Nuclear Structure at the Limits

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Abstract

One of the frontiers of today’s nuclear science is the “journey to the limits”: of atomic charge and nuclear mass, of neutron-to-proton ratio, and of angular momentum. The tour to the limits is not only a quest for new, exciting phenomena but the new data are expected, as well, to bring qualitatively new information about the fundamental properties of the nucleonic many-body system, the nature of the nuclear interaction, and nucleonic correlations at various energy-distance scales. In this talk, current developments in nuclear structure at the limits are discussed from a theoretical perspective.

I. INTRODUCTION

The atomic nucleus is a fascinating many-body system bound by strong interaction. The building blocks of a nucleus – protons and neutrons – are themselves composite aggregates of quarks and gluons governed by quantum chromodynamics (QCD) – the fundamental theory of strong interaction. In the description of low-energy nuclear properties, one often simplifies the problem by replacing the complex subnucleonic structures with “effective” nucleons interacting through “effective” forces. But even in this limit, the dimension of the nuclear many-body problem is overwhelming. Nuclei are exceedingly difficult to describe; they contain too many nucleons to allow for an exact treatment, and far too few to disregard finite-size effects. Also, the time scale characteristic of collective nuclear modes is close to the single-nucleonic time scale. This means that many concepts and methods applied successfully to other many-body systems, such as solids and molecules, cannot be blindly adopted here.

A. Nuclear Forces

The common theme for the whole field of nuclear structure is the problem of force: the one acting between two colliding nucleons, the one which produces exotic topologies in light nuclei, and the one giving rise to the collective motion in heavy nuclei. The main challenges in understanding the nuclear force are shown in Fig. 1, in the context of the hadronic and nucleonic many-body problem.
The low-energy interaction between nucleons has complicated spin-isospin dependence dictated by the hadron's substructure. One of the main challenges of nuclear science, indicated by the first bridge in Fig. 1, is the derivation of a nucleon-nucleon (NN) interaction from the underlying quark-gluon dynamics of QCD. Experimentally, the NN force can be studied by means of NN scattering experiments. Examples of phenomenological parameterizations based on the NN scattering data are the Bonn, Paris, Nijmegen, and Argonne potentials. While the long-range part of these free (but still effective!) NN forces is well described by the one-pion exchange potential, their short-range behavior is not fully understood. Here the quark-gluon degrees of freedom must be considered explicitly.

While the very light nuclei can nowadays be described as A-body clusters bound by a free NN force (including higher-order interactions, such as a three-body force), the conceptual framework of larger nuclei is still that of the nuclear shell model. Here, the basic assumption is that the nucleons are moving almost independently in a mean potential obtained by averaging out the interactions between a single nucleon and all remaining A-1 protons and neutrons. The “effective” NN interaction in the heavy nucleus used to determine the mean potential differs considerably from the free NN force. In principle, it should be obtained by means of the complicated Brückner renormalization procedure which corrects the free NN interaction for the effects due to the nuclear medium, and there has been some progress in this area. This challenging task is represented by the second bridge in Fig. 1. Many features of the effective interaction, such as short range; strong dependence on spins, isospins, and relative momenta of interacting nucleons; and the reduction of mass in the nuclear interior, have been extracted from experimental data.

Figure 1 illustrates the intellectual connection between the hadronic many-body problem (quark-gluon description of a nucleon) and the nucleonic many-body problem (nucleus as a system of Z protons and N neutrons). The free NN force can be viewed as a residual interaction of the underlying quark-gluon dynamics of QCD, similar to the intermolecular forces that stem from QED. Similarly, the effective NN force in heavy nuclei can be derived from the effective free NN interaction. It probably would be very naive to think of the behavior of a heavy nucleus directly in terms of the underlying quark-gluon dynamics, but, clearly, the understanding of the bridges in Fig. 1 will make this goal qualitatively possible.

B. Recent Developments in the Nuclear Many-Body Problem

The best NN force parameterizations not only describe the two-body on-shell properties but have been used in few-body and many-body calculations. Probably the most advanced few-body calculations today are the quantum Monte Carlo calculations with the Argonne-Urbana interaction for nuclei with A≤7 [1].

What about the shell-model treatment of heavier nuclei? In the past, shell-model calculations utilizing the concept of valence nucleons interacting in a restricted configuration space were limited to medium-mass nuclei, owing to the rapid growth of the size of the model space. Today, this is still the case, although the conventional shell-model calculations employing realistic NN interactions [2,3] are becoming more and more efficient in handling large configuration spaces. The state-of-the-art shell-model studies of the A=47 and 49 fp nuclei in the full 0ω space [4] set the new standard in this area, although future progress is strongly limited by present-day computer resources. Actually, it took as long as two gener-
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ations of hardware and software development to extend shell-model calculations from $A=44$ to $A=48$. (For the latter nucleus, the model spaces have dimensions of several millions.) The recently developed shell-model Monte Carlo method [5] overcomes the dimension barrier; it represents the interaction between valence nucleons by auxiliary fields. Applications of the Monte Carlo approach to the nuclear shell model have been remarkably successful in describing many structural properties of heavy nuclei such as $^{64}$Ge [6] or $^{124}$Xe [7]. (Here, the dimension of the model space is of the order of $10^{13}$.)

For nuclei close to the magic ones, the low-energy properties depend primarily on the behavior of a few valence nucleons. However, for nuclei with many valence particles, the concept of valence nucleons is less useful, and the valence and inner-shell nucleons have to be treated on an equal footing. Many properties of these heavy systems are well described by means of self-consistent theories with the density-dependent effective $NN$ interaction. Here, the static mean-field description, based on the Hartree-Fock (HF) and Hartree-Fock-Bogolyubov (HFB) methods, or relativistic mean field (RMF) theory, provides a useful starting point. Thanks to developments in computational techniques, the approaches employing microscopic effective interactions are now widely used and, in terms of their predictive power, favorably compare with results of more phenomenological macroscopic-microscopic models. Examples of recent large-scale self-consistent mean-field calculations include HF [8], HFB [9], and RMF [10] studies of ground-state nuclear properties.

C. Nuclear Structure at the Limits

What are the frontiers of nuclear structure today? For light nuclei, one of such frontiers is physics at subfemtometer distances where the internal quark-gluon structures of nucleons overlap. For heavier nuclei, the frontiers are defined by the extremes of (i) the $N/Z$ ratio (Sec. II), (ii) atomic charge and nuclear mass (Sec. III), and (iii) angular momentum (Sec. IV). The tour to the limits is not only a quest for new and unexpected phenomena (sometimes dubbed as a “fishing expedition”), but the new data are expected, as well, to bring qualitatively new information about the effective $NN$ interaction and hence about the fundamental properties of the nucleonic many-body system. By exploring exotic nuclei, one can magnify certain terms of the Hamiltonian which are small in “normal” nuclei, and thus difficult to test. The hope is that after probing these important interactions at the limits, we can later improve the description of normal nuclei (at ground states, close to the valley of beta stability, etc.).

