DOE/ER/25183--T1

FINAL REPORT

Parallel Supercomputing: Advanced Methods, Algorithms, and Software for Large-Scale Linear and Nonlinear Problems

DOE NO. DE-FG03-93ER25183

Period Covered: 8/15/93 - 2/28/96

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July 1996

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Final Report

Parallel Supercomputing: Advanced Methods, Algorithms and Software for Large-Scale Linear and Nonlinear Problems

G. F. Carey and D. M. Young

This report describes research progress on Methods, Algorithms and Software for Large Scale Parallel Supercomputer Applications. The focus is large scale applications of interest to DOE such as coupled viscous flow and heat or mass transport, and energy related applications such as 3D petroleum and gas reservoir simulations on massively parallel systems. The work has been carried out jointly by Drs. Carey and Young as Principal Investigators and Drs. Sepehrnoori and Kincaid as Associate Investigators. This interdisciplinary research involved collaboration in numerical mathematics and computer science (Young and Kincaid) with methodology and large-scale engineering applications (Carey and Sepehrnoori) in the respective colleges of Natural Sciences and Engineering. The interdisciplinary collaboration has been effective since it enhances the development of new iterative schemes for complex problems important to DOE. For example, we have made significant advances with modified forms of generalized gradient methods and multigrid methods for viscous flow and reservoir problems. The work has also involved collaboration with colleagues at the DOE National Laboratories.

Several graduate students have participated in the research and the investigators have collaborated closely with the students on related research projects. For example, Dr. Young (Mathematics and Computer Sciences) and Dr. Sepehrnoori (Petroleum Engineering) were dissertation co-advisors for one of the students and Dr. Carey served on the student's committee. Some of our students have

been involved as interns at the DOE National Laboratories during the course of this project.

As part of the work, a new class of parallel element-by-element spectral schemes was developed for the stream function-vorticity formulation of the Navier-Stokes equations. The algorithm involved the use of generalized iterative methods recast at the element subdomain levels. The schemes were implemented on distributed parallel supercomputers. Parallel performance studies were conducted on representative Navier-Stokes problems. These results also provide the most accurate flow benchmark studies to date for the recirculation zone in the "backward facing step" problem. Some interesting phenomenological results on the growth of the separation zone with increasing Reynolds number were obtained. A paper from this has been submitted to a journal and a copy is enclosed. Two other related papers dealing with scalability and performance studies were also published. This work is also described in the dissertation by E. Barragy completed under this project. (see attached publication summary). Dr. Barragy is now the parallel applications analysis for Intel at Sandia.

Our research on massively parallel domain decomposition strategies has been very successful. This work was initiated jointly with Sandia's Doug Cline and John Shadid and a paper describing the recent results of the research has been published in the AIAA journal (see publication list). A color print of a cross-plane section of the flow field computed for a conical profile is enclosed with this report. This calculation was made on a massively parallel distributed system by one of the students supported by the DOE contract (Alan Stagg) during an internship at Sandia. Extensions of this work involved implementation on the CRAY T3D at JPL. Alan Stagg completed his PhD during the course of the project, worked with CRAY applications at JPL, and has recently taken a

position with the parallel supercomputing group at the Waterways Experiment Station.

We have also developed new multigrid strategies for parallel applications to semiconductor problems. Bruce Davis is completing his Ph.D. research with Dr. Carey and they have developed a new hybrid multigrid multilevel spectral finite element scheme. The problem of interest is an augmented energy balance model for the semiconductor device equations. During the course of the project Bruce Davis was resident for two semesters at Los Alamos Lab as an intern to implement this model on the CM5 and T3D. Results from this work were presented at the Colorado Multigrid Conference in April 1995. This work also intersects closely with the new Los Alamos initiative on semiconductor modeling sponsored by DOE. (Dr. Carey is the university liaison between the DOE National Laboratories on this CRADA, the semiconductor research Consortium and industry on the grid aspects of that project). Mr. Davis is completing his dissertation and is scheduled to defend in October of this year.

Under sponsorship of this project, Dr. Young and Mr. Chen have been working on the development of iterative methods for solving large sparse nonsymmetric linear systems. A new method, called MGMRES, which can be viewed as a generalization of GMRES, has been developed for solving linear systems of the form $Au = b$ for cases where a nonsingular symmetric matrix Y is available such that Y and YA are symmetric (but not necessarily symmetric and positive definite.) By the application of MGMRES to an augmented linear system related to the Lanczos method one can derive LANGMRES, a method which can be regarded as a combination of the biconjugate gradient method and GMRES. LANGMRES is designed to combine the numerical stability of GMRES with the short recurrence properties of the biconjugate gradient method.

