Title: On the Rossi-α Measurements of $\beta_{\text{eff}}$ in Reflected Reactors

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On the Rossi-$\alpha$ Measurement of $\beta_{\text{eff}}$

in Reflected Reactors

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In unreflected reactors, the probability of detecting chain-related counts is given by the well known Rossi-$\alpha$ expression as originally derived by Feynman.\(^1\)\(^2\)

\[
R(t) = C \int dt + \frac{D}{2F} \cdot \frac{k^2 C}{\tau^2 |\alpha|^2} \cdot (1 - \beta)^2 \cdot e^{-|\alpha|^2 t} dt ,
\]

(1)

where

- $C =$ detector count rate,
- $D =$ adjoint-weighted neutron dispersion factor (i.e., $D = g \Gamma$),
- $F =$ total fission reaction rate,
- $k =$ effective multiplication factor,
- $\tau =$ adjoint-weighted neutron removal lifetime, and
- $|\alpha| =$ magnitude of the alpha-eigenvalue, defined by

\[
|\alpha| = \frac{1 - k (1 - \beta)}{\tau} ,
\]

(2)

where $\beta =$ the effective delayed neutron fraction.

The integral of the correlated part of Eq. (1) is given by

\[
S = \frac{D}{2F} \cdot \frac{k^2 C}{\tau^2 |\alpha|^2} \cdot (1 - \beta)^2 .
\]

(3)

Using the definition of alpha from Eq. (2) and using the definition of the magnitude of the reactiv-
ity of the system (in units of dollars),

\[ |\rho_s| = \frac{1-k}{\beta k}, \quad (4) \]

Eq. (3) can also be written as

\[ S = \frac{DC (1-\beta)^2}{2F} \cdot \frac{1}{\beta^2 (1+|\rho_s|)^2} \cdot \quad (5) \]

Solving for \( \beta \) in Eq. (5) yields the following expression.3

\[ \frac{1}{\beta} = 1 + (1+|\rho_s|) \sqrt{\frac{2F}{D}} \cdot \frac{S}{C}. \quad (6) \]

We shall now demonstrate that this expression for \( \beta \) is equally applicable for a reflected system in which two alphas are experimentally observed. We begin by assuming the Rossi-\( \alpha \) solution for a simple reflected system as derived by Kistner.4 That is,

\[ \mathcal{R} (t) \, dt = Cdt + A_1 e^{-|\alpha_1| t} \, dt + A_2 e^{-|\alpha_2| t} \, dt, \quad (7) \]

The integral of the correlated part is given by

\[ S = \frac{A_1}{|\alpha_1|} + \frac{A_2}{|\alpha_2|} = \frac{DC (1-\beta)^2}{2F} \cdot \frac{k_c^2}{\tau^2 \left( \lambda_c - \frac{\lambda_{cr} \lambda_{rc}}{\lambda_r} \right)^2}, \quad (8) \]

where

\[ \lambda_c = \frac{1-k_c (1-\beta)}{\tau_c}. \quad (9) \]
In Eqs. (8) through (12), $k_c$ is the number of neutrons produced in the core per neutron lost from the core; $k_{cr}$ is the fraction of core neutrons that leak from the core into the reflector; $k_{rc}$ is the fraction of reflector neutrons that leak back into the core; and $\tau_c$ and $\tau_r$ are the core and reflector lifetimes, respectively.\(^5\) Substituting Eqs. (9) through (12) into Eq. (8) leads to

\[
S = \frac{DC (1 - \beta)^2}{2F} \cdot \frac{k_c^2}{[1 - k_c (1 - \beta) - k_{cr}k_{rc}]^2} .
\]

As derived by Spriggs et al.,\(^6\) the effective multiplication factor, $k$, is related to $k_c$, $k_{cr}$, and $k_{rc}$ by

\[
k = \frac{k_c}{1 - k_{cr}k_{rc}} ,
\]

where the product $k_{cr}k_{rc}$ is the fraction of core neutrons that return to the core after having leaked into the reflector. When Eq. (14) is inserted into Eq. (13), we again obtain Eq. (5). Hence, Eq. (6) is just as valid for reflected systems as it is for unreflected systems.
References


3. (In Russian. Needs to be translated.)


5. (In Russian. Needs to be translated.)