

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SF2900Q(8-81)

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Printed January 1998

Distribution
Category UC-705

Tolerance Analysis and Variational Solid Geometry

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Abstract

The fields of tolerancing and assembly analysis have depended for decades on ad hoc, shop floor methods. This causes serious problems when subjecting toleranced designs to automated, analytical methods. This project attempted to further the formalization and mathematization of tolerancing by extending the concept of the Maximum Material Part. A software system was envisioned that would guide designers in the use of appropriate tolerance specifications and then create software models of Maximum Material Parts from the toleranced nominal parts.

Contents

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Introduction

This report is being written to document efforts made to further the mathematization of tolerance analysis of three dimensional mechanical parts and assemblies and to describe efforts and plans to incorporate expected results into designer's systems. The goal of this and related work is to answer fundamental questions about mechanisms without the necessity of building and testing a (potentially large) number of samples. Ideally, without creating any physical instances of a design, we want to predict the answer to such questions as: Can each part be manufactured? If all the parts are manufactured within tolerances, will it assemble? Once assembled, will it function?

The concept of specifying tolerances and methods of measurement is as old as engineering (at least as old as the pyramids of Egypt). The field of tolerancing of mechanical parts has grown up from the shop floor over the last dozen decades or so. As a result, it is full of ad hoc methods, as practiced principles and physical inconsistencies. The first major improvement to such a situation is to standardize the symbology and semantics used in the field. The American Society of Mechanical Engineers (ASME) produces and periodically revises standards for tolerancing. The current standard is ASME Y14.5M-1994. This is an enormous step forward as it makes possible and facilitates the exchange of information between different groups of designers and between designers and different groups in manufacturing.

In the never ending quest for "faster, better, cheaper", it is necessary to push tolerances and manufacturing processes as far as possible. In order to do this in the modern age, automated (computer) analysis methods must be used. It is here that the field of tolerancing is currently facing large problems. For all their value, the standards are still just codifications of the shop floor practice. They still are just a collection of practices, perhaps unrelated and inconsistent. There are no mathematical models on which to develop theory or write analysis systems.

This lack has been apparent for some time. In 1988, a National Science Foundation workshop identified a need for a mathematical definition for the current tolerancing standards. In response, ASME produced ASME Y15.5.1M-1994, "Mathematical Definition of Dimensioning and Tolerancing Principles" which has become an ANSI standard as well. The effort to provide concepts, definitions and theories for this field has only just begun. We are still very far from being able to answer the questions posed above: can we make the parts? can we put it together? will it work? While creating what computer analysis tools we can, we must also be building and firming up the mathematical foundations of the field of tolerancing.

The effort reported here attempts to make progress in the bringing of mathematical rigor to the field of tolerance analysis. The work done here attempts to build on the work done by Steffan Parratt in his PhD thesis at Cornell University. There he introduced the concept of a Maximum Material Part for one dimension. This work attempts to extend that to three dimensions. Not all kinds of tolerances will lead to Maximum Material Parts so, a major part of this effort was to examine the ASME Y14.5-1994 and document the kinds of tolerances and their relationships that will produce Maximum Material Parts.

The first section of this report introduces the standard "stack-up" method of tolerance analysis and demonstrates its inadequacies. The second section explains what a Maximum Material Part is. The third section talks about efforts to encapsulate the benefits of Maximum Material Part descriptions into tolerance analysis. Following that are results and conclusions.

Stack-Up and Assembly Analysis

The first difficulty with tolerance and assembly analysis is deciding what is in tolerance and what is out. As an example of the kind of problem that can arise, consider the fundamental problem of picking the frame of reference from which to measure compliance.

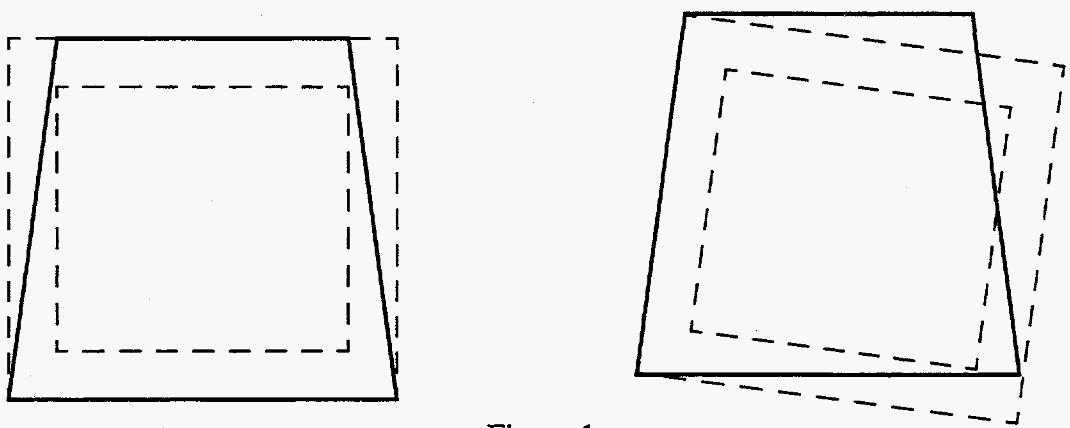

Figure 1

The above figure shows the tolerance zones in which a manufactured square is supposed to lie and a sample of the square from the manufacturing process. Whether the manufactured "square" is within tolerance or not depends on the frame of reference chosen to measure it. On the left, the frame is parallel to the top of the "square" and the "square" lies in the tolerance zone. On the right, the frame is parallel to the left side of the "square" and the "square" is out of tolerance. We shall leave this and like problems (such as the fact that manufactured parts do not have perfect form, i.e. do not have perfectly flat, straight or smooth surfaces) to be dealt with as much as possible in ASME Y14.5.1M-1994, Mathematical Definition of Dimensioning and Tolerancing Principles. This work will assume parts of perfect form and obvious orientations.

