MATHEMATICAL ANALYSIS OF HE DATA

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JULY - SEPTEMBER 1971

Normal Process Development
Endeavor No. 230

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The purpose of this project is to aid in the analysis of experimental data by the application or development of mathematical techniques.

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MATHEMATICAL ANALYSIS OF HE DATA

ABSTRACT

A new approach to the analysis of capacitor discharge unit (CDU) current-time traces is discussed. The method involves the formulation of a mathematical model based on physical considerations of the system.

DISCUSSION

Recent effort for this project has been the development of a technique for analysis of capacitor discharge unit (CDU) current-time traces, with special emphasis on non-linear transient behavior patterns. As stated in a previous report(1), these traces have been difficult to analyze because of deviation from the ideal case for an LRC circuit. Early attempts to characterize the circuit included transient and Fourier analysis methods. These techniques, while useful for 2nd and 3rd cycle data, failed to adequately describe the first cycle, the primary region of interest. A new approach to the problem has been developed which involves formulation of a different model for the circuit and curve fitting the model to the current-time data. From the curve fit parameters, meaningful calculations can be made for the characterization of the capacitor bank.

The major objective in the analysis of a CDU current trace is the calculation of inductance (L) and resistance (R) of the system. Previous attempts at analysis of the current were based on these assumptions:

1. L, R, and C are constant for the system.
2. The circuit may be represented as a simple LRC circuit with no voltage source.

The differential equation describing this system is

$$L\dot{I} + RI + \frac{1}{C} + \int I dt = 0 \quad (1)$$

This equation results from an application of Kirchoff's circuit laws. Differentiating both sides of the equation with respect to time results in

$$L\ddot{I} + R\dot{I} + \frac{I}{C} = 0 \quad (2)$$

(1) R. W. Ashcraft, Mathematical Analysis of HE Data, Quarterly Report for April, May, June, 1971.

a linear second order differential equation with two initial conditions necessary for solution. The general solution for this equation (assuming the damped oscillatory case) is

$$I(t) = e^{-\frac{Rt}{2L}} \left(A_1 \sin \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} t + A_2 \cos \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} t \right) \quad (3)$$

where A_1 and A_2 are arbitrary constants to be determined by the initial conditions $I(0) = I_0$ and $\dot{I}(0) = \dot{I}_0$. These conditions lead to

$$I(t) = e^{-\frac{Rt}{2L}} \left(\frac{\dot{I}_0 + \frac{R}{2L} I_0}{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}} \sin \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} t + I_0 \cos \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} t \right) \quad (4)$$

Ordinarily, I_0 is equal to zero, and the current reduces to

$$I(t) = e^{-\frac{Rt}{2L}} \left(\frac{\dot{I}_0}{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}} \sin \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} t \right) \quad (5)$$

By introducing the substitutions

$$\alpha = \frac{R}{2L}, \quad \beta = \frac{\dot{I}_0}{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}}, \quad \gamma = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}},$$

the current can finally be expressed as

$$I(t) = \beta e^{-\alpha t} \sin \gamma t \quad (\text{the ideal case}) \quad (6)$$

Since γ is 2π divided by the period and α is the damping factor, it is relatively simple to obtain values for these parameters by measuring the two current peaks in the first cycle and the time duration of this cycle. Then L and R may be computed from the relations

$$L = \frac{1}{C(\alpha^2 + \gamma^2)} \quad (7)$$

$$\text{and } R = 2\alpha L$$

(8)

It has been found, however, that the current does not behave in the manner described by equation (6), hence, the values of L and R as obtained by equations (7) and (8) are of doubtful accuracy. Equation (6) implies that the frequency of the current is constant, but readings taken from several records indicate a variable frequency (as much as 3% variation between half cycles of data). For the observed cases, this variation is most pronounced in the first cycle due to a "ramp" in the current which starts at $t = 0$ and lasts for about 0.2 μsec . It is felt that the ramp is caused by ionization effects in the spark gap. This ramp tends to elongate the duration of the first half cycle and causes the behavior of $\frac{dI}{dt}$

to differ considerably from the behavior predicted by equation (6). To analytically describe the data, it was necessary to formulate new assumptions about the system and arrive at a different model. These assumptions were:

1. L, R, and C are constant for the system.
2. The circuit may be represented as a simple LRC circuit with a voltage source due to the spark gap (an unknown function of time, $E(t)$).

Applying Kirchoff's circuit laws leads to

$$V_L + V_R + V_C = E(t) \quad (9)$$

which is equivalent to the differential equation

$$L\dot{I} + RI + \frac{Q}{C} = E(t) \quad (10)$$

Differentiating both sides of the equation with respect to time results in

$$L\ddot{I} + R\dot{I} + \frac{I}{C} = \frac{dE}{dt} \quad (11)$$

This equation is a nonhomogeneous version of equation (2), which implies that its general solution will be the sum of the general solution to equation (2) and a particular solution determined by the functional form of $E(t)$. To determine a possible form of $E(t)$, a particular data case was selected for detailed study. The current-time trace was generated(2) by applying a load voltage of 2KV to a

(2) Experiment was performed by R. D. Worley, Mason & Hanger, PX.

