INSTITUTE FOR FUSION STUDIES

DOE/ER/54346--80K

DE-FG03-96ER-54346-804

IFSR #804

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January 1998

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Received _____; accepted _____

ABSTRACT

Dynamics of magnetic flux tubes in the fast rotating stellar atmosphere is studied. We focus on the effects and signatures of the instability of the flux tube emergence influenced by the Coriolis force. We present the result from a linear stability analysis and discuss its possible signatures in the course of the evolution of G-type and M-type stars. We present a three dimensional magnetohydrodynamical simulation of local isolated magnetic flux tubes under a magnetic buoyancy instability in co-rotating Cartesian coordinates. We find that the combination of the buoyancy instability and the Coriolis effect gives rise to a mechanism to twist the emerging magnetic flux tube into a helical structure. The tilt angle, east-west asymmetry and magnetic helicity of the twisted flux tubes in the simulations are studied in detail. The linear and nonlinear analyses provide hints as to what kind of pattern of large spots in young M-type main-sequence stars might be observed. We find that young and old G-type stars may have different distributions of spots while M-type stars may always have low latitudes spots. The size of stellar spots may decrease when a star becomes older, due to the decreasing of magnetic field. A qualitative comparison with solar observations is also presented.

1. Introduction

The introduction (Vogt and Penrod 1983) and use of the Doppler imaging techniques allow the reconstruction of surface luminosity patterns of rapidly rotating stars. A surprising result provided by these Doppler images was the presence of large, dark starspots appearing at high latitudes. In contrast, sunspots are restricted to a latitude band within 30° of the solar equator and relatively small in size. The Zeeman and magnetic Doppler images suggest that stellar spots are magnetic in nature, just like sunspots (Donati et al. 1992; Saar et al. 1994). One might naturally ask why sunspots are different from some starspots, both in size and distribution. Two possible explanations for the large high latitude starspots have been proposed. One is by Schussler and Solanki (1992): the Coriolis force deflects magnetic flux tube rising through the stellar convection zone towards higher stellar latitudes. The other is by Chou et al. (1997): the magnetic buoyancy of flux tubes at low latitudes is suppressed by the Coriolis force, while flux tubes at high latitudes remain unstable and their longer wavelength perturbations grow faster, forming large active regions. Schussler et al. (1996) made a further analysis on the emergence of flux tubes in stars of $1M_\odot$ ranging over different ages and at different rotational speeds. They concluded that the young Sun may have had high latitude spots. However, the shape and distribution of starspots for other types of young stars, which may have magnetic fields and rotational periods different from those of the Sun, remain unknown.

Lower main-sequence stars (spectral types G, K, and M) have deep convection zones. The magnetic activity in these cool stars is believed to be associated with the combination of convection and rotation of the stars (Weiss, 1994) and is similar in essence to, though possibly different in details from, that found in the Sun. It has been known for many years that the Ca II emission is closely correlated with magnetic fields on the Sun. Skumanich et al. (1975) deduced a linear proportionality of Ca^+ emission to magnetic field strength at

the surface. It therefore provides a good measure to estimate the surface magnetic field. On the other hand, Rosner and Weiss (1985) studied the rotational history of the Sun (or G type stars), and concluded that the Sun had a rapid acceleration of rotation due to the decreasing of moment of inertia while it collapsed gravitationally onto the main sequence, followed by a spin-down to its current rotational speed through magnetic braking owing to the torque exerted by the stellar (solar) wind. The maximum angular velocity occurs at the age of 3×10^7 years and may have reached to a speed 100 times faster than its current value. Moreover, it has become apparent that there is a strong positive correlation between the stellar rotation rates and overall chromospheric activities (the Ca II emission levels). Skumanich (1972) studied the relations of the Ca⁺ emission luminosity and the rotation of Sun-like stars to their age, and found that the rotational temporal decay curve followed the same rule as that of the Ca⁺ emission. This means that the surface magnetic field and the rotation of the sun-like stars decay at the same temporal rate. For stars of other spectral types Noyes et al. (1984) studied the relations of magnetic activity to the rotation and to the spectral type. They found that the mean chromospheric emission is rather well characterized by a single parameter, the Rossby number. The Rossby number in rotational hydrodynamics is the ratio of the rotation period to the convective overturn time, characterizing the thermal instability of the rotating fluid. The Rossby number is defined as

$$Ro = P/\tau_c, \quad (1)$$

where P is the rotation period and τ_c is the convective overturn time. In Noyes et al. (1984) they used the observed rotation period P and the estimated convective overturn time τ_c near the bottom of the convection zone. Their study indicates that τ_c is a function of the spectral type alone. The mixing length theory predicts an increasing influence of rotation on convection by decreasing the Rossby number, which may in turn lead to a greater α -effect for late type stars (Schussler 1983). The magnetic field generated by this dynamo

process is determined by the Rossby number. For stars of the same rotational periods P (and hence, the same age), later spectral type stars have deeper convection zones, and thus have larger τ_c and stronger magnetic fields, which may result in higher Ca^+ emission.

