Enhanced Safeguards
Via Solution Monitoring

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ENHANCED SAFEGUARDS VIA SOLUTION MONITORING

by

Tom Burr and Larry Wangen

ABSTRACT

This report describes some of the enhancements that solution monitoring could make to international safeguards. The focus is on the quantifiable benefits of solution monitoring, but qualitatively, solution monitoring can be viewed as a form of surveillance. Quantitatively, solution monitoring can in some cases improve diversion detection probability. For example, we show that under certain assumptions, solution monitoring can be used to reduce the standard deviation of the annual material balance, $\sigma_{MB}$, from approximately 17 kg to approximately 4 kg. Such reduction in $\sigma_{MB}$ will not always be possible, as we discuss. However, in all cases, solution monitoring would provide assurance that our measurement error models are adequate so that we have confidence in our estimate of $\sigma_{MB}$. Some of the results in this report were generated using data that was simulated with prototype solution monitoring software that we are developing. An accompanying document describes that software.

1. Introduction, Summary, and Recommendations

1.1 Introduction

The purpose of this document is to describe quantitatively how solution monitoring can enhance safeguards. The motive for our effort has been the fact that the International Atomic Energy Agency (IAEA) will soon be safeguarding a large reprocessing plant in Japan at the Rokkasho Reprocessing Plant (RRP). One safeguards measure will be to apply traditional near-real-time accounting (NRTA) at RRP. Traditional NRTA means that material balances (MBs) are computed approximately once per month in hopes of meeting the IAEA goal for abrupt (over one month or less) and protracted (over one year) diversion of Pu.
However, it has been known for about 20 years that traditional NRTA applied to large-scale reprocessing plants will probably not meet the IAEA loss detection goal for protracted diversion of 8 kg of Pu over one year. For an explanation, see Appendix A.

As we show in Appendix A, because of the large throughput planned (8000 kg per year) at RRP, and because of the notoriously troublesome systematic errors, the annual $\sigma_{MB}$ based on traditional monthly MB accounting will be too large (by a factor of nearly 10) to meet the IAEA protracted loss detection goal. Our view is that if such a goal is met by a facility, then the IAEA can justify using reduced inspection efforts and reduced contaminant/surveillance (C/S) measures. But, because we do not think RRP will meet that goal, we anticipate that the IAEA will pursue one or both of the following options:

1) Implement various forms of enhanced containment/surveillance.
2) Implement methods for reducing $\sigma_{MB}$.

Solution monitoring plays a role in both options (1) and (2).

To show why, we first define solution monitoring as follows:

*essentially continuous monitoring of solution level, density, and temperature in all tanks in the process that contain, or could contain, safeguards-significant quantities of nuclear material.*

For safeguards purposes these measurements should be authenticated and independently verified. Instrumentation is expected to include dip tubes for measuring density and level, and thermocouples for measuring temperature. Figure 1 illustrates a standard dip tube arrangement with dip tubes for reference, density, and level. **In-tank Pu measurements would be very beneficial, as we will discuss, but in-line measurements are not essential.** Concerning the frequency of measurements, Ref. 3 suggests that the measurement frequency be the minimum of the time required to transfer 8 kg of Pu and the time required to allow at least 10 snapshots during a typical tank transfer. Applying that suggestion to RRP, we expect measurements **every 1–3 minutes.**

Concerning option (1) above, solution monitoring is a C/S method because tank events are monitored essentially continuously, so each tank is under constant surveillance. See Figs. 2 and 3, in which even an untrained eye can detect the shipments and receipts in each tank (for more detail, see Ref. 1). Therefore, we consider solution monitoring to be an attractive form of enhanced C/S.
Fig. 1. A tank with three dip tubes, each measuring pressure at a different height. If solution level falls below the density pressure dip tube then no density measurement is available. The separation between the density pressure dip tube and the level pressure dip tube is known.

