ABSTRACT

I review the status and future prospects of searches for forbidden and highly kaon suppressed decays.

1. Introduction

This workshop comes as we are poised at the threshold of a new generation of rare $K$ decay experiments. There are new experiments running or about to run at KEK, BNL, FNAL, and CERN. In another year or so these will be joined by the KLOE experiment at DAΦNE. The good news is that it's a very exciting time. The bad news, at least for a reviewer, is that there aren't too many new results. Thus I'll be giving a little more attention than usual to what the experimenters expect to do.

My discussion of rare $K$ decays covers processes that are forbidden in the Standard Model, those that highly suppressed and to a smaller extent, those that are merely discouraged.

2. Forbidden Decays

Most of the recent work in forbidden decays has concerned those that violate separate lepton flavor (LFV). Although this sort of current is not allowed in the Standard Model, we don't know any really fundamental principle that forbids it, and in fact it's quite commonly predicted by proposed extensions$^1$ of the SM. Such processes have a lot of appeal, since they typically have good experimental signatures, and probe very high mass scales in generic models. The now canonical example of this is $K_L \rightarrow \mu e$ via a horizontal gauge boson, $X^0$, compared in Fig.1 to ordinary $K\mu2$ decay.

![Fig. 1.](image)

Assuming a V-A interaction, one can readily show:

$$B(K_L \rightarrow \mu e) = \left(\frac{g_{sd}g_{\mu e}^X}{g^2\sin\theta_C}\right)^2 \left(\frac{M_W}{M_X}\right)^4 \frac{\tau_{K_L}}{\tau_{K^+}} B(K^+ \rightarrow \mu^+\nu)$$

$$= 3.3 \times 10^{-11} \left(\frac{91 \text{ TeV}}{M_X}\right)^4 \left(\frac{g_{sd}g_{\mu e}X}{g^2}\right)^2$$

Since the latest limit$^2$ on $B(K_L \rightarrow \mu e)$ is in fact $3.3 \times 10^{-11}$, assuming that $g_{sd}^X \approx g_{\mu e}^X \approx g$, one is already probing scales near 100 TeV for an LFV interaction. As we have heard from Jack Ritchie,$^3$ AGS-871 is going extremely well, and will bring the sensitivity to this process to $\sim 10^{-12}$/event in the next year or so.

Tony Pich$^4$ reminded us that the process $K^+ \rightarrow \pi^+\mu^+\nu$ is complementary to $K_L \rightarrow \mu e$ in the sense that it can probe a purely vector interaction, whereas that latter probes only
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axial-vector and pseudoscalar interactions. Also, a three-body process can give information on the dynamics of a new interaction, whereas the two-body process yields only a single number. Finally, studying a suite of LFV processes can shed light on whether selection rules such as the conservation of generation number (G) obtain. Mike Zeller has given us an update on the excellent progress of AGS-865 in its pursuit of $K^+ \rightarrow \pi^+ \mu^+ e^-$. There, too, sensitivity in the ball-park of $10^{-12}$ is expected within a couple of years. This is to be compared with the current upper limit on this decay, $2.1 \times 10^{-10}$, which was obtained by the predecessor to this experiment, AGS-777. At this workshop, Steve Schnetzer reported a new result of FNAL-799 for the closely related process $K_L \rightarrow \pi^0 \mu e$. They obtained $B(K_L \rightarrow \pi^0 \mu e) < 3.2 \times 10^{-9}$ at 90% c.l. This corresponds to a limit of $\sim 7.4 \times 10^{-10}$ for $B(K^+ \rightarrow \pi^+ \mu^+ e^-)$, because of the smaller $K_L$ decay rate. He also noted that this mode is sensitive to a $G$-violating decay, unlike $K^+ \rightarrow \pi^+ \mu^+ e^-$. Thus, for example, if the LFV interaction completely ignores $G$, the above limit on $K_L \rightarrow \pi^0 \mu e$ corresponds to $B(K^+ \rightarrow \pi^+ \mu^+ e^-) < 3.7 \times 10^{-10}$. Schnetzer also remarked that since the leading background to $K_L \rightarrow \pi^0 \mu e$ seems to be $K_L e4$, it is likely to be cleaner than $K_L \rightarrow \mu e$ for which the corresponding background is the $10,000 \times$ larger $K_L e3$. He envisions getting down to a single event sensitivity of $10^{-11}$ in KTeV.

