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Suppression of Acoustic Streaming in Tapered Pulse Tubes

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ABSTRACT

In a pulse tube cryocooler, the gas in the pulse tube can be thought of as an insulating piston, transmitting pressure and velocity from the cold heat exchanger to the hot end of the pulse tube. Unfortunately, convective heat transfer can carry heat from the hot end to the cold end and reduce the net cooling power. Here, we discuss one driver of such convection: steady acoustic streaming as generated by interactions between the boundary and the oscillating pressure, velocity, and temperature.

Using a perturbation method, we have derived an analytical expression for the streaming in a tapered pulse tube with axially varying mean temperature in the acoustic boundary layer limit. Our calculations showed that the streaming depends strongly on the taper angle, the ratio of velocity and pressure amplitudes, and the phase between the velocity and pressure, but it depends only weakly on the mean temperature profile and is independent of the overall oscillatory amplitude. With the appropriate tapering of the tube, streaming can be eliminated for a particular operating condition. Experimentally, we have demonstrated that an orifice pulse tube cryocooler with the calculated zero-streaming taper has more cooling power than one with either a cylindrical tube or a tapered pulse tube with twice the optimum taper angle.

INTRODUCTION

In a simplified view of the operation of the orifice pulse tube refrigerator,¹ the gas in the pulse tube acts as a long (and slightly compressible) piston, transmitting pressure and velocity oscillations from the cold heat exchanger to the orifice at higher temperature. The gas in the pulse tube must thermally insulate the cold heat exchanger from higher temperatures. Unfortunately, convective heat transfer within the pulse tube can carry heat from the hot heat exchanger to the cold heat exchanger and thereby reduce the net cooling power. Such convection can be steady or oscillatory, and has causes as mundane as gravity or as subtle as jetting due to inadequate flow straightening at either end of the pulse tube.

Here, we consider convection driven by streaming. In the acoustics literature, streaming² denotes steady convection which is driven by oscillatory phenomena. In the context of the pulse tube, this driving can occur in the oscillatory boundary layer at the side wall of the pulse tube: in this layer, both viscous and thermal phenomena are important.

Lee et al.³ first suggested that tapering a pulse tube slightly, as illustrated in Figure 1, might suppress such streaming-driven convection. A general method for calculating boundary-layer streaming was published by Rott⁴ for the case of axially varying wall temperature, with standing wave phasing between pressure and velocity and with constant tube cross-sectional area. However, standing wave phasing is a poor assumption for the pulse tube in an orifice pulse tube refrigerator, because significant acoustic power flows along the pulse tube. We recently published⁵ a more general method for calculating streaming, following Rott's approach but incorporating variable cross section and arbitrary phase between pressure and velocity. This method yielded a prediction for the taper angle that suppresses streaming. We also presented measurements confirming these calculations.

Here, we briefly summarize our previously work in order to bring it to the attention of a wider audience, and we elaborate on some aspects.

SUMMARY OF OUR PREVIOUS WORK

We restricted attention to ideal gases, so that $p = \rho R_g T$ and $\rho a^2 = \gamma p$, where p is the pressure, ρ is the density, R_g is the gas constant, T is the temperature, a is the sound speed, and γ is the ratio of specific heats. The coordinates used are shown in Figure 1. The relevant variables were expanded to second order; for example:

$$u(x, y, t) = \operatorname{Re}[u_1(x, y)e^{i\omega t}] + u_{2,0}(x, y) \quad (1)$$

$$T(x, y, t) = T_m(x) + \operatorname{Re}[T_1(x, y)e^{i\omega t}] + T_{2,0}(x, y) \quad (2)$$

$$p(x, t) = p_m + \operatorname{Re}[p_1(x)e^{i\omega t}] + p_{2,0}(x) \quad (3)$$

where u is the axial component of the velocity, x is the axial position, y is the lateral distance

Figure 1 Schematic representation of pulse tube refrigerator with tapered pulse tube. The coordinate system used for the equations is shown. The angle ϕ of the conical pulse tube has been greatly exaggerated.

from the wall, t is the time, ω is the angular frequency of the oscillations, and $\text{Re}[z]$ denotes the real part of z . The lateral component of velocity v has the same form as u ; the dynamic viscosity μ and the density ρ have the same form as T .

In this notation, the variables with subscript "m" are steady-state mean values, without time dependences; these represent the values the variables would have if there were no oscillating pressure or velocity. The mean temperature profile $T_m(x)$ is assumed to be known, and leads to the x dependences of ρ_m and μ_m . We omit the subscript "m" on constant properties (such as γ) and on variables for which terms of order higher than mean are unimportant in this work (such as a).