Fig. 2. Fifteen days of level readings at five-minute intervals for nine tanks.
Option (2) is the main topic of this paper (especially Section 3). Concerning option (2), one way to reduce $\sigma_{MB}$ is to use both Tank 1 (input accountability tank) and the first downstream tank (Tank 2) to estimate the inputs (assuming that the cumulative shipper-receiver difference (SRD) between Tank 1 and Tank 2 was acceptably small). By using both tanks to estimate input, we reduce the uncertainty in the input measurement. More importantly, we can also reduce the effect of systematic errors by using solution monitoring. To understand how, consider Fig. 3, where volume changes are plotted versus time (data is reported every five minutes in Figs. 2 and 3). A measurement of the volume change in a given tank during a static mode (no transfer occurring) will involve the random error only because the systematic error will cancel. Therefore, detecting loss during a tank’s static period when no transfers occur is easier than detecting loss during a tank-to-tank transfer. However, with more effort, and less-but-still-noteworthy success, we can also detect loss during tank-to-tank transfers. We give more detail later (Section 3), including another way to reduce $\sigma_{MB}$, which will involve the use of bias corrections. To apply bias corrections, we require that some reasonable number of transfers (20, for example) are known to have no true loss and we then either (1) apply a formal bias correction to reduce the systematic error
variance (a type of poor-man’s calibration) or we (2) use the average SRD between the two tanks during the 20 no-loss transfers as the “target” SRD, around which all future shipper-receiver differences (SRDs) are assumed to randomly vary with a variance that depends only on the random error variances involved. In either method, we are reducing the effect of systematic errors so that \( \sigma_{MB} \) is reduced. The effectiveness of the technique will depend on the relative sizes of the random and systematic errors involved, as we will demonstrate. **We cannot claim that this technique will work in all cases.** For example, if there are large variations in true (legitimate) temporary losses like pipe or pump hold-up, or in permanent losses like evaporation, either of which might be hard to model (for the purpose of estimating measurement errors), then we could not accept the simple measurement error models given in Appendix B or Section 3, and we would not expect to reduce \( \sigma_{MB} \).

We also mention here that Ref. 4 is one key safeguards-related reference that relates to Section 3. Reference 4 has perhaps given the idea that more frequent material balance (MB) closures or smaller material balance areas (such as individual tanks) cannot improve protracted diversion (over one year) detection above what can be achieved by an annual MB around the entire facility. In one sense, that is the case, but only if bias corrections are not applied. **That is, Ref. 4 does not consider the possibility of reducing \( \sigma_{MB} \) by applying bias corrections.** We do so in Section 3.2, but we impose two conditions: (1) we require access to several tank-to-tank transfers that have no true loss and (2) we require that nuisance effects due to fluctuating pipe hold-up be negligible.

Further, our Section 3.2 result is for a simplified three-tank system in which all tanks operate in batch-ship and batch-receipt modes, with Tank 1 shipping to Tank 2, which ships to Tank 3. The simplified system is adequate for present purposes, but at RRP we must consider the effect of having the purification and separation cycles separating some pairs of main process tanks. Main process tanks that are separated by the purification cycles can have nonnegligible fluctuations in the hold-up (“hold-up” means \( \text{Pu} \) in the purification cycles) between the two tanks, and can operate in either continuous ship or continuous receipt mode. It is more difficult to estimate the amount of material received by a tank that is shipping continuously. These are potential problems that could limit the reduction in \( \sigma_{MB} \), but we are still optimistic about the possible reduction in \( \sigma_{MB} \).

Because of the large benefits of solution monitoring and because of the quite small inspection effort required to implement solution monitoring, in our view the only plausible facility objection to implementing solution monitoring for safeguards would be disagreement over what constitutes
proprietary information. To our knowledge, the only proprietary information involves specific operational details of the pulsed columns. We are not expecting the IAEA to have access to measurements in any tanks within the pulsed column operation so there should be no problem concerning proprietary information.

1.2 Summary

The main motive for investigating the benefits of solution monitoring to safeguards at large reprocessing plants is that traditional NRTA alone (monthly MBs) cannot meet IAEA loss-detection goals for protracted diversion. And, traditional NRTA does not allow for the possibility of monitoring tanks for loss during nonevent modes or of applying bias corrections to effectively monitor tank-to-tank transfers for loss during transfer modes, whereas solution monitoring, as we define it, does allow for those possibilities.

The remainder of this paper is organized as follows: in Section 2 we list some qualitative benefits of solution monitoring which complement the benefits given in Ref. 3. In Section 3 we describe the quantitative benefits of solution monitoring. Section 4 is a summary. Appendix A compares traditional NRTA to the very-near-real-time accounting that is possible with solution monitoring and Appendix B gives a bias-correction example.

1.3 Recommendations

We strongly recommend that solution monitoring techniques and the associated information management need be developed. It is easy to quantify some of the benefits and the qualitative benefits alone justify the effort. We are not in position to recommend how the logistics be handled such as whether a commercial expert system or some native language like Visual C++ be used to implement solution monitoring. We have been developing a prototype system in an object-oriented, graphical and statistical programming language (Ref. 1), which we have found to be useful to rapidly experiment with candidate data analysis methods.