Although theoretical interest in kaon LFV results is still reasonably strong, this type of experiment may become the victim of its own success. The strong constraints placed on BSM models have motivated theorists to keep well out of the way of $K_L \rightarrow \mu e$ and its ilk. The most popular versions of supersymmetry do predict some LFV, but mainly in the non-strangeness changing sector, e.g. in $\mu \rightarrow e\gamma$.

Table 1 gives summarizes searches for lepton flavor violation in the kaon, pion, and muon systems. One can observe that the sensitivity to LFV attained in muon decays is quite comparable to that reached in $K$ decays, and that this field is still very active.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Current u.l.</th>
<th>Experiment</th>
<th>ref.</th>
<th>Date</th>
<th>(Near-)future aim</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^+ \rightarrow \pi^+ e^- \mu^+$</td>
<td>$2.1 \times 10^{-10}$</td>
<td>AGS777</td>
<td>7</td>
<td>1990</td>
<td>$3 \times 10^{-12}$ (AGS865)</td>
</tr>
<tr>
<td>$K_L \rightarrow \mu e$</td>
<td>$3.3 \times 10^{-11}$</td>
<td>AGS791</td>
<td>2</td>
<td>1993</td>
<td>$2 \times 10^{-12}$ (AGS871)</td>
</tr>
<tr>
<td>$K_L \rightarrow \pi^0 \mu e$</td>
<td>$3.2 \times 10^{-9}$</td>
<td>FNAL799</td>
<td>8</td>
<td>1996</td>
<td>$2 \rightarrow 3 \times 10^{-11}$ (KTeV)</td>
</tr>
<tr>
<td>$\pi^0 \rightarrow \mu e$</td>
<td>$8.6 \times 10^{-9}$</td>
<td>FNAL799</td>
<td>12</td>
<td>1994</td>
<td></td>
</tr>
<tr>
<td>$\mu^+ \rightarrow e^+ \gamma$</td>
<td>$4.9 \times 10^{-11}$</td>
<td>Xtal Box</td>
<td>13</td>
<td>1988</td>
<td>$7 \times 10^{-13}$ (MEGA)</td>
</tr>
<tr>
<td>$\mu^+ \rightarrow e^+ e^- e^+$</td>
<td>$1.0 \times 10^{-12}$</td>
<td>SINDRUM</td>
<td>14</td>
<td>1988</td>
<td></td>
</tr>
<tr>
<td>$\mu^- T_i \rightarrow e^- T_i$</td>
<td>$8.5 \times 10^{-13}$</td>
<td>SINDRUM II</td>
<td>15</td>
<td>1996</td>
<td>few $\times 10^{-14}$</td>
</tr>
<tr>
<td>$\mu^+ e^- \rightarrow \mu^- e^+$</td>
<td>$8 \times 10^{-9}$</td>
<td>PSI R-89-06</td>
<td>16</td>
<td>1996</td>
<td>$3 \times 10^{-9}$</td>
</tr>
</tbody>
</table>

There are a number of other recent or on-going BSM searches in the $K$ system. AGS-871 expects to have the sensitivity to observe $K_L \rightarrow e^+ e^-$ at the SM level, thus either observing or closing the window on new physics in that process. AGS-787 is sensitive to the decay $K^+ \rightarrow \pi^+ X^0$, where $X^0$ is a weakly interacting light particle, such as a familon. As mentioned by Takao Shinkawa, the 90% c.l. limit on this process has recently been improved to $5.2 \times 10^{-10}$ for $m_X \approx 0$. Data already collected by AGS-787 should bring this below $10^{-10}$, with an eventual
goal of $\sim 10^{-11}$. Schnetzer\textsuperscript{8} reported a limit on the decay $K_L \rightarrow e^+ e^- \mu^+ \mu^-$ ($< 3.2 \times 10^{-9}$). This is the kind of process that no one expects, so therefore has the maximum power to confound.

3. Suppressed Processes

The current interest in highly suppressed $K$ decays is due to their potential for improving our knowledge of the CKM matrix and elucidating its relation, if any, to CP-violation. Fig. 2, which exhibits the relevant modes, their relation to the unitarity triangle, and to each other, will serve as a framework for my remarks. Note that the $K$ decays depicted don’t directly determine the sides of the usual version of this triangle, but yield equivalent information.

Fig. 2. Processes needed to determine the unitarity triangle from the $K$ sector. Quantities adjacent to the sides of the triangle are the measurables directly sensitive to CKM parameters. Those on the periphery are needed to determine backgrounds, allow subtraction of long-distance effects, etc.