In the framework provided by empirical models, nuclear physics has much in common with other subfields such as condensed matter physics where understanding the effective degrees of freedom provides the essential insight into the behavior of many-particle systems. The theoretical techniques used in nuclear physics are shared by many other subfields. Some intersections between the nuclear many-body problem and general many-body theory are discussed in Sec. V.
II. LIMITS OF EXTREME $N/Z$ RATIOS

Physics of radioactive nuclear beams (RNB) is one of the main frontiers of nuclear science today. Experimentally, thanks to technological developments, we are on the verge of invading the territory of extreme $N/Z$ ratios in an unprecedented way [11,12].

The nuclear landscape – the territory of the RNB physics – is shown in Fig. 2. Black squares indicate stable nuclei; there are less than 300 stable nuclei, or those long-lived, with half-lives comparable to or longer than the age of Earth. Some of the unstable nuclei can be found on Earth, some are man-made (actually, as many as $\sim$2,200 nuclei have been produced in nuclear laboratories), and several thousand nuclei are the yet-unexplored exotic species. The yellow color indicates man-made nuclei that have been produced in laboratories and that live a shorter time. Moving away from the yellow area by adding either protons or neutrons, one finally reaches the particle drip lines where the nuclear binding ends. Many thousands of exotic radioactive nuclei with very small or very large $N/Z$ ratios are yet to be explored. In the $(Z,N)$ landscape, they form the “terra incognita” indicated in green. The nuclei beyond the drip lines are unbound to nucleon emission; that is, for those systems the strong interaction is unable to cluster $A$ nucleons as one nucleus. (Exceptions are neutron stars lying far far away from the neutron drip line. These giant nuclei, having masses of $\sim$1.4 of the mass of the Sun and radii of $\sim$5 miles, exist thanks to gravitation – an interaction which can safely be neglected in the description of “normal” nuclei!)

The uncharted regions of the $(N,Z)$ plane contain information that can answer many questions of fundamental importance for science: How many protons and neutrons can be clustered together by the strong interaction to form a bound nucleus? What are the properties of very short-lived exotic nuclei with an extreme neutron-to-proton ratio $N/Z$? What is the effective nucleon-nucleon interaction in the nucleus having a very large neutron excess? There are also related questions in the field of nuclear astrophysics. Since radioactive nuclei are produced in many astrophysical sites, knowledge of their properties is crucial for our understanding of the underlying processes.

Where lie the particle drip lines? As seen in Fig. 2, the proton drip line is placed relatively close to the line of beta stability; hence it is easy to reach experimentally. (Actually, it has been delineated up to bismuth.) However, since neutrons do not repel each other by the Coulomb force, many neutrons can be added to nuclei starting from the valley of stability. As a result, the “lever arm” separating the neutron drip line from the valley of stability is large and difficult to probe experimentally; except for the lightest nuclei, the bounds of the neutron stability are not known. The uncertainty in the limit of nuclear stability is illustrated by showing the unknown drip line as a dashed curve.

Nuclear life at extreme $N/Z$ ratios is different from that around the stability line. The unique structural factor is the weak binding; hence the closeness to the particle continuum.

A. Theoretical Aspects of Physics at Extreme $N/Z$ Ratios

From a theoretical point of view, spectroscopy of exotic nuclei offers a unique test of those components of effective interactions that depend on the isospin degrees of freedom [13]. Effective interactions used to describe heavy nuclei are usually approximated by means of density-dependent forces with parameters that are usually fitted to stable nuclei and to
selected properties of the infinite nuclear matter. Hence, it is by no means obvious that the isotopic trends far from stability, predicted by commonly used interactions (such as Skyrme or Gogny forces), are correct. In the models aiming at such an extrapolation, the important questions asked are: What is the $N/Z$ behavior of the two-body central force and the one-body spin-orbit force? Does the spin-orbit splitting strongly vary with $N/Z$? What is the importance of the effective mass (i.e., the non-locality of the force) for isotopic trends? What is the role of the medium effects (renormalization) and of the core polarization in the nuclear exterior (halo or skin region) where the nucleonic density is small?

Exotic nuclei are wonderful laboratories to study superconducting correlations. Here, the main questions pertaining to the problem of pairing force are: What is the microscopic origin of the pairing interaction? How can properties of the pairing force be tested? These questions are of considerable importance not only for nuclear physics but also for nuclear astrophysics and cosmology [14]. For instance, a better understanding of the density dependence of the nuclear pairing interaction is important for theories of superfluidity in neutron stars.

As mentioned above, the main theoretical challenge is the correct treatment of the particle continuum. For weakly bound nuclei, the Fermi energy lies very close to zero, and the decay channels must be taken into account explicitly. As a result, many cherished approaches of nuclear theory, such as the conventional shell model, the pairing theory, or the macroscopic-microscopic approach, must be modified. But there is also a splendid opportunity: the explicit coupling between bound states and continuum, and the presence of low-lying scattering states invite strong interplay and cross-fertilization between nuclear structure and reaction theory.

**B. Physics of Large Neutron Excess**

The intense current interest in experimental exploration of the neutron drip-line region is driven not only by the substantial uncertainties in theoretical predictions of its location, but also by the expectation that qualitatively new features of nuclear structure will be discovered in this exotic territory. What makes neutron-rich nuclei so unusual? Firstly, they have very large sizes, as implied by their weak binding. Secondly, they are very diffused; their properties are greatly dominated by surface effects. Thirdly, they are very superconducting; the close-lying particle continuum provides a giant reservoir for scattered neutron Cooper pairs [15,16]. So, roughly speaking, they are large, fuzzy superconductors.

Just before the neutron drip line, neutrons occupy orbits outside the nuclear core that are spacially extended and, from Heisenberg’s uncertainty principle, have low momentum. These states, called halos [11], have radii that are up to several times that of the core (see Fig. 3). The halo region is a zone of weak binding in which quantum effects play a critical role in distributing nuclear density in regions not classically allowed. The heaviest neutron halo found so far is $^{19}$C, having 6 protons and 13 neutrons. Nuclei with two neutrons in their halo, such as $^{11}$Li, are unique objects because they break apart on removing one neutron. The attractive interaction between the two neutrons in the halo of these “Boromean” nuclei is essential for their binding. In the limit of extremely small binding energies of a pair of neutrons to a core nucleus, one will encounter giant halos with radii an order of magnitude larger than that of any stable nuclei!
In the heavier, neutron-rich nuclei, where the concept of mean field is better applicable, the separation into a “core” and “valence nucleons” seems less justified. However, also in these nuclei the weak neutron binding implies the existence of the neutron skin (i.e., a dramatic excess of neutrons at large distances). An example of neutron skin in $^{100}$Zn is shown in Fig. 4.

