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Tune Spread in TEV33**

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“Electron Lens” to Compensate Bunch-to-Bunch Tune Spread in TEV33

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Abstract

In this article we discuss an electron beam lens for compensation of bunch-to-bunch tune spread in the Tevatron antiproton beam. Time-modulated current of an electron beam can produce defocusing forces necessary to compensate effects caused by parasitic beam-beam interactions with proton beam. We estimate main parameters of the electron beam and consider resulting beam footprint. Emittance growth rate due to the electron current fluctuations is discussed.

1 Introduction

An “Electron Compressor” is a device for compensation of the beam-beam effects in proton-(anti)proton colliders with use of electron beam. Since the electron charge is opposite to the proton charge, its impact can in principle counteract the electromagnetic force due to proton beam. The “electron compressor” for Tevatron collider is described in detail in Ref.[1]. Here we consider an extension of the idea with a goal to compensate a bunch-to-bunch tune spread.

The Tevatron beam parameters which we used in our simulations are presented in Table 1, see column “TEV33” [2, 3]. Note that number of bunches and the rms bunch length can be changed if a short batch kicker and superconducting RF system are installed.

Table 1: The Tevatron Upgrades

Parameter		Run II	TEV33
Beam Energy,	E, GeV	1000	1000
Luminosity	$L, s^{-1}cm^{-2}$	$2.1 \cdot 10^{32}$	$1.16 \cdot 10^{33}$
N of bunches ($p, \bar{p}$),	N_b	36	90→140
Min. bunch spacing	τ, ns	396	132
Protons/Bunch	N_p	$3.3 \cdot 10^{11}$	$2.7 \cdot 10^{11}$
Antiprotons/Bunch	$N_{\bar{p}}$	$0.75 \cdot 10^{11}$	$6 \cdot 10^{10}$
p -Emittance rms,	$\epsilon_{np}, mm \cdot mrad$	3.3	3.3
$\bar{p}$ -Emittance rms,	$\epsilon_{n\bar{p}}, mm \cdot mrad$	2.5	3.3
Number of IPs	N_{IP}	2	2
Interaction focus	β^*, cm	35	35
Bunch length	σ_s, cm	37	37 (→14)

There are several beam dynamics issues caused by beam-beam forces not only from the two head-on interaction points (IPs, at CDF and D0 experiments), but also from an additional $2 \times (N_b - 1) \sim$ hundred(s) of parasitic crossings of proton and antiproton bunches. It is to be noted that the design value of the total tune shift for antiprotons (pbars) is approximately equal to 0.02 that is about maximum experimentally achieved value for proton colliders. The “footprint area” of the $\bar{p}$ beam with such a tune shift is large enough to cause an increase of particles losses due to higher order lattice resonances

Other beam-beam induced effects include the variation of the betatron tunes along the bunch train and $x - y$ coupling due to skew component of the beam-beam kick [3]. The maximum spread

$\delta\nu_{max}$ of vertical and horizontal $\bar{p}$ bunch tunes is estimated to be about 0.003 during Run II and about 0.01 in TEV33 as quoted in Table I. This spread and the estimated skew-kicks for TEV33 are expected to be a problem for collider operation if uncorrected. While the proton beam intensity is supposed to be several times the antiproton one, one expects the beam-beam effects are severe for antiprotons. Thus, in this article we discuss the beam-beam compensation for antiproton beam only.

In Section 2 we give a brief overview of the “electron compression” and requirements on the electron beam. Section 3 is devoted to the bunch-to-bunch tune spread compensation with “electron lenses”. The electron beam modulation and current stability are discussed in Sections 4 and 5 correspondingly. Final conclusions are given in Section 6.

2 “Electron compression” of tune space

Fig.1 from [1] presents a general view of the “electron compression” device. Its electron beam travels in the direction opposite to the antiproton beam and interacts with an antiproton bunch via its electric and magnetic forces.

Figure 1: Layout of “electron compression” device.