Parker (1955, 1966) was the first to propose that magnetic activities observed in the Sun originate from the emergence of magnetic flux embedded in the convection zone. The sunspots are thought to be the foot points of emerging magnetic flux tubes. The physics of emergent flux tubes in the Sun has been discussed in detail in the recent literature using the so-called thin flux tube approximation (see review by Moreno-Insertis 1993; also Caligari et al. 1995; Moreno-Insertis et al. 1994; Fan et al. 1993, 1994; D'Silva & Choudhuri 1993; Wang & Sheeley 1991; Chou & Fisher 1989; Choudhuri 1989; Choudhuri & Gilman 1987). Compared with the current Sun, the young G-type stars, at the age of 3×10^7 years, rotate much faster (100 times faster) and possess much stronger magnetic fields. Young M-type stars have even stronger magnetic fields than young G-type stars. There are evidences showing that stars with strong magnetic fields have large (up to 60% of the surface area) starspots (Byrne 1992). Therefore, the thin flux tube approximation may not apply to these stars and flux tubes with finite dimensions must be considered in order to understand the formation of large spots. To our knowledge, no one has ever carried out of a 'fat' magnetic flux tube appropriate for such a study in three dimensional magnetohydrodynamical (3D MHD) simulation. In what follows we present a linear theoretical analysis and a nonlinear MHD simulation of this problem. The advantage of using a fat flux tube is that it has a radius instead of a thread-like ring so that we can study its internal structure, and it more likely corresponds to large spots of stars with strong magnetic fields. In this paper, we study the dynamics of local isolated emergent flux tubes (Matthews et al. 1995), in particular in its nonlinear stage, under the influence of the Coriolis force. We set the tube in a local co-rotating Cartesian frame, and we neglect the magnetic tension force produced by the curvature of the ring, and assume an isothermal background. This work provides

general ideas how the Coriolis force affects a fat buoyant flux tube, including a theoretical understanding for the patterns of active regions in a young M-type star and a comparison with the solar observations and simulations using thin flux tube approximation.

2. Basic Equations and Linear Stability Analysis

The basic equations governing the plasmas in a rotating stellar atmosphere are:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0, \quad (2)$$

$$\frac{\partial(\rho \mathbf{V})}{\partial t} + \nabla \cdot \left(\rho \mathbf{V} \mathbf{V} + p \mathbf{I} - \frac{\mathbf{B} \mathbf{B}}{4\pi} + \frac{\mathbf{B}^2}{8\pi} \mathbf{I} \right) - \rho \mathbf{g} + 2\rho \boldsymbol{\Omega} \times \mathbf{V} = 0, \quad (3)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}), \quad (4)$$

$$\begin{aligned} & \frac{\partial}{\partial t} \left(\rho U + \frac{1}{2} \rho \mathbf{V}^2 + \frac{\mathbf{B}^2}{8\pi} \right) \\ & + \nabla \cdot \left[\left(\rho U + p + \frac{1}{2} \rho \mathbf{V}^2 \right) \mathbf{V} + \frac{c}{4\pi} \mathbf{E} \times \mathbf{B} \right] - \rho \mathbf{g} \cdot \mathbf{V} = 0, \end{aligned} \quad (5)$$

where

$$U = \frac{1}{\gamma - 1} \frac{p}{\rho}, \quad (6)$$

$$\mathbf{E} = -\frac{1}{c} \mathbf{V} \times \mathbf{B}, \quad (7)$$

γ is the adiabatic gas constant, $\boldsymbol{\Omega}$ is the rotational angular frequency, and the other letters have their usual meanings in magnetohydrodynamics. Compared with the MHD equations in the inertial frame, the only change in the current set is that the momentum equation (3) has an additional Coriolis force term $\boldsymbol{\Omega} \times \mathbf{V}$. The centrifugal force term is neglected since in the stellar case it is small compared with the gravity term. The resistivity, viscosity, and thermal diffusivity are neglected.

A simplified linear analysis considering the stabilization effect on buoyancy (Parker) instability by the Coriolis force was carried out by Chou et al (1997). Here we briefly

summarize the analyses and the key results. We adopt isothermal hydrostatic equilibrium with a constant gravitational field g in the negative z -direction and a magnetic field of constant β in the x -direction. The perturbation is assumed to be in the form

$$\delta\rho = \delta\rho_0(z)\exp(i\omega t - ik_x x - ik_y y). \quad (8)$$

After an appropriate manipulation similar to what have been carried out by Parker (1979), we obtain the linear dispersion relation by the determinant of a 8×8 matrix,

$$\begin{vmatrix} \frac{i\omega}{4\pi} & 0 & 0 & 0 & -ik_y V_A^2 & 0 & 0 & 0 \\ 0 & \frac{i\omega}{4\pi} & 0 & 0 & ik_x V_A^2 & 0 & 0 & 0 \\ 0 & 0 & \frac{i\omega}{4\pi} & 0 & 0 & 0 & ik_x V_A^2 & 0 \\ \frac{ik_x}{4\pi} & 0 & \frac{1}{8\pi H} & i\omega & -2\Omega_z & 0 & 2\Omega_y & -ik_x \\ 0 & \frac{ik_x}{4\pi} & 0 & 2\Omega_z & i\omega & 0 & -2\Omega_x & -ik_y \\ \frac{1}{4\pi C_s^2} & 0 & 0 & 0 & 0 & 1 & \frac{i(\gamma-1)}{\omega \gamma H} & \frac{-1}{C_s^2} \\ 0 & 0 & \frac{ik_x}{4\pi} & -2\Omega_y & 2\Omega_x & g & i\omega & \frac{-1}{2H} \\ 0 & 0 & 0 & -ik_x & -ik_y & i\omega & \frac{-1}{2H} & 0 \end{vmatrix} = 0, \quad (9)$$