In the skin region of heavier, very neutron-rich nuclei, one may find the opportunity to study in the laboratory nearly pure neutron matter at densities much less than the normal nuclear density. In addition, the existence of a neutron skin should lead to new collective vibration modes in such nuclei, in which, for example, the neutron skin may oscillate out of phase with a well-bound proton-neutron core. Some of these modes are schematically represented in Fig. 5.

C. Nuclear Shell Structure Far From Stability

The structure of nuclei is expected to change significantly as the limit of nuclear stability is approached in neutron excess. Due to the systematic variation in the spatial distribution of nucleonic densities and the increased importance of the pairing field, the average nucleonic potential is modified [17]. This results in a new shell structure characterized by a more uniform distribution of normal-parity orbits and the unique-parity intruder orbit which reverts towards its parent shell (see Fig. 6).

The quenching of shell effects manifests itself in the behavior of two-neutron separation energies $S_{2n}$. This is illustrated in Fig. 7, which displays the two-neutron separation energies for the $N=80, 82, 84,$ and $86$ spherical even-even isotones calculated in the HFB model with the SkP and SLy4 effective interactions. The large $N=82$ magic gap, clearly seen in the nuclei close to the stability valley and to the proton drip line, gradually closes down when approaching the neutron drip line. As discussed in, e.g., Refs. [18,19] and in Fig. 8, this has important consequences for the r-process and the stellar nucleosynthesis.

D. Physics of Proton-Rich Nuclei: Nuclear Life at and Beyond the Drip Line

On the proton-rich side of the valley of stability, physics is different than in nuclei with a large neutron excess. Because of the Coulomb barrier which tends to localize the proton density in the nuclear interior, nuclei beyond the proton drip line are quasibound with respect to proton emission. However, in spite of the stabilizing effect of the Coulomb barrier, the effects associated with the weak binding are also present in proton drip-line nuclei. They are not as dramatic as on the other side of the stability valley, but nevertheless important [13].

The doubly magic $N=Z=50$ nucleus $^{100}$Sn is a paradigm of RNB physics on the proton-rich side. Although it was found experimentally three years ago [20,21], it took more than two years to roughly determine its mass [22], and it will probably take several years to find its first excited state. Actually, the question, “What is this state?”, constitutes an unresolved problem which is a challenge for theoretical predictions.

A unique aspect of proton-rich nuclei with $N=Z$ is that neutrons and protons occupy the same shell-model orbitals. Consequently, due to the large spatial overlaps between
neutron and proton single-particle wave functions, the proton-rich \( N=Z \) nuclei are expected to exhibit unique manifestations of proton-neutron (\( pn \)) pairing [23,24] carried by isotropic (\( S=0, T=1 \)) and anisotropic, doughnut-shaped [25] (\( S=1, T=0 \)) proton-neutron Cooper pairs (see Fig. 9).

At present, it is not clear what the specific experimental fingerprints of the \( pn \) pairing are, whether the \( pn \) correlations are strong enough to form a static pair condensate, and what their main building blocks are. So far, the strongest evidence for enhanced \( pn \) correlations around the \( N=Z \) line comes from the measured binding energies. An additional binding (the so-called Wigner energy) found in these nuclei manifests itself as a spike in the isobaric mass parabola as a function of \( T'=i(N-Z) \) [26,27]. Recent calculations [28], displayed in Fig. 9 (right), have revealed the rather complex mechanism responsible for the nuclear binding around the \( N=Z \) line. In particular, it has been found that the Wigner term cannot be solely explained in terms of correlations between the proton-neutron \( J=1, T=0 \) (deuteron-like) pairs. The \( pn \) correlations are also expected to play a role in beta decay [29,30], deuteron transfer reactions [31], and structure of high spins [24,32].

Proton-rich nuclei also offer the unique opportunity to study life beyond the drip line. Although the protons in this region are strictly not bound to the nuclear core, nonetheless their escape is impeded by the Coulomb force which is not present for neutrons. Quantum barrier tunneling allows these nuclei to decay by proton emission, but with lifetimes ranging from microseconds to a few seconds – long enough that one can measure their spectroscopic properties [33,34]. Experimentally, a number of proton emitters have now been discovered in the mass regions \( A \sim 110, 150, \) and \( 170 \) [35]. However, it is anticipated that new regions of proton-unstable nuclei will be explored in the near future using RNBs.

Proton radioactivity is an excellent example of the elementary three-dimensional tunneling. Experimental and theoretical investigations of proton emitters (or theoretically predicted ground-state di-proton emitters) will open up a wealth of exciting physics associated with the residual interaction coupling between bound states and extremely narrow resonances in the region of very low density of single-particle levels. In general, proton emission half-lives depend mainly on the proton separation energy and orbital angular momentum, but rather weakly on the details of the intrinsic structure of proton emitters, e.g., on the parameters of the proton-core potential. This suggests that the lifetimes of deformed proton emitters will provide direct information on the angular momentum content of the associated Nilsson state, and hence on the nuclear shape. Figure 10 shows, schematically, a possible scenario of proton-gamma competition in a deformed proton emitter. Of course, the energy window for such a process is expected to be fairly narrow. A slightly different phenomenon – gamma-delayed proton emission – has been recently observed in the nucleus \( ^{58}\text{Cu} \) [36]. Here the strongly deformed excited intruder band gamma-decays to the lower-lying states, but its proton-unstable bandhead directly feeds into the excited state in \( ^{57}\text{Ni} \).

E. Radioactive Nuclear Beams

Many of the current limitations in nuclear physics studies promise to be lifted by the developing field of radioactive nuclear ion beam experimentation. Advances in accelerator, ion source, and mass separator technology, have, over the past 20 years, greatly improved our ability to produce, separate, and accelerate radioactive ions. One of the indications of the
potential of this field is the large international interest in the development of facilities with RNB capabilities. At present there are only a few laboratories with radioactive ion beam capabilities. However, the prospects for new experiments and the success of the current programs have lead to a number of RNB facilities under development and a number of further proposals. Figure 11 illustrates the worldwide effort in radioactive nuclear beams. Facilities which are now operational are labeled with a star (for in-flight separation, IFS) or full circle (for isotope separation on line, ISOL), while facilities under construction are illustrated with an open circle. Other facilities which have been proposed, or are under discussion, are indicated by a small closed circle.