Such an interaction shifts the antiproton beam tunes and distorts the tune footprint in a way which depends on the transverse electron charge distribution. For example, consider the footprint due to “head-on” collisions of round equal size proton and antiproton beams at the IPs. The “electron compression” can in principle shrink the $\bar{p}$ footprint to a point if the electron beam a) has the same transverse charge distribution as the proton beam; b) the $\bar{p}$ beam distribution at the “electron compressor” is the same as at the IPs (just scaled), i.e. zero dispersion; and c) its total charge eN_e on the path of the $\bar{p}$ beam (e.g. over the length L of the central solenoid in Fig.1) satisfies the equality condition of beam-beam shifts. This equality condition for protons and electrons is:

$$\frac{N_p r_p}{4\pi\epsilon_n} N_{IP} \equiv \xi_p^p = -\xi_p^e \equiv \frac{N_e r_p (1 + \beta_e)}{4\pi\epsilon_n} \mathcal{F}, \quad (1)$$

here N_p is the number of protons per bunch, $N_{IP} = 2$ is the number of IPs, $r_p = e^2/m_p c^2 = 1.53 \cdot 10^{-18} \text{m}$ is the proton classical radius, ϵ_n is the rms normalized proton bunch emittance, β_e is longitudinal velocity of the electron beam divided by speed of light v_e/c , and we assume that transverse sizes of all beams are the same $\sigma_e = \sigma_p = \sigma_{\bar{p}} = \sigma$. The numerical factor $\mathcal{F}$ depends on

the relative length of the electron beam L and the pbar bunch σ_s . For example, $\mathcal{F} \approx 1$ if $L \gg \sigma_s$ and $\beta_e \ll 1$, and $\mathcal{F} \approx 1/2$ if $L \simeq \sigma_s$ and $\beta_e \approx 1$. For simplicity, we assume equal horizontal and vertical emittances and beta functions for antiprotons at the “compressor”, and we consider a round electron beam with $L \gg \sigma_s$, which gives $\mathcal{F} \approx 1$. From Eq.(1) one gets

$$N_e = N_{IP} N_p / (1 + \beta_e), \quad (2)$$

or $N_e \approx 2 \cdot N_p = 4.5 \cdot 10^{11}$ for $\beta_e = 0.2 \ll 1$ and $N_{IP} = 2$, $N_p = 2.7 \cdot 10^{11}$ in TEV33. The extent of compensation in the real case depends on the control of the electron current, the transverse distribution, the separation of the electron beam from the $\bar{p}$ orbit, the angular separation between the beams, and the choice of the horizontal/vertical antiproton beta-functions in the electron beam region.

For effective “electron compression” of the tune area the electron beam size must be about the same size as $\bar{p}$ beam which has an rms size of $\sigma_{\bar{p}} = \sqrt{\beta \varepsilon_n / \gamma_{\bar{p}}}$. In the ultimate case of small emittance at TEV33 $\varepsilon_n = 3.3 \cdot 10^{-6} \text{m}$ one gets $\sigma_{\bar{p}} = 0.6 \text{mm}$ with the beta function $\beta_{x,y} = 108 \text{m}$ (as at the A0 straight section of Tevatron), or $\sigma_{\bar{p}} = 0.9 \text{mm}$ at $\beta_{x,y} = 250 \text{m}$ (if one decide to modify the existing lattice and provide high-beta, zero-dispersion region in Tevatron).

For numerical estimates we consider the electron beam with radius of $a_e = 1 \text{mm}$ and constant transverse distribution. Having the electron charge already defined in Eq. (2), one can estimate the electron beam current J_e necessary for the compression:

$$J_e = \frac{2\beta_e e N_p c}{L(1 + \beta_e)}, \quad (3)$$

where L is the length of the beam-beam interaction. Taking $L = 3 \text{m}$ and $N_p = 2.7 \cdot 10^{11}$ we get $J_e [\text{A}] \approx 8.64 \frac{\beta_e}{1 + \beta_e}$, for example, $J_e = 1.44 \text{A}$ for $\beta_e = 0.2$ (the latter corresponds to 10 kV electron beam).

The maximum current of space-charge limited diode electron gun is given by Child-Langmuir law

$$J_e = P \cdot U^{3/2} \propto \beta_e^3,$$

where P (perveance) is gun-geometry dependable constant proportional to the cathode area. Therefore, higher electron beam energy $U = \beta_e^2 m_e c^2 / 2$ seems preferable if one needs to get the required current from smaller area. From the other hand, the required gun power grows with U as $\propto U^{3/2}$ and it might limit the gun energy unless power recuperation scheme is introduced.

Taking all the above into consideration we conclude that the current of $J_e = 1 - 2 \text{A}$ of $U = 10 - 20 \text{kV}$ electrons seems appropriate for compensation of $p\bar{p}$ collisions in two IPs, although exact optimum can be found after more detailed studies.

