FRACTION OF SPACE DEBRIS COLLISIONS THAT ARE CATASTROPHIC

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Analytic calculations estimate the fraction of catalog collisions that are catastrophic by a modification of the collision rates. Most catalog collisions are catastrophic. Impactors of 60 kg or larger participate in about half of the catastrophic collisions. Analytic estimates give accurate values for catastrophic collisions, which are complicated numerically.

Analytic calculations are used to estimate the fraction of catalog collisions that are catastrophic, i.e., that convert all of the target and impactor mass into fragments. The fraction of the fragments produced by impactors of a given size or larger is addressed by a modification of the collision rates for catastrophic collisions. For nominal conditions, most catalog collisions are catastrophic. Impactors of 60 kg or larger would participate in about half of the catastrophic collisions, as estimated by others. It appears that analytic estimates quite accurate values for catastrophic collisions, which are complicated for numerical integrations.

The rate of catastrophic collisions between debris objects of mass $x$ and impactors of mass $y$ can be written as

$$C = \frac{1}{W} \int_R^U dx \int_{f_x}^{x} dy \frac{n(x)n(y)}{V} \sigma V,$$

where $n(x)$ and $n(y)$ are their number densities, $W$ is the volume of LEO that contains objects, $V$ is their relative velocity, and $w = W/V$. The collision rate is restricted to catalog target particles by taking $R = 0.6$ kg and $U = 6$ tons, the smallest and largest masses in the catalog, as the limits of integration. For the impactors, the limits of integration of $y$ are taken to be $f_x$ and $x$, where $f_x = 0.001$ is the smallest impactor to target mass ratio that can produce complete fragmentation. The cross section $\sigma$ is roughly geometric.

Calculation. For a power law dependence of the cumulative number of objects on mass, with exponent $c$, the cumulative object density varies as $N(m) = N(R/m)^c$, where $N$ is the total number of objects with mass greater than $R$, so the differential density is

$$n(m) = NcR^c/m^{c+1},$$

The cross section is $\sigma = K(x^{s/2} + y^{s/2})^2$, but it is assumed that $x >> y$ throughout, so $\sigma = Kx^s$, where $s = 1$ to 1.1 and $K = 0.017$ m$^2$/kg are empirical constants. Then, Eq. (1) becomes

$$C = (K/w)(NcR^c)^2 \int_R^U dx 1/x^{c+1} x^s \int_{f_x}^{x} dy 1/y^{c+1}$$

$$= (K/w)c(NR^c)^2 \int_R^U dx x^{s-c-1}/(fx)^c$$

$$= (Kc/wf^c)(NR^c)^2U^{s-2c}/(s-2c).$$

The total rate of collisions between catalog objects is

$$R_{cat} = (K/w)MN = (K/w)N^2R^cU^{1-c}/(1-c),$$

so the overall fraction of collisions that are catastrophic is
$$C/R_{\text{cat}} = [(1-c)/(s-2c)](R/fU)^c,$$

which is about

$$[C/R_{\text{cat}}]_{\text{nominal}} = (1-1/4)/(1-1/2))(0.6\text{kg}/0.001\times6\text{ton})^{1/4} = 84\%,$$

so for nominal conditions, c and s, most collisions are catastrophic.

**Impactor mass.** A related question is the fraction of the fragments produced by impactors of a given size L or larger, which can be evaluated by modifying Eq. (1) to

$$C(L) = (1/W) \int_{\text{max}(R,L)} U dx \int_{\text{min}(L/f,U)} dy n(x) n(y) \sigma V.$$  

The modification of the limits of integration over x assures that the target is larger than the impactor, as assumed in formulating Eq. (1). The modification of the lower limit of the integration over y assures that the impactor is \( \geq L \). The modification of the upper limit of integration assures that the kernel of the integral is zero for impactors larger than the largest objects in the catalog. If L is restricted to \( L > 10R \), the limits reduce to

$$C(L) = (1/W) \int_L U dx \int_L U dy n(x) n(y) \sigma V$$

$$= (K/w) \int_L U dx \int_L U dy n(x) n(y) x^s$$

$$= (K/w) \int_L U dx \int_L U dy n(x) n(y) x^{s-1}$$

$$= (K/w)c(nRc)^2 \int_L U dx x^{s-c-1} 1/L^c$$

$$= [c/(s-c)w]K(nRc)^2 U^{s-c}L^{-c},$$

from which the ratio of catastrophic collisions by impactors of size greater than L to the total number of catastrophic collisions is