The subscript 1 indicates the first-order part of each variable, which accounts for oscillation at angular frequency ω . The first-order variables are complex quantities, having both magnitude and phase to account for their amplitudes and time phasing. The oscillating pressure p_1 and the lateral spatial average $\langle u_1 \rangle$ of the oscillating axial velocity u_1 are assumed to be known, as they are experimentally accessible through measurements of oscillating pressure in the pulse tube and mass flow through the orifice. Expressions for the other oscillating variables (temperature, density, etc.) in terms of p_1 and $\langle u_1 \rangle$ are well known.^{4,6} The tube radius is assumed to be much larger than the boundary layer thickness. Surprisingly, the oscillatory part of the viscosity cannot be neglected; we assume it to be independent of pressure and to depend on the oscillatory temperature via the temperature dependence of the viscosity, which takes the form $\mu(T) = \mu_0 (T/T_0)^b$.

Second-order, time-independent parts of variables are indicated by the subscript "2,0". This includes the axial mass flux density $\dot{m}_{2,0}(x, r)$ which is of greatest interest here. With this notation, the streaming mass flux density just outside the boundary layer is given by

$$\begin{aligned} \dot{m}_{2,w} = & \frac{|p_1| |\langle u_1 \rangle|}{a^2} \left[\left(\frac{3}{4} + \frac{(\gamma-1)(1-b\sigma^2)}{2\sigma(1+\sigma)} \right) \cos \theta + \left(\frac{3}{4} + \frac{(\gamma-1)(1-b)\sqrt{\sigma}}{2(1+\sigma)} \right) \sin \theta \right] \\ & + \frac{\rho_m |\langle u_1 \rangle|^2}{\omega} \left[\frac{3}{4} \frac{dA/dx}{A} + \frac{(1-b)(1-\sqrt{\sigma})}{4(1+\sigma)(1+\sqrt{\sigma})} \frac{dT_m/dx}{T_m} \right] \end{aligned} \quad (4)$$

where θ is the phase angle by which $\langle u_1 \rangle$ leads p_1 , and σ is the Prandtl number. This mass flux (which is outside the boundary layer but still effectively "at" the wall compared to R), acts like a forced slip boundary condition, driving the streaming profile shown in Figure 2a. This streaming profile away from the wall is then given by $\dot{m}_{2,0}(r) = \dot{m}_{2,w} (2r^2/R^2 - 1)$, where r is the radial coordinate in the pulse tube. (To display the boundary-layer behavior clearly, the Figure does not have $R \gg \delta_v$, and hence the maximum of $\dot{m}_{2,0}$ falls slightly short of $\dot{m}_{2,w}$ in the Figure. However, our calculations are only valid for $R \gg \delta_v$, when the maximum of $\dot{m}_{2,0} \equiv \dot{m}_{2,w}$.)

Setting Eq. (4) equal to zero, we find that streaming is eliminated in a tapered pulse tube if

$$\begin{aligned} \frac{1}{A} \frac{dA}{dx} = & - \frac{\omega |p_1|}{\gamma \rho_m |\langle u_1 \rangle|} \left[\left(1 + \frac{2(\gamma-1)(1-b\sigma^2)}{3\sigma(1+\sigma)} \right) \cos \theta + \left(1 + \frac{2(\gamma-1)(1-b)\sqrt{\sigma}}{3(1+\sigma)} \right) \sin \theta \right] \\ & - \frac{(1-b)(1-\sqrt{\sigma})}{3(1+\sigma)(1+\sqrt{\sigma})} \frac{dT_m/dx}{T_m} \end{aligned} \quad (5)$$

$$\approx - \frac{\omega |p_1|}{\rho_m |\langle u_1 \rangle|} (0.75 \cos \theta + 0.64 \sin \theta) - 0.0058 \frac{dT_m/dx}{T_m} \quad (6)$$

In Eq. (6), we have used numerical values for low-temperature helium gas: $\gamma = 5/3$, $\sigma = 0.69$, and $b = 0.68$. Note that for a conical tube, $1/A \, dA/dx = -2 \tan(\phi/2) / R$, where ϕ is the taper angle (see Figure 1) and R is the pulse tube radius. For this value of dA/dx , the parabolic part of the velocity profile shown in Figure 2a is eliminated; the only nonzero streaming occurs at distances from the wall comparable to the penetration depths, as shown in Figure 2b.