In trying to see the phenomena of a "new physics", we should ask the fundamental question of "how far is far"? Experiments with radioactive beams are going to be long and difficult, and many examples of nuclear exotica discussed in this paper (especially those concerning very neutron-rich systems) are clearly out of reach, even assuming most optimistic experimental scenarios. The hope is, however, that some of the effects associated with the loose binding will be seen as deviations from smooth systematic trends or will show up at higher excitation energies closer to the particle threshold, as in the example of the analog states [37,38]. Theoretically, we are bound to adopt the strategy of going to the extreme values of $N/Z$ in order to identify the qualitatively new phenomena, and then back down to experimentally achievable regions to see whether these phenomena can actually be observed. There is very little doubt that we are on the verge of the most fascinating fishing expedition; a lot of exciting physics will probably be caught already at the beginning of this journey.

III. LIMITS OF MASS AND CHARGE

The dawn of the nuclear age dates back to one hundred years ago when Henri Becquerel and the Curies discovered the phenomenon of radioactivity. This laid the foundation for nuclear chemistry, and changed the face of science, technology, and medicine. The modern periodic table as of 1997, shown in Fig. 12, ends with the 112th element now known. But the search is still going on; we may be due for a major expansion at the upper end of the periodic table [39].

The discovery of the properties of a brand new set of chemical elements can answer questions of fundamental importance for science: The discovery of the properties of a brand new set of chemical elements can answer questions of fundamental importance for science: What is the maximum charge and mass that a nucleus may attain? What are the proton and neutron magic numbers of the new elements? Where are the closed electron shells of the new atoms?

Due to the strong Coulomb repulsion between protons, the nuclear liquid drop would fission immediately for $Z>104$. However, the quantal shell effects allow very heavy elements to exist. Since the first theoretical predictions of the region of superheavy elements in the mid-sixties, and Herculean experimental efforts in a number of the world's leading nuclear laboratories, it is only now that the new heavy elements with $Z\geq110$ have been produced [40-43]. These isotopes decay predominantly by groups of $\alpha$-chains. The experimental chain of $\alpha$ decays that identifies the heaviest known element with $A=273$ and $Z=112$ is indicated in Fig. 12.
All the heaviest elements found recently are believed to be well deformed. Indeed, the measured α-decay energies, along with the complementary syntheses of new neutron-rich isotopes of elements \( Z=106 \) and \( Z=108 \), have furnished confirmation of the special stability of the deformed shell at \( N=162 \) [44-46]. Still heavier and more neutron-rich elements are expected to be spherical and even more strongly stabilized by shell effects; they form the long predicted region of the superheavy elements. The discovery of these elements would provide a critical test of not only nuclear models, but also of relativistic quantum chemistry. According to relativistic calculations, the velocity of the inner-shell electrons is predicted to approach the velocity of light as the atomic number of a nucleus grows up to 173. The resulting relativistic effects cause deviations from the periodicity of chemical properties that characterize the periodic table for lighter elements.

Where is the center of superheavy shell stability expected to fall? For the neutrons, most calculations predict an increased stability at \( N=184 \). However, theorists are not unanimous with regard to the position of the magic proton number. Indeed, in the region of the super-heavies, the Coulomb interaction cannot be treated perturbatively, and the self-consistent coupling between nuclear and electromagnetic parts of the Hamiltonian becomes essential. While earlier macroscopic-microscopic calculations yielded the value \( Z=114 \) [47-49], modern models favor \( Z=126 \) or \( Z=120 \) [50,51]. According to recent HF calculations of Ref. [50], the valley of shell stability extends from the deformed region around \( Z=108, N=162 \) towards the spherical doubly magic nucleus \( A=310, Z=126 \) which is actually predicted to decay via α-, β-, p-radioactivity, and fission!

The contour map of shell energy calculated in the HF+Sly7 model of Ref. [50] is displayed in Fig. 12 (bottom). The calculation clearly shows the presence of strong shell stability around the "doubly magic" nucleus \( ^{310}_{126} \). This result markedly differs from the most macroscopic-microscopic approaches where the island of shell stability is concentrated around \( Z=114 \). The HF prediction is consistent with the recent results of the experimental systematics of \( B(E2) \) rates [52], according to which \( Z=126 \) is presumably the dominant proton shell closure when the global properties of nuclei in the trans-actinide region are parameterized in terms of numbers of valence particles.

IV. LIMITS OF ANGULAR MOMENTUM

Considering that the atomic nucleus is, in a good approximation, an aggregation of strongly interacting nucleons (or pairs of nucleons) moving solo in all directions; the very existence of nuclear collective motions such as rotations or vibrations, with all particles moving in unison, is rather astonishing. In molecular physics, for example, the fast electronic motion is strongly coupled to the equilibrium position of slowly moving ions. Consequently, the adiabatic assumption is justified due to different time scales. In atomic nuclei, the total \( A \)-body wave function cannot, in general, be expressed in terms of slow and fast components. This is because (i) the collective nuclear coordinates are auxiliary variables which depend, in a complex way, on fast nucleonic degrees of freedom, and (ii) the nuclear residual interactions are large.

How good is the time separation between single-particle and collective nuclear motion? The typical single-particle period (i.e., the average time it takes a neutron or a proton to go across the nucleus), \( T_{s.p.}=4R/\nu_F \) [where \( R \) is the nuclear radius and \( \nu_F \) is the Fermi
velocity (~0.29 c), is approximately $3 \cdot 10^{-22}$ sec. The typical period of nuclear rotation ($T_{\text{rot}} \approx 10^{-21}$ sec) is only ~30 times greater than $T_{s.p.}$, and for nuclear vibrations the period of oscillations is only slightly greater than the single-particle period. It is truly amazing that these relatively small differences in time scales seem to be sufficient to create rotating or vibrating potentials, common for all nucleons! Indeed, as remarked earlier in Ref. [53]: “In contrast with the molecular case, there are here no heavy particles to provide the necessary rigidity of the structure. However, nuclear matter appears to have some of the properties of coherent matter which makes it capable of types of motion for which the effective mass is large, as compared with the mass of a single nucleon.”

One of the outstanding challenges in nuclear structure is to understand the mechanism governing the nature of nuclear collective excitations. By studying nuclear rotations and vibrations, one is probing the details of the nuclear force in the strongly interacting medium.

A. Territory of High-Spin Physics

Rotation is a common phenomenon in nature; most objects in the universe, including very small and very large, rotate. The typical rotational velocities decrease rapidly with the system size. Atomic nuclei, with their typical dimensions of several femtometers and rotational periods ranging from $10^{-21}$ to $10^{-20}$ sec, are the champions of fast rotation (see Fig. 13). Additional effects coming from quantum mechanics, shell structure, and superconductivity make the nuclear rotational motion uniquely interesting.