Solenoidal magnetic field of several kilogauss helps to keep the electron beam stable under defocusing forces of self space charge and oncoming antiproton beam [1]. Stability of the antiproton beam propagating through the electron beam may require an order of magnitude stronger solenoidal field [4].

Required current density in 2 mm diameter beam is $j_e = \frac{J_e}{\pi a_e^2} \approx 280 \frac{\beta_e}{1 + \beta_e} \text{A/cm}^2$, or about 46 A/cm^2 for 10kV beam. This is somewhat larger than 10 A/cm^2 that oxide cathodes usually pro-

Figure 2: Tevatron with two “electron lenses”.

vide. To overcome the cathode current density limit, one can use an adiabatic magnetic compression when the beam is born on the cathode with larger radius a_c in weak field B_c and transported to the region of the higher magnetic field B with conservation of adiabatic invariant of $B_c a_c^2 = B a_e^2$.

3 Bunch-to-bunch tune spread compensation with electron beams.

In the multibunch operation regime proton and antiproton beams are separated around the Tevatron ring except two IPs at B0 and D0 and beam-beam interaction at numerous parasitic crossings concludes in bunch-to-bunch variation of the tune shifts and couplings if the proton beam has gap(s) or significant bunch-to-bunch charge variations. The “electron compressor” allows, in principle, to avoid such detrimental effects and equalize tune shifts and coupling of different bunches by modulation of the electron current in time providing different quadrupole kicks on different antiproton bunches. In order to distinguish the electron beam set-up for non-linear beam-beam compensation and the modulated electron beam device which is a subject of this paper, we will call the latter as an “electron lens”.

Long-range beam-beam interactions at multiple parasitic crossings do shift horizontal and vertical tunes in opposite directions $\Delta\nu_x^{LR} \approx -\Delta\nu_y^{LR}$, while the head-on electron compressor with round electron beam¹ shifts the tunes in the same direction $\Delta\nu_x^{EC} \approx \Delta\nu_y^{EC}$. To compensate the long-range interaction, one may install two “electron compressors-electron lenses”: one at the location with horizontal beta-function larger than vertical one $\beta_x \gg \beta_y$ and another one at $\beta_x \ll \beta_y$ -see Fig.2 Consequently, the first produce larger tune shift in horizontal plane, and the second in vertical plane.

¹ the requirement of having round electron beam looks natural since electrons travel in accompanying solenoidal magnetic field

Figure 3: **a** – left figure – Currents in two electron lenses to compensate bunch-to-bunch tune spread in 90×90 bunches scenario - see text, ; **b** – right figure – Resulting pbar bunch tune shifts (core particles only) with 10% error of the compensation (see text).

Table 2: Bunch-to-bunch electron beam lenses

Parameter		Location 1	Location 2
Horiz. beta-function,	$\beta_{1,2}^x$, m	101.67	59.0
Vert. beta-function,	$\beta_{1,2}^y$, m	30.89	110.1
Dispersion,	$D_{1,2}$, m	1.92	1.69
Hor. $\bar{p}$ beam size, rms	$\sigma_{1,2}^x$, mm	0.61	0.47
Vert. $\bar{p}$ beam size, rms	$\sigma_{1,2}^y$, mm	0.31	0.6
Round e -beam size, rms	$a_{e,1,2}$, mm	0.61	0.6
Length of the e -beam,	L , m	2	2

For our numerical simulations we chose two locations at the Tevatron for two “electron lens” devices – one at so-called location 48, another at the upstream end of C0 section. Parameters of these locations and the two corresponding electron beams are presented in Table 2. Everywhere we assume round Gaussian electron beams with the rms size equal to $1 \times$, $2 \times$ or $3 \times$ the maximum of the $\bar{p}$ sizes at the corresponding location.

Originally, without the “electron lenses”, the tunes of different bunches $\Delta\nu_{x,y}^{\bar{p}}(i)$ are calculated with use of other code [5] under conditions of the TEV33 with $136 \mu\text{rad}$ crossing angle at IP in both planes, 132 ns minimum bunch spacing and some particular separations along the ring. We use these data to demonstrate operation of the compensation technique.