2. Qualitative Benefits of Solution Monitoring

The proposed purposes of solution monitoring from a safeguards perspective include the following:

- Provide data for verifying that each tank or vessel is operating in a manner consistent with operator declarations.
• Provide assurance that no unauthorized alterations have been made to tanks (related to design verification).
• Enable consistency checks to be made on solution transfers. These can be made automatically in real time if desired.
• Maintain a running inventory of nuclear material for each vessel. This requires a means for estimating nuclear material concentrations.

In addition to these, other uses of solution monitoring measurements have also been suggested.
• Monitor the quality of measurements.
• Identify all normal events affecting the solution occurring in a tank, such as, mixing or sparging.
• Estimate and partially remove bias in tank calibrations, thereby reducing $\sigma_{MB}$ (the theoretical standard deviation of the MB which includes all sources of measurement error), and increasing loss detection probability.
• Estimate hold-up in pipes between tanks.
• Identify abnormal events.
• Partially validate measurement error models. Many safeguarded facilities do not document their measurement control information well enough to fully support their measurement error models. In such cases, when a large MB occurs, the first thought is often that $\sigma_{MB}$ is understated. We will show in Section 3 that solution monitoring can provide considerable assurance that measurement error models are acceptable, and therefore, provided the variance propagation is done correctly, the estimate of $\sigma_{MB}$ should be quite good.

3. Quantifiable Benefits of Solution Monitoring

We assume that all tank transfers are identified and checked (in software) for compliance with either historical behavior or expected behavior. The quantitative benefits of solution monitoring can be placed into three categories. Solution monitoring

1) provides partial measurement-error model validation; SRDs in terms of volume or mass between any pair of communicating tanks should exhibit a variance that is within expectations based on the random errors involved, and should exhibit a mean that is within expectations based on the systematic errors involved.

2) provides an internal consistency check that operator declarations match plant operations. For example, if the flow sheets or operator declarations suggest that 10 transfers from
Quantifiable Benefits of Solution Monitoring

Tank 1 to Tank 2 should have occurred during a particular 10 days, we can confirm the 10 transfers with our software.

3) can improve loss detection. It should be obvious that abrupt loss detection is dramatically improved by solution monitoring because less material is involved in each tank-to-tank transfer than in, for example, monthly throughput. Solution monitoring can also improve protracted loss detection by reducing the effect of systematic errors on the overall uncertainty; this is the most technically challenging benefit to quantify, and we will give three example calculations to illustrate the benefit. Unless we use in-line concentration measurements and use better measurement methods than currently planned for the intermediate main process tanks (tanks between the input and output accountability tanks in the flow stream) then we still cannot formally met the IAEA detection goal for diversion of 8 kg over one year. The protracted loss detection is improved, however, in a sense to be described, and it is obvious that the abrupt loss detection is dramatically improved.

We think that quantitative benefits in categories 1 and 2 are obvious, so we now provide more detail about the quantitative benefits in category 3.

3.1 Improved Loss-Detection Probability

We expect about 15 main process tanks at RRP. Figures 4 and 5 represent simulated results for level measurement data at RRP. These figures are intended to be somewhat realistic, but they do not represent the real situation. For example, the level units remain unspecified, but we do expect that solution monitoring data will include tank level, density, and temperature (L, D, T) at approximately 1–5 minute intervals. The L, D, T snapshots are every five minutes in Figs. 4 and 5.

About half of the tanks operate in batch-ship and batch-receipt mode (B/B) and about half of the tanks operate in either batch ship and continuous receipt (B/C) or vice versa (C/B) modes. The tanks that feed the pulsed columns ship continuously and the tanks that receive from the pulsed columns receive continuously. The same is true for the tanks that feed or receive from the evaporator. One technical issue will involve the measurement of shipments or receipts by tanks in B/C or C/B mode. Here we will simplify the discussion and consider only a three-tank problem having all B/B tanks, such as the first three tanks in Fig. 4.
Fig. 4. Level versus time (every 5 minutes) for first 9 tanks at RRP.