Figure 14 displays, schematically, the territory of nuclear high-spin science. The preferred gamma-ray pathways observed in the de-excitation process can often be associated with specific nuclear deformations. Some spectacular examples of intrinsic shapes are illustrated in Fig. 14: pear-like shapes characteristic of parity-breaking in the intrinsic system, superdeformed (SD) and hyperdeformed (HD) shapes corresponding to huge elongations of nuclear density, magnetic deformations represented by shears bands, or static pairing deformations that give rise to the nuclear Josephson effect and diabolic pair transfer.

One of the most exciting recent discoveries is the link between SD bands and previously known less-deformed states in several nuclei around $^{192}$Hg [54,55]. This direct connection makes it possible to determine the absolute binding energy of the nuclear SD state and hence the magnitude of shell effects (including proton and neutron separation energies) in the SD configuration. This information is critical for the further discrimination between various effective interactions, previously applied successfully to high-spin states [56].

Another important achievement has been the identification of single-particle excitations in the rotating SD well through their electric and magnetic properties. This has been accomplished by measurements of intraband magnetic transitions [57,58] and relative electric quadrupole moments [59–61]. Discussed below are some of the bullets shown in Fig. 14.

B. Superdeformation: Spectroscopy at a keV Scale

The observation of SD states constitutes an important confirmation of the shell structure of the nucleus. Quantum-mechanically, the remarkable stability of SD states can be attributed to strong shell effects that are present in the average nuclear potential at very
elongated shapes [62–64]. For the oscillator potential, this happens when the frequency ratio is 2:1. (For more realistic average potentials, strong shell effects appear even at lower deformations.) The structure of single-particle states around the Fermi level in SD nuclei is significantly different from the pattern at normal deformations. Indeed, the SD shells consist of states originating from spherical shells having different principal quantum numbers, and hence having very different spatial character. This unusual situation produces new effects in nuclear structure.

Superdeformed states have been discovered in several mass regions. These are fission isomers in the actinides, high-spin bands around $^{152}$Dy [65], $^{192}$Hg [66], $^{83}$Sr [67], and $^{62}$Zn [68]; and “molecular” (cluster) configurations in light nuclei [69]. Intrinsic configurations of SD states are well characterized by the intruder orbitals carrying large principal oscillator numbers [70,71]. Because of their large intrinsic angular momenta and quadrupole moments, these orbitals strongly respond to the Coriolis interaction and to the deformed average field.

The increased precision of experimental tools of gamma-ray spectroscopy has allowed us to look much more closely at very weak effects which are, energetically, at an eV or a keV scale. Among the most startling new phenomena is that of identical bands, i.e., the observation of sequences of ten or more identical gamma rays associated with rotational bands in different nuclei [72]. Another recent discovery in SD nuclei is the observation of very small (tens of eV!), but systematic, shifts in the energy levels of certain bands ($\Delta I=2$ staggering) [73,74].

A satisfactory explanation of both phenomena is still lacking [72]. It is rather clear that the puzzle of identical bands is ultimately related to the questions normally addressed in the context of large-amplitude collective motion such as: What are the “strong” quantum numbers that stabilize collective rotation? Indeed, recent studies of moments of inertia and quadrupole moments in SD bands around $^{152}$Dy [75] show that nuclear systems at very large deformations and high angular momenta are the best examples of an almost-undisturbed single-particle motion (i.e., extreme one-body picture) [76].

### C. Magnetic Rotation

In nearly spherical nuclei, sequences of gamma rays were observed reminiscent of collective rotational bands, but consisting of unusually strong magnetic dipole transitions [77]. The basic mechanism leading to the unusually large magnetic collectivity is explained theoretically [78] in terms of a gradual alignment of valence protons and neutrons along the axis of the total spin (shears mechanism). Since quadrupole deformations of these bands are extremely small, the rotational-like pattern observed cannot be attributed to the rotation of the electric charge distribution. It is believed that the collective properties of shears bands are linked to magnetic-type static intrinsic deformations. However, the exact nature of the intrinsic magnetic fields responsible for this magnetic rotation is not fully understood at present.
D. Complete Spectroscopy and Quasicontinuum: From Chaos to Order

Gamma-ray spectroscopy with the new generation of detector systems offers a unique possibility to probe quantum chaos, roughly defined as a regime where the quantum numbers that may be used to characterize low-lying (cold) states of a many-body system are gone. Today, the important issues under study in the so-called “complete spectroscopy” experiments are: At what energy does chaos set in? What are the unique fingerprints of the transition from regular to chaotic motion? The signatures for the onset of chaos can be observed not only in the distribution of energy levels but also in the properties of electromagnetic transition intensities.

There are many splendid examples of an interplay between chaos and order in nuclear spectroscopy. At low excitation energies (low temperatures), there exist intrinsic quantum numbers that can well isolate and characterize nuclear configurations. At high excitation energies, because of a very large level density and dramatic configuration mixing, the concept of quantum numbers virtually breaks down. However, even at high excitation energies, there still survive well-ordered states, dubbed “symmetry scars”, that can be beautifully described by stable regular mean fields with well-defined quantum numbers: superdeformed states, high-K isomers, coexisting structures, and analog states. One of the most important tasks for experiment and theory is to identify such islands of stability, and to understand the underlying quantum numbers, that is, to find order in chaos.

E. High-Spin Physics: Theoretical Developments

Unexpected experimental data from large arrays have generated a great deal of theoretical interest and activity. As a result, many new insights have been gained regarding the behavior of atomic nuclei at high spins.

The new-generation high-spin data tell us a lot about effective $NN$ interaction, and about the pairing force in particular. For instance, self-consistent HF studies of moments of inertia of SD bands indicate the importance of higher-order pairing interactions. Interestingly, the question of the density dependence of pairing interaction is also of great interest for physics of nuclear radii, deep hole states, and properties of drip-line nuclei [16]. That is, this important problem has also surfaced in a different corner of nuclear structure.