There are many questions associated with the assembly of mechanical parts. For example, there is the question of assembly sequence – what parts must be placed first; or is it even possible to maneuver a part into place at all? This is known as assembly planning. The question addressed here is one of assembly analysis which determines if all the parts will fit together. In particular, we are concerned with determining if an assembly is an *interchangeable assembly.* An assembly is an interchangeable assembly if none of the constituent parts interfere with each other in their assembled positions for any possible set of parts that are manufactured to within specified tolerances. In other words, if you had an interchangeable assembly widget composed of three parts, A, B and C, you could randomly get an instance of each part from its respective bin and be guaranteed you could assemble a widget.

If the widget were not an interchangeable assembly, part C might not fit. You would have to rummage around in bin C for a part that would fit. On the assembly line floor, the costs could be large. A machine might jam, causing a line halt. There might be considerable wastage as non-fitting parts are discarded. Or, a large amount of time might be spent finding combinations that would assemble.

One of the standard methods used to do this kind of assembly analysis is the stack-up method. To employ stack-up, one selects a linear dimension of the assembly, adds up the minimum and maximum tolerances for each feature and checks for interference of the overlaps in the tolerance ranges for the mating parts. The following example of a hinge (Figure 2) will illustrate. Upon initial inspection, it

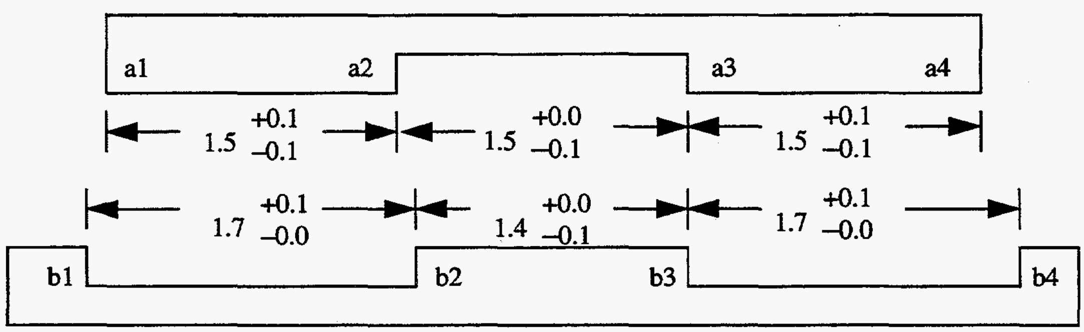

Figure 2

would appear that these two parts will assemble easily. But much depends on whether this is a static or floating assembly.

In a static assembly, one pair of features (surfaces) is required to be in contact. Perhaps they will be glued or welded. Let us assume that features a1 and b1 are required to be in contact. Then, to perform stack-up analysis, we work left to right to construct where the possible zones are for each of the other features. If none of the other features can overlap (interfere) with each other, the assembly is interchangeable.

For the hinge example, we see from the shaded regions in Figure 3 that there is a potential (in this case probable) interference between a3 and b3. That is, it is possible for parts a and b to be individually within tolerance and yet the two would not assemble. At first glance, it would appear that part A might be moved to the right to avoid the a3-b3 interference. However, a4 and b4 would then start to interfere. It seems, at this point, that the parts won't assemble interchangeably at

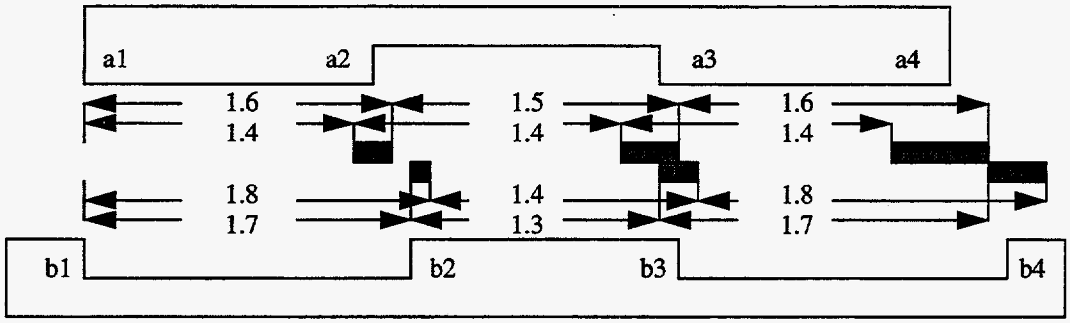

Figure 3

all.

The apparent problem stems from the overly restrictive stack-up method and what can be termed the "conditionality" of the tolerance specifications. The stack-up method is overly restrictive in that it doesn't take advantage of a floating assembly's positional flexibility. The "conditionality" comes into play when one part is fixed with respect to the other and tolerance zones are stacked-up from there. Note in Figure 3 that feature a3 cannot be at its minimum position simultaneously with a4 being at its maximum position. Indeed, max(a4) - min(a3) is 1.9. Considerably out of specification. This a conditional tolerance specification since the range of positions for a4 is conditional on the actual position of a3.

This can be demonstrated in the extreme by considering a piano hinge. A piano hinge might have several dozen slots and tabs. If the tolerances on each were specified as in this example, the actual length of the hinge could vary by several inches and still be within spec. This is not only unacceptable, it is inconsistent with reality. In the manufacture of a piano hinge, the length is controlled by a single cut. The slots and tabs are then formed within that piece.

Returning then to the current hinge, if this were a static assembly, we would have a problem. We would either have to accept the potentially large costs described above for an assembly that wasn't interchangeable or we would have to redesign the parts, possibly creating very sloppy assemblies for the average case.

However, this is to be a hinge and is a floating assembly (i.e. none of the features are required to be in contact although, quite probably, one or more will be). A commonly used assembly analysis technique for floating assemblies is to select all possible pairs of features that could come in contact, force them to be in contact and apply stack-up at each. This method is called successive stack-up. It is possible one of the pairs of features will yield a stack-up analysis that guarantees interchangeable assembly.

