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ESTIMATES OF SEVERAL CORRELATED MEANS

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SIMULTANEOUS MONTE CARLO ZERO-VARIANCE ESTIMATES OF SEVERAL CORRELATED MEANS

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I. INTRODUCTION

Zero-variance procedures have been in existence since the dawn of Monte Carlo. Previous¹ works all treat the problem of zero-variance solutions for a single tally. One often wants to get low variance solutions to more than one tally. When the sets of random walks needed for two tallies are similar, it is more efficient to do zero-variance biasing for both tallies in the *same* Monte Carlo run, instead of two separate runs.

II. ZERO-VARIANCE BIASING FOR TWO TALLIES

The "state" of a particle is its physical location in phase-space and any partial information about its next physical location in phase-space. Two examples are:

1. The particle may have been split into collided and uncollided parts at a geometric cell boundary (the "forced collision technique"). Just before the collision site of the collided particle is sampled, the state of the collided particle is its current physical phase-space location on the boundary *and* the fact that the particle is known to collide in the cell.

2. The particle has entered a collision and three random numbers have already been used toward specifying the outcome of the collision. The state of the particle is its present physical phase-space location entering collision *and* the three random numbers representing partial information about the outcome of the collision.

If two tallies are identical (and started with the same seed of course), the random walks in two separate Monte Carlo calculations will be identical. Furthermore, if the tallies are *almost* the same, the random walks in two separate Monte Carlo calculations will be *almost* identical. This suggests using a correlated sampling technique that optimizes the coincidence of the random walks. That is, the sampling is treated as two Monte Carlo runs occurring simultaneously. Particles with a "1" tag contribute only to tally 1 and particles with a "2" tag contribute only to tally 2. The correlated sampling tracks both particles simultaneously until their random walks diverge.

Consider two particles that are currently coincident with weights w_1 and w_2 . Let the next sampling advance a particle from its current state x to its next state y . Let $I_1(y)$ and $I_2(y)$ be the expected score (per unit weight), subsequent to departing x , for a particle whose next state is y . That is, $I_j(y)$ includes any immediate score generated in going from x to y plus the expected score subsequent to arriving at y . Let $p(x \rightarrow y)$ be the unbiased probability of going from x to y . The zero-variance biasing for tally j is (biasing proportional to expected subsequent score)

$$b_j(x \rightarrow y) = \frac{p(x \rightarrow y)I_j(y)}{\int p(x \rightarrow y)I_j(y)dy} \quad (1)$$

The new weight after sampling from the biased probability density is

$$w'_j = \frac{p(x \rightarrow y)}{b_j(x \rightarrow y)} w_j \quad (2)$$

Let $a_j > 0$ be constants and define

$$I(y) \doteq \max(a_1 I_1(y), a_2 I_2(y)) \quad (3)$$

and

$$f(y) = \frac{p(x \rightarrow y)I(y)}{\int p(x \rightarrow y)I(y)dy} \quad (4)$$

Although any a_j will do, I have in mind

$$a_j^{-1} = \int I_j(y)dy \quad (5)$$

because the average values of $a_1 I_1(y)$ and $a_2 I_2(y)$ are then the same. In particular, if tallies 1 and 2 only differ by a constant multiple then it makes sense to preserve the perfect correlation in the tracking by setting $I(y) = a_1 I_1(y) = a_2 I_2(y)$.

The next event point y is sampled by rejection as explained below; Figure 1 illustrates the details. The procedure for simultaneously zero-variance biasing *both* tallies is:

1. Sample a point uniformly in area under the curve $I(y)p(x \rightarrow y)$ (i.e., from $f(y)$ in Eq. 4)
2. Accept the sample for tally j if the sampled point is under the curve $a_j I_j(y)p(x \rightarrow y)$. That is, y will be accepted for tally j with probability

$$h_j(y) = \frac{a_j I_j(y)p(x \rightarrow y)}{I(y)p(x \rightarrow y)} \quad (6)$$

There are two possibilities:

- a. Both samples are accepted and continue their random walk together with weights multiplied as per Eq. 2.

Fig. 1. Rejection Sampling of Next Event Point.

- b. One sample is accepted and one sample is rejected and the random walks of the samples do not continue together. The accepted sample's weight is multiplied as per Eq. 2. For the rejected sample (say j), a new point is sampled from the biased density of Eq. 1 and weighted accordingly by Eq. 2.

Note from Eq. 6 that the tracks will tend to stay coincident whenever their $a_j I_j(y)$ are similar. Thus, if the tallies are estimating similar quantities, getting zero-variance estimates for two tallies will take marginally longer than getting a zero-variance estimate for a single tally.

III. CONCLUSION

The theory presented here correlates the random walks of particles by the similarity of their tallies. Particles with dissimilar tallies rapidly become uncorrelated whereas particles with similar tallies will stay correlated through most of their random walk. The theory herein should allow practitioners to make efficient use of zero-variance biasing procedures in practical problems.

REFERENCES

1. Iván Lux and László Koblinger, "*Monte Carlo Particle Transport Methods: Neutron and Photon Calculations*," CRC Press, Inc. (1991). (see page 250)

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