Fig. 5. Level versus time (every 5 minutes) for tanks 10–15 at RRP.
Tank 1 ships to Tank 2 which ships to Tank 3. In this section, we assume that software such as that described in Ref. 1 has been used to locate all shipments and receipts by the three tanks during one 200-day "year." We can then divide all tank snapshots (L, D, T readings) into three categories for a given tank: wait mode, ship mode, or receipt mode. Having done that, in this section, we consider four solution monitoring activities:

1) Monitor each of the three tanks for volume loss during all wait modes. "Wait mode" means that no abrupt level changes occurred, though there could have been a slow evaporation or other event that slowly affected the level.

2) Monitor each of the three tanks for mass loss during all wait modes.

3) Monitor each of the two SRD pairs for volume loss during all transfer modes.

4) Monitor each of the two SRD pairs for mass loss during all transfer modes.

First, remember that unless we have an in-line Pu concentration measurement, we cannot completely monitor these three tanks. However, because we do have a density measurement, we can monitor the three tanks for loss of mass as we will describe. That means that any statistically detectable diversion would have to be concealed by replacing the lost solution with proper density solution. What we accomplish by this monitoring is that the adversary must work hard to conceal the diversion. The same type of calculations would apply if we used an in-line Pu concentration measurement, and in that case there would be no way for the adversary to conceal a statistically detectable diversion of plutonium.

The advantages of solution monitoring in any of the above four activities includes the following:

- dramatically improved abrupt loss detection, and
- improved protracted loss detection by allowing improved treatment of systematic errors.

To compare our results to traditional NRTA, consider an alternative safeguards strategy that only computed an annual MB as $MB = T_{in} - T_{out}$, which simply compares the annual output $T_{out}$ to the annual input $T_{in}$. We show in Appendix A that if both the Tank-1 and Tank-3 measurements were as good as can be expected, then for an 8280-kg throughput facility, we expect $\sigma_{MB} \approx 17$ kg. (If Tank 3 made 40 shipments per 200-day year, then we would get the same result as Eq. (A-2) in Appendix A). To meet the IAEA protracted diversion detection goal, we require $\sigma_{MB} \approx 2.42$ kg.

We now compare this result ($\sigma_{MB} = 17$ kg) to what we can do with solution monitoring. Monitoring wait mode is easier than monitoring transfer mode, which means that the optimal diversion time is during transfers. During a wait mode, we need only check for a significant change in mass.
If some legitimate but long-term event like evaporation is occurring, then the volume will decrease but the density will increase so that the mass will remain approximately constant. In fact, with evaporation the mass will decrease slightly because some evaporate has been removed, and this source of variability in the estimated mass loss would have to be investigated.

For simplicity here we assume
- input batches to Tank 1 take 1 hour and occur once every 32 hours (20,000 L at 2.76 g/L, approximately 150 batch shipments per 200-day “year”);
- the same is true for batch shipments to Tank 2;
- output batches from Tank 3 take 1/3 hour and occur once approximately every 6.4 hours (4000 L at 2.76 g/L, approximately 750 batch shipments per 200-day year); and
- SAME measurement errors as given in Appendix A—both types of systematic error (systematic error for volume-to-level relation and for Pu assay) have standard deviations of 0.1% as do the three types of random error (random error for volume measurement, for Pu assay, and for sampling error associated with $M = VC$ calculation).

In monitoring wait modes, to check for a significant volume change, we can ignore systematic errors because we are simply doing change detection, which means that systematic errors will nearly cancel (we assume here that they exactly cancel). Also, monitoring volumes is easier than monitoring mass or Pu concentration because the effect of sampling errors is much smaller with volume measurements.

**Activity 1: volume loss during wait modes**

Now we calculate the volume needed to divert a quantity $SQ$.

$$V_{needed} = \frac{(SQ)}{(conc)} = 8000 \text{ g} / 2.76 \text{ g/L} = 2898 \text{ L} \text{ if } SQ = 8 \text{ kg}.$$ 

For each wait mode section, we compute $\Delta V = V_{final} - V_{initial}$, which has a random error variance, $\sigma^2_{RV}$, equal to $V^2_{initial} \times 2 \times 0.001^2$ (which is the total error variance for a given wait mode). We assume that Tank 1 transfers essentially all of its volume during each shipment to Tank 2. Therefore, we need only monitor for loss during the 150 wait modes at the full (20,000 L) status. If we check for volume loss during the 150 wait modes at full status, we will have a measurement error standard deviation, $\sigma_{\Delta V}$, equal to approximately 347 L. Because $2898 / 347 = 8.4$, a diversion of 2898 L over one year would constitute an “8.4 sigma” event, which is easily detectable. Even if we operated this test for all three tanks, we could simply adjust
the per-tank false alarm rate to be 0.05/3 (using the conservative Bonferroni correction for multiple testing) and thereby maintain a less than 0.05 false alarm probability over all three tanks while easily achieving the 0.95 detection goal. The same statement holds even for 15 tanks, but if some of the tanks operate in B/C or C/B mode then the measurement error for change detection will be larger, but we believe it will still be small enough to easily detect a diversion of sufficient volume to accumulate 8 kg of Pu.