Unfortunately, as will be shown below, successive stack-up is not a conclusive analysis. It is merely convenient. It is straightforward to apply successive stack-up to the hinge example and note that it fails to find a pair of mating features that will guarantee interchangeable assembly. To demonstrate that this is, in fact, an interchangeable assembly, we can do an algebraic analysis. What follows is a condensed version of what appears in Steffen Parratt's thesis [PAR94].

Figure 4 shows the hinge redrawn with the distances between neighboring features on each part represented as variables x_i and y_i and the distance between features a1 and b1 described as a variable p.

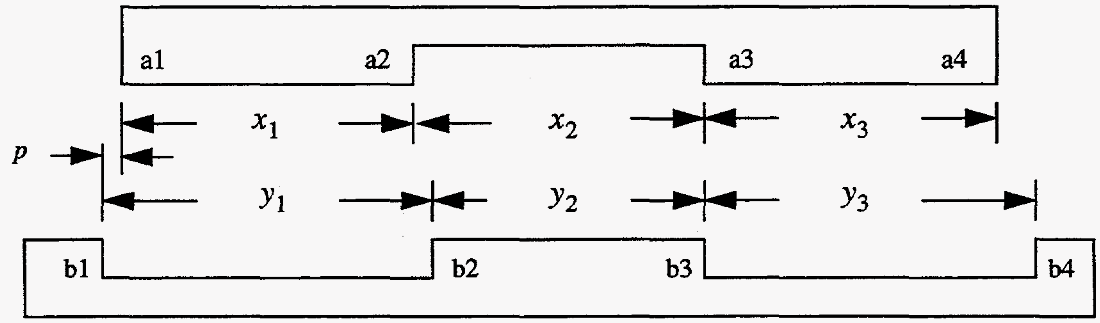

Figure 4

From this diagram we can write down the following inequalities that will guarantee interchangeable assembly if they can be satisfied.

$$p \geq 0$$
$$y_1 \geq p + x_1$$
$$p + x_1 + x_2 \geq y_1 + y_2$$
$$y_1 + y_2 + y_3 \geq p + x_1 + x_2 + x_3$$

Rearranging we get:

$$p \geq 0$$
$$p \geq y_1 + y_2 - x_1 - x_2$$
$$p \leq y_1 - x_1$$
$$p \leq y_1 + y_2 + y_3 - x_1 - x_2 - x_3$$

We have isolated p. If we explicitly write all brackets on p, we have:

$$y_1 - x_1 \geq p \geq 0$$

$$y_1 - x_1 \geq p \geq y_1 + y_2 - x_1 - x_2$$

$$y_1 + y_2 + y_3 - x_1 - x_2 - x_3 \geq p \geq y_1 + y_2 - x_1 - x_2$$

$$y_1 + y_2 + y_3 - x_1 - x_2 - x_3 \geq p \geq 0$$

Now, we can remove p and rewrite to get:

$$y_1 \geq x_1$$

$$x_2 \geq y_2$$

$$y_3 \geq x_3$$

$$y_1 + y_2 + y_3 \geq x_1 + x_2 + x_3$$

If all these inequalities are satisfied, we know there exists some p for which all the original inequalities are satisfied and the assembly is an interchangeable assembly. To find out if these inequalities hold for all x_i and y_i, we must check the extremes of each side. That is, for each, check to see that the minimum of the left side is greater than or equal to the maximum of the right side. Fetching the numbers from Figure 3 we have:

$$1.7 \geq 1.6$$

$$1.4 \geq 1.4$$

$$1.7 \geq 1.6$$

$$4.7 \geq 4.7$$

At this point, we can see that all the inequalities are satisfied and the hinge is an interchangeable assembly. This kind of approach has been generalized in one dimension and an algorithm developed in [PAR94].

Note the focus, in the analysis, on the extremes of the features. There would be many advantages if a model of a part could be constructed that incorporated all the extremes. This would remove all inequalities in the analysis and provide other benefits as well. These will be discussed in the next section which introduces the Maximum Material Part.

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The Maximum Material Part

For any given feature such as a hole, a slot, a tab or a protrusion or void of any sort, the concept of a maximum material condition has existed for some time. This is the obvious state wherein that particular feature is composed of as much material as possible while keeping the feature within tolerance specifications. We will expand this concept to the Maximum Material Part.

Definition: A *Variational Class* is the set of all solids satisfying a tolerance specification.

Definition: A *Maximum Material Part* (MMP) of a variational Class V is a solid M such that M is an element of V and for any other element S of V, there exists a rigid rotation and translation which, when applied to S, will place S so that no point in S is outside of M.

This satisfies the intuitive definition that an MMP contains all other parts that are also within tolerance. Note that not all variational classes have an MMP. We will come back to that shortly. There is an analogous definition for a Least Material Part (LMP). The usefulness of such a part will be discussed in the next section.

Consider an assembly where all the parts have an MMP associated with the variational class defined by the toleranced part descriptions. If the set of MMPs are non-interfering, then all possible sets of within-tolerance parts will assemble and it is an interchangeable assembly. This is true since the MMPs are non-interfering and all instances of a part can be contained within its MMP. This remove the inequalities and conditionality of the tolerance analysis. The problem of interchangeability can now be attacked with configuration space methods. These methods search for positions and orientations of the parts so that they will be in their desired relative positions and not interfere with one another.

Configuration space methods are also used extensively in assembly planning. Assembly planning is normally done with nominal parts. Difficulties arise when using toleranced parts due to the exponentially exploding number of possibilities and due to the conditionality mentioned previously. Conditionality poses problems since most assembly planning relies on goal states, i.e. known positions to which a part must be moved. When the uncertainty of tolerances are introduced, the goal position is variable. When conditionality occurs, the goal position depends on the placement of any number of previous parts.

Using MMPs eliminates these problems for floating assemblies. If assembly analysis shows that the parts do not interfere, then any position inside of a part can be arbitrarily picked as its goal state. This goal state is then independent of all variation and placement of other parts.