3.1. The hypotenuse

The hypotenuse of the triangle of Fig. 2 is given by the branching ratio for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. This mode gives theoretically very clean information, since it is entirely short-distance dominated,\textsuperscript{20} the hadronic matrix element is well determined from that of $K e 3$ decay, and the QCD corrections have been calculated to NLLO\textsuperscript{21} and are very well under control. In fact recently there have been calculations of few percent corrections like isospin-violating mass effects.\textsuperscript{22} AGS-787 has been in pursuit of this decay mode for several years now, and has recently published\textsuperscript{19} a 90\% c.l. upper limit of $B(K^+ \rightarrow \pi^+ \nu \bar{\nu}) < 2.4 \times 10^{-9}$. While this represents a factor $\sim 60$ improvement over previous data, it is still about a factor 20 short of the top of the range of
SM prediction given at this workshop by Andrzej Buras. As described by Shinkawa, in the years between 1991 and 1994, both the 787 beam and detector were very significantly improved, and the experiment has been running for two years at much higher intensity than in the 1989-91 period. It is estimated that the sensitivity collected in these two years is five or six times that of the earlier runs. It is planned to run the experiment at increasingly higher $K$ fluxes over the next few years.

This should enable an observation of the process to be made, but unless the branching ratio is near (or above!) the top of the range of current SM prediction, only a handful of events can be expected. Since there’s clearly a strong incentive to make a more precise measurement than this would allow, the question of how to make further progress on $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ was taken up at the recent AGS-2000 Workshop. This workshop was organized to explore the use of the AGS for external beam experiments after it becomes an injector to RHIC. At the moment the AGS can deliver $6 \times 10^{13}$ protons every 3 seconds in slow beam mode. By the year 2000, it is anticipated that the machine will be able to deliver $10^{14}$ protons/cycle. Since the AGS’s duties as an injector require only a couple of hours a day at most, and any external beam running could be done on a very economical marginal cost basis, it seems well worthwhile to explore what experiments could best be done there. In the case of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ the approach was to see what could be accomplished without a major upgrade to the present detector.

Analysis of data taken in 1995 confirmed that the greatest barrier to increasing E787’s sensitivity/hour is the high random rate in the detector. It is estimated that the overall effect of this rate on the 1995 sensitivity is a reduction by a factor of almost two. Clearly one can’t just turn up the wick. The trick is to increase the sensitivity without increasing the instantaneous rate. One straightforward way to accomplish this is to increase the duty factor of the AGS. It currently spills protons out over a 1.6 second interval, every 3.6 seconds. If, for example, this could be increased to 3.2/4.2 seconds, the sensitivity/hour could be increased by a factor 1.7, at the cost of increasing the proton intensity/spill by a factor 2 beyond the current $1.5 \times 10^{13}$. One can also exploit the fact that the rates in the detector are proportional to the flux of hadrons striking the detector, as are most of the potential backgrounds. In 1995, only about 1 in 7 of the $K$’s incident on the degrader decayed at rest in the stopping target. The other 6/7 were available to do mischief (by interacting in the degrader or decaying in flight). One way to improve this ratio is to reduce the beam energy. There are fewer $K^+$/proton produced at lower energy, but a higher fraction of those incident on a degrader will stop beneficially. If one has protons to spare, it’s a good thing to do. In fact, during the workshop, a conceptual design for a beam with a yield of 550 MeV/c $K^+$/proton equal to that of the present LESB3 was laid out. This would result in an increase in sensitivity/hour of a factor 1.5. If the AGS beam were tightly bunched, it would be possible to align what are presently randoms so that they come before the $K$ decays. It is estimated that one could then afford to increase the flux by a factor 2. Finally, in the RHIC era, it is estimated that one could run about twice as long per year as has been run in recent times at the AGS. This would allow $5 - 10$ times more sensitivity per year than at present.

As we have heard from Bob Tschirhart at this workshop, a group at Fermilab is working
on a proposal to measure $K^+ \rightarrow \pi^+\nu\bar{\nu}$ in flight at the Main Injector. They would use a “velocity spectrometer” in which RICH’s with photomultiplier tube readout would be used both for picking $K^+$ out of a 300 MHz unseparated 22 GeV/c beam (they are about 5%), and for identifying and measuring the daughter pion velocity. Muons would be further identified in a range stack, and there would be a series of $\gamma$ vetoes. This would be a real departure for $K^+ \rightarrow \pi^+\nu\bar{\nu}$, since all previous experiments have used stopped $K^+$’s. Table 2 compares the two approaches.