A general result is available for this type of calculation. Assume that at steady state all tanks process the same amount of Pu per hour (approximately 1725 g/hour), let the random measurement error standard deviation for volume be $\sigma_{RV}$, and let the number of cycles per year be $n$. Then the ratio of the volume needed to accumulate 8 kg, $V_{\text{needed}}$, to the standard deviation of the cumulative measurement error for the volume change, $\sigma_{\Delta V, \text{cumulative}}$ over one year is

$$V_{\text{needed}}/\sigma_{\Delta V, \text{cumulative}} = (8 \times \sqrt{n})/(\sigma_{RV} \times 1.725 \times \sqrt{2} \times 200 \times 24).$$

Equation (1) assumes a 200-day “year”. The $\sqrt{2}$ arises because in subtracting one measured volume from another, we double the random error variance; the 24 is the hour/day conversion. Note that increasing the number of cycles per year, $n$, increases the “signal” due to a volume loss, because smaller volumes are implied if $n$ increases and error models for the volume are assumed to be proportional to the true volume. For tanks 1, 2, and 3, Eq. (1) gives: 8.4, 8.4, and 18.7, respectively, so we could easily detect a removal of $V_{\text{needed}}$ from any of the 3 tanks. Equation (1) is useful to determine the required $\sigma_{RV}$ for a given $n$.

Also, Eq. (1) is useful in monitoring for lost volume over one year during the wait modes. A diverter would have to replace the volume or it would be detected.

Activity 2: mass loss during wait modes
We give an equation similar to Eq. (1), but replace $\sigma_{RV}$ by $\sigma_{RM}$, where $\sigma_{RM}$ is the random measurement error standard deviation for mass. We then have

$$M_{\text{needed}}/\sigma_{\Delta M, \text{cumulative}} = (8 \times \sqrt{n})/(8280 \times \sigma_{RM} \times \sqrt{2}).$$

In Eq. (2), the factor 8280 is the kilograms of Pu per year through the facility. The random measurement error relative standard deviation for the mass, $\sigma_{RM}$, is larger than that for volume, and so far we have not discussed error models for the in-line density measurement. We assume here that the random error for the density measurement is 0.2%, which gives us
\[ \sigma_{RM} = \sqrt{(\sigma^2_{RV} + \sigma^2_{RD})} = 0.0022. \] For tanks 1, 2, and 3, Eq. (2) gives 3.8, 3.8, and 8.5, respectively. Not surprisingly, we cannot detect mass loss as well as we can detect volume loss, but we still do far better than the annual MB test described in Appendix A. Also, currently we have less information about error models for density than we do for other error models, so our assumption of 0.2% total relative standard deviation for density should be considered with that in mind. Finally, the ability to detect a mass loss of 8 kg of solution implies a much greater ability to detect a mass loss of 8 kg of Pu, at least in Tanks 1–3 where the solution has low Pu concentration.

To summarize the wait mode activities, there is reason to be very optimistic about the ability of solution monitoring to detect a volume or mass loss that corresponds to 8 kg of Pu. Because the in-line density measurement is not specific for Pu, we simply worked with mass of the solution, which gives us a very conservative result.

The “transfer mode” activities raise the issue of bias corrections. A major advantage in the “wait mode” analysis was the cancellation of systematic errors. There is no such cancellation during the “transfer mode” analysis, but there is the possibility of reducing overall measurement error variance by doing bias corrections.

Historically, the subject of bias corrections for safeguards has been controversial (for good reasons). It is rarely clear that a bias-corrected result is better than an non-bias-corrected result, if for no other reason than there is uncertainty in the bias correction itself, so the performance of a bias-corrected result can actually be worse than that of the non-bias-corrected result. We give a specific example in Appendix B.

Note that if we perform only monthly or annual material balances around the entire tank system, there is no opportunity to attempt bias corrections because there is no communication between the input and output accountability tanks (Tanks 1 and 3 in our present simplified example).