The question, then, is how do we obtain MMPs from a tolerance specification? Unfortunately, as noted above, not all tolerance specifications lead to MMPs. In fact, normal methods frequently don't. Recall the discussion following the stack-up analysis in Figure 3. Using feature a1 as the origin, an MMP encompassing all possible extents for a3 and a4 (the lower part in Figure 5) would have a width of 1.9 from a3 to a4. This is not in tolerance.

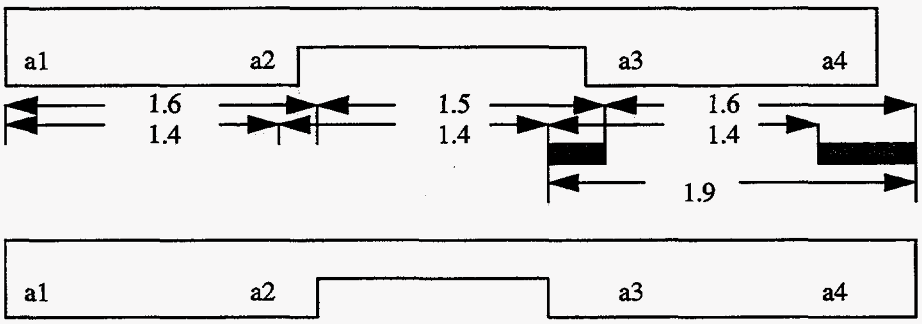

Figure 5

What is needed is a class of tolerance specifications from which MMPs can be created. In the hinge case, consider the implications when all the position and tolerance specifications are relative to a fixed origin (in this case, a2 in Figure 6) rather than relative to each other as they have been.

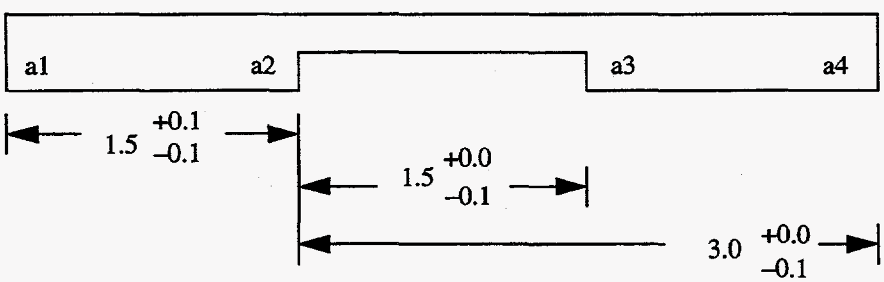

Figure 2

Each of the specified dimensions has the same range when looked at in isolation but we can now construct an MMP which would be an in-tolerance instance of the part. This may look like some sleight of hand, but the difference is that the distances a2a3 and a3a4 cannot vary independently. The minimum of a2a3 cannot occur simultaneously with the minimum of a3a4. In a similar fashion, we can create a tolerance specification for the bottom part of the hinge that will produce an MMP. We can then use configuration space methods to see if the two parts fit together. These methods work much the same as a human would with these parts, they attempt to move one relative to the other until then fit together.

The difference between using MMPs and other instances of the parts is that if the MMPs are found to assemble, we can be sure that all in-tolerance parts will also assemble. Furthermore, if the MMPs assemble, we can use their relative positions as goal positions in assembly planning.

So far, the discussion has applied only to a single dimension. To be sure, there are problems for which this is a useful result. However, to really have an impact, we must be able to expand the results to three dimensions. Central to this effort is the identification of tolerance specifications that can produce MMPs. There are two thrusts to this effort. On the one hand, we must identify the tolerance specifications in current standards (ASME Y14.5M-1994) and their interactions and conditions that can produce MMPs. Also, to encourage progress in the field, we must determine where small changes to the standards would have a large impact in building MMPs.

The other major effort is to build systems that will encourage use of MMP tolerance specifications by designers and construct computer models of the MMP itself to be used in various analysis codes.

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The Design System

Another major focus of this project was the creation of a software system to examine designs of mechanical assemblies to promote the use of Maximum Material Parts. This system would inspect all tolerance specifications and determine if they could lead to an MMP. Where possible, the MMP for each part would be constructed. This new assembly composed of MMPs could then be passed to other software analysis systems which could then do assembly analysis, assembly planning and functional analysis.

Since not all tolerance specifications can be used to generate MMPs, it would be important to educate designers as to which kinds of tolerance specifications lead to MMPs and the advantages of using them. Since it is difficult for an education to be complete and difficult for old habits to be replaced, the software system should interactively advise the designers about their choice of specifications and offer alternatives to be considered.

This system would also produce an assembly consisting of Least Material Parts. LMPs are useful in calculating the maximum relative displacement. This is a measure of how much the parts will move relative to each other, in other words the slop in the assembly. This can be important in determining whether the assembly will function as required.

In fact, since the relationship between MMPs, LMPs and functionality is unknown, it would be quite useful to do the functionality analysis for both extremes and for a large number of samples in-between. The software system could produce specified or "random" instances of the design parts upon request.

This ability to produce variations of part geometry within tolerances could also be useful in assembly analysis, assembly planning and functional analysis techniques on assemblies that do not have MMP and LMP representations. Each could be varied independently through its tolerance ranges, thereby building a "random" variation of the part and hence the entire assembly.

The analysis engine could ask for an instantiation of each part, perform the desired analysis and iterate. The analysis engine could ultimately, after a large number of trials, provide a statistical analysis of the properties of the parts and/or assembly. For example, this Monte Carlo method could predict the probability the parts will assemble or the likelihood it will function.

The variation of each toleranced feature could be controlled using a linear probability function. Alternately, the probability function could mimic the statistical properties of geometric tolerances as characterized by the collaborative work done for this project at Cornell (see appendix A). The Cornell work has developed mathematical models which can generate the appropriate distributions.

The choice of method would depend on the question the analysis is attempting to answer. If the analysis is trying to provide insight about the assembly over the entire tolerance range, then a linear probability function would be in order. If the analysis is trying to determine properties about the assembly as it will likely be manufactured, then the Cornell mathematical models should be used.

Results

Along the way, this project suffered several major setbacks. Consequently, it achieved far less than it set out to.

