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THE INFRARED BEHAVIOUR OF α_S FROM
MANDELSTAM'S APPROXIMATION TO THE GLUON
DYSON-SCHWINGER EQUATION[†]

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Using Mandelstam's approximation to the gluon Dyson-Schwinger equation we calculate the gluon self-energy in a renormalisation group invariant fashion. We obtain a non-perturbative β function. The scaling behaviour near the ultraviolet stable fixed point is in good agreement with perturbative QCD. No further fixed point for positive values of the coupling is found, i.e., α_S increases without bound in the infrared. The non-perturbative result for an infrared singular quark interaction can be used to relate the string tension σ to the QCD scale Λ .

The infrared behaviour of the running coupling in strong interactions, α_S , is of great interest, since it may provide an understanding of confinement. Its study is an intrinsically non-perturbative problem. One suitable framework to address this problem is provided by the Dyson-Schwinger equations of QCD. Studies of this infinite tower of equations rely on specific truncation schemes. In the present paper we focus on an approximation to the gluon Dyson-Schwinger equation originally proposed by Mandelstam.¹ This yields a simplified equation for the inverse gluon propagator in Euclidean momentum space,

$$D^{-1\mu\nu}(k) = D_0^{-1\mu\nu}(k) + \frac{g_0^2 N_C}{32\pi^4 k^2} \int d^4q \Gamma_0^{\mu\rho\alpha}(k, q-k, -q) \times D^{\alpha\beta}(q) D_0^{\rho\sigma}(k-q) \Gamma_0^{\beta\sigma\nu}(q, k-q, -k) , \quad (1)$$

where D_0 and Γ_0 are the bare gluon propagator and the bare three-gluon vertex. The use of a second bare vertex in the gluon loop is combined with one of the gluon propagators being bare. This is to account for some of the dressing of the vertex as entailed by its Slavnov-Taylor identity.¹

Solutions to this non-linear integral equation for the gluon self-energy have been studied and approximated with various numerical and analytic methods² showing that it results in an infrared singular gluon propagator diverging as

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Figure 1: The strong coupling $\alpha_s(\mu^2)$ with its asymptotic limits and the non-perturbative β function of the Mandelstam approximation for a momentum subtraction scheme.³

$1/k^4$ for $k^2 \rightarrow 0$. It can be shown that in the Mandelstam approximation the product of the coupling and gluon propagator, $gD_{\mu\nu}(k)$, is a renormalisation group invariant, which is obtained as a selfconsistent solution to (1), and the infrared singular result allows to identify a physical scale, the string tension.³

The non-perturbative and renormalisation group invariant result relates the string tension σ to the QCD scale (the only parameter) by $\sigma = 2\Lambda^2$. Fixing σ to its phenomenological value (it may be extracted from heavy quarkonium spectra or the slope of Regge trajectories) gives a value for Λ of about 600 MeV. Alternatively, from $\alpha_s(m_Z^2) \simeq 0.108$ we find $\Lambda \simeq 640$ MeV. For a momentum subtraction scheme with $N_f = 0$ this is the correct order of magnitude.

The scaling behaviour of the solution near the ultraviolet fixed point is determined by the coefficients $\beta_0 = 14$ and $\gamma_A^0 = 7$ which are reasonably close to the perturbative values for $N_f = 0$, i.e., $\beta_0 = 11$ and $\gamma_A^0 = 13/2$. The running coupling, which is obtained for arbitrary scales from the renormalisation condition, resembles the two-loop perturbative form asymptotically (see fig. 1). It increases without bound in the infrared, no further fixed point exists.

References

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