Presentations were given by Dr. Young during the course of this project on MGMRES and LANGMRES at the Householder Conference at Lake Arrowhead, California, in 1993, at the Lanczos Conference at North Carolina State University in 1993, at the Colorado Conference at Breckenridge Colorado in 1994 and at a Workshop at the University of Washington in 1995..

Dr. Young and Dr. Shengyou Xiao have worked on parallel multigrid methods. Frederickson and McBryan showed parallel multigrid to be effective when applied to certain discrete periodic problems. In some cases, it is possible to transform certain discrete nonperiodic problems into discrete periodic problems so that parallel multigrid methods can be rigorously applied. This work is described in Xiao's Ph.D. dissertation, Xiao [1994], which was supervised by Dr. Young and Dr. Sepehrnoori; see also the paper by Young, Xiao, and Baker [1995].

Young and Kincaid, together with Wan Chen, have worked on parallel alternating-type methods, including the alternating direction implicit (ADI) method and the symmetric and nonsymmetric SOR methods (USSOR and SSOR methods). As shown by Young and Kincaid [1996], under certain conditions, one can carry out several single iterations in parallel and achieve the same result as would be achieved by carrying out a (smaller) number of iterations in sequence. This can be done, for example, for the ADI method, for certain separable Dirichlet problems in two space dimensions, and for the USSOR/SSOR methods, for certain discrete periodic problems in rectangular regions. The validity of the procedure for some cases has been verified by numerical experiments; see Young and Kincaid [1996].

An important aspect of our work has been the development of several research-oriented software packages developed as part of the ITPACK Project. This

software is available over the Internet through the WEB site at the Center for Numerical Analysis and also through the Netlib Distribution System that is maintained at the Oak Ridge National Laboratory. In addition, a WEB site has established in the Computational Fluid Dynamics Laboratory with pointers to a package for solving linear systems on parallel computers. User can obtain our software without charge.

Related DOE Publications and Presentations

Publications

1. Barragy, E., "Parallel Finite Element Methods and Iterative Solution Techniques for Viscous Incompressible Flows," Ph.D. Dissertation, The University of Texas at Austin, 1993.
2. Barragy, E. and G. F. Carey, "Stream Function Vorticity Solution Using High Degrees (p) Finite Elements and Element-by-Element Techniques," *CNME*, 9, 387-395, 1993.
3. Barragy, E., G. F. Carey and R. Van de Geijn, "Parallel Performance and Scalability for Block Preconditioned Finite Element (p) Solution of Viscous Flow," *IJNME*, Submitted October 1993.
4. Runnels, S. and G. F. Carey, "A Domain Decomposition Strategy for Finite Element Simulation of Phase Change," *Numerical Heat Transfer*, Part B, Vol 24, No. 2, 181-189, 1993.
5. Wang, K. C. and G. F. Carey, "A Least Squares Finite Element Method for Viscoelastic Fluid Flow Problems," *IJNMF*, 17, 943-953, 1993.
6. Barragy, E., G. F. Carey and R. Van de Geijn, "Performance and Scalability of Finite Element Analysis for Distributed Parallel Computation,"

J. of Parallel and Distributed Computing, In Press 1994.

7. Rahman, M. and G. F. Carey, "Modeling Separated Forced Convection in Laminar Flow Past Cavities," *Numerical Heat Transfer*, Part A, Vol 25, 85-101, 1994.
8. Rahman, M. and G. F. Carey, "Nonlinear Dynamics of Heat Transfer Enhancement using Eddy Promoters," *J. of Numerical Heat Transfer*, Part A, Vol 25, 117-133, 1994.
9. Wu, H., G. F. Carey and M. E. Oakes, "Numerical Simulation of AC Plasma Arc Thermodynamics," *Journal of Comp. Phy.*, In Press 1994.
10. Xiao, Shengyou [1994]. "Multigrid Methods With Applications in Reservoir Simulation," Ph.D. Dissertation, The University of Texas at Austin, Austin, Texas.
11. Joubert, W. and G. F. Carey, "Embedded Gradient Iterative Solution of a Class of Nonlinear PDE's on the Connection Machine," *Int'l Journal of High Speed Computing*, Vol 6, No. 2, 277-286, 1994.
12. Carey, G. F., A. Pehlivanov and P. S. Vassilevski, "Least-Squares Mixed Finite Element for Non-Selfadjoint Elliptic Problems: II. Performance of Block-ILU Factorization Methods," CNA Report 267, May 1994.
13. Pehlivanov, A., G. F. Carey and P. S. Vassilevski, "Least-Squares Mixed Finite Element for Non-Selfadjoint Elliptic Problems: I. Error Estimates," CNA Report 271, July 1994.
14. Carey, G. F. and Y. Shen, "Simulation of Fluid Mixing using Least-Squares Finite Element and Particle Tracing," *Int'l J. of Num. Meth. for Heat and Fluid Flow*, Vol. 5, 549-573, 1995.