Steffan Parratt's thesis work at Cornell produced two major new concepts – Generalized Feature of Size and Maximum Material Part – which showed the potential to have enormous impact in the fields of tolerance analysis, assembly analysis and assembly planning. Parratt obtained his Phd and a position at Sandia in early 1994. In 1995 he conceived this project. Its primary goals were to extend the MMP concept to three dimensions, identify which tolerance specifications in the current standards could produce MMPs and to test it in a real engineering environment. This work was to be done in collaboration with the research group at Cornell that Parratt had come from. This group is headed by Professor Herb Voelcker. Of the research to extend Parratt's work, Dr. Voelcker says "it is probably the important research we do." Dean Robinson, who was at Cornell and now works at GE, was also a collaborator.

Unfortunately, near the end of the first year of this project, Parratt left Sandia to begin work on an MBA at the Wharton Business School at the University of Pennsylvania. He had been working with Dean Robinson and Cornell but up to that point, little had been accomplished. This eliminated one of the major contributors to the entire project and left Sandia with nothing to contribute in the theoretical arena. Management of the project was assumed by Peter Watterberg. Many discussions were held to determine how the project should continue. It was felt that the software project that could produce variations of a part from a nominal, toleranced model could still be useful.

Up to this time, systems to do assembly analysis, assembly planning and functional analysis have been, for the most part, constrained to working on nominal part models. If these systems could be provided with parts which would vary in form based on the tolerance specifications, they could produce reports on the sensitivities of various aspects of the assembly to these variations and ranges. These would be the first automated analyses that could comment on how the tolerances would affect the final assembly.

In the fall of 1996, Watterberg went to Cornell to hold discussions with them on the future of the project. Dean Robinson reported at that time that the criteria for standard geometric tolerances to produce MMPs are extremely complicated and he was essentially abandoning that effort. He would henceforth concentrate on how the tolerancing rules and guidelines might be changed so that MMPs may be guaranteed. The meeting concluded with general agreement that Cornell should continue in their efforts which supported this project in other areas (see appendix A) and that the direction for the software system suggested by Sandia was a promising pursuit.

In early 1997, other personnel defections necessitated the transfer of Watterberg to other projects. He maintained oversight of this one but the primary workload was assigned to David Darras. Darras began learning what was necessary and assembling the tools to build the software system to provide sample parts. One of those parts was a tolerancing husk for ACIS that had been written by people at Allied Signal in Kansas City.

Parratt had originally discovered the Allied Signal work and identified it as a useful piece for the system. Watterberg had gone to Kansas City in the fall of 1996 to discuss use of their system. The system was brought to Sandia and it was left to Darras to implement it. The plan was to take assemblies which are designed using PRO/E, translate them to ACIS and use ACIS functions to manipulate the geometry. ACIS is the geometry environment most commonly used by local analysis packages. ACIS had no native facility for holding tolerance information. Hence the need for the Kansas City software.

By the time Darras had come up to speed on the project and assembled the pieces, it was becoming apparent to him that the software from Kansas City had many bugs and was going to be difficult to work with. It was by now, more than half way through the final year of the project. There was little room for changing directions again. Hence, it was decided that Darras would push on and find out what he could, given the time and poor software environment, about varying the geometry of ACIS parts. In view of the dire situation the project was in, it was decided to cut back the overall level of effort and return $30,000 to the LDRD office. Given the difficulty of the environment, Darras was unable to accomplish anything significant in the remainder of the project.

The only significant results of this project have come from the collaborative work at Cornell. This work is documented in appendix A which is a report from Cornell on the work done there.

Conclusions

This project failed to make any significant progress towards the goals that were set forth. This is due primarily to the change in personnel but also to the poor choice of software environment and to some extent the difficulty of the task in some areas. However, the initial concepts upon which this project were based still have all the promise they ever did. In fact, the work at Cornell and GE, although not as productive as was hoped, has served to enhance the possibilities of the concepts of Maximum Material Part and Generalized Feature of Size. Since the departure of Steffan Parratt, Sandia probably has little to contribute in this area. It is still hoped, however, that progress will continue to be made and these ideas can be brought to their full potential. The fields of tolerancing and assembly analysis will be significantly advanced.

The concept of a software system to produce "random" or statistically bound variations of designed assemblies is still one that could be important to work at Sandia. The results of our efforts in this area have convinced us that a better approach would be to build an interface with PRO/E to modify the geometries at that stage instead of in the ACIS world. This work could and perhaps should be furthered at Sandia but will have to find new champions since most of the people who would have pushed for such a system have left.

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Appendix A

Final report from Cornell on collaborative work.

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FINAL REPORT
on
SANDIA CONTRACT AS-2874

"Research in Dimensional Tolerancing"

Year 1: 14-May-96 through 30-Sep-96
Year 2: 1-Oct-96 through 30-Sep-97

by

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December 12, 1997

Summary

The research reported here was intended originally to provide theoretical and background support for a new tolerancing project at Sandia. However, the scale of the Sandia project was reduced due to personnel changes just as the Cornell work was getting underway, and early in Year 2 of the contract the Sandia project was postponed indefinitely. Cornell's research followed the goals set originally, which are consistent with our long-term research objectives, and good – in some cases excellent – progress was made in all aspects of the work. The research is summarized below in terms of four themes. All results are accessible through technical reports and published papers, and a few of the results are experimentally usable or testable now.

Contents

1. ORGANIZATIONAL CHARACTER OF THE RESEARCH

The research reported here was intended originally to provide theoretical and background support for a new CAD modeling and tolerancing project at Sandia – "Solid Variant Geometry Modeling" (LDRD 96-0511) – directed by Dr. Steffen Parratt. Dr. Parratt had been awarded a Cornell Ph.D in January of 1994 for basic research in one-dimensionaol (1-D) tolerancing for assembly, and had joined Sandia in February of 1994. One of its main objectives of Parratt's project was to test experimentally whether two major new concept that had emerged from his Ph.D research – Generalized Feature of Size (GFS) and Maximum Material Part (MMP) – are viable in 'real' engineering design. The Cornell research group known as CPA (Cornell Programmable Automation), in which Parratt had done his doctoral research, was engaged under Contract AS-2874 to support his experiments per the Year-1 Work Statement reproduced in this report's first Appendix.