where

$$C_s^2 = \frac{\gamma P}{\bar{\rho}}, \quad (10)$$

$$H = \frac{C_s^2}{\gamma g} \left(1 + \frac{1}{\beta}\right) \quad (11)$$

$$\beta = \frac{8\pi P}{B^2} \quad (12)$$

$$V_A^2 = \frac{B_x^2}{4\pi\rho} (= \frac{2}{\beta\gamma} C_s^2). \quad (13)$$

From this dispersion relation we find that the magnetic flux instability (or buoyancy) at low latitudes is stabilized when the rotational angular velocity Ω is large enough. The

critical rotational angular velocity Ω_c that stabilizes the buoyancy of equatorial toroidal magnetic fields depends on the field strength. The solid thick line in Figure 1 shows the critical Ω_c as a function of the field strength, or the plasma β . The region above the line (upper-right) is the stable region, where low latitude spots are prohibited and high latitude spots are expected. The region below the line is the region where equatorial magnetic fields are unstable. It is interesting to compare this critical curve to that estimated by Schussler and Solanki (1992). They suggested a minimum field strength $B_{\min}$ for a significant influence of the buoyancy force. When buoyancy dominates the Coriolis effect (i.e. $B > B_{\min}$), a radial eruption occurs instead of poleward motion. In terms of solar values (see the next paragraph), they have

$$B_{\min} = 10^5 \left(\frac{H}{H_{\odot}} \right) \left(\frac{\rho}{\rho_{\odot}} \right)^{1/2} \left(\frac{\Omega}{\Omega_{\odot}} \right) \text{G}. \quad (14)$$

We find in Fig. 1 that this equation approximately fits with the critical curve Ω_c we discussed earlier. A flux tube in the upper-right region will move poleward and will generate high latitude spots, and this is the same assertion of our linear analysis. The agreement of these two analyses is expected since both of them are dealing with the same physical process: the emerging of a magnetic flux tube affected by the Coriolis force.

Adopting the values from Spruit's solar convection zone model (Spruit 1974), at the bottom of the convection zone the temperature T is $2.2 \times 10^6 \text{K}$, sound speed $C_s = 2.2 \times 10^7 \text{cm/s}$, the density $\rho_0 = 0.2 \text{g/cm}^3$, the pressure $6 \times 10^{13} \text{dyn/cm}^2$, and the scale height $H = 5.6 \times 10^9 \text{cm}$. The magnetic field in this region is estimated to be 10^5G (D'Silva and Choudhuri 1993). The angular velocity Ω is then $7 \times 10^{-4} [H/C_s]$, and the magnetic $\beta = 1.6 \times 10^5$. A young G-type star at its age of 3×10^7 years (a "zero-age" G-type star) rotates 100 times faster than the current Sun, and the magnetic field (at the bottom of the convection zone) is estimated to be 100 times stronger. Using the values calculated by stellar models adopted by Schussler et al. (1996), we estimate that the pressure at the bottom of convection zone in the zero-age G-type star is about 10 times larger than that

in the current Sun. The corresponding values for the current Sun and the zero-age G-type star are marked in Fig. 1, and an estimated evolution trace is drawn by a dotted line. The positions of the Sun at its age of 10^8 and 10^9 years characterized by its rotational speeds relative to the current Sun (Rosner and Weiss 1985) are also marked on this trace. Extrapolating from the relation of $\tau_c(B - V)$ in Noyes et al. (1984) to spectral type M stars, assuming that young M stars rotate as fast as young G stars of the same ages, and making another extrapolation to estimate $B(Ro)$, we estimate the strength of magnetic field in a M-type star is one order of magnitude greater than that in a G-type star. Thus the zero-age M-type star has parameters indicated by the dark triangle in Fig. 1 ($\beta \sim 1, \Omega \sim 0.1$), and these are the values we adopt in our simulation. An estimated trace of evolution for a M-type star is drawn by another dotted line assuming that it is parallel to the evolution of a G-type star. We find that M-type stars stay in the unstable region during their evolution, meaning that magnetic fields embedded in a M-type star erupt radially at low latitudes regardless of its age. In contrast, the evolution path of a G-type star crosses the critical Ω_c curve so that a zero-age G-type star (in which low latitudes spots are prohibited) and the Sun have different spots distributions. Caution should be exercised because we have made a rough extrapolation and assumption about the magnetic fields and rotational speed of M-type stars. However, since the M-star evolution curve (the dotted line) is away from the critical Ω_c curve (the thick line) at least one and a half orders smaller in β , the conclusion that M-type stars always stay in the unstable region probably remains valid even if we had underestimated β to one order of magnitude. The dashed line in Fig. 1 corresponds to the relation $B \propto \Omega$ as if the stellar structure (pressure, scale height, etc) were not changed during the evolution. In another word this line corresponds to the quantity $\Omega H/V_A = 0.25$ remaining the same value as the Sun. The physical interpretation is that every point in this line has the same ratio of the Coriolis force to the magnetic buoyancy force.

