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# Enhanced Harmonic Spontaneous Radiation Using a Novel Undulator

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In a typical free electron laser (FEL), the electron beam interacts with a "dipole" undulator that has a sinusoidal magnetic field variation; the electron motion is sinusoidal in the plane transverse to this field and emits odd-numbered harmonic radiations along the axis. However, one need not limit the choice of undulator field profile to the sinusoid, providing other profiles result in significant advantages. In connection with the IFEL accelerator, in the past we have pointed out [1] that the use of an undulator profile that approximates a "square wave" will result in an enhanced acceleration gradient, by as much as a factor of two (equivalent in effect to an increase of laser drive intensity by a factor of four). This improvement (essentially at the fundamental FEL resonance) results largely from the fact that, for a given peak undulator field amplitude, the rms electron acceleration obtained from the square wave undulator is larger than that from the sinusoid; furthermore, the electron orbit is stable as well. In this paper, we find additional advantages that should result particularly at harmonic numbers f >1 if the undulator field profile is nearly "square wave": namely, a large enhancement of the harmonic spontaneous power radiated, together with enhanced FEL gain. The modification of undulators to enhance FEL gain has been examined in the past[2,3,4,5], usually with a particular design in mind, but with similar conclusions.

A purely square-wave undulator field profile is unphysical, however it is possible to develop a strong nonlinearity of the sinusoidal pattern in an electromagnet undulator containing ferromagnetic material by operating the device at a field where the material becomes saturated. As saturation progresses, the square wave profile could become a limiting case. Our analysis consists in a calculation of the spontaneous (single-electron) radiation in an undulator having N periods, where the axial magnetic field profile is approximated by the first few Fourier components of a square wave (we have computed the cases n = 1, 3, 5, and 7, but present here only the example where we use just the first three Fourier components; Fig. 1). This provides a more physical approximation to the undulator field that actually could be produced; but more importantly, it permits us to extend a straightforward calculation originally made by Colson [6] which expands the electron orbit in harmonics of the undulator period and gives an expression for the radiated power in terms of a series of Bessel functions. We retain the "undulator approximation", namely that not only is the amplitude of the motion $K/\gamma$ small but also $K< 1$: then the radiation will have sharp lines at the harmonics on the axis since the radiation cone (width $\sim 1/\gamma$) overlaps the orbit.

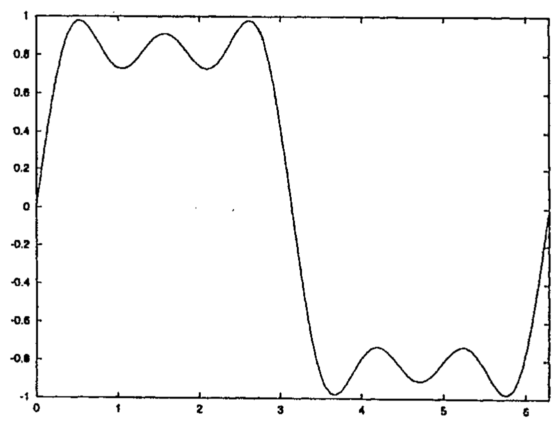


*Fig. 1: Representation of a "square-wave" undulator field by the first three Fourier components. Ordinate, normalized magnetic field; abscissa, axial distance measured in radians over one period ($2\pi$).*

In a long undulator, the spectrum becomes sharply peaked at frequencies satisfying $\omega_{rf} = \omega_0 f / (1 - \beta_0 \cos\theta) = f\omega_{r1}$, where $\omega_0 = k_0 c$ (the undulator wavenumber times the speed of light), f is the harmonic number, θ is the angle from the axis of motion along the undulator, $\omega_{r1}$ is the FEL resonance frequency for f=1, and $\omega_{rf}$ is the resonance frequency for the $f^{th}$ harmonic. The resonance line widths are all ~ $\omega_{r1}/N$. K is the normalized magnetic vector potential, $eB_0/k_0mc^2$, and $B_0$ is the peak undulator field. The undulator is the planar dipole type, and we have computed only radiation directed along the axis. We point out that taking higher n in the undulator expansion will smooth the top of the square wave, but will also increase the slope of the jump. The overall plan of the computation has been presented in detail in [3,6] and due to lack of space here, we will only present certain results.

The spontaneous power was computed numerically, and in Fig. 2 we show a typical result where we have taken K = 0.64 (note that $\omega_B/\omega_0 = K/\gamma \sim \beta_x$) and $\gamma = 80$ (40MeV). Only the peak power emission data point is plotted at each harmonic, all intermediate points are close to zero and are not plotted, and we compare the sinusoidal undulator with two approximations to the square wave undulator, where we include respectively only the n = 1 and 3 components, or the n= 1,3, and 5 components. The striking feature is the very substantial enhancement of spontaneous power emitted at the higher harmonics. That there should be *some* enhancement of radiation is apparant from the fact that electron radiation depends on the square of the electron speed, and the latter is proportional to the integral of the undulator field. The ratio of emission from the "square-wave" undulator to the "sinusoid" is 2.0 for f=1; this is the ratio of the mean square motion of the electron in these two different undulators that have equal peak field amplitudes. For the "approximate" square-wave undulator here, this ratio is about 1.3. However, discounting this factor of 1.3, there is still a remaining factor ~ 10 in enhanced radiation at the fifth harmonic, and larger enhancements (~100) at the higher harmonics. The cause for the power enhancement is in the axial oscillation motion, where the component of the motion driven by the large n=1 term of the field mixes with the components driven by the weaker n harmonics; this arises from the relation $\beta_z \approx \beta_0 - \beta_x^2/4\beta_0$. Harmonic enhancement would be clearly identifiable in a simple experiment.

The enhancement of harmonic spontaneous emission using the "square-wave" undulator profile has further implications. From Madey's theorem[7], the FEL gain will be proportional to the derivative of the spontaneous spectrum. But, the linewidth (~1/N) of this radiation does not depend on the details of the undulator field, just the number of periods. ; hence the FEL gain should be proportional to the spontaneous power emitted at the various harmonics. The enhanced gain has been the subject of previous work[2-5] where specially-devised undulators, not of this type, are described.

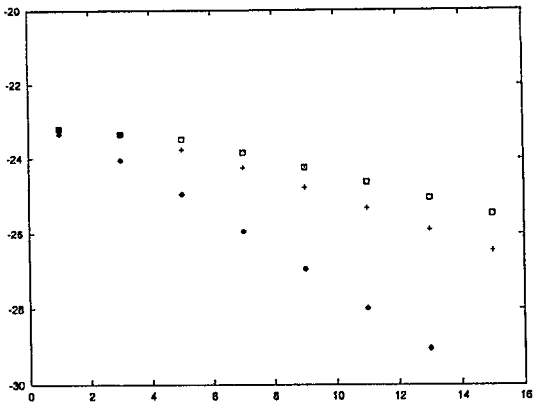


*Fig. 2: Logarithm to base 10 of {dW/dΩdω}, in {watt/ster.-sec$^{-1}$}, versus f, the harmonic number. The diamonds are the sinusoidal undulator; the crosses and squares are for the square-wave undulator approximated by the terms n =1,3; and by n = 1,3,5.*

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