If we denote the currents in the two “e-lenses” as $I_1(t)$ and $I_2(t)$, then the core particles tune shifts due to electrons are equal to:

$$\Delta\nu_{x,y}(t) = \beta_1^{x,y} \cdot I_1(t) \cdot C1 + \beta_2^{x,y} \cdot I_2(t) \cdot C2. \quad (4)$$

The constants are $C_{1,2} \approx \frac{2.2 \cdot 10^{-5} \cdot L[m]}{a_{e,1,2}^2[mm]}$ where $a_{e,1,2} = \max(\sigma_{1,2}^x, \sigma_{1,2}^y)$. If one requires 100% compensation of the core particles tune spread from bunch to bunch, then the necessary currents are solutions of two linear equations $\Delta\nu_{x,y}(t) = -\Delta\nu_{x,y}^{\bar{p}}(i)$. These currents I_1 and I_2 for the

Figure 4: **a** – left figure – Tune spread of 14cm long pbar bunches # 02,17,21 in 90×90 bunches scenario. Two electron beams with the rms sizes $1 \times \sigma_{\bar{p}}$ are used for “electron compression”; **b** – right figure – The same for the rms electron beam sizes $= 2 \times \sigma_{\bar{p}}$.

TEV33 operation scenario with 90 p bunches and 90 $\bar{p}$ bunches are shown in Fig.3a. Due to 3-fold symmetry, only 30 different current values are presented. Note, that due to two gaps the maximum bunch number in Fig.3a is 34, although the number of bunches is 30, i.e. four bunches are missing. Again, the bunch spacing is 132 ns. One can see the maximum necessary current does not exceed 1.4 A. The result of implementation of such compressors would be that all core particle tune of all the bunches become the same. Fig.3b shows the initial bunch-to-bunch tune spread for core particles (30 circles in the right upper corner of the plot) and the resulting bunch tunes under assumption of 10% compensation error. Such an error may be due to current mismatch, inadequate beam-beam model or not-precise single bunch tune diagnostics (without these errors, the result of compensation will look like a point in Fig.3b).

Fig.4a shows transformation of the footprints of bunches # 02, 17 and 21 with the “electron compressors”. Particles with up to 3 times rms betatron amplitudes are plotted. The amplitudes a_x, a_y are varied from 0 to $3 \sigma_a$ with a step of $0.5 \sigma_a^{x,y}$ where $\sigma_a^{x,y}$ is the rms betatron oscillations amplitude. Each point (a_x, a_y) is presented by a point, while the core (zero amplitude) tunes are marked by larger circles. One can see that the application of two electron compressors about halves the tune spread of all particles in the bunches. Even better compression of the tune area can be achieved if the electron beam size is twice the $\bar{p}$ size and the currents $I_{1,2}(t)$ are four times larger (i.e. maximum current will be about $4 \times 1.4 = 5.6$ A) – see Fig.4b. Corresponding calculations for pbar bunch length of $\sigma_s = 37$ cm and a_e twice and three times of $\sigma_{x,y}^{\bar{p}}$ (and corresponding electron currents four and nine times of what is shown in Fig.3a) are presented in Fig.5a and 5b, respectively. Again, compensation of bunch-to-bunch tune spread gives about two-fold reduction of the tune area covered by the beam. Note, that as the current density remains the same, then the compressors will require the same magnetic field to maintain e -beam stability. The solenoidal field required for “electron lens” is somewhat stronger than what is needed for “electron compressor” in order to avoid two additional interfering effects: a) variation of the beams size while the electron current varies; and b) distortion of the electron beam transverse charge distribution under

Figure 5: **a** – left figure – Tune spread of 37cm long pbar bunches # 02,17,21 in 90×90 bunches scenario. Two electron beams with the rms sizes $2 \times \sigma_{\bar{p}}$ are used for “electron compression”; **b** – right figure – The same for the rms electron beam sizes $= 3 \times \sigma_{\bar{p}}$. (see text).

impact of the azimuthally non-uniform electric force of the elliptic pbar beam. We estimate that the solenoidal field of some 2-4T is enough for that, although detailed simulations are under way.