Figure 2 (a) The net drift near the wall, illustrated in (c) and (d), affects the entire pulse tube, causing an offset parabolic velocity profile which leads to a second-order mass flux density $\dot{m}_{2,0}$. For the signs chosen here for illustration, the gas moving downward in the center of the pulse tube is hotter than the gas moving upward around it, so that heat is carried downward. The coordinate r measures the distance from the center of the pulse tube, of radius R . The calculations here are for $R \gg \delta_v$, but, for clarity, δ_v is exaggerated in the figure. (b) Mass flux density for the case where the streaming is suppressed by tapering the tube. Although there is still some streaming near the wall and a correspondingly small offset flux in the rest of the tube, they carry negligible heat. (c) Illustration of net drift caused by one process within a penetration depth δ_v of the pulse tube wall. A parcel of gas is shown in three consecutive positions. Here we imagine that the temperature dependence of the viscosity is the only important effect, and assume that the temperature is lower during upward motion than during downward motion. (d) Illustration of net drift caused by a different process, as discussed in the text.

To verify Eq. (6), we tested an orifice pulse tube refrigerator with three pulse tubes: a right-circular cylinder, a truncated cone with the optimum angle determined by Eq. (6), and a second cone with roughly twice the optimum angle. The double-angle cone should induce the same amount of streaming as the cylindrical pulse tube, but in the opposite direction, and so should exhibit roughly the same performance as the cylindrical pulse tube.

The pulse tube refrigerator, shown schematically in Figure 1, was filled with 3.1 MPa helium and driven at 100 Hz. The optimum-angle cone pulse tube (see below) had a total included angle ϕ of 0.049 rad = 2.8°, with the cold end larger than the hot end. This is a small enough angle that it is barely discernible by eye. The double-angle pulse tube had an angle of 0.103 rad. In both cases, the internal diameter at each end was calculated in order to make the volume of the pulse tube the same as in the cylindrical case.

The results of our measurements are shown in Figure 3. The data corresponding to the cylindrical pulse tube are represented by the circles, while those of the optimum-angle conical pulse tube are shown as triangles and those of the double-angle cone as squares. For all measured values of applied heat, the temperature corresponding to the optimum cone pulse tube was at least 5 K colder than the temperatures with either the cylindrical or the double-angle pulse tubes, indicating that the optimum cone performed significantly better than the other pulse tubes. From an alternative point of view, it appears that the streaming-driven convective heat load on the cold heat exchanger was 3–5 W greater with the cylindrical and double-angle pulse tubes than with the optimally tapered pulse tube.

DISCUSSION

The complexity of Eqs. (4) and (5) is due to many nonlinear interactions among oscillatory pressure, velocity, and temperature in the fundamental equations of hydrodynamics and heat transfer. One such interaction is easily imagined and is illustrated in Figure 2c. Consider a small parcel of gas oscillating up and down along the wall, at a distance from the wall of the order of

Figure 3 Experimental results, showing cooling power as a function of the cold temperature for three different pulse tubes with equal volumes and lengths. The conical pulse tube (triangles), designed according to Eq. (13), had more cooling power than either a cylindrical one (circles), or one with twice the optimum cone angle (squares). The lines are only guides to the eye.

the relevant boundary layer: the viscous penetration depth $\delta_v = \sqrt{2\mu / \omega\rho}$. On average, the gas between the parcel and wall will have a different temperature during the parcel's upward motion than during its downward motion, due to imperfect thermal contact with the wall's temperature gradient and to the adiabatic temperature oscillations with time phasing between oscillatory motion and oscillatory pressure. Since the viscosity depends on temperature, the moving parcel will experience a different amount of viscous drag during its upward motion than during its downward motion, and hence will undergo a different displacement during its upward motion than during its downward motion. After a full cycle, the parcel does not return to its starting point; it experiences a small net drift. This process is represented by terms proportional to the product of b , T_1 , and u_1 in our starting nonlinear equations, and is responsible for the presence of b in Eqs. (4) and (5).

Such drifting gas close to the wall has a profound effect on the entire pulse tube, because it drags gas farther from the wall along with it. For sufficiently slow streaming, and with pulse tube radius much larger than the penetration depth, the resulting velocity profile is essentially parabolic, as shown in Figure 2a, with the gas velocity near the wall equal to the drift velocity just outside the penetration depths, and the velocity in the center of the pulse tube determined by the requirement that the net mass flux along the tube must be zero. In turn, this parabolic streaming profile convects heat along the pulse tube.