In the microscopic theory of a rotating nucleus, the average nucleonic field is obtained self-consistently from the nucleonic density. Consequently, in the presence of a large angular momentum, the intrinsic density is strongly polarized, i.e., the nucleus shows phenomena and behaviors characteristic of condensed matter in the magnetic field: ferromagnetism, the Meissner effect, and the Josephson effect. The nuclear magnetism is caused by the time-odd components in the average potential [79–81]. The understanding of the structure of these terms will rely mostly on the expected increase of information on high-spin properties and on magnetic properties. Here, a good example are recent calculations based on a RMF theory; as a consequence of broken time-reversal invariance, the spatial contributions of the vector meson fields have a strong influence on the moments of inertia [82].
V. NUCLEUS: A FINITE QUANTUM MANY-BODY SYSTEM

While the number of degrees of freedom in heavy nuclei is large, it is still very small compared to the number of electrons in a solid or atoms in a mole of gas, and as such the nucleus presents one of the most challenging many-body problems. In the past, many fundamental concepts and tools of nuclear theory, such as the treatment of nuclear superfluidity and of nuclear collective modes, were brought to nuclear physics from other fields. Today, because of its wide arsenal of methods, nuclear theory contributes significantly to the interdisciplinary field of finite many-body systems. The intersection with theory of metallic clusters and mesoscopic systems is a particularly illustrative example. Selected examples of connections between the nuclear many-body problem and other areas of physics are discussed below.

A. From Atoms and Nuclei to Clusters: Shells and Collective Phenomena

The existence of magic numbers is a consequence of independent particle motion. Nucleons having identical or similar energies are said to belong to the same shell, and different shells are separated by energy gaps (see Fig. 15). The way the energy bunchings (called shell structure) occur depends on the form and the shape of the average potential in which particles are moving.

The small clusters of metal atoms (typically made up of thousands of atoms or fewer) represent an intermediate form of matter between molecules and bulk systems. When the first experimental data on the cluster’s structure were obtained [83], it was immediately realized that the shell-model description could be applied to valence electrons in clusters [84]. Since metal clusters can be made neutral, there is no limitation to their size, in contrast to nuclei. Magic numbers corresponding to filled shells up to $\sim 3000$ electrons have been seen [85].

The effect of the bunchiness in the energy spectrum can be illuminated by looking at the quantal shell energy, i.e., the contribution to the total energy of the system resulting from the shell structure. The nuclear shell energy and the shell energy for small sodium clusters are shown in Fig 16. In both cases, the same nuclear theory technique of extracting the shell correction has been used. The sharp minima in the shell energy correspond to the magic gaps. Nuclei and clusters which are not magic have non-spherical shapes. In Fig. 16 the deformation effect is manifested through the reduction of the shell energy for particle numbers that lie between magic numbers.

How can one see whether the clusters are spherical or not? The deformation of the clusters can be deduced by studying a collective vibrational mode, the so-called dipole surface plasmon, which corresponds to an almost-rigid oscillation of the electrons with respect to the positive ion background. This is a direct analog of the nuclear giant dipole resonance which is an oscillation of protons with respect to neutrons [86]. The occurrence of the deformation of the cluster implies that the dipole frequency will be split. This and many other properties of collective plasmon states in clusters have been initially predicted by nuclear theorists, and their existence has been subsequently confirmed experimentally. Other applications of nuclear methods to clusters include the application of the theory of nuclear rotation to cluster magnetism [87] and the application of experimental and theoretical nuclear reaction
techniques to cluster fragmentation [88]. The experience accumulated in nuclear physics plays an important role in understanding many of the properties of atomic clusters.

**B. Nuclear Shell Model in Condensed Matter and Molecular Physics**

Interacting electrons moving on a thin surface under the influence of a strong magnetic field exhibit an unusual collective behavior known as the fractional quantum Hall effect. At special densities, the electron gas condenses into a remarkable state—an incompressible liquid—while the resistance of the system becomes very accurately quantized. These special densities (or fillings), measured in units of the elementary magnetic flux penetrating the surface, correspond to rational numbers. Although the fractional quantum Hall effect seems quite different from phenomena in nuclear many-body physics—the electron-electron interaction is long-range and repulsive rather than short-range and attractive—it was shown recently that these incompressible states have a shell structure similar to light nuclei [89].

Nuclear and atomic physicists recognized very early that the behavior of complex many-body systems is often governed by symmetries. Some of those symmetries reflect the invariance of the system with respect to fundamental operations such as translations, rotations, inversions, exchange of particles, etc. Other symmetries can be attributed to the features of the effective interaction acting in the system. Although they are more difficult to visualize, the "interaction symmetries" (dynamical symmetries) can often dramatically simplify the description of otherwise very complicated systems. The natural language of symmetries is mathematical group theory. In this language, dynamical symmetries are associated with a special type of group, a *Lie group*. These methods have made it possible to describe the structure and dynamics of molecules in a much more accurate and detailed way than before [90]. In particular, it has been possible to describe the rotational and vibrational motion of large molecules, such as buckyballs ($^{60}$C).

**C. From Chaos to Order**

A topic of great interest is the signatures of classical chaos in the associated quantum system, a sub-field known as quantum chaos. A nuclear physics theory, *random matrix theory*, developed in the 1950's and 60's to explain the statistical properties of the compound nucleus in the regime of neutron resonances [91], is now used to describe the universality of quantum chaos. At the excitation energies of the neutron resonances (several MeV), the density of states (i.e., the number of states per energy unit) becomes very high, and a statistical description is appropriate. Today, the random matrix theory is the basic tool of the interdisciplinary field of quantum chaos, and the atomic nucleus is still a wonderful laboratory of chaotic phenomena. Other excellent examples of an interplay between chaotic and ordered motion in nuclei are: parity-violation effects amplified by the chaotic environment [92], and the appearance of very excited nuclear states (symmetry scars) well characterized by quantum numbers [93]. The study of collective behavior, of its regular and chaotic aspects, is the domain where the unity and universality of all finite many-body systems is beautifully manifested.
D. Mesoscopic Physics: From Compound Nuclei to Quantum Dots

Recent remarkable advances in materials science permit the fabrication of new systems with small dimensions, typically in the nanometer to micrometer range. Mesoscopic physics (meso originates from the Greek word mesos, middle) describes an intermediate realm between the microscopic world of nuclei and atoms, and the macroscopic world of bulk matter. Quantum dots are an example of mesoscopic microstructures that have been under intensive investigation in recent years; they are small enough that quantum and finite-size effects are significant, but large enough to be amenable to statistical analysis. For "open" dots, electrons can move classically into the dot. On the other hand, for "closed" dots, the movement of electrons at the dot-leads interfaces is classically forbidden, but allowed quantum mechanically by a quantal tunneling. Tunneling is enhanced when the energy of an electron outside the dot matches one of the resonance energies of an electron inside the dot.

The conductance of quantum dots displays rapid variations as a function of the energy of the electrons entering the dot. These fluctuations are aperiodic and have been explained by analogy to a similar phenomenon in nuclear reactions (known as Ericson fluctuations). The electron's motion inside the closed dot is generically chaotic. Recently, the same nuclear random matrix theory that was originally invoked to explain the fluctuation properties of neutron resonances has been used to develop a statistical theory of the conductance peaks in quantum dots [94]. That is, a quantum dot can be viewed as a nanometer-scale compound nucleus!