3. Nonlinear Simulation

3.1. Initial and Boundary Conditions

We adopt the normalization $\rho(z=0) = C_s = H = 1$ in the following analysis, where C_s is the sound speed and H is the scale height. In the simulation, we assume an ideal gas. Cartesian coordinates are used with z in the vertical direction and a cylindrical flux tube lying along the x direction. The gravitational acceleration is $\mathbf{g} = -g\hat{e}_z$, where g is assumed to be constant. The equilibrium density and pressure distribution without magnetic field is derived from the equation of hydrostatic balance assuming isothermal atmosphere.

The magnetic field is embedded inside a cylinder of radius R and parallel to the x -direction with distribution

$$B_x = \left[\frac{8\pi p(z)}{\beta + 1} \right]^{1/2}, \quad (15)$$

and $\mathbf{B} = 0$ outside the tube. Inside the tube, the magnetic pressure $B^2/8\pi$ is subtracted from the pressure $p(z)$, so that this equilibrium state satisfies the force balance, and hence the temperature inside the tube is lower than that of the surroundings. At the edge of the tube, we smooth the edge by using a hyperbolic tangent function:

$$\mathbf{B}'(r) = \mathbf{B}(r) \times \left[\frac{1}{2} \tanh\left(\frac{r - R}{\Delta}\right) \right], \quad (16)$$

in order to avoid abrupt change of the field and thus numerical errors. Here $r = \sqrt{y^2 + z^2}$ is the distance from the center of the tube, R is the radius of the tube, and Δ is the grid size.

The code uses the modified Lax-Wendroff scheme with artificial viscosity to avoid numerical instability. The accuracy of such an MHD code is described in Shibata (1983). Some 3D tests using this code are described in Matsumoto & Shibata (1992) and Matsumoto et al. (1993). The periodic boundary condition is imposed at $y = y_{min} = -4H$, $y = y_{max} = 4H$, $x = 0$, and $x = L_x = 20H$ (where L_x is the length of the flux tube), while the symmetric boundary condition at $z = z_{min} = -4.32H$, and the free boundary at

$z = z_{max} = 8.64H$ so that waves transmit through the top boundary. The simulation starts with a straight cylindrical magnetic flux tube with radius $R = H$. To initiate the dynamics, a small velocity perturbation is added inside the tube,

$$V_x = 0.05 \sin[k_x(x - L)] \cos(k_y y), \quad (17)$$

where the perturbation wavelength $k_x = \frac{2\pi}{L_x/2}$ and $k_y = \frac{2\pi}{L_y}$.

We adopt nearly isothermal value of the adiabatic index $\gamma = 1.05$. In an isothermal atmosphere which is imposed in our simulation to keep the analysis simple, a larger γ has a stabilizing effect of Parker instability. In reality, in the boundary between the convection zone and the radiation zone where we expect magnetic twists are accumulated, the background atmosphere is not isothermal but has temperature gradient slightly below marginal stable for convection. In such a case, we can use a larger γ . The combined effect of the temperature gradient and the Coriolis twisting on buoyant flux tubes will be studied in future works.

3.2. Simulation Results

We show the results from two models: in model 1, the angular velocity Ω is in the z-direction, which corresponds to the flux tube sitting at the north pole. In model 2, the x-direction is stellar west and the y-direction is north, and Ω lies in the y-z plane having an angle of 45° with the z-axis. This corresponds to the case that a toroidal magnetic flux tube sitting in the latitude of north 45° . We also have a testing model to check our code by setting $\Omega = 0$ and compare the result with the existing studies and have successfully reproduced simulations of magnetic buoyancy in a static inertial frame, similar to the results by Matsumoto et al.(1993)

Figure 2 shows isosurfaces of magnetic field energy density $\|\mathbf{B}\|^2 = 0.5[\rho_0 C_s^2]$. In model

1 (Fig. 2a), we see the tube is twisted by the Coriolis force. It looks like a rubber tube wrung from two ends, and the magnetic field lines are wound along the twisted tube. The reason for such a twist is that the buoyant flux tube expands at a higher level so that the rising tube has $\pm V_y$ velocity component on two different sides of the tube, hence the directions of Coriolis forces are different on these two sides and cause the twist. When the flux tube is in the southern hemisphere, the tube is twisted in an opposite direction hence the tilt angle and the helicity have opposite signs compared to those in the northern hemisphere. On the other hand, when the rotational angular frequency Ω does not have a z-component, which corresponds to the tube sitting at the equator, we do not find twisting. We conclude that it is the z-component (anti-parallel to gravity) of Ω that causes the twisting of flux tube. In Fig. 2(b), Ω has both z and y components, so the Coriolis force not only serves as a twisting agent, but also makes the emerging portion and foot point develop east-west asymmetry because the rising portion of the flux tube is shifted toward the x-direction.