15. Joubert, W.D., G. F. Carey, N. A. Berner, A. Kalhan, H. Kohli, A. Lorber, R. T. McLay and Y. Shen, "PCG Reference Manual, A Package for the Iterative Solution of Large Sparse Linear Systems on Parallel Computers," CNA Report 274, January 1995.
16. Lorber, A., G. F. Carey and W. Joubert, "ODE Recursions and Iterative Solvers for Linear Equations," *SIAM J. of Scientific Computation*, Vol. 17, No. 1, 66-77, 1996.
17. Lorber, A. and G. F. Carey, "A Vector-Parallel Scheme for Navier-Stokes Computations at Multi-Gigaflop Performance Rate," *IJNMF*, Vol. 21, 445-466, 1995.
18. Barragy, E. and G. F. Carey, "Driven Cavity Benchmark Solution Using p Finite Elements," *Computers and Fluids*, Submitted June 1996.
19. Young, David M. and Jen-Yuan Chen [1993], "LANGMRES: An Alternative to the Biconjugate Gradient Method for Solving Large Sparse Nonsymmetric Linear Systems", appeared in the *Proceedings of the Cornelius Lanczos International Centenary Conference*, edited by J. D. Brown, Moody T. Chu, Donald C. Ellison, and R. J. Plemmons, SIAM, Philadelphia, PA, 279-281.
20. Young, David M., and Jen-Yuan Chen [1994] "MGMRES: A Generalization of GMRES for Solving Large Sparse Nonsymmetric Systems," Volume 2 of the *Proceedings of the Colorado Conference on Iterative Methods*, Breckinridge, Colorado, April 5-9, 1994.
21. Young, David M., and B. Vona, "Parallel Multilevel Methods," *Studies in Computer Science*, John Rice and Richard Demillo, ed., Plenum Press, New York, 1994.

22. David M. Young and David R. Kincaid. Parallel implementation of a class of nonstationary alternating-type methods. In D. Bainov and V. Covichev, editors, *Proceedings of the Third International Colloquium on Numerical Analysis*, pages 219–222. VSP, Utrecht, The Netherlands, 1995.
23. David R. Kincaid and David M. Young. Linear stationary second-degree methods for solution of large linear systems. In Th. M. Rassias, H. M. Srivasiava, and A. Yanushauska, editors, *Topics in Polynomials of One and Several Variables and Their Applications*, pages 609–629. World Scientific Publishing Co., River Edge, NJ, 1993.
24. David M. Young and David R. Kincaid. A new class of parallel alternating-type iterative methods. *Journal of Computational and Applied Mathematics*, 1996. To appear.
25. Thomas C. Oppe and David R. Kincaid. Iterative BLAS. *Journal of Applied Science & Computations*, 1(3):494–520, 1995.
26. David R. Kincaid. Stationary second-degree iterative methods. *Applied Numerical Mathematics*, 16:227–237, 1994.
27. David R. Kincaid, Linda J. Hayes, and David M. Young. ITPACK: Then and now. In *IMACS 13th World Congress on Computational and Applied Mathematics*, 1994.
28. David R. Kincaid, Linda J. Hayes, and David M. Young. ITPACK: Past, present, and future. In *Colorado Conference on Iterative Methods*, volume 1, Boulder, CO, 1994. University of Colorado & Front Range Scientific Computations, Inc.