In mid-1996 (about two months into the 4.5-month abbreviated Year 1 of the contract), Dr. Parratt resigned his Sandia appointment to undertake full-time graduate study in Management, and Dr. Peter Watterberg became the leader of the Sandia 96-0511 project. Dr. Watterberg paid a constructive visit to Cornell a few months later; he was briefed on our work and provided with some tutorial materials to aid him on the Sandia side, and the original goals for Cornell's research were reaffirmed in slightly sharpened form – see the Year-2 Work Statement in the second Appendix to this report.

Unfortunately several additional members of Sandia's research staff departed at the end of 1996, and Dr. Watterberg acquired new responsibilities that led to his postponing indefinitely all work on the Sandia side of the LDRD project. Cornell continued its research under the terms of the work statements, and good – in some cases excellent – technical progress was made on all fronts. The technical character and results of Cornell's work are summarized below, in Section 3, under four themes. These themes differ somewhat from the goals listed in the work statements and are logically tidier; a cross-reference table between the themes below and those in the work statements can be provided.

The research and research results reported below cannot be attributed solely to Sandia support, because Sandia's funds were, in effect, pooled with other external funds to support CPA's work in tolerancing and metrology. The 'other external funds' included two one-year grants under the Ford Motor Company's University Research Program that were commensurate in size and tenure with the Sandia contract, and a portion of our Year 3 and Year 4 funding under NSF Grant MIP-93-17620. (Sandia funds were not used to support research in CPA's other main line of research – meshless computational methods for solving boundary-value problems; that work is funded wholly from our NSF grant.) We can probably produce, if required under the Sandia contract, a table showing specific efforts and results per Sandia dollar by reviewing our records of actual salary allocatiions versus the work done by the supported individuals (Voelcker, Morse, Braun).

2. REFERENCE MATERIAL FOR THE RESEARCH REPORTED BELOW

The technical substance of each of the topics summarized below is conveyed in technical reports and papers cited in the style [9n-m], e.g. [94-1], which means Report CPA94-1 ... the first entry in the references listed in Section 5 below. Any or all of these documents will be provided to Sandia, *on request,* on paper and/or electronic media.

CPA does not, as a matter of policy, disseminate documents unless they are requested explicitly by an individual or organization, and CPA does not post documents on the World Wide Web or similar openly accessible electronic media. This policy is intended to reduce 'data proliferation and pollution'. In blunt terms, we believe that the technical community is already swamped with unsolicited paper and electronic documents, with the result that the value of all documents – even the most meritorious – is being reduced progressively. The policy also provides some measure of control over our documents, in that we know who has copies and may be interested in the work.

The second section of the reference list in Section 5 cites five magazine articles different in character from traditional technical reports and papers. There were written, on request, for a high quality trade magazine – *mfg.* – published by the Brown & Sharpe Manufacturing Company (America's largest supplier of dimensional metrology apparatus). They address succinctly some major contemporary technical issues in a conversational but technically correct style. They are directed to a much broader audience than are traditional technical reports and papers. Such articles are not easy to write, but they have proved to be surprisingly effective, in terms of reader comments and questiions, in informing the technical community at large.

3. TECHNICAL THEMES

[97-3], and to a lesser extent [95-4], provide useful tutorial overviews for all of the topics summarized below, and for several others as well.

3.1 TOLERANCING FOR ASSEMBLY: NEW CONCEPTS AND APPROACHES

Dimensional tolerances are the primary tools used in mechanical design to control 'form' (loosely, shape and size) variability of parts and products. They are assigned to permit as much variability as possible while insuring interchangeable assembly on one side of a tolerancing limit, and 'functionalism' on the other. An assembly of just two moderately complex parts may involve several or many 'feature matings'; each must be checked for interference individually, and 'stationary states' must be found for the collection as a whole ... an expensive, error-prone process that is nearly impossible to automate effectively in today's tolerancing technology.

Parratt's research [94-2, 97-4, Mfg 97a] exposed an alternative and potentially revolutionary procedure: if worst cases (called Maximum Material Parts – MMPs) can be found for each part in an assembly, one can then design just these for mating and eliminate all of the case-by-case analysis associated with the traditional method. Parratt's results were restricted to 1-D tolerancing, and left open a host of issues on 'deep implications' of the results and necessary generalizations to 2- and 3-D tolerancing.

Our assembly research for the past three years has been focused on extending Parratt's work; it is probably the important research we do.. Robinson has been seeking MMP criteria for parts carrying 2- and 3-D geometric tolerances, and Morse and Voelcker have been attacking the 'deep implications' area. Robinson has concluded that MMP criteria for parts carrying geometric tolerances assigned under today's rules, which apply only locally, are extremely complicated and are likely to be satisfied by very few if any 'real' parts; however, MMP-ness can be guaranteed from the outset of a design if tolerances are assigned within moderately restrictive guidelines that Robinson is developing [98-1]. At present we are optimistic: Robinson's approach appears to be practically viable and potentially important.

Morse and Voelcker have proved (disproved in a few cases) some of the conjectures in Parratt's thesis, and are now breaking new ground. An early, still tentative result: the MMP approach to tolerancing for assembly is quite (sometimes very) conservative; there seem to be classes of parts whose tolerances carry the advantages of MMP-ness but are looser and hence cheaper to attain. We are trying to characterize these classes sharply.