Figure 3 and Figure 4 are top views of the flux tube at a specific altitude ($z = 4.86$). Here we show (a) the current in z direction (J_z), (b) the magnetic field in z direction (B_z), (c) the current field, and (d) the local helicity pattern, defined by $\alpha = J_z/B_z$. Figure 3 is for model 1 (tube at the north pole) and Figure 4 is for model 2 (tube at north 45 degrees), respectively. These top views give us a hint what we can expect if we have enough power to resolve a stellar surface. The thin line in each J_z figure indicates the tilting of the flux tube viewed from above. Note that the flux tube is twisted in such a way that its foot points (corresponding to large B_z areas, denoted by "preceding" and "following" in figures of B_z) align in the direction of upper-left to lower-right if the tube is in the northern hemisphere. This result shows that the alignment of the stellar spots is essentially the same as that of the sun, in which case the thin flux tube simulation has obtained a consistent result to the solar observation. In Fig. 3, we see that preceding and following foot points have the

same pattern and are symmetric to each other. However, in Fig. 4, we lose this symmetry. Instead, the field in the preceding spot is slightly greater than that in the following one. We conclude that this east-west asymmetry is due to the presence of the y -component of rotational angular velocity Ω_y . This is because the Coriolis force, the cross-product of the rising velocity V_z with Ω_y , is in the negative x -direction. The preceding spot which locates on the right hand side (positive x) of the following one has a lower gas pressure since the plasma has been pushed away towards the following spot. Thus the preceding spot has stronger magnetic field. From Fig. 4 we also note that there is no noticeable poleward (y -direction) motion of the flux tube upon its emerging to the surface.

In order to compare the influence of plasma β on the flux tube dynamics, we set a Model 3 whose configuration is identical to Model 1 except for the plasma β value ($\beta = 10$ for Model 3 and $\beta = 1$ for Model 1). In Fig. 2a (and 2b as well) we note that the flux tube does not deform itself into smaller filaments but rather rises as a whole. Figures 5 (a) (velocity field) and (b) (isocontour of B_x) show the YZ cross section at the center of the flux tube ($x = 10$) for Model 1 at $t = 29.09$. The flux tube expands itself into a mushroom shape as it rises toward a lower density layer. Since the radius of the flux tube is relatively large (one scale height), we conclude that this flux tube should produce large spots on the stellar surface after its buoyancy. Figure 5 (c) shows the isocontour of B_x for Model 3 ($\beta = 10$) at the same cross section as in (a) and (b) at $t = 88.8$. The velocity field for Model 3 is not shown because the buoyant motion has ceased and the velocity becomes small. In contrast to Fig. 5 (b), Fig. 5 (c) shows the flux tube has split into smaller filaments. Tajima and Shibata (1997) pointed out that the general morphology of plasmas with high β tends to be filamentary while that of low β plasmas is cellular or sheet-like. The simulation result shown here agrees with this tendency: filamentary for high β case (Model 3) and cellular for the low β (Model 1). The value of β in the overshoot region of the current Sun is large ($\beta = 10^5$), hence we can surmise that the thin flux tube simulation applies. However, for

zero-age M-stars whose β at the bottom of the convection is estimated to be on the order of unity, it is possible that flux tubes are fat. This is consistent with the observation that large spots are inferred in stars with strong magnetic fields. As young (G and M types) stars evolve, the size of the stellar spots may become smaller since the overshoot region may have thinner flux tubes due to increasing of β .

4. Discussion

Since the Sun is the only star that we can observe its surface magnetic features in detail, it is of interest to qualitatively compare the simulation result with the solar observation. The solar soft X-ray pictures taken by the *Yohkoh* satellite show a sequence of regularly spaced, S-shaped active regions at the low latitudes of the northern and southern hemispheres (Acton et. al. 1992; Shibata 1994; Matsumoto et. al. 1996; Chou et. al. 1996). In general sunspots come in a pair with opposite polarities. In the absence of rotation it is expected that the axis of the pair lines up parallel to the equator and the strength of each spot of the pair is approximately equal. However, the optical images and magnetograms show that the sunspots have a definite arrangement: the preceding polarity of an active region tends to be in the form of large well-formed sunspots with stronger magnetic field near the equator, whereas the following polarity tends to appear less organized and more fragmented with a weaker magnetic field farther from the equator (Moreno-Insertis 1993). Thus actual sunspots show tilt and asymmetry, where the tilt refers to the axis of the pair veers away from a latitudinal line and the asymmetry refers to the differences of appearance (including the shape, and the field magnitude) in the preceding spot and following one.

There have been two schools of thoughts to study the tilt and asymmetry features of sunspots. First is based on the thin flux tube approximation (Moreno-Insertis 1993, et al.). These authors claim that the rotation, more specifically, its Coriolis force, causes

the east-west asymmetry and the tilting of magnetic flux tubes. Another set of authors (Rust & Kumar 1996; Moreno-Insertis & Emone 1996; Matsumoto et al 1996, 1997) studied magnetic flux tubes helically kinked prior to their emergence rise to the surface of the Sun. Ideally one would like to simulate the full evolution of a rising magnetic flux tube with off-axis structures. However, such a real 3D fat flux tube simulation adopting solar parameters is still beyond the current computing power. The simulation shown in this paper is the first one to adopt a real 3D fat flux tube, though it does not adopt solar parameters. Still, it is interesting to make a qualitative comparison of our results with the the solar observation. We compare the latitudinal dependence of the tilt angle and that of the east-west asymmetry in our “fat” tube simulation with those from solar observations. The magnetic helicity, which cannot be studied in thin tube approximation, is also discussed.