29. Asha Nallana and David R. Kincaid. A Cray performance study. Report CNA-283, University of Texas at Austin, Center for Numerical Analysis, May 1996.
30. David M. Young and David R. Kincaid. A new class of parallel alternating-type iterative methods. Report CNA-282, University of Texas at Austin, Center for Numerical Analysis, May 1996.
31. David R. Kincaid and David M. Young. A note on parallel alternating-type iterative methods. Report CNA-276, University of Texas at Austin, Center for Numerical Analysis, June 1995.
32. David M. Young and David R. Kincaid. On the parallel implementation of alternating-type iterative methods. Report CNA-277 (revised), University of Texas at Austin, Center for Numerical Analysis, August 1995.
33. Young, David M., Shengyou Xiao, and Karen R. Baker [1995], "Parallel Generated Iterative Methods for Solving Elliptic Equations," *Applied Numerical Mathematics*, 19, 375-387.
34. Xiao, Shengyou and David M. Young [1995] "Multiple Coarse Grid Multigrid Methods for Solving Elliptic Equations," to appear in the *Proceedings of the Colorado Conference on Multigrid Methods*, held at Breckenridge, Colorado, April 1995.

Presentations

Carey Presentations

- Carey, G. F., R. McLay, D. Hu and Y. Shen, "Transport, Sedimentation and Mixing, Workshop on Finite Element Modeling of Environmental Problems," March 4-5, 1994, Thompson Conference Center, Austin, TX.

- "MPE-Iterative Methods for Solving Reaction-Diffusion Problems," *Fifth SIAM Conf. on Applied Linear Algebra*, Snowbird, Utah, June 15-18, 1994 (with R. Abbasian).
- "Time-Iterative Recursion with Domain Decomposition for Viscous Flows," *IMACS Conference*, July 11-15, 1994, Atlanta (with A. Lorber).
- "PCG: A Software Package for the Iterative Solution of Linear Systems on Scalar, Vector, and Parallel Computers," *Supercomputing '94*, Nov. 14-18, 1994, Washington, DC (with W. Joubert).
- "Maximizing Sparse Matrix Vector Product Performances in MIMD Computers," *Conf. on Iterative Methods*, April 1-9, 1994, Breckenridge, CO (with R. McLay and S. Swift).
- "Multi-Gigaflop Performance for Navier Stokes Problems on Cray Supercomputers," *Cray-Day*, Cray Research Grant Program Recipients, May 10, 1994, Austin, TX (with Alfred Lorber).
- "Parallel Element-by-Element Performance for Navier-Stokes Computations," Poster Session, *7th SIAM Conf. on Parallel Processing for Scientific Computing*, Feb 15-17, 1995, San Francisco, CA (with E. Barragy).
- "Implementing a Parallelized Navier-Stokes Flow Solver on the Cray T3D," Poster Session, *7th SIAM Conf. on Parallel Processing for Scientific Computing*, Feb 15-17, 1995, San Francisco, CA (with A. K. Stagg and D. D. Cline).
- "Parallel Element-by-Element Spectral Multi-Level Techniques for Finite Elements," Poster Session, *7th SIAM Conf. on Parallel Processing for Scientific Computing*, Feb 15-17, 1995, San Francisco, CA (with B. Davis).

- "A Parallel Multilevel Spectral Element Scheme," *Copper Mtn. Conf. on Multigrid Methods*, Copper Mtn, CO, April 2-7, 1995 (with B. Davis).
- "Development and Applications of a Parallel Portable System Solver for PDE's," *1996 SCS Simulation Multiconference, High Performance Computing '96*, April 8-11, 1996, New Orleans, LA (with W. Joubert, R. McLay, A. Lorber and Y. Shen).
- "Improving Matrix-Vector Product Performance and Multi-Level Preconditioning for the Parallel PCG Package," *Copper Mtn. Conf. on Iterative Methods*, April 9-13, 1996, Copper Mtn., CO (with R. McLay).
- "Accelerated Solution of Non-Linear Navier-Stokes Problems using Chebyshev Iteration Polynomial Based RK Recursions," *Copper Mtn. Conf. on Iterative Methods*, April 9-13, 1996, Copper Mtn., CO (with A. Lorber, S. Bova and C. Harle).
- "Iterative Solution of High Order Compact Systems," *Copper Mtn. Conf. on Iterative Methods*, April 9-13, 1996, Copper Mtn., CO (with B. Spotz).