VI. SUMMARY

The main objective of this brief review was to discuss various facets of today's nuclear structure "at the limits". There is very little doubt that an excursion into nuclear exotica will offer many excellent physics opportunities, both in experiment and in theory. We are only beginning to explore many unusual aspects of the nuclear many-body problem offered by systems with extreme $N/Z$ ratios, extreme angular momenta, and extreme nuclear charges and masses.

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FIG. 1. From hadrons to heavy nuclei: main challenges in understanding the nuclear force.

FIG. 2. The nuclear landscape: territory of radioactive ion beam physics. Some of the important physics bullets are indicated schematically.

FIG. 3. Loosely bound halo nuclei such as \(^{11}\)Li and \(^{19}\)C are unique few-body systems. A paradigm of the unexpected phenomena and unusual topologies that may occur in the vicinity of the neutron drip line is the nucleus \(^{11}\)Li (3 protons and 8 neutrons), understood as a three-body halo consisting of two neutrons and a well-bound \(^9\)Li core. Interestingly, while all three constituents of \(^{11}\)Li form a bound system when placed together, the nuclear potential is not strong enough to bind any two of them separately. Hence the name borromean nuclei. (The name “borromean” comes from the Borromeo family of Renaissance Italy, who used three interlocking rings with the property that if any one of them is removed, then all three separate, as their family coat of arms.) The heaviest neutron-rich carbon isotope \(^{19}\)C has 6 protons and 13 neutrons. That is, it has five more neutrons than the naturally occurring \(^{14}\)C! The nucleus \(^{19}\)C is the heaviest neutron “halo” known. Due to very weak binding, the last neutrons in \(^{11}\)Li and \(^{19}\)C are spread throughout a volume of similar size to that occupied by the far heavier nucleus \(^{208}\)Pb. The extension of the most weakly bound neutrons so far from the nuclear core would not be allowed if the nucleons were governed by classical laws of motion; it occurs by grace of quantum mechanics.

FIG. 4. Proton and neutron density distributions in the A=100 nuclei calculated in the self-consistent Hartree-Fock-Bogolyubov theory [17]. Top: \(^{100}\)Sn (proton rich, \(N/Z=1\)); Bottom: \(^{100}\)Zn (neutron rich, \(N/Z=2.33\)). Note how the neutrons extend much farther out in \(^{100}\)Zn. This effect of skin is clearly seen in the logarithmic-scale plots in the insets. The small excess of neutrons in the interior of \(^{100}\)Sn is compensated by the small excess of protons in the surface region. The diffused neutron density in \(^{100}\)Zn gives rise to a very shallow shell-model potential.

FIG. 5. Schematic illustration of collective modes associated with the neutron skin. The left diagram shows the low-energy electric isovector dipole mode, dubbed “pygmy” resonance, associated with vibrations of the neutron skin (or halo) with respect to the core (indicated by a light dotted line). The middle diagram shows the system with very different quadrupole deformations of skin and core (here the core is deformed while the skin is spherical). The right diagram illustrates the angular vibrations of the deformed skin with respect to the deformed core (skin “scissors” mode).

FIG. 6. Left: shell structure characteristic of nuclei close to the valley of stability. Middle: new shell structure characteristic of very shallow single-particle potentials in drip-line nuclei. It corresponds to a more uniform distribution of normal-parity orbits, and the unique-parity intruder orbit that reverts towards its parent shell. In the absence of a spin-orbit splitting (a significant reduction of spin-orbit coupling in neutron-rich nuclei has actually been predicted by some calculations), this single-particle spectrum is expected to approach the limit of the spherical harmonic oscillator (shown in right panel). (From Ref. [16].)
FIG. 7. Two-neutron separation energies for the $N=80, 82, 84,$ and $86$ spherical even-even isotones calculated in the HFB+SkP and HFB+SLy4$^\alpha$ models as functions of the proton number. The arrows indicate the proximity of neutron and proton drip lines for small and large proton numbers, respectively. (From Ref. [13].)

FIG. 8. The very neutron-rich drip-line nuclei cannot be reached experimentally under present laboratory conditions. On the other hand, these systems are the building blocks of the astrophysical $r$-process; their separation energies, decay rates, and neutron capture cross sections are the basic quantities determining the results of nuclear reaction network calculations. Consequently, one hopes to learn about properties of very neutron-rich systems by studying the $r$-process component of the solar-system abundances of heavy elements. The black squares with error bars indicate the experimentally-deduced $r$-process abundances for nuclei with mass numbers greater than $A=100$. The theoretical abundances, marked by red and blue, were obtained in the recent $r$-process network calculations [19]. They are based on microscopic mass formulae which assume that the spherical shell effects towards the neutron drip line are either similar to these in stable nuclei (red curve) or significantly quenched (blue curve). It is seen that a quenching of magic gaps at $N=82$ and $N=126$ is required in order to understand the experimental solar abundances around $A=118$ and 178.

FIG. 9. Superconductivity in solids is a well-known and well-understood phenomenon. It arises from the strong interaction between pairs of electrons. Less known is that superconductivity is also realized in atomic nuclei. Nucleons can form four types of Cooper pairs, shown in the left panel, each of which can be in a state with net orbital angular momentum zero and thus well correlated in space. To date, only the nuclear superconducting phases associated with Cooper pairs of like nucleons – neutrons with neutrons (a) and protons with protons (b) – have been achieved. Since the total spin of these pairs is zero, they are nearly isotropic. A unique aspect of heavy proton-rich nuclei with $N=Z$, accessible with radioactive nuclear beams, is that neutrons and protons occupy the same shell-model orbitals. Consequently, due to the large spatial overlaps between neutron and proton wave functions, the proton-rich $N=Z$ nuclei are expected to exhibit the superconducting phases that arise from proton-neutron Cooper pairs (c) and (d). While the pair (c) has spin zero and isospin $T=1$ is isotropic, the pair (d) having spin $S=1$ and isospin $T=0$ is expected to be strongly anisotropic (donut-shaped). Right panel: the Wigner energy obtained in the recent shell-model calculations of Ref. [28]. The magnitude of the Wigner term is well reproduced theoretically. On the other hand, if the proton-neutron ($T=0$) interaction is excluded, Wigner energy is dramatically reduced.

FIG. 10. Competition between gamma and proton decays in a deformed proton emitter.