Figure 6 shows the tilt angle of the emergent portion of flux tubes as a function of the latitude in both northern and southern hemispheres. The tilt angle is defined as the angle of the thin line in Fig. 3 or Fig. 4 to the latitudinal line (x-direction). Positive values mean that such a thin line is rotated counterclockwise from the horizontal orientation. The tilt angle has a different sign in each hemisphere. In the northern hemisphere, the angle is negative, meaning that a pair of spots are positioned at upper left and lower right; the arrangement is opposite in the southern hemisphere. The absolute value of the tilt angle ranges from zero at the equator, to about 10° at north and south 45° latitude. These results suggest that the correlation of the tilting angle and the latitude of stellar spots in a young M-type star is similar to that in the Sun.

Our simulation results show that the ratio of maximum $|B_z|$ in the preceding foot to that in the following one is slightly greater than unity at low latitudes, ranging from 1.0 to 1.25, indicating that the field strength is just slightly greater in the preceding foot. Two reasons may explain why the ratio is near unity. First, our simulation does not consider the

effects the flux tube would encounter while it goes through the convection zone. Second, Moreno-Insertis (1993) and Fan et al. (1993) point out that if the flux tube has magnetic field as large as 1×10^5 G, or larger, then the ratio of field strength between the preceding and following wings is close to unity. Because our simulation employs magnetic field larger than this value, thus the east-west asymmetry is not pronounced. However, we do have a correct tendency of the asymmetry expected by the argument of the previous section.

Pevtsov et al (1995) studied the average magnetic helicity of the Sun, using a 1988-1994 data set of vector magnetograms. They concluded that statistically the average magnetic helicity of the Sun has a sign difference in opposite hemispheres. Intuition might also attribute the sign difference in north and south hemispheres to the rotation. We would like to analyze the helicity patterns caused by the Coriolis force, which cannot be seen from the thin flux tube simulation. The magnetic helicity density is usually defined by $\mathbf{B} \cdot \mathbf{A}$, where $\mathbf{A}$ is vector potential with Coulomb gauge and $\nabla \times \mathbf{A} = \mathbf{B}$. However, it is difficult to obtain this quantity from observation. On the other hand, it would be convenient to define α as

$$\alpha = J_z / B_z, \quad (18)$$

then the sign of α would be the same as that of local magnetic helicity density ($\mathbf{B} \cdot \mathbf{A}$), and the quantities J_z and B_z could be easily extracted from observation. Note that magnetic field is not force-free in photosphere where magnetic field is observed. However, we are analyzing the same α as that in the observation (Petsov et al. 1995), referring it as an indication of helicity *because of its simplicity*. Figure 7 shows the dependence of α (integrated on $z = 4.86$ plane) on the latitude. The absolute value of α indicates how strong the magnetic field lines are twisted. The fact that the absolute value of α is larger at higher latitude could be expected since the flux tube at higher latitude is twisted more than it is near the equator where $\alpha = 0$ (no twisting). From our simulation, the sign of α is positive in the north and negative in the south. This is, however, opposite to the observation by

Pevtsov et al. (1995). It does not help if we reverse the magnetic field in the simulation, because the patterns, the asymmetry, and especially the helicity α remain the same, which follows directly from the basic MHD equations that a sign reversal of B keeps the equation unchanged. This suggests that the helicity observed by Pevtsov et al. (1995) may not be produced by Coriolis twisting of rising-expanding flux tubes, but instead may be the result of the dynamo action with convection column (Kageyama et al. 1995, Yoshizawa & Yokoi 1996). Another possibility is that adopting real solar parameters in the simulation would generate the same helicity results, but it is highly skeptical.

One way to check the accuracy of the code is to compare the linear growth rate with that calculated by linear analysis. Figure 8 shows the time history of $\ln |V_x|$ in Model 1, where V_x is the x-component of velocity at a fixed grid point (chosen at the center of the flux tube). There is a linear growth stage during $7 < t < 15$, and a nonlinear (saturation) stage thereafter. The linear growth rate, which is the slope of the straight line, in our simulation is 0.42. The current existing linear analysis in the literature calculates the growth rate of a 'thin' flux tube (e.g. Spruit & van Ballegoijen 1982, which does not include the effect of rotation, and Ferriz-Mas & Schussler 1994, which includes rotation), an isolated flux sheet (Matsumoto et al. 1993), or continuous field distributions (Chou et al. 1997, and the references cited), but the analysis of a 'fat' flux tube is absent. Comparing the value in our simulation to these linear analysis, we find that the growth rate is identical to that calculated in Matsumoto et al.(1993) and Chou et al.(1997) with wavelength $k_x = 0.6$ and $k_y = 3$ which is adopted as the parameters in our simulation. Therefore, we confirm the accuracy of our MHD code. This growth rate is, however, greater than the value in Spruit & van Ballegoijen (1982). The reason may be that the growth rate of Parker instability is enhanced by interchange modes, but this effect is not included in thin flux tube approximation.