Young Presentations

- "Parallel Alternating-Type Methods," Distinguished Lecture, Department of Computer Science, University of Utah, March 14, 1996.
- "On the Implementation of Parallel Implicit USSOR Methods for Solving Discrete Periodic Problems," *Colorado Conference on Iterative Methods*, Copper Mountain, Colorado, April 3, 1996.
- "Parallel Iterative Methods and Multigrid Methods," *Conference on Algebraic Multilevel Iterative Methods and Applications*, University of Nijmegen, The Netherlands, June 13, 1996.

- "Parallel Iterative Methods," *Householder Symposium*, Pontresina, Switzerland, June 20, 1996.
- "Parallel Alternating-Type Iterative Methods," *Conference on the Mathematics of Finite Elements and Applications (MAFELAP)*, Brunel University, Uxbridge, England, June 28, 1996.
- *DOE/OSC Applied Mathematics Workshop*, Albuquerque, NM, February 27–March 1, 1995 (poster).
- "Multiple Coarse Grid Multigrid Methods for Solving Elliptic Problems," *Colorado Conference on Multigrid Methods*, Copper Mountain, Colorado, April 1995.
- "On the Parallel Implementation of Alternating-Type Iterative Methods," Conference held at The University of Texas at Austin under the sponsorship of the Texas Institute of Computational and Applied Mathematics (TICAM), April 1995.
- "Orthogonal-Based Iterative Methods for Nonsymmetric Linear Systems," *1995 AMS-IMS-SIAM Summer Research Conference: Linear and Nonlinear Conjugate Gradient-Related Methods*, University of Washington, Seattle, WA, July 9–13, 1995.
- "LANGMRES: An Alternative to the Biconjugate Gradient Method for Solving Large Nonsymmetric Linear Systems," Conference held at North Carolina State University in honor of Cornelius Lanczos, December 1993.
- "MGMRES: A Generalization of GMRES for Solving Large Sparse Nonsymmetric Linear Systems," *Colorado Iterative Methods Conference*, Breckenridge, CO, April 1994.

- "Periodically Generated Iterative Methods for Solving Elliptic Problems,"
IMACS 13th World Congress, Georgia Tech, Atlanta, GA, July 1994.
- *DOE/OSC Applied Workshop*, Albuquerque, NM, February 3-5, 1993
(poster).
- "Lanczos-Type Methods for the Solution of Large Sparse Linear Systems,"
Householder Conference on Numerical Linear Algebra, Lake Arrowhead,
CA, June 17, 1993.

Kincaid Presentations

- *Algebraic Multilevel Iterative Methods and Applications*, Nijmegen, The
Netherlands, June 13-15, 1996.
- *Householder Symposium*, Pontresina, Switzerland, June 17-21, 1996 (poster).
- *First Workshop on Numerical Analysis and Applications* Rousse, Bulgaria,
June 24-26.
- *Approximation Theory Conference*, Texas A&M University, January 10,
1995 (banquet speaker).
- *DOE/OSC Applied Mathematics Workshop*, Albuquerque, NM, February
27-March 1, 1995 (poster).
- *Computer and Applied Mathematics Conference*, UT Austin, May 20-22,
1995 (session chair).
- *IMACS Iterative Methods Conference*, Blagoevgrad, Bulgaria, June 17-
20, 1995.

(member international organizing committee, chair session, invited talk).

- *AMS-IMS-SIAM Summer Research Conference: Linear and Nonlinear Conjugate Gradient-Related Methods*, University of Washington, Seattle, WA, July 9–13, 1995.
- *Colorado Iterative Methods Conference*, Breckenridge, CO, April 6, 1993.
- *Cray Research Review*, UT Austin, May 10, 1994.
- *Mathematics Colloquium*, Utah State University, June 14, 1994.
- *SIAM Linear Algebra Conference*, Snowbird, UT, June 18, 1994.
- *IMACS 13th World Congress*, Georgia Tech, Atlanta, GA, July 12, 1994.
- *Third International Colloquium on Numerical Analysis*, Plovdiv, Bulgaria, August 15, 1994.
- *IMACS International Symposium on Scientific Computing and Mathematics Modeling*, Bangalore, India, December 7–11, 1992 (chair session, invited talk).
- *DOE/OSC Applied Workshop*, Albuquerque, NM, February 3–5, 1993 (chair session, poster).
- *1993 International High-Performance Computing Conference & Exhibition*, Taiwan National Center for High-Performance Computing, Hsin-Chu, Taiwan, April 20–22, 1993 (3 hour invited lecture, two panel sessions).
- *Computer Sciences Dept*, National Chiao-Tung University, HsinChu, Taiwan, April 22, 1993.
- *Mathematics Department*, Tamg Chiang University, Tanshui, Taiwan, April 27, 1993.