$$C(L)/C = [(s-2c)/(s-c)](u/L)^c.$$  

For \( L = R \), the ratio is 1; or \( L = 60 \text{ kg} = 100 R \), the ratio is about

$$[C(L)/C]_{\text{nominal}} = [(1-1/2)/(1-1/4)](10^{-3}6000\text{kg}/60\text{kg})^{1/4} = 40\%,$$

so that impactors of 60 kg or larger would participate in about 40% of the catastrophic collisions, as estimated by others. Figure 1 shows \( C(L)/C \) as a function of L for \( c = 0.20, 0.25, \) and 0.30. For small L the curves are within a factor of 0.5/0.7 = 0.7. As L approaches U, the curves diverge by a factor of 3. However, the nominal value of c = 1/4 is accurate throughout the catalog, so the middle curve is of primary interest. It falls from \( L = 0.6 \text{ at 10 kg} \) to 0.12 for \( L = U \). As \( C(L)/C \) is the fraction of catastrophic collisions involving impactors larger than L, its fall with L means that larger impactors play a decreasing role in catastrophic collisions.

The reason is that in Eq. (7), the second integral (over y) is the number of objects larger than L, which decreases strongly as L increases. The first integral (over x) is essentially the total mass of the distribution, which depends most strongly on U and varies little with L. The large objects provide the target area; the smaller catalog objects provide the flux that strikes them. Of the two effects, the decrease in the number of impactors as L increases is the stronger.

**Fragment production** in catastrophic collisions can be estimated by inserting an additional factor for the cumulative number of fragments produced per collision. The number of
fragments larger than mass \( z \) produced in a collision that fractures both the impactor and target masses is approximately

\[
C(z) = A[(x + y)/z]^B,
\]

where \( B = 2/3 \) is an empirical constant and \( A = 1/B - 1 \) by conservation of mass. The number of catalog fragments is \( C(R) = A[(x + y)/R]^B \); thus, the rate of production of catalog fragments in collisions between catalog objects and impactors larger than \( L \) is

\[
Q(L) = (1/W) \int_L^U dx \int_L^U dy n(x) n(y) \sigma V A[(x + y)/R]^B
\]

\[
= (AK/wR^B) \int_L^U dx n(x) x^{s+B} \int_L^U dy n(y),
\]

which amounts to multiplying the integral in Eq. (7) by \( A \) and changig the exponent \( s \) to \( s + B \), so

\[
Q(L) = A[c/(s+2B-c)]K(NRC)^2Us+BcL-c.
\]

The total catalog collision rate is

\[
Q(R) = [c/(s+B-2c)]A(U/R)^B-2c(US)V
\]

so their ratio is

\[
Q(L)/Q(R) = (s+B-2c)/(s+B-c) (U/L)^c,
\]

which is \([s+B-2c]/(s+B-c)][(s-C)/(s-2c)]C(L)/C. \) For nominal conditions, \( (s+B-2c)/(s+B-c) = 0.83 \), so the fraction of fragments is comparable to the fraction of collisions. Figure 2 shows \( Q(L)/Q(R) \) for \( B = 0.6, 0.7, \) and \( 0.8 \), although there is very little difference between them. The fraction of fragments produced in collisions by impactors under 10 kg is about 70%. At 100 kg it is 40%; at 1 ton it is 12%. Thus, a majority of the fragments are produced in collisions with the smaller catalog objects. The reason is that in Eq. (12), the second integral is still the number of objects larger than \( L \), which decreases strongly as \( L \) increases. The first integral is essentially the mass of the distribution with an additional weighting by a factor of \( x^B \), but that just weights the large masses somewhat more heavily; it does not change its strong scaling on \( U \) and weak scaling on \( L \). Again, the large objects provide the target area, and the smaller catalog objects provide the flux that strikes them, which is the more decisive.

**Summary and conclusions.** Analytic estimates are used to assess the fraction of catalog collisions that are catastrophic. The fraction of the fragments produced by impactors of a given size or larger is addressed by modifying the collision rates for all collisions and catastrophic collisions. For nominal conditions, most catalog collisions are catastrophic, and impactors of 60 kg or larger participate in about half of the catastrophic collisions, as estimated by others. Analytic calculations can also bound the fraction of fragments produced by impactors larger than a given size. Again about half of the fragments are produced in collisions in which the impactor is on the order of 100 kg. Analytic estimates can give quite accurate values for catastrophic collisions, which involve complicated boundary conditions for numerical integrations.
References


2. G. Cleghorn, Orbital Debris (Washington, National Research Council, 1995). Fig. 4-1


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Fig. 1. fraction of objects in catastrophic collisions versus impactor mass for various distribution exponents.