Installation of a short batch injection kicker at the Tevatron with 132 ns rise-time will allow to consider more bunches in TEV33, smaller gaps, and consequently smaller bunch-to-bunch tune spread. Fig.6a shows calculated currents in the two “electron lenses” for operation with 140 proton and 121 antiproton colliding bunches. One can see that the required currents are approximately twice less than what is needed in 90×90 bunches operation. Fig.6b demonstrates the compensation achieved with two compressors and 10% error in the electron currents. Footprints of bunches # 86,108,150 with and without “electron compressors” are shown in Fig.7a. Again, the degree of the tune area compression is better with wider electron beams (rms e -beam size is twice the pbar beam size) – see Fig.7b. From that, one can conclude that wider beam is preferable, because in that case electron beam works much like a time-variable linear defocusing lens.

4 Electron current modulation

Now we consider the time structure of the quadrupole kick produced by electron beam. The operation of the electron lens look very similar to a traveling wave kicker [6].

Now we consider the time structure of the defocusing kick (or the tune shift) produced by the traveling wave kicker. Fig.8 demonstrates the effect of a step-like current modulation with the pulse duration of t_p (presented at the upper plot) on the antiproton bunches. Let us denote $t = 0$ the moment when the front of the electron pulse enters the interaction section. As the antiproton beam passes through the oncoming electron current pulse, the maximum deflection will be seen by test particles which at $t = 0$ are distanced by $2(1 + \beta_e)l/\beta_e$ from the input end of the device. We will call the corresponding time value of $\tau_g = 2(1 + \beta_e)l/c\beta_e$ as “kick growth time”. The

Figure 6: **a** – left figure – Currents in the two electron lenses to compensate bunch-to-bunch tune spread in 140×121 bunches scenario - see text, ; **b** – right figure – Resulting of the antiproton bunch tune shifts (core particles only) with 10% error of the compensation (see text).

maximum kick lasts over time interval of $t_f = t_p - \tau_g$ which is supposed to be synchronized with the bunch arrival (see lower diagram in Fig.8). Behind that bunch, the kick amplitude vanishes over the growth time. Analytical expression for the tune shift is as follows:

$$\Delta\nu(t) \propto \frac{1}{2} \int_{t_s}^t J(t') dt', \quad t_s = -t + 2 \max(0, t - (1 + \beta_e)l/c\beta_e). \quad (5)$$

Let the required flat top of the pbar kick be about $t_f = 5$ ns, and the required “no-impact time” to be the same $t_n = 5$ ns; then, summarizing all times in Fig.8, the condition of $264 \text{ ns} > t_f + 2L(1 + \beta_e)/c\beta_e + t_n$ must be satisfied in order to have no impact on preceding and following bunches. That gives $\beta_e > 0.08$ or kinetic energy of the electrons $U > 1.6 \text{ kV}$. I.e., if one needs to modulate the electron current in order to equalize the bunch-by-bunch tune shift, then the electrons have to be fast enough to provide different quadrupole kicks on different bunches. We choose β_e of the order of 0.2 that is far beyond the requirement.

One can make two remarks: firstly, if the current pulse duration is less than the growth time $t_p < \tau_g = 2(1 + \beta_e)l/c\beta_e$, then the electron beam does not work in full strength; secondly, if the bunch spacing in the ring is equal to τ , then the electron current pulse duration must be less than $t_p < 2\tau - 2(1 + \beta_e)l/c\beta_e$ otherwise neighbor bunches will be defocused too. As the result, one can conclude, that the rectangular pulse duration of $t_p = \tau_g$ corresponds to the maximum device’s strength. The length of the electron beam has to be less than $L < c\beta_e\tau/2(1 + \beta_e)$ because the current pulse shape can not be exactly rectangular, besides that, as we mentioned above, some flat top of the kick is required.

Making numerical example for the TEV33 with $\tau = 132$ ns and $\beta_e = 0.2$, we choose $L = 2$ m (that satisfies condition of $\tau_g = 2(1 + \beta_e)L/c\beta_e = 80$ ns $< \tau$) and the requirements on the pulse length is $t_p \leq 264 - 80 = 184$ ns. In fact, as the pulse shape of the current modulation can not be exactly rectangular, than the one should require the pulse FWHM to be somewhat smaller

Figure 7: **a** – left figure – Tune spread of 14cm long pbar bunches # 86,108,150 in 140×121 bunches scenario. Two electron beams with the rms sizes $1 \times \sigma_{\bar{p}}$ are used for “electron compression”; **b** – right figure – The same for the rms electron beam sizes = $2 \times \sigma_{\bar{p}}$. (see text).

(but still longer than τ_g), e.g. 100-120 ns.