- Mathematics Department, Fu Jen University, Taipei, Taiwan, April 28, 1993.
- Mathematics Department, National Chung Hsin University, Taichung, Taiwan, April 29, 1993.
- Department of Mathematics, National Central University, Chung-Li, Taiwan, May 4, 1993.
- Department of Mathematics, Fu Jen University, Taipei, Taiwan, May 5, 1993.
- Department of Mathematics, University of Tsukuba, Tsukuba Science City, Ibaraki, Japan, May 7, 1993.
- *Householder Symposium*, Lake Arrowhead, CA, June 17, 1993.
- *SIAM Conference on Linear Algebra in Signals, Systems, and Control*, Seattle, WA, August 18, 1993.

Workshops and Short-courses Organized and Taught

- Carey, G. F., "Finite Elements in Fluids and Heat Transfer," October 18-20, 1993, Thompson Conference Center, University of Texas, Austin, TX. (with D. Gartling, Sandia).
- Carey, G. F., "Workshop on Finite Element Modeling of Environmental Problems," March 4-5, 1994, Thompson Conference Center, University of Texas, Austin, TX.
- Carey, G. F., "Grid Generation, Adaptive Refinement and Redistribution," May 2-4, 1994, Thompson Conference Center, University of Texas, Austin, TX.

- Carey, G. F., D. M. Young, D. Kincaid, and W. Joubert, "Workshop on Iterative Methods and Software," as part of the "Colorado Conference on Iterative Methods," Breckenridge, Colorado, April 1994.

Dissertations Completed or in Progress

Ph.D.

- Barragy, E., "Parallel Finite Element Methods and Iterative Solution Techniques for Viscous Incompressible Flows." Fall 1993. (Supervisor: Carey)
- Stagg, A., "Scalable, Parallelized Navier-Stokes Computation for Parallel, Distributed Memory Architectures." Spring 1995. (Supervisor: Carey)
- Spatz, W., "High-Order Compact Finite Difference Schemes for Computational Mechanics." Fall 1995. (Supervisor: Carey)
- Lorber, A., "Parallel-Vector Computer Simulation of Navier-Stokes Problems Using a Novel Runge-Kutta Recursion." Summer 1996. (Supervisor: Carey)
- Davis, M., "Parallel Multilevel Solution of Iteratively Decoupled Transport Problems," in progress (Supervisor: Carey)
- Sheng-You Xiao, "Multigrid Methods With Applications in Reservoir Simulation," Spring 1994. (Supervisor: Young and Sepehrnoori)
- Jen-Yuan Chen, "Lanczos-Type Methods for Solving Large Nonsymmetric Systems," in progress. (Supervisor: Young)

Master's

- Sirman, M., "Parallel Partitioning of Unstructured Grids for Heat Transfer and Potential Problems." Spring 1996. (Supervisor: Carey)
- Berner, A., "Parallel Iterative Solutions of Nonlinear Diffusion Problems on Workstation Clusters." Spring 1996. (Supervisor: Carey)
- Asha Nallana, "Cray T3D Performance Study. " Fall 1995. (Supervisor: Young and Kincaid)
- Wan Chen, " Parallel Implementation of an Alternating-Type ADI Method." Summer 1995. (Supervisor: Young and Kincaid)
- Chih-Chuan Chen, "A Parallel SSOR Method for a Class of Periodic Elliptic Problems." Summer 1995. (Supervisor: Young and Kincaid)

ENCLOSURES

FINAL REPORT DOE GRANT # DE-FG03-93ER25183

Carey, G. F. and Y. Shen, "Simulation of Fluid Mixing using Least-Squares Finite Element and Particle Tracing," *Int'l J. of Num. Meth. for Heat and Fluid Flow*, Vol. 5, 549--573, 1995.

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Xiao, Shengyou [1994]. "Multigrid Methods With Applications in Reservoir Simulation," Ph.D. Dissertation, The University of Texas at Austin, Austin, Texas.

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Asha Nallana and David R. Kincaid. A C ray performance study. Report CNA--283, University of Texas at Austin, Center for Numerical Analysis, May 1996.

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