5. Summary

Young stars, which rotate much faster and possess much stronger magnetic field than old stars like the Sun, have different spots from sunspots in their sizes and distributions. The large size of stellar spots is considered to be an outcome of a strong magnetic field, that is, stars with smaller β at their bottom of the convection zone may form a fatter flux tube and produce larger spots when the fat tube emerges to the surface. The distribution of stellar spots depends on the ratio of buoyancy force to the Coriolis force, that is, on the position in the plane of β and Ω in Fig. 1. Possible evolution traces for G and M type stars are also shown in Fig. 1. The trace for G-type stars crosses the critical Ω_c threshold so that the distribution of stellar spots for young and old G-type stars are different. On the other hand, M-type stars stay in the unstable side of the threshold and we expect that starspots at low latitudes may be observed in M type stars of all ages. However, because the value of plasma β increases during the evolution, the size of starspots is expected to be large and cellular in young stars (of M-type, and probably of G-type as well) but to become small and filamentary in old stars.

In a rotating system, the Coriolis force twists the buoyant magnetic flux tube to form a helical structure. This paper presents a 3-D MHD simulation of a fat tube for the first time to test this idea. The z-component, anti-parallel to the direction of gravitational force, of angular velocity causes this twisting. Except for the differences in size and the latitudinal location, the simulation shown in this paper indicates that stellar spots are qualitatively similar to sunspots in two aspects: the helical structure, and the latitudinal dependence of the tilt angle of the emergent twisted flux tube. On the other hand, we expect the y-component of Ω , which is perpendicular to the direction of the magnetic field, makes the Coriolis force acquire a negative x-component upon emerging toroidal flux tube in the northern hemisphere and thus makes E-W asymmetry. Our simulation adopting a flux tube

with strong magnetic field (plasma $\beta = 1$) shows this tendency, but it is not significant. Our 3-D fat tube simulation shows the current field from top view. This result is new and can not be obtained by the thin flux tube simulation. We are looking for an observational signature that corresponds to this current field.

The signs of magnetic helicity α obtained in our simulation are opposite in each hemisphere, indicating that the direction of twisting is opposite in each hemisphere. However, these signs are opposite to the existing solar observations. This discrepancy suggests that the observed helicity patterns in the solar surface may not be due to the Coriolis twisting of emerging magnetic flux tubes, and hence other models such as the dynamo action may be needed.

This work was supported by NSF ATM9401128.

REFERENCES

- Acton, L. et al. 1992, *Science* 258,618.
- Caligari, P., Moreno-Insertis, F., and Schussler, M. 1995, *ApJ* 441,886.
- Chou, D.-Y. and Fisher, G. H. 1989, *ApJ* 341,533.
- Chou, W., Tajima, T., Matsumoto, R., and Shibata, K. 1996, in *Proc IAU Colloquium 153, Magnetodynamic Phenomena in the Solar Atmosphere-Prototypes of Stellar Magnetic Activity*, ed Uchida, Y., Kosugi, T and Hudson H. S. (Kluwer, Dordrecht) p613.
- Chou, W., Tajima, T., Matsumoto, R., and Shibata, K. 1997, *PASJ* 49, 389.
- Choudhuri. A. R. and Gilman, P. A. 1987, *ApJ* 316, 788.
- Choudhuri. A. R. 1989, *Solar Phys.* 123,217.
- Donati, J.-F., Brown, S.F., Semel, M. et al. 1992, *A&A* 265, 682.
- D'Silva, S. and Choudhuri, A. R. 1993, *A&A* 272, 621.
- Fan, Y., Fisher, G. H. and DeLuca, E. E. 1993, *ApJ* 436, 390.
- Fan, Y., Fisher, G. H. and McClymont, A.N. 1994 *ApJ* 436, 907.
- Ferriz-Mas, A. and Schussler, M 1994, *ApJ* 433, 852.
- Kageyama, A., Sato, T., and the Complexity Simulation Group 1995, *Phys. Plasmas* 2(5),1421.
- Matthews, P. C., Hughes, D. W., and Proctor, M. R. E. 1995, *ApJ* 448,938.
- Matsumoto, R. and Shibata, K. 1992, *PASJ* 44, 167.

- Matsumoto, R., Tajima, T., Shibata, K. and Kaisig, M. 1993, ApJ 414,357.
- Matsumoto, R., Tajima, T., Chou, W., and Shibata, K. 1996, in Proc IAU Colloquium 153, *Magnetodynamic Phenomena in the Solar Atmosphere-Prototypes of Stellar Magnetic Activity*, ed Uchida, Y., Kosugi, T and Hudson H. S. (Kluwer, Dordrecht) p355.
- Matsumoto, R., Tajima, T., Okubo, A., Chou, W., and Shibata, K. 1997, ApJ Letter (submitted).
- Moreno-Insertis, F. 1993, in *Solar Magnetic Field*, ed M. Schussler and W. Schmidt (Cambridge University Press, Cambridge) p117.
- Moreno-Insertis, F., Caligari, P., and Schussler, M. 1994, Solar Phys. 153,449.
- Moreno-Insertis, F. and Emone, T. 1996, ApJ Letter, 472,153.
- Noyes, R.W., Hartmann, S.L., Ducan, D.K., and Vaughan, A.H. 1984, ApJ 279, 763.
- Parker, E. N. 1955, ApJ 122,293.
- Parker, E. N. 1966, ApJ 145,811.
- Parker, E. N. 1979, *Cosmical Magnetic Fields* (Clarendon Press, Oxford) pp 325-331.
- Pevtsov, A. A., Canfield, R. C., and Metcalf, T. R. 1995, ApJ Letter 440, 109
- Rosner, R. and Weiss, N.O. 1985, Nature 317, 790.
- Rust, D. M. and Kumar, A. 1996, ApJ Letter 464, 199.
- Sarr, S.H., Piskunov, N.E., and Tuominen, I. 1994, in *Cool Stars, Stellar Systems, and the Sun VIII*, ed Caillault, J.-P., Astro. Soc. Pacific Conf. Ser. Vol. 64, p.661.