3.2 GEOMETRIC TOLERANCES: STANDARDS AND SEMANTICS

Geometric tolerances are governed by elaborate national and international standards, but the definitions in the standards are informal – cast in prose and graphics, and based on special-case examples. The American standard, Y14.5-1994, was partially 'mathematized' in an enumerative manner in 1990-94 with the results reported in a new companion standard, Y14.5.1-1994. However, inconsistencies and gaps remain, and there is no set of underlying mathematical principles from which the standard can be generated. We are one of only three 'theory' groups working closely with the Y14.5.1 Committee to fill the gaps, rationalize the inconsistencies, and adduce some foundation principles. It is slow, picky, but necessary work. Thus far we have made significant contributions to clarifying the national of 'size' [94-1, 94-6, 95-6, Mfg 95], disambiguating the rules for datum declaration (a technical report on this topic is being written), and moderating – not always successfully – some of the more radical (read ill-conceived) recent initiatives launched in 1996-7 within the ISO standards community [Voel 96].

3.3 STATISTICAL TOLERANCES: EXPERIMENTAL STUDIES; MODELS & ANALYSIS; STANDARDS

Statistical tolerancing evolved contemporaneously with, but largely independently of, geometric tolerancing. To date no American standards, and only one largely ignored DIN (German) standard, have been adopted for statistical tolerances – for interesting and quite subtle reasons; applications, which are burgeoning, have been covered by *ad hoc* company standards. Two years ago ISO launched a major 'crash' effort to develop some standards, and to include coverage of geometric tolerances within said standards. This endeavor raises a host of issues, and we are deeply involved in some of the most important.

The first is developing semantic (mathematical) interpretations of what the draft ISO standards mean, and also elucidating some alternatives to the methods embodied in the draft ISO standards: see [97-10, Mfg 97b]. The second and third Cornell thrusts are centered on statistical interpretations of geometric tolerances. Essentially nothing is known about the statistical properties of geometric tolerances. For example: are they naturally normal, i.e. do they exhibit Gaussian statistics? We launched an ambitious, exploratory measurement program to obtain sample distributions for representative geometric position, orientation, circularity, and runout tolerances ... and found that their statistics are not naturally normal [97-2, 97-7]. To complement the experimental program we launched a mathematical modeling program that has yielded phenomenological models from which we can derive (predict) distributions that are mathematically interesting and agree quite well with the experimental data. The model-based distributions allow us to apply rationally the draft ISO toleracing standards, which assume normality, to intrinsically non-normal geometric tolerances [97-5, Mfg 96]. We believe this work is important, and at the moment we are clearly leading the world in this area.

3.4 A FRAMEWORK FOR RATIONALIZATION

Classical parametric (plus/minus limit) tolerancing, zone-based geometric tolerancing, statistical tolerancing, Taguchi quality criteria ... a growing collection of seemingly independent techniques for controlling dimensional ('form') variability, with no clear interconnections, translation rules, or assessments of global validity. Can some kind of unifying framework be induced?

Six months ago we proposed, as a 'strawman' approach to rationalization, the notion that a *tolerancing scheme* should be viewed, or 'taxonomized', as a triple [97-8]:

(representation scheme for variability, criteria, composition rules) .

The known schemes can be mapped by their variability representations schemes and criteria into the 2-D table below. Adding a third dimension to cover composition rules (for variability and criterion metrics) is considerably more difficult, mainly because the composition rules for the various known schemes are so ill-defined, disparate, and 'quirky'. We don't know whether this approach will be genuinely productive, but it is the first and only proposal on the table in an area that desperately needs rationalization.

		REPRESENTATION SCHEMES FOR VARIABILITY				
↓ *CRITERIA* ↓		MINIMAL PARAMETRIC	EXTENDED PARAMETRIC	GEOMETRIC	SURROGATE GEOMETRY	
DETERMINISTIC	WC (Worst Case)	Simple Standards Declining Use		Elaborate Standards Widespread Use	Emerging in European circles	
	L^p Norms (≈ Taguchi)	(In limited use?)				
	... other ?					
STATISTICAL	CFR (C_p, C_{pk}, ...)	No Standards Used in Industry		Under study in ISO		
	CDF (Distri. Bounds)	(Under study)		Under study in ISO		
	L^p Norms (≈ Taguchi)			Under study in ISO		
	... other ?					

Table 1: A classification of tolerancing schemes.

4. WHAT'S USABLE NOW?

Almost all of our work should be viewed as 'research in progress'. There are no 'final results', but there are some intermediate results that are either directly usable or experimentally testable now. The most promising candidates, in our opinion, are the following.

1) Our statistical models for some important classes of geometric tolerances seem to be 'right', or at least right enough to be useful now in industrial process control and for developing numeric norms for the new ISO statistical tolerancing standards that will come into use soon. They are the only results currently available for these applications.

2) Our interpretations of the semantics of the new ISO standards, and also the alternative approaches we have proposed, also seem to be right, or at least 'not wrong and practically plausible' ... and again are almost the only such results available now.

3) Our soon-to-be-published proposal for disambiguating datum declarations in geometric tolerancing is simple and, in our opinion, considerably more sensible than the alternatives being developed within the Y14.5 and ISO/TC-213 Committees. It should be accepted, tuned, and put into service as soon as it appears – but may well be squelched for minor technical and major political reasons.

4) Robinson's new guidelines for insuring MMP-ness in geometric tolerancing are not yet complete enough to be used, even experimentally, on a broad range of individual parts, but they can be (and should be) tested on selected sets of relatively simple parts.

5. REFERENCES

TECHNICAL REPORTS AND PUBLISHED PAPERS

CPA94-1 K. Suresh & H. B. Voelcker, "Notes on Size: 1 – Issues raised by the Requicha/Srinivasan (spine-based) definition of size", January 1994 (release to ASME Y14.5.1); rev. May 1994 & December 1994. Report

