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Numerical and Asymptotic Studies
of Complex Flow Dynamics

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If one has computed a stationary state of an evolution equation, a major question is if the state is stable, and many different techniques have been developed to discuss this important question. Together with H.-O. Kreiss I completed a survey article on nonlinear stability for time dependent PDEs [8]. We give an account of the Lyapunov technique and the resolvent technique to study stability questions. Our interest in the resolvent technique was partially motivated by recent work on pseudospectra.1 Further work on the relation between pseudospectra and nonlinear stability is in progress [11].

It has been known for some time that the strength of the resolvent technique is the control of the small-wave-number projection of the solution, whereas Lyapunov’s technique is good for high wave numbers. In [6] we succeeded in combining the two techniques, using the strength of each for the appropriate projection of the solution. I believe this is a major achievement in stability theory. Applications to viscous conservation laws are given [6].

Together with T. Hagstrom, I have studied somewhat related questions for flows at low Mach number [4]. We show all time existence of classical solutions when the initial data are almost incompressible. Our result is stronger than Hoff’s2 in that we can allow for a ball of slightly compressible data with a radius independent of the Mach number $M$. In contrast, the radius of Hoff’s ball shrinks to zero as $M \to 0$.

Together with H.-J. Schroll, RWTH Aachen, I worked on conservation laws with stiff source term [2,3,5,9]. From a mathematical point of view, these are singular perturbation problems for which the reduced problem is singular. In [2] we introduce the notion of stiff well-posedness, which — in the linear constant coefficient case — characterizes all hyperbolic systems with stiff source term $\frac{1}{\epsilon}Bu$ whose solutions converge for $\epsilon \to 0$. There are many applications. For example, a technological process, now commonly used

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in Norway, is to pump water with polymer into an oil field to enhance production. The polymer increases the viscosity, which delays breakthroughs of water into the oil wells. Due to the different time scales of the water/oil flow and the adsorption of polymer into the sea ground, mathematical modeling leads to a hyperbolic system with stiff relaxation term. In [9] we prove that the corresponding model can be treated using our theory of stiff well-posedness, but there is no entropy in the sense of Chen, Levermore, and Liu.³

A rather new field, which has received considerable attention in the physics community, is self-organized criticality. Bak gives a popular account.⁴ The aim is to study the self-organization of systems towards a critical state and to understand the scaling properties at criticality. Together with my students I have analyzed a model and have performed numerical simulations [7].

Description of Plans for 1998. In accordance with my new proposal Applied Dynamical Systems: Analytical and Numerical Aspects I plan to work in the following areas:

PDEs whose solutions vary on different time and space scales

Invariant tori in dynamical systems

Numerical simulation of self-organized criticality

For PDEs I want to concentrate on stability issues. Trefethen has raised the important question of the practical relevance of eigenvalues for stability analysis and the possible role played by non-normality of the linearized problem. Considering convection dominated flow⁵ he shows that — in the left half-plane — the resolvent grows exponentially with Reynolds number. What is the implication of this for nonlinear stability? Can one only allow exponentially small perturbations? These questions are not directly answered in Trefethen’s work and, to my believe, the exponential growth of the resolvent in the left half-plane does not imply that perturbations have to be exponentially small. Using the techniques suggested in my proposal, I plan to show that the behaviour of the resolvent in the left half-plane is largely irrelevant. What is relevant, however, is the behaviour of the resolvent on the imaginary axis.

Concerning invariant tori, I want to concentrate on analytical and computational issues for Lyapunov type numbers. These play an important role in understanding torus breakdown.

⁴Bak, How nature works, Springer Verlag, 1996.
In the area of self-organized criticality I will concentrate on computational aspects. The numerical codes, which have been developed by my students in MATLAB, shall be translated to run on the parallel hardware of the Maui High Performance Computing facilities. The extensive (sequential) computations that we have performed already clearly show that an increase in computing power is necessary to answer the relevant statistical questions.

Education and Human Resource Development. I continued to work with the graduate students Steven Jackett and Wangguo Qin from UNM.

W. Qin continued his Ph.D. work on Numerical computation of invariant curves with spiral points and also was involved in my project on self-organized criticality [7]. Qin contributed considerably to our analysis of the Markov matrices corresponding to the self-organization process.

Steven Jackett’s work was mostly numerical. He has completed UNM’s Certificate Program in Scientific Computing and is now employed part time as research assistant at UNM’s High-Performance Computing Education & Research Center.

Publications