Table 2. Possible future $K^+ \rightarrow \pi^+\nu\bar{\nu}$ experiments

<table>
<thead>
<tr>
<th>Experiment</th>
<th>BNL-787+</th>
<th>FNAL-CKM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_K$</td>
<td>0</td>
<td>22 GeV/c</td>
</tr>
<tr>
<td>timescale</td>
<td>2000+</td>
<td>2000+</td>
</tr>
<tr>
<td>sensitivity</td>
<td>5 - 10 events/year</td>
<td>74 events/year</td>
</tr>
<tr>
<td>extrapolation of technique</td>
<td>10</td>
<td>$10^{12}$</td>
</tr>
<tr>
<td>cost</td>
<td>$3M$?</td>
<td>$10M$?</td>
</tr>
<tr>
<td>duty factor</td>
<td>0.76</td>
<td>0.33</td>
</tr>
<tr>
<td>$K$ decays/live sec</td>
<td>$2.5 \times 10^6$</td>
<td>$2.7 \times 10^6$</td>
</tr>
<tr>
<td>live sec/year</td>
<td>$10^7$</td>
<td>$10^7$</td>
</tr>
<tr>
<td>geometrical acceptance</td>
<td>$\sim 0.03$</td>
<td>0.047</td>
</tr>
<tr>
<td>$\pi^+$ vetoing inefficiency</td>
<td>$&lt; 10^{-6}$</td>
<td>$2 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\mu$ ID method</td>
<td>$K^+$ separation method</td>
<td>$\pi^+$ scattering background</td>
</tr>
<tr>
<td>uncertainties</td>
<td>fraction of rates prompt</td>
<td>$\mu$ backgrounds</td>
</tr>
<tr>
<td></td>
<td>tails under battle conditions</td>
<td>rates from $\pi^+$ interactions</td>
</tr>
</tbody>
</table>

Note the arrow on Fig. 2 from $K^+ \rightarrow \pi^+\nu\bar{\nu}$ to $K^+ \rightarrow \pi^+\gamma\gamma$. This symbolizes the fact that the latter process is a possible background to the former. As reported by Shinkawa,\textsuperscript{18} AGS-787 has now observed about 30 events of $K^+ \rightarrow \pi^+\gamma\gamma$. Since the yield and spectrum shape are roughly compatible with the predictions of chiral perturbation theory\textsuperscript{26} (ChPT), it is now clear that the photon veto inefficiency necessary to deal with $K\pi2$ background is more than adequate for $K^+ \rightarrow \pi^+\nu\bar{\nu}$.

3.2. Direct CP-violating decays

As has been emphasized many times,\textsuperscript{26,23,27} the decay $K_L \rightarrow \pi^0\nu\bar{\nu}$ is unique in that its branching ratio unambiguously determines CKM CP-violation in a single measurement. The problems with this decay are entirely experimental. The present experimental bound from FNAL-799,\textsuperscript{33} $5.8 \times 10^{-5}$, falls at least seven orders of magnitude short of what’s necessary to measure the branching ratio at the expected level\textsuperscript{23} ($\sim 3 \times 10^{-11}$). The E799 result is based on detecting the small Dalitz-pair converted minority among the $\pi^0$'s. According to Schnetzer,\textsuperscript{8} this technique will be pushed down to a sensitivity of $2.5 \times 10^{-8}$ by KTeV. Beyond this, one will probably have to detect the $\pi^0 \rightarrow 2\gamma$ decay. The FNAL group plans to pursue an observation ($\sim 10$ events/year) at KAMI.\textsuperscript{8} This experiment would use a pencil beam to define a transverse vertex, then detect unaccompanied pairs of $\gamma$’s emanating from it. Forcing the $\pi^0$ mass on the $\gamma$’s determines the longitudinal coordinate of the vertex, and thus the $p_T$. One can then demand that this $p_T$ be greater than that possible for $\pi^0$'s from the 30 million-fold more copious
Although the cost in acceptance is considerable. One of course also vetoes on the extra γ's. A similar approach, although at much lower energy, has recently been proposed at KEK.  