CPA94-2	S. W. Parratt, "A theory of one-dimensional tolerancing for assembly", Ph.D dissertation, The Sibley School of Mechanical & Aerospace Engineering, Cornell University; May 1994 (registered February 1994); CPA report version April 1994.	Report
CPA94-6	K. Suresh and H. B. Voelcker, "New challenges in dimensional metrology: A case study based on 'Size'", *ASME Manufacturing Review,* vol. 7, no. 4, pp. 291-303, December 1994.	Preprint
CPA95-4	H. B. Voelcker, "Dimensional tolerancing today, tomorrow, and beyond", Ch. 1 of **Advanced Tolerancing Techniques**, Ed. H. C. Zhang, pp. 3-11: John Wiley & Sons, New York, 1997; first published in *Proceedings of the 1995 International Mechanical Engineering Congress & Exposition,* Session DE-8; San Francisco, CA, November 1995.	Reprint
CPA95-6	K. Suresh, H. B. Voelcker, and E. P. Morse, "Notes on Size: 2 – Weaknesses in the Y14.5.1M - 1994 definition of 'size tolerance' and some remedial modifications", October 1995; Working Paper prepared for the ASME Y14.5.1 Standards Committee.	Report
CPA97-2	P. R. Braun, E. P. Morse, and H. B. Voelcker, "Research in statistical tolerancing: Examples of intrinsic non-normalities, and their effects", *Proc. 5th CIRP Seminar on Computer Aided Tolerancing,* Ed. H. ElMaraghy, pp. 1-12; University of Toronto, Toronto, Canada, April 1997. See also **<title>**, Ed. H. ElMaraghy, pp. <in press>; Chapman & Hall, London, 1998.	Preprint; abridged version of CPA96-2
CPA97-3	H. B. Voelcker, "The current state of affairs in dimensional tolerancing", *Proc. 1997 International Conference on Manufacturing Automation,* Eds. S. T. Tan, T. N. Wong, and I. Gibson, pp. 30-45; University of Hong Kong, Hong Kong, April 1997.	Preprint
CPA97-4	S. W. Parratt, "Models and methods for analyzing one-dimensional toleranced assemblies", March 1997.	Report
CPA97-5	E. P. Morse, "Short Communication: More on the effects of non-normal statistics in geometric tolerancing", July 1997; to appear in **<title>**, Ed. H. ElMaraghy, pp. <in press>; Chapman & Hall, London, 1998.	Preprint
CPA97-7	P. R. Braun, "Statistical properties of geometric tolerances: Models and experiments", M.S. dissertation, The Sibley School of Mechanical & Aerospace Engineering, Cornell University; registered August 1997; CPA report version September 1997.	Report
CPA97-8	H. B. Voelcker, "Short Communication: Remarks on the essential elements of tolerancing schemes", July 1997; to appear in **<title>**, Ed. H. ElMaraghy, pp. <in press>; Chapman & Hall, London, 1998.	Preprint
CPA97-10	H. B. Voelcker, "New standards and criteria for statistical tolerancing", *Notes – 3rd Seminar on Tolerancing and Assembly Modeling,* Ed. J. Iannuzzi, Univ. of Michigan & VSA, Inc.; Ann Arbor, MI, October 1997.	Preprint
CPA98-1	D. M. Robinson, "Geometric tolerancing for assembly", Ph.D dissertation, The Sibley School of Mechanical & Aerospace Engineering, Cornell University; registered January 1998; CPA report version March/April 1998.	Report
[Voel 96]	H. B. Voelcker, "Proposed Modifications to Concepts and Terminology in ISO/CD 14660-1:1995(E)", Version 1.0; Working Paper of the ASME/ANSI Standards Committee Y14.5.1, May 1996.	Reprint

INVITED MAGAZINE ARTICLES

[Mfg 94] H. Voelcker, "Some consequences of the 1980s' Metrology Crisis", *mfg.* (a new magazine published by the Brown & Sharpe Manufacturing Co., of Kingstown, RI), vol. 1, no. 2, pp. 40-41, October 1994. Reprint

[Mfg 95] H. Voelcker, "Let's talk about size ...", *mfg.* (a magazine published by the Brown & Sharpe Manufacturing Co.), vol. 2, no. 1, pp. 40-41, April 1995; reprinted in *Tooling and Production,* pp. 17-19, September 1995. Reprint

[Mfg 96] Ed Morse and Herb Voelcker, "A tale of two tails", *mfg.* (a magazine published by the Brown & Sharpe Manufacturing Co.), vol. 3, no. 1, pp. 46-47, April 1996. Reprint

[Mfg 97a] Steffen Parratt and Herb Voelcker, "How do you tolerance a hinge?", *mfg.* (a magazine published by the Brown & Sharpe Manufacturing Co.), vol. 4, no. 1, pp. 42-43, Winter 1997. Reprint

[Mfg 97b] Herb Voelcker and Ed Morse, "De-mystifying C_p and C_{pk}", *mfg.* (a magazine published by the Brown & Sharpe Manufacturing Co.), vol. 4, no. 2, pp. 40-42, Spring/Summer 1997. Reprint

6. APPENDICES: WORK STATEMENTS

Year-1 Work Statement

The purpose of this contract is to provide support for Cornell Programmable Automation's (CPA) program of research in assembly tolerancing. The Sandia Laboratory Directed Research & Development (LDRD) "Solid Variant Geometry Modeling" (96-0511) is especially interested in those aspects of CPA's research that address

- application of statistical methods to geometric tolerances and Maximum Material Parts (MMPs);
- fundamental issues in dimensional tolerancing, such as generalization and mathematization of the notion of "size";
- definitions of Virtual and Resultant conditions;
- provides bridges to the ASME's Y14.5.1 Standard Committee and ISO's technical committees.

Year-2 Work Statement

Year 2 of this contract will continue support for research at Cornell in the broad area of assembly tolerancing. The following tasks will be prosecuted, some independently at Cornell, with results to be communicated to Sandia. and some in collaboration with Sandia's Dr. Peter Watterberg. This list may be modified, in consultation with Dr. Watteberg, in the early months of Year 2.

- Extension and refinement, in one-dimensional tolerance domains, of the Generalized Feature of Size (GFS) and Maximum Material Part (MMP) concepts developed by Dr. Steffen Parratt.
- Development of two- and three-dimensional versions of these concepts.
- Continuing development of statistical models and criteria for parametric and geometric tolerances, process control, and assembly tolerancing.
- Continuing liaison with, and contributions to, the ISO TC213 and ASME H213 and Y14.5.1 tolerance standards committees.

— *End of Report* —