The effect of a taper on streaming can also be imagined, with reference to Figure 2d. In general, a gas parcel close to the wall will be farther from the wall during, say, its upward motion than during its downward motion, due to, for example, the compressibility of the gas in the boundary layer and the phasing between oscillatory motion and pressure. Hence, again the moving parcel will experience a different amount of viscous drag during its upward motion than during its downward motion, and so the parcel will again fail to return to its starting point after a full cycle. This effect is represented in our starting equations by terms proportional to the product of u_1 and v_1 . However, the boundary-layer continuity equation couples v_1 and du_1/dx , while the y-averaged continuity equation $i\omega A \langle \rho_1 \rangle + d \langle \rho_m A u_1 \rangle / dx = 0$ couples du_1/dx to dA/dx . Hence the process shown in Figure 2d is controlled in part by the taper dA/dx .

Including all such second-order streaming effects in our calculations allowed us to determine the conditions under which they all add to zero, represented by Eq. (5).

Note that all the variables in the right hand side of Eq. (5) or (6) should be known during the design of a pulse tube refrigerator and are experimentally accessible. The most challenging of these variables are the velocity $\langle u_1 \rangle$ and phase difference θ , but these can be calculated from other known or measured quantities: The pressure amplitude and phase in the compliance, its volume, and the adiabatic compressibility of the gas yield the volumetric velocity and phase of the gas entering the compliance, which is essentially that at the hot end of the pulse tube. The volumetric velocity and its phase at other x in the pulse tube can then be estimated from its geometry and the adiabatic compressibility of the gas.

We generally use these methods to calculate the magnitudes and relative phases of p_1 and $\langle u_1 \rangle$ at both ends and in the middle of the pulse tube, substituting these values into Eq. (6) to determine the optimum dA/dx at the ends and in the middle of the pulse tube. In all cases we have considered to date, the values have been such that a simple cone is a reasonable approximation to the ideal shape, making more difficult fabrication unnecessary.

The fact that streaming in a pulse tube can be suppressed so simply and conveniently is the result of a remarkable series of fortunate coincidences. First, there was no a priori guarantee that tapering the pulse tube would have a large enough effect on streaming to cancel streaming's other causes. Second, it might have turned out that rather large taper angles (say, greater than 10 or 20 degrees) were required; in this case, flow separation of the high-Reynolds-number first-order velocity from the pulse tube wall would have invalidated the entire laminar, boundary-layer approach. Third, it is fortunate that most pulse tubes operate in the "weakly turbulent" regime of

oscillatory flow, and with tube surface roughnesses much smaller than the boundary-layer thicknesses, so that laminar analysis is adequate in the boundary layer.

Fourth, the perturbation expansion upon which this calculation is based is only valid for zero or extremely weak streaming—the very situation we are most interested in. This point is subtle. Strong streaming (but nevertheless with streaming velocity small compared to the oscillatory velocity) distorts the axial temperature profile of the pulse tube significantly, contradicting our fundamental assumption that the time-averaged temperature, density, etc in the boundary layer are well approximated by their zero-oscillation values $T_m(x)$, $\rho_m(x)$ etc. This fundamental assumption requires that the streaming be so weak that the temperature profile in the pulse tube is unperturbed by the streaming—or, equivalently, that the streaming is so weak that it carries negligible heat! Hence, the calculation self-consistently predicts the conditions of zero streaming, but it cannot be used to accurately predict the magnitude of strong streaming.

Fifth, there are numerous other fourth-order energy flux terms in addition to $\dot{m}_{2,0} c_p T_{2,0}$ which would in principle have to be considered to obtain a formally correct fourth-order result. Fortunately, the only other large term, $\rho_{2,0} c_p \text{Re}[T_1 \langle \tilde{u}_1 \rangle]$, is zero at the same taper angle that makes $u_{2,0}$ zero, while the remaining terms, such as those involving products of first and third order quantities, are small for all angles. Hence, the suppression of $u_{2,0}$ is sufficient to suppress streaming heat transport.

Sixth, as a practical matter, it is extremely convenient that the streaming-suppression taper is independent of oscillatory amplitude. Seventh, it is also convenient that the streaming-suppression taper is only weakly dependent on temperature gradient, so that streaming is suppressed over a broad range of T_{cold} and of T_{hot} . Finally, at least in our experience so far, a simple conical section is an adequate approximation to the ideal streaming-suppression taper shape $A(x)$.

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