At the AGS-2000 Workshop, an alternative scheme was pursued. Fig. 3 shows the detector. The approach is to measure everything possible in $K_L \rightarrow \pi^0 \nu \bar{\nu}$, so that redundant techniques of background suppression can be applied. An intense 24 GeV proton beam is used to make a large aperture neutral beam at 45°. The unusually large production angle has several benefits. It minimizes the high energy neutrons that can produce background $\pi^0$'s off residual gas, windows, apertures, etc. It also mitigates the potential problem of $\Lambda$s. It softens the $K_L$ spectrum, so that the $\pi^0$ opening angle is large, allowing good vertex determination (γ directions as well as energies are measured). Most important, this soft spectrum also makes it possible to measure the $K_L$ momentum to a few percent by timing the γ arrivals with respect to the time the proton beam strikes the production target. For this to work, the beam must be tightly bunched. This is accomplished on extraction, by forcing the beam through the interstices in a train of empty RF buckets. A recent series of tests of this idea at the AGS have achieved RMS bunch widths of $< 300$ ps with 10 nsec spacing, and it is is expected that this technique can eventually produce RMS widths of $\sim 150$ ps. Determining the $K_L$ momentum makes it possible to transform to the $K_L$ cm system, so that the $\pi^0$ from the primary background, $K_L \rightarrow \pi^0 \pi^0$ has a unique energy and so can be suppressed kinematically without excessive signal loss. This reduces the load on photon vetoing, allowing one to design the veto on the basis of previous working systems, with confidence that it will be adequate. The bunched beam structure also puts much potential background out of time with the signal. Photon directions are measured in a preradiator designed to determine the direction of the earliest electrons in the shower. Photon vetoing is nearly hermetic, including a hadron-blind "catcher" in the beam itself.

Table 3 compares the three experiments. In the case of such a difficult and important measurement, it would seem prudent to have at least two complementary experiments.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>KEK</th>
<th>BNL-926</th>
<th>FNAL-KAMI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_p$</td>
<td>12 GeV</td>
<td>24 GeV</td>
<td>120 GeV</td>
</tr>
<tr>
<td>timescale</td>
<td>2000+</td>
<td>2000+</td>
<td>2002+</td>
</tr>
<tr>
<td>$\theta_{prod}$</td>
<td>6°</td>
<td>45°</td>
<td>8°</td>
</tr>
<tr>
<td>n/K</td>
<td>10</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>$\Delta \Omega$</td>
<td>12.6 $\mu$sr</td>
<td>500 $\mu$sr</td>
<td>0.1 $\mu$sr</td>
</tr>
<tr>
<td>$\bar{p}_K$</td>
<td>2.7 GeV/c</td>
<td>0.65 GeV/c</td>
<td>20 GeV/c</td>
</tr>
<tr>
<td>$K_L$/pulse</td>
<td>$2.4 \times 10^6$</td>
<td>$2.5 \times 10^8$</td>
<td>$8 \times 10^7$</td>
</tr>
<tr>
<td>useful $K$ decays/live sec</td>
<td>$2.1 \times 10^5$</td>
<td>$2 \times 10^7$</td>
<td>$7 \times 10^5$</td>
</tr>
<tr>
<td>live sec/year</td>
<td>$1.25 \times 10^6$</td>
<td>$5 \times 10^6$</td>
<td>$10^7$</td>
</tr>
<tr>
<td>Acceptance</td>
<td>0.102</td>
<td>0.016</td>
<td>0.067</td>
</tr>
<tr>
<td>single evt. sensitivity/yr</td>
<td>$4 \times 10^{-11}$</td>
<td>$1.2 \times 10^{-12}$</td>
<td>$2.2 \times 10^{-12}$</td>
</tr>
<tr>
<td>required vetoing inefficiency/γ</td>
<td>$\sim 10^{-4}$</td>
<td>$10^{-2} - 10^{-4}$</td>
<td>$10^{-3} - 10^{-6}$</td>
</tr>
</tbody>
</table>

Since $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is so hard to measure, until very recently experimenters have concentrated on the closely related, but experimentally much more tractable, $K_L \rightarrow \pi^0 \ell^+\ell^-$ decays. In $K_L \rightarrow \pi^0 e^+e^-$, very similar one-loop diagrams produce a direct CP-violating con-
tribution to the rate roughly an order of magnitude smaller than that of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ (i.e. $B(K_L \rightarrow \pi^0 e^+ e^-)_{dir} \sim 2 \times 10^{-12}$). For $K_L \rightarrow \pi^0 \mu^+ \mu^-$, the corresponding contribution is about 5 times smaller still. These are to be compared with the current upper limits from FNAL-79$^9$ of $4.3 \times 10^{-9}$, and $5.1 \times 10^{-9}$, for $K_L \rightarrow \pi^0 e^+ e^-$ and $K_L \rightarrow \pi^0 \mu^+ \mu^-$ respectively. Y. Fuku~hima$^{35}$ reported to this workshop that the KEK-162 experiment currently in progress is expected to improve the $K_L \rightarrow \pi^0 e^+ e^-$ sensitivity by a further factor of $\sim 3$. Thus a $3 - 4$ order of magnitude improvement in sensitivity is required before the direct CP-violating contribution to these decays can be observed. For reasons discussed below, probably another order of magnitude beyond this will be necessary before these contributions can be usefully isolated. We have heard at this workshop$^8$ that KTeV expects to get to single event sensitivities of $3 \times 10^{-11}$ for the electron and $10^{-11}$ for the muonic mode. Sensitivities adequate to observe and study these modes are unlikely to be available until the KAMI program is well underway.