5 Electron Current Stability

Fluctuations of the electron current from turn to turn cause time variable quadrupole kicks which lead to a transverse emittance growth of the antiproton bunches. In “the electron lens” the current has to be modulated rather fast although periodically, and thus, the issue of how stable is the current at one-turn scale may be of a great importance.

Emittance growth rate due to fluctuations of a gradient δG of a lens with length l is given by [7]:

$$\frac{d\varepsilon}{dt} = f_0^2 \frac{\varepsilon}{16} \left(\frac{el\beta_0}{Pc} \right)^2 \sum_{n=-\infty}^{\infty} S_{\delta G}(f_0|2\nu - n|), \quad (6)$$

where f_0 is the revolution frequency, β_0 is beta function at the lens location, Pc is the antiproton kinetic energy, ν is the machine tune, and $S_{\delta G}(f)$ is the power spectral density (PSD) of the gradient fluctuations. The PSD we use is defined for positive frequencies f . One can see that only some particular frequencies contribute into the emittance growth, the lowest of them is twice the betatron frequency $2\Delta\nu f_0$. If one assumes that the current ripple is a “white noise” with a constant PSD $S_{\delta G}$, then, the rms value of the ripple δG relates to the PSD as

$$\delta G^2 = (1/2)f_0 S_{\delta G},$$

and therefore, taking into account that there are two electron beams² on the pbar orbit, one gets:

²with presumably uncorrelated current fluctuations

Figure 8: Electron current modulation scheme.

$$\frac{d\varepsilon}{dt} = f_0 \frac{\varepsilon}{16} \left(\left(\frac{\beta_1}{F_1} \right)^2 + \left(\frac{\beta_2}{F_2} \right)^2 \right) \left(\frac{\delta J_e}{J_e} \right)^2 = 2\pi^2 f_0 \varepsilon (\Delta\nu_1^2 + \Delta\nu_2^2) \left(\frac{\delta J_e}{J_e} \right)^2, \quad (7)$$

where $F_{1,2}$ is focal lengths of two electron beam, $\delta J_e/J_e$ is the rms value of relative current fluctuation, $\Delta\nu_{1,2}$ is the value of tune shift produced by electron lenses 1 and 2 respectively (proportional to the current) and we used relation of $\Delta\nu = (1/4\pi)\beta_0/F$.

From Eq.(7) one immediately gets the emittance evolution equation:

$$\varepsilon(t) = \varepsilon_0 \exp(t/\tau_e), \quad (8)$$

where characteristic growth time is equal to

$$\tau_e = \frac{1}{4\pi^2 f_0 (\Delta\nu_1^2 + \Delta\nu_2^2) \left(\frac{\delta J_e}{J_e} \right)^2}. \quad (9)$$

The growth time is different for different bunches, e.g. τ_e is smaller for the bunches which experience larger currents J_e and, therefore, tune shifts $\Delta\nu$. These bunches (named PACMAN bunches) are usually located near the gaps (see bunches 33, 53, 86, 106 in Fig.6). Let us take for example bunch number 33 with $\Delta\nu = 0.01$ and $\Delta\nu_2 = 0.002$, then, requirement of

$$\tau_e > 10 \text{ hrs}$$

concludes in $\frac{\delta J_e}{J_e} < 0.53 \cdot 10^{-3}$. If one assumes constant distribution function of the ripple³, then the value above corresponds to peak-to-peak current fluctuations of

$$\frac{\Delta J_e}{J_e} \approx 1.8 \cdot 10^{-3}. \quad (10)$$

For non-PACMAN bunches (inside the batch) similar requirement is somewhat less stringent

$$\frac{\Delta J_e}{J_e} < 3.2 \cdot 10^{-3}.$$

Transverse motion of the electron beam may also cause direct antiproton emittance growth. Indeed, if the electron beam displacement is equal to δX , then the dipole kick experienced by antiprotons is $\delta\theta = \delta X/F$, where F is the focal length of the defocusing electron lens. Coherent $\bar{p}$ betatron oscillations occur and after some decoherence time they conclude in the antiproton emittance growth. The emittance grows *linearly* in time and its growth rate is equal to [8]:

$$\frac{d\varepsilon}{dt} = \frac{f_0^2}{4} \sum_{\text{sources}} \frac{\beta}{F^2} \sum_{n=-\infty}^{\infty} S_{\delta X}(f_0|\nu - n|). \quad (11)$$

Note, that now the frequencies of interest $f_0|\nu - n|$ starts from the betatron frequency of the Tevatron $\Delta\nu f_0 \approx 20 \text{ kHz}$.