**Activity 3: volume loss during transfer modes**

Here is a typical error model for the difference between the shipped and received volume from Tank 1 to Tank 2 during one shipment.

\[ V_1 - V_2 = V_{1T} \times ((\varepsilon_{V_{S1}} - \varepsilon_{V_{S2}}) + (\varepsilon_{V_{R1}} - \varepsilon_{V_{R2}})), \] (3)

where \( V_{1T} = V_{2T} \) is the true volume shipped by Tank 1 and received by Tank 2, and \( \varepsilon_{V_{S1}} \) is the systematic volume error for Tank 1 and similarly for the other terms. Note that we use a simplified...
model that assumes the absolute errors are proportional to the true volume shipped rather than a more correct model that considers the final and initial volumes of both the shipper and receiver separately. Because we expect a consistent operating mode in which the final and initial levels of the shipper (or receiver) tank tend to be approximately the same for each batch shipment, the more correct error model is not needed here. See Appendix B for further details about Eq. (3).

From Eq. (3) we see that each volume SRD between Tank 1 and Tank 2 should exhibit a variance that is determined by the random error variance of the volume measurements in each tank, and should exhibit a mean that depends on systematic volume measurement errors for each tank (assumed to be constant between calibrations). The bad news is that until we have some example SRDs from Tank 1 to Tank 2, we cannot estimate the difference between the Tank-1 and Tank-2 systematic volume measurement errors so we cannot improve the loss detection ability based on an annual MB as is described in Appendix A. We will show why momentarily, but first we mention that the “good news” is that if we can assume we have several no-loss transfers from Tank 1 to Tank 2 from which to estimate the difference between the Tank-1 and Tank-2 systematic volume measurement errors, then we can again mitigate the effect of systematic errors on the total measurement-error variance, and thereby improve the loss detection ability based on an annual MB.

In the following two cases we assume that Eq. (3) describes the measurement error involved in estimating the volume loss (or gain) in each shipment from Tank 1 to Tank 2. We also assume that any true volume change due to pump heating or pump and pipe hold-up fluctuations are negligible.

Case 1: no bias correction

Assume that we monitor the Tank-1-to-Tank-2 shipments for cumulative (over one year) volume loss, and do not apply any bias correction method. It is simple to show that the cumulative (annual) measurement error standard deviation is

$$\sigma_{\Delta V, \text{cumulative}} = V_{Total} \times \sqrt{(\sigma_S^2 + \sigma_R^2 / n_{\text{shipments}})}.$$  (4)

where $V_{Total}$ is the total true volume shipped during the year, $\sigma_S^2$ is the total systematic error variance (sum of volume measurement systematic error variances for both Tank 1 and Tank 2), and similarly for $\sigma_R^2$. Eq. (4) is the same concept as is given by Eq. (A-1) in Appendix A, but Eq. (A-1) deals with mass rather than volume. For Tank-1-to-Tank-2 transfers (150 transfers of 20,000 L per year), Eq. (4) gives $\sigma_{\Delta V, \text{cumulative}} = 4256.8$ L. From the previous wait mode discussions,
we know that the volume needed to accumulate one SQ is \( V_{\text{needed}} = \frac{\text{SQ}}{\text{(conc)}} = \frac{8000}{2.76} = 2898 \). Therefore, the needed volume is only about \( 0.68 \times \sigma_{\Delta V, \text{cumulative}} \). We cannot detect removal of one SQ with an acceptable probability without bias correction.

**Case 2: bias correction**

Now assume that we monitor the Tank-1-to-Tank-2 shipments for cumulative (over one year) volume loss, and that we apply a bias correction to the volume measurements given in Appendix B. The “catch” is that we must have at least one shipment from Tank 1 to Tank 2 in which we know there was no true loss or change in volume. Here we assume the first 24 shipments (one shipment per 32 hours) from Tank 1 to Tank 2 are known to have no true volume change. Then Eq. (4) still applies, but \( \sigma_x^2 \) reduces from \( 2 \times 0.001^2 = 0.000002 \) in Case 1 to \( 2 \times 0.001^2/24 = 6.67 \times 10^{-8} \). The new Eq. (4) result is \( \sigma_{\Delta V, \text{cumulative}} = 848.5 \) so that the needed volume is now approximately \( 3.4 \times \sigma_{\Delta V, \text{cumulative}} \), and we can detect this volume removal with greater than 0.95 probability while maintaining a 0.05 false alarm probability. (See Appendix B for details on these calculations.)