The theoretical situation has been discussed many times.$^{36,37}$ Besides, the direct CP-violating piece, there are three possible contributions to any apparent signal.

1. The indirect CP-violating term due to state mixing, $B(K_L \rightarrow \pi^0 \ell^+ \ell^-)_{ind} = |\epsilon|^2 \frac{T_{K_L}}{T_{K_S}} B(K_S \rightarrow \pi^0 \ell^+ \ell^-)$. This is expected to be of the same order as the direct term. There are arguments$^{36}$ connecting $B(K_S \rightarrow \pi^0 e^+ e^-)$ to the well measured $B(K^+ \rightarrow \pi^+ e^+ e^-)$, but they are less than air-tight.$^{37}$ It would be best to measure $B(K_S \rightarrow \pi^0 e^+ e^-)$. At this workshop Gordon Thomson$^{38}$ reported on a new upper limit on this decay, i.e. $B(K_S \rightarrow \pi^0 e^+ e^-) < 3.9 \times 10^{-7}$ at 90% c.l., which corresponds to $B(K_L \rightarrow \pi^0 \ell^+ \ell^-)_{ind} < 1.2 \times 10^{-9}$. Although this is a
factor three improvement over the previous limit, it still falls about three orders of magnitude short of what is needed.

2. A CP-conserving contribution mediated by $K_L \rightarrow \pi^0\gamma\gamma$. This is also expected to be of the same order as the direct CP-violation. A few tens of $K_L \rightarrow \pi^0\gamma\gamma$ events have been observed, so that the absorptive part of this contribution can be roughly calculated, and it appears to be $< 5 \times 10^{-12}$. Future data from KTeV and NA48 should allow this to be refined considerably. However there's also a possible dispersive part that is more problematic. It may not be known in advance of a high-statistics $K_L \rightarrow \pi^0e^+e^-$ experiment, how large the CP-conserving contribution is.

3. Background from $K_L \rightarrow \gamma\gamma e^+e^-$. This occurs at the few $\times 10^{-7}$ level (FNAL-799 obtains $(6.5 \pm 1.2 \pm 0.6) \times 10^{-7}$ and NA31 obtains $(8.5 \pm 1.4 \pm 0.4) \times 10^{-7}$), but $m_{\gamma\gamma}$ falls in the $\pi^0$ mass region often enough for this to be a problem for $K_L \rightarrow \pi^0e^+e^-$. This was explored in detail by Greenlee, who found that given practical experimental resolution, it is difficult to beat this background down beyond the few $\times 10^{-11}$ level. This is about an order of magnitude larger than the expected direct CP-violating signal, so that to reach a given sensitivity in the latter, one would require roughly three times the number of events needed in the absence of background. This assumes that the background can be estimated with negligible uncertainty, which may be possible, given a good measurement of $K_L \rightarrow \gamma\gamma e^+e^-$ — preferably in the same experiment that studies $K_L \rightarrow \pi^0e^+e^-$. This complex situation makes the attempt to extract $\eta$ from $K_L \rightarrow \pi^0e^+e^-$ a challenge, and it is not yet clear how successful it will be.