5.76 μ f capacitor bank and allowing the unit to "ring down." The shape of the "ramp" in the first 0.2 μ sec of data suggested a modified exponential form for E, that is,

$$E(t) = B_1 + B_2 t + B_3 e^{B_4 t} \quad (12)$$

Time differentiation of this model gives

$$\frac{dE}{dt} = B_2 + B_3 B_4 e^{B_4 t} \quad (13)$$

or combining constants,

$$\frac{dE}{dt} = A_1 + A_2 e^{-A_3 t} \quad (13a)$$

thus, equation (11) becomes the differential equation

$$L \ddot{I} + R \dot{I} + \frac{I}{C} = A_1 + A_2 e^{-A_3 t} \quad (11a)$$

with initial conditions $I(0) = I_0$ and $\dot{I}(0) = \dot{I}_0$.

The general solution for the underdamped case of this equation is

$$I(t) = e^{-\frac{Rt}{2L}} \left(B_1 \sin \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} t + B_2 \cos \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} t \right) + CA_1 + \frac{A_2}{LA_3^2 - RA_3 + \frac{1}{C}} e^{-A_3 t} \quad (14)$$

where B_1 and B_2 are arbitrary constants to be determined by the initial conditions. Using these substitutions,

$$\alpha = \frac{R}{2L},$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}},$$

$$B_3 = CA_1,$$

$$B_4 = \frac{A_2}{LA_3^2 - RA_3 + \frac{1}{C}},$$

$$\text{and } B_5 = A_3,$$

equation (14) may be expressed more simply as

$$I(t) = e^{-\alpha t} (B_1 \sin \omega t + B_2 \cos \omega t) + B_3 + B_4 e^{-B_5 t} \quad (14a)$$

Applying the initial conditions leads to

$$B_2 = I_0 - B_3 - B_4 \quad (15)$$

$$\text{and } B_1 = \frac{\dot{I}_0 + \alpha B_2 + B_4 B_5}{\omega} \quad (16)$$

Summarizing these results, equation (11a) has lead to a new model for the current which can be written

$$I(t) = e^{-C_1 t} (C_3 \sin C_2 t + C_4 \cos C_2 t) + C_5 + C_6 e^{-C_7 t} \quad (17)$$

Where, in terms of circuit parameters $L, R, C, A_1, A_2, A_3, I_0$ and $\dot{I}_0$,

$$C_1 = \frac{R}{2L} \quad (18)$$

$$C_2 = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \quad (19)$$

$$C_3 = \frac{\dot{I}_O + \frac{R}{2L} \left(I_O - CA_1 - \frac{A_2}{LA_3^2 - RA_3 + \frac{1}{C}} \right) + \frac{A_2 A_3}{LA_3^2 - RA_3 + \frac{1}{C}}}{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}} \quad (20)$$

$$C_4 = I_O - CA_1 - \frac{A_2}{LA_3^2 - RA_3 + \frac{1}{C}} \quad (21)$$

$$C_5 = CA_1 \quad (22)$$

$$C_6 = \frac{A_2}{LA_3^2 - RA_3} \frac{1}{C} \quad (23)$$

$$C_7 = A_3 \quad (24)$$

This model was fitted to the first cycle of the 2KV data with excellent results. Fig. 1 is a plot of the first half cycle data and the fitted curve. The parameters from the curve fit were

$$C_1 = 0.1763$$

$$C_2 = 1.0870$$

$$C_3 = 12.6913$$

$$C_4 = -3.4778$$

$$C_5 = -0.0333$$

$$C_6 = 3.4395$$

$$C_7 = 2.9219$$

The standard error of estimate was 0.08286, the sum of the squared residuals was 0.30207, and the average squared residual was 0.00592. The curve fit parameters

Fig. 1. First Half Cycle of 2KV Data Case with Fitted Curve of the Form

gave for the values of L and R, respectively, 143 nH and 50.5 mΩ. Based on the goodness of fit of the model to the data, it is felt that these estimates of L and R are highly representative of the system.

FUTURE WORK; COMMENTS; CONCLUSIONS

The model given by equation (17) gave an excellent representation for the first cycle of the 2KV data case and produced reasonable estimates of L and R for the capacitor bank circuit. The addition of a voltage source to the differential equation serves as a correction factor to describe the behavior of the spark gap. Future effort will include analysis of more data cases (a wider range of load voltage on the capacitor bank), possible modification of $E(t)$, and an extension of the model to include 2nd and 3rd cycle behavior of the circuit.

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