FIG. 11. Worldwide efforts in radioactive nuclear beams.
FIG. 12. Top: the Mendeleev’s Periodic Table of the elements as of 1997. It contains the heaviest elements synthesized by man: Rutherfordium (Rf, Z=104), Dubnium (Db, Z=105), Seaborgium (Sg, Z=106), Bohrium (Bh, Z=107), Hassium (Hs, Z=108), Meitnerium (Mt, Z=109), and the recently synthesized elements 110, 111, and 112 which are yet to be named. The transactinide elements have been extended up to element 112, but the chemical properties have been investigated only up to Seaborgium [95]. The highlighted elements 107-112 form chemical “terra incognita”. The strong relativistic effects cause deviations from the periodicity of chemical properties that characterize the periodic table for lighter elements. Indeed, examples of such chemical deviations have already been observed for elements of Rutherfordium and Dubnium whose properties differ from trends observed for the lighter members of their chemical families. The deviations are expected to increase with Z, and it is likely that the elements 112 and 114, respectively below Mercury and Lead in the periodic table, would behave like gases. Bottom: contour map of the calculated shell energy for the heaviest elements [50]. According to theoretical models, the region of the transactinide nuclei is connected with the region of superheavy shell stability through the valley of deformed nuclei around N=164 and Z=110. The indicated experimental chain of α-particle decays that identifies the currently heaviest known element with Z=112 and N=165 [41] goes through this valley. While for the neutrons, most calculations predict an increased stability at N=184, theorists are not unanimous with regard to the position of the magic proton number. While earlier calculations yielded the value Z=114, modern self-consistent models [50,51] favor Z=126 or Z=120.

FIG. 13. Rotation is a common phenomenon in nature; most objects in the universe, including very small and very large, rotate. The largest and slowest rotors are galaxy clusters. The rotation of the Andromeda Galaxy, the nearest major galaxy to our Milky Way, can be inferred from its giant spiral-shaped disk containing a few million stars. Saturn is a great example of a deformed oblate rotator; its shape deformation is caused by a very large centrifugal force. Among stellar bodies, pulsars are by far the fastest rotors; the Crab pulsar makes one revolution every 33 msec! One of the dizziest mechanical man-made objects are ultra-centrifuges used for isotope separation. With some modifications, the concept of rotation can be applied to very small, microscopic systems such as molecules, nuclei, and even hadrons, viewed as quark-gluon systems. Atomic nuclei with their typical dimensions of several femtometers and rotation periods ranging from $10^{-20}$ to $10^{-21}$ sec, are among the giddiest systems in nature. What makes the nuclear rotation very special and uniquely interesting are quantal effects due to the nuclear shell structure and superfluid correlations.

FIG. 14. Territory of nuclear high spins. Some of the important physics bullets are indicated schematically.
FIG. 15. Left: shell structure of the single-particle potential. The energy bunchings (shells) are separated by large energy gaps. Right: the absence of shells in the spectrum. Magic gaps and positions and sizes of shells are governed by the strict rules of quantum mechanics. But, amazingly enough, the origin of shell effects can be traced back to the geometry of periodic orbits of the corresponding \textit{classical} Newton equation. That is, there exists a close link between the nuclear shell structure and the dynamics of the corresponding classical problem. The classical closed trajectories are counterparts of quantum orbits that are bunched in shells. The absence of closed trajectories indicates the disappearance of the bunchiness in the spectrum; hence the presence of chaos.

FIG. 16. Top: experimental and calculated nuclear shell energy as a function of the neutron number [96]. The sharp minima at 20, 28, 50, 82, and 126 are due to the presence of nucleonic magic gaps. Bottom: experimental and calculated shell energy of sodium clusters [87] as a function of the electron number. Here, the magic gaps correspond to electron numbers 58, 92, 138, and 198. In both cases, the shell energy has been calculated by means of the same nuclear physics technique (macroscopic-microscopic method), assuming the individual single-particle motion (of nucleons or electrons) in an average potential. The reduction of the shell energy for particle numbers that lie between the magic numbers is due to deformation (the \textit{Jahn-Teller} effect).
Nuclear Forces
and
The Nuclear Many-Body Problem

light nuclei

nucleon

quarks & gluons

free NN force

effective NN force

medium-mass and heavy nuclei
Physics of Radioactive Nuclear Beams

stable or long-lived

known nuclei

terra incognita

proton emitters

shell gap melting

neutron stars

protons

pn pairing

halos

neutrons
Halo Nucleus

$^{11}\text{Li}$: Borromean Halo Nucleus

$^{19}\text{C}$: The Heaviest Known Halo Nucleus

$^{208}\text{Pb}$: Well Bound Heavy Nucleus

The Borromean Rings
Shapes and Excitation Modes of Halos & Skins

“pygmy” resonance

skin

core

spherical skin
deformed core

scissors
vibrations
Nuclear Shell Structure

Around the valley of β-stability

Very diffuse surface neutron drip line

Harmonic oscillator
N=82 shell gap quenching

![Graph showing the relationship between proton number and S_{2n} (MeV) for SLy4 and SkP models.](image)

- **SLy4**
  - N=80
  - N=82
  - N=84
  - N=86

- **SkP**
  - N/Z
  - Proton Number

- Neutron drip line
- Proton drip line
Proton emission from deformed nuclei

\[ (N, Z+1) \rightarrow (N, Z) \]

\[ 19/2^+ \rightarrow 15/2^+ \rightarrow 11/2^+ \rightarrow 7/2^+ \rightarrow 3/2^+ \]

\[ 21/2^+ \rightarrow 17/2^+ \rightarrow 13/2^+ \rightarrow 9/2^+ \rightarrow 5/2^+ \rightarrow 1/2^+ \]

\[ \gamma \rightarrow \gamma \rightarrow \gamma \rightarrow \gamma \rightarrow \gamma \rightarrow \gamma \]

\[ p \rightarrow p \rightarrow p \rightarrow p \rightarrow p \]

\[ 4^+ \rightarrow 2^+ \rightarrow 0^+ \]
Major World Wide Radioactive Beam Facilities

- ANL
- GANIL
- GSI
- Dubna
- RIKEN
- TISOL
- Louvain-la-Nuve
- CERN
- Lanzhou
- Catania
- Munich
- MSU
- ORNL
- ASIA
- INS

The World

- ● ISOL
- ★ IFS
- ○ construction or upgrade

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Physics with Large Gamma-ray Arrays

- Rotational continuum, chaos, complete spectroscopy
- Giant resonances
- Hyper deformation
- Superdeformation
- Yrast line
- Diabolic pair transfer
- Shears bands
- Identical bands
- Octupole and dipole modes
- Angular momentum

Diagram includes:
- 149Gd, 146Eu
- Gamma-ray energy
- Staggering
- ΔI=2

Energy axis
<table>
<thead>
<tr>
<th>Pronounced shell structure</th>
<th>Shell structure absent</th>
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closed trajectory (regular motion)  trajectory does not close (chaos)