- Schussler, M. 1983, in IAU Sym 102, *Solar and Stellar Magnetic Fields: Origins and Coronal Effects*, ed Stenflo, J.O. (Reidel, Dordrecht) p213.
- Schussler, M. and Solanki, S. K. 1992, A&A 264, L13.
- Schussler, M., Caligari, P., Ferriz-Mas, A., Solanki, S.K., and Stix, M 1996, A&A 314, 503.
- Shibata, K. 1983, PASJ 35, 263.
- Shibata, K. 1994. in Proc. of IAU Colloquium no. 143, *The Sun as a Variable Star*, ed J.M. Pap, C. Frohlich, H.S. Hudson and S.K. Solanki (Cambridge University Press, Cambridge) p89.
- Skumanich, A. 1972, ApJ 171, 565.
- Skumanich, A., Smythe, C., and Frazier, E.N. 1975, ApJ 200, 747.
- Spruit, H. C. 1974. Solar Phys. 34, 277.
- Spruit, H. C. and van Ballegooijen, A. A. 1982, A& A 106, 58.
- Tajima, T. and Shibata, K. 1997, *Plasma Astrophysics*, (Addison-Wesley, New York), Chap2.
- Vogt, S.S. and Penrod, G.D. 1983, PASP 95, 565.
- Weiss, N.O. 1994, in *Lectures on Solar and Planetary Dynamos*, ed Proctor, M.R.E. and Gilbert, A.D. (Cambridge University Press, Cambridge) p.59.
- Wang, Y.-M., and Sheeley, N. R. JR. 1991, ApJ 375,761.
- Yoshizawa, A. and Yokoi, N. 1996, Phys. Plasmas 3(10),3604.

Figure Captions

Figure 1 (Solid line) The critical angular velocity Ω_c that stabilized equatorial magnetic flux as a function of the plasma β . It is obtained by the linear analysis and coincidentally identical to $B_{\min}$ (see the text for detail) obtained by Schussler and Solanki (1992). The area to the upper-right is the stable region, that is, the buoyancy of flux tubes at low latitudes are largely suppressed by the Coriolis force and hence high latitude spots are expected. The corresponding position for the Sun, young G-type stars at the age of 3×10^7 yr (zero-age G star), 10^8 yr, 10^9 yr, and zero-age M-types stars are marked. The dotted lines show possible evolutions of G-type stars and M-type stars. Note that the trace of the evolution line for G-types stars crosses the critical Ω_c curve while the trace for M-type stars stays on the unstable side. The dashed line shows constant $\Omega H/V_A$, which means the ratio of magnetic buoyancy force to the Coriolis force are the same as that in the Sun.

Figure 2 Isosurface of magnetic field energy density $\|\mathbf{B}\|^2 = 0.5[\rho_0 C_s^2]$, for (a) Model 1 (a flux tube at the north pole) at time=29.09 [H/C_s], and (b) Model 2 (a toroidal flux tube at north 45°) at time=28.37 [H/C_s].

Figure 3 The cross section at $z = 4.86$ plane for Model 1 (a tube at the north pole) at time=29.09 [H/C_s], showing (a) the current in the z-direction (J_z), the thin line indicates the tilt angle; (b) the magnetic field in the z-direction (B_z); (c) the current field in this plane; and (d) $\alpha = J_z/B_z$.

Figure 4 The XY-cross section for Model 4 (a toroidal tube at north 45°) at time=28.42 [H/C_s]. The meanings of the four figures (a) through (d) are the same as those in Figure 3.

Figure 5 The YZ cross section at $x = 10$ plane (middle of the flux tube) for Model 1

($\beta = 1$) at $t=29.09$ and Model 3 ($\beta = 10$) at $t=88.8$, showing (a) the velocity field for $\beta = 1$, (b) isocontour of the magnetic field B_x for $\beta = 1$, (c) isocontour of B_x for $\beta = 10$. Note that the flux tube becomes filament-like in (c) compared with (b).

Figure 6 The tilt angle at $z = 4.86$ plane as a function of the latitude. Positive angle is defined by counterclockwise veered from the latitudinal line.

Figure 7 The magnetic helicity $\alpha = J_z/B_z$ integrated over $z = 4.86$ plane as a function of the latitude.

Figure 8 Time history the perturbative quantity V_x at the center of the flux tube (10,0,0). The straight line shows the linear growth stage with growth rate (slope) 0.42.

Figure 1

Figure 2

Figure 3

Figure 4

(a) $\beta=1$

VELOCITY

$\rightarrow = 0.71C_s$

(b) $\beta=1$

BX

max=2.494 step=1.593

(c) $\beta=10$

BX

max=1.065 step=0.063

Figure 5

Figure 6

Figure 7

Figure 8