3.3. Reality testing

In principle the real axis of the triangle in Fig. 2 can be well-determined by the branching ratio for $K_L \rightarrow \mu^+\mu^-$. However, with respect to the $K \rightarrow \pi\nu\bar{\nu}$ modes discussed above, the experimental and theoretical situations are reversed. AGS-871 expects to achieve a $1 - 2\%$ measurement of the branching ratio within a year or two, but the prospect for extracting short-distance information from this number is rather murky. If there were no other contributions to this decay, $B(K_L \rightarrow \mu^+\mu^-)$ would be roughly $10^{-9}$. But it is observed to be around seven times larger ($\gamma\gamma$), which is in fact dominated by an absorptive contribution, which is mediated by $K_L \rightarrow \gamma\gamma$, and can be calculated to be $(6.8 \pm 0.3) \times 10^{-9}$. The precision of this calculation is limited by the present precision on $B(K_L \rightarrow \gamma\gamma)$, which will probably be improved markedly by the next round of $K \rightarrow \pi\pi$ experiments. Unfortunately, this is not the only issue in extracting $\rho$ from $B(K_L \rightarrow \mu^+\mu^-)$. Aside from the absorptive contribution, long distance contributions to the real part of the amplitude are possible, and these can interfere with the short distance contribution (which is also real). The problems that this occasions are discussed in some detail and the apposite references given in Ref. 36. Very briefly, to calculate the dispersive contribution from $K_L \rightarrow \gamma\gamma$, it is necessary to know the amplitude off the mass shell. This is beyond the state of the art of ChPT, and it is controversial whether any extant model can be trusted for such a calculation. To a certain extent this amplitude can
be measured in processes such as \( K_L \rightarrow \ell^+\ell^-\gamma, \ K_L \rightarrow e^+e^-e^+e^- \), and \( K_L \rightarrow e^+e^-\mu^+\mu^- \), which accounts for their presence in Fig 2. There has been recent progress in such measurements (including the observation by FNAL-799 of \( K_L \rightarrow e^+e^-\mu^+\mu^- \) reported to this workshop\(^{47,48}\)), and more is expected in the next few years. However, these measurements can probe only to \( m_{\gamma} < m_K \) at best, and outside of particular models, it is not possible to be certain that this is sufficient (one must integrate to infinity).

Now if it does turn out to be possible with future data on these modes to get sufficient confidence in a model, then one could hope to get quite a good determination of \( \rho \) from \( B(K_L \rightarrow \mu^+\mu^-) \). To illustrate this, I choose the model of Paul Singer and collaborators,\(^{49,50}\) which has been successful in predicting the form factor in \( K_L \rightarrow e^+e^-\gamma \). Let's imagine, after AGS-871, \( B(K_L \rightarrow \mu^+\mu^-) = (7.85 \pm 0.10) \times 10^{-9} \). Further imagine, after NA48's measurement of \( B(K_L \rightarrow \gamma\gamma) \), that \( B(K_L \rightarrow \mu^+\mu^-)_{abs} = (6.85 \pm 0.06) \times 10^{-9} \). Then \( B(K_L \rightarrow \mu^+\mu^-)_{dep} = (1.00 \pm 0.12) \times 10^{-9} \).

Now in the model, \( B(K_L \rightarrow \mu^+\mu^-)_{long distance} = 0.076 \times 10^{-9}(1 + \alpha_K/27) \), where \( \alpha_K \) is a parameter whose absolute value is predicted to be about 0.27. Finally imagine, that with the anticipated 20,000 \( K_L \rightarrow \mu^+\mu^- \) events, KTeV determines \( \alpha_K = -0.27 \pm 0.013 \) (note that with 200 events, FNAL-799 determined\(^{51}\) \( \alpha_K \) to \( \pm 0.13 \)). Then

\[
A(K_L \rightarrow \mu\mu)_{short distance} = (3.16 \pm 0.19) \times 10^{-5} - (0 \pm 0.13) \times 10^{-5}
\]

\[
= (3.16 \pm 0.23) \times 10^{-5}
\]

i.e., \( A^2(\rho_0 - \rho) \) is determined\(^{52}\) to 7.3%. If \( \alpha_K \) turns out to be \(-0.15\) instead of \(-0.27\), this deteriorates to 12%, still not bad at all. The moral is that if we knew which model to use, in a few years, through the efforts of a number of different experiments, we'd have sufficient data to fully exploit \( K_L \rightarrow \mu^+\mu^- \).

There are two other measurements in the \( K \) system that can in principle give clean determinations of \( \rho \), but these are much more difficult experimentally than \( B(K_L \rightarrow \mu^+\mu^-) \). One which has been discussed for a few years is that of the parity-violating \( \mu^+ \) polarization asymmetry in \( K^+ \rightarrow \pi^+\mu^+\mu^- \). This is predicted\(^{53,54}\) to be \( \sim 0.1\% \). Unfortunately, as Shinkawa reports,\(^{18}\) AGS-787 has recently verified the theoretical expectation that the branching ratio for this process should be only a few times \( 10^{-8} \), which makes the polarization measurement a very tall order.