Using the same transformations as above, one gets for two electron lenses:

$$\frac{d\varepsilon}{dt} = 8\pi^2 f_0 \delta X^2 \left(\frac{\Delta\nu_1^2}{\beta_1} + \frac{\Delta\nu_2^2}{\beta_2} \right), \quad (12)$$

where δX now stands for the rms electron beam vibration amplitude.

Let us apply constrain on the emittance growth rate to be less than $\varepsilon/10$ hours, $\varepsilon = 3.3 \text{ mm}\cdot\text{mrad}/\gamma_{\bar{p}} = 3.3 \cdot 10^{-9} \text{ m}$. Then, for the PACMAN bunches we get requirement on the rms electron beam turn-to-turn position stability

$$\delta X \leq 0.16 \mu\text{m}. \quad (13)$$

For the bunches in the middle of the bunch train, the requirement is about $0.24 \mu\text{m}$.

The obtained values are several orders of magnitude less than vibrations of the Tevatron quadrupoles at high frequencies, e.g. accordingly to [9], rms amplitude of the Tevatron quadrupole magnet at frequency of 450 Hz is about $2 \text{ nm} = 0.002 \mu\text{m}$, and the amplitude goes down with frequency rapidly.

³for such distribution the rms value is $1/\sqrt{12}$ of the peak-to peak value

If the electron beam and the antiproton beam are not properly aligned with respect to each other and they collide off-center with displacement equal to ΔX , then the electron current ripple at betatron frequencies causes dipole kicks on antiprotons and can also lead to the transverse emittance growth. The tolerance can be easily estimated from Eq.(13) as:

$$\frac{\Delta J_e}{J_e} \Delta X \approx \delta X. \quad (14)$$

Making estimate for $\Delta X = 0.25\sigma_{\bar{p}} = 0.15$ mm, one gets the rms current ripple tolerance for the PACMAN bunches

$$\frac{\Delta J_e}{J_e} < 1.1 \cdot 10^{-3},$$

or about 0.37% peak-to-peak, and about $1.6 \cdot 10^{-3}$ (0.52 % peak-to-peak) for non-PACMAN bunches. These requirements are somewhat loose in comparison with the quadrupole kicks effect (see above in this Section), although depend on the straightness of the electron beam in the interaction region which is determined by the solenoid field quality.

6 Conclusion

Interaction with electrons can substantially reduce tune variation from one Tevatron antiproton bunch to another. Such a task can be fulfilled with two round electron beam with specially programmed time-variable electron currents. One of the setups has to be installed at location where vertical beta-function is larger than horizontal one $\beta_y > \beta_x$ and, therefore, will affect mostly vertical $\bar{p}$ tune; another requires opposite relation $\beta_y < \beta_x$ for mostly horizontal tune change.

We found that for better tune area compression the size of the electron beam should be two-three times the rms size of the antiproton beam.

Electron current needed is periodic with Tevatron revolution frequency. The current waveform (amplitude and modulation) depends on particular colliding bunches pattern, bunch intensities, crossing angle and orbit separation.

We considered the time structure of the defocusing force due to electron current and estimated, that 132 ns bunch spacing in TEV33 will require 100-120 ns current modulation time in 2 m long 10kV electron lenses.

Electron current fluctuations from turn to turn (more precisely, at frequencies about double the betatron frequency) should be less than $\Delta J_e/J_e < (2 - 3) \times 10^{-3}$ peak-to-peak, otherwise variable defocusing kicks may lead to significant transverse antiproton emittance growth. Consideration of the transverse emittance growth caused by dipole kick due to displaced electron beam has shown somewhat less stringent requirement on the current ripple, though depending on how well the electron and the antiproton beams are centered at the interaction region. Direct emittance growth with ideally centered beams due to the electron beam vibrations is predicted to be negligible.

Finally, we would like to note that time-variable electron beam can, in principle, be used to raise threshold of the coupled bunch instabilities in accelerators and storage rings (e.g. due to

resistive walls or another narrow band impedance), since it produces tune spread from bunch to bunch which results in stabilization if larger than increment of the corresponding instability.

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