To summarize, we can reduce the annual measurement error standard deviation if we apply bias corrections, but the “catch” is that we must have several (we used 24 for example) Tank-1-to-Tank-2 shipments that are known to have zero true volume loss.

This kind of bias reduction is only possible between pairs of communicating tanks such as Tank 1 and Tank 2. Another type of bias reduction is possible even with annual MBs that compared the cumulative output from Tank 3 to the cumulative input to Tank 1. However, we would have to wait too long (several years) to accumulate enough historical MBs to do the bias corrections, and we would have to insist that there be no tank volume-to-level recalibrations during the years. This is not realistic, but does illustrate the concept involved in this kind of bias corrections. Intuitively, we realize that the observed variability in volume changes in the Tank-1-to-Tank-2 transfers is due to random errors (by definition of systematic error—they do not change during the year unless a tank is recalibrated), but the mean volume change is determined by the systematic errors that are in effect. As soon as we accumulate enough volume-change data we can estimate the mean volume change well enough for bias corrections to be worthwhile.

**Activity 4: mass loss during transfer modes**

The mass loss case is the same as the volume loss but the measurement errors must include the density measurement error.
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Case 1: no bias correction
Assume that we monitor the Tank-1-to-Tank-2 shipments for cumulative (over one year) mass loss, and do not apply any bias correction method. It is simple to show that the cumulative (annual) measurement error standard deviation is

\[
\sigma_{\Delta M, \text{cumulative}} = M_{\text{Total}} \times \sqrt{\sigma_s^2 + \sigma_R^2 / n_{\text{shipments}}}.
\]  

\(M_{\text{Total}}\) is the total true mass shipped during the year, \(\sigma_s^2\) is the total systematic error variance (sum of the mass measurement systematic error variances for both Tank 1 and Tank 2), and similarly for \(\sigma_R^2\). We must include the systematic error standard deviation of the density measurement error, which we will assume to be 0.2% (same as its random error standard deviation). Equation (4) is the same concept as is given by Eq. (A-1) in Appendix A. For the Tank-1-to-Tank-2 transfers (150 transfers of 55.2 kg per year), Eq. (5) gives \(\sigma_{\Delta M, \text{cumulative}} = 26.3\) kg. Again, if we are trying to detect a loss of 8 kg then we will not have a sufficiently large detection probability because \(8/26.3 = 0.3\).

Case 2: bias correction
Now assume that we monitor the Tank-1-to-Tank-2 shipments for cumulative (over one year) volume loss, and do apply the bias correction method given in Appendix B. Using the same assumptions as in the volume loss case under activity 3, we find \(\sigma_{\Delta M, \text{cumulative}} = 5.8\) kg. This is a considerable improvement over the no-bias-correction case, but is not quite as small as we need to detect an 8 kg loss of solution with a 0.95 probability (for that, we need \(\sigma_{\Delta M, \text{cumulative}} = 8/3.3 = 2.2\)). However, recall that it will take considerably more than 8 kg of solution to acquire 8 kg of Pu in the “upstream” tanks where the Pu concentration is low.

3.2 Solution Monitoring Can Reduce \(\sigma_{MB,\text{annual}}\)
The problem with assessing loss detection probability with MBs at any time interval is that the detection probability depends on the loss scenario. Abrupt losses are easy to detect only if MBs are computed very frequently, such as after each tank-to-tank transfer as we recommend in solution monitoring. Protracted losses are always harder to detect, which is why we considered protracted loss in Section 3.1 and in Appendix A. See Appendix A for more detail about \(\sigma_{MB}\) based on an annual MB. Though it leads to overly pessimistic conclusions, it is very convenient to simply compute the \(\sigma_{MB,\text{annual}}\) as a quick check for whether the facility could meet the worst case
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(protracted loss) scenario. For the main chemical processing area of RRP, in Appendix A we calculate $\sigma_{MB,annual} = 16.9 \text{ kg}$, using some arbitrary but reasonable measurement-error assumptions. Now what impact will solution monitoring have on the 16.9-kg estimate?

For simplicity assume that we have only a two-tank system (an input accountability tank and an output accountability tank). Because we have a two-tank system, we can compare each measured input to the corresponding measured output. In effect, we are in position to accumulate long sequences of facility MBs and thereby exploit bias corrections as described in Section 3.1 to reduce $\sigma_{MB}$.