A second quantity sensitive to \( \rho \) is the branching ratio of \( K_L \rightarrow \pi\pi\nu\bar{\nu} \). This has recently been discussed by Geng, et al.\(^{55}\) and by German Valencia and myself.\(^{56}\) This reaction is related to \( K\epsilon_4 \), much as the reaction \( K \rightarrow \pi\nu\bar{\nu} \) is related to \( K\epsilon_3 \), so that there's no need to calculate hadronic matrix elements. The possible long distance effects are known to be small.\(^{57}\) In principle, \( K_L \rightarrow \pi^+\pi^-\nu\bar{\nu} \) is sensitive to \( \eta \) and \( \rho \) separately, through the study of the angular distribution of the the pions in their c.m. system (see Fig.4). However the coefficient of the \( \eta \) term is much smaller than that of the \( \rho \) term, so that one could not make practical use of this technique, particularly in a decay with a branching ratio in the region of a few \( \times 10^{-13} \). Perhaps the more practical version of this process is \( K_L \rightarrow \pi^0\pi^0\nu\bar{\nu} \). It is a few times smaller \(([(1.37 - \rho)^2] \times 10^{-13} \) vs. \([1.8(1.37 - \rho)^2 + \eta^2] \times 10^{-13}\) \), but one could concentrate entirely on
photon detection, rather than having to optimize for both particle ID and kinematic resolution at the required level. The all-neutral mode is likely to be the first version of this process to be bounded, since a trigger for it could easily be added in experiments searching for $K_L \rightarrow \pi^0 \nu \bar{\nu}$. Perhaps the main utility of $K_L \rightarrow \pi \pi \nu \bar{\nu}$ at the moment is to make the search for $K_L \rightarrow \pi^0 \nu \bar{\nu}$ look easy by comparison.

Fig. 4. $d\Gamma(K_L \rightarrow \pi^+ \pi^- \nu \bar{\nu})/d\cos \theta_{\nu \bar{\nu}}$. $\theta_{\nu \bar{\nu}}$ is the angle between the $\pi^+$ momentum and the sum of $\nu$ and $\bar{\nu}$ momenta evaluated in the $\pi^+ \pi^-$ center of mass. The two terms corresponding to the $P^2$ (marked as $\rho$-only) and $G^2$ (marked as $\eta$-only) contributions are shown. The distribution given by each term has been normalized to unit area.

4. Conclusions

In the next few years, experiments probing lepton flavor violation in $K$ decay will approach $10^{-12}$. Windows for for new physics in $K_L \rightarrow e^+ e^-$ and $K^+ \rightarrow \pi^+ X^0$ will be closed. If nothing new is found, there will probably be a pause in initiating dedicated BSM experiments, although such searches will continue as by-products of other work. Any new physics that is found will pretty much come as a surprise to everyone. Non-SM CP-violation, however, will probably retain the power to motivate new experiments.

The emphasis in rare $K$ decays will shift toward SM studies. If we don’t flag in our efforts, these experiments can compete with results from the $B$ sector. In any case, $K$’s complement $B$’s, and a comparison of results in the two sectors again probes BSM physics.

Fig. 5 summarizes the history of K decays. Various milestones in this history are indicated on a scale of decay rates, along with the reach of recent, current, and next-generation experiments. The figure makes it clear that this reach is very large on a historical scale; smaller increments have yielded watershed results. I believe we are overdue for a big discovery.

5. Acknowledgments

Fig. 5. Rates of various $K$ decay modes, giving an historical perspective. The solid vertical lines show the progress of recent and current experiments. The dashed extensions show the expected reach of the current experiments. The dotted extensions show the expected reach of proposed experiments. The short horizontal lines show the approximate upper limit of Standard Model prediction for those decays that are allowed.

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References


3. J. Ritchie, This workshop.


5. $G$ is an additive quantum number that $= +1$ for first generation fermions, $-1$ for first generation anti-fermions, $+2$ for second generation fermions, etc.

6. M. Zeller, this workshop.


8. S. Schnetzer, this workshop.


18. T. Shinkawa, this workshop.


23. A. Buras, this workshop.

24. In fast extraction mode, the AGS can cycle in 1.7 seconds.

25. R. Tschirhart, this workshop.


29. T. Inagaki, et al., Measurement of the $K^0 \rightarrow \pi^0 \nu \bar{\nu}$ decay, KEK proposal, 7 June 1996.


35. Y. Fukushima, this workshop.


38. G. Thomson, this workshop.


42. J. Scheidt, New results on rare $K^0$ decays from the NA31 experiment, this workshop.

52. \( B(K_L \rightarrow \mu^+\mu^-) \approx 10^{-9}A^4(\rho_0 - \rho)^2 \), where \( A \) and \( \rho \) are the usual CKM parameters. More exact formulae can be found in Ref. 21. After the NLLO calculation of Ref. 21, the residual “intrinsic” theoretical uncertainty in the extracted \( \rho \) is \( \sim 10\% \).