First, note that in our high-level summary MB, we will measure inputs to Tank 1 rather than outputs from Tank 1. We then compare inputs to Tank 1 to outputs by Tank 2. In doing so, we cover both the “wait mode” loss detection and the “transfer mode” loss detection that was separated in Section 3.1. If we detect a potential loss, we would follow up with separate “wait mode” and “transfer mode” calculations to check whether the loss appeared to be during a wait mode or during a transfer mode.

Assume the following:

1. One hundred and fifty shipments during the year from Tank 1 to Tank 2.
2. Measurement errors are 0.1% relative.
3. We have an in-line concentration measurement of Pu.*
4. As with the bias correction results given in 3.1, we require some small number of transfers from Tank 1 to Tank 2 that are known to have zero true loss. Here, we assume we have 24 transfers that are known to have zero true loss.

Using assumptions 1-4 and the same error models as in Appendix A, we obtain a $\sigma_{MB,annual} = 4.2$, which is a substantial reduction from the 16.9 result. This improvement is achieved by reducing the systematic error variance. This reduction relies heavily on assumption 4. How can we ever know that, for example, the first 24 transfers have no true loss? We concede that it will be necessary to closely monitor those transfers in some reasonable way, which requires increased inspection time early in the campaign. We think that is reasonable to request.

*Note: Assumption 3 is for convenience so we can have a convenient comparison to the $\sigma_{MB,annual} = 16.9$ result in Appendix A. As mentioned, an in-line Pu concentration measurement would be nice but it is not essential. However, only with an in-line Pu concentration measurement could we have a chance to formally achieve the goal loss detection probability for diversion of 8 kg of Pu over one year. Recall that with L, D, T measurements we can build a strong case for solution monitoring because a diversion of Pu would have to be accompanied with replacement of proper density solution. Thus, solution monitoring has “added another hurdle” for a potential diverter to contend with.
Summary

The purpose of Section 3.2 has been to illustrate with some example numbers the impact that bias corrections can have on $\sigma_{MB,\text{annual}}$. If the random error variance is large compared to the systematic error variance (not usually the situation, fortunately), then we would not achieve such a reduction in $\sigma_{MB,\text{annual}}$ by doing bias corrections. However, it is typical for systematic errors to be an important contribution to the overall $\sigma_{MB,\text{annual}}$ at a reprocessing plant, so there is reason to expect bias corrections to be effective. By using this simple two-tank system, we highlight three things that solution monitoring can achieve.

1) We can possibly reduce $\sigma_{MB,\text{annual}}$ as shown.
2) We can compare the “first principles,” variance propagation based estimate of $\sigma_{MB,\text{annual}}$ to a purely empirical estimate. This is possible because we exploit the direct communication between Tank 1 and Tank 2. That is, we have both the theoretically anticipated $\sigma_{MB}$ for each Tank-1-to-Tank-2 transfer based on measurement-error models, and we also have the first 24 observed MBs from which to empirically estimate $\sigma_{MB}$. This is a specific instance of what we meant in our solution monitoring benefit number (10) given in Section 2: we can build confidence in our measurement error models this way.
3) We can apply the bias correction based on the first 24 MBs to all remaining MBs during the year, or nearly equivalently, compare each remaining MB to the average MB during the first 24 transfers and use the observed standard deviation of the first 24 MBs to set decision thresholds. That is, we could put our faith in the $\sigma_{MB,\text{annual}} = 4.2$ or we could use the observed standard deviation during the first 24 transfers. This option is simply not possible using traditional monthly MBs, which ignore the direct connections between communicating tanks.

4. Summary

We considered simplified three-tank and two-tank systems in which all tanks operated in batch ship/batch receipt (B/B) mode. Because our main detection probability concern is the protracted loss of 8 kg over one year, we compared traditional monthly NRTA and the associated annual MB test to a reduced-systematic error method that relied on having solution monitoring data. In doing so, we considered two main tank modes: wait and transfer. The wait mode is easier to think about and to handle, but in either the wait or transfer mode, solution monitoring data can be used in such a way that systematic errors at least partially cancel, to reduce the total measurement error standard deviation. In the transfer mode, a typical bias correction is made to achieve partial systematic error cancellation, but the method requires: (1) strong faith in our error models, and (2) having at
least one tank-to-tank transfer for which it is known that no volume or mass was changed during the transfer. By achieving the partial cancellation in systematic errors, the measurement error standard deviation can be reduced significantly, at least in our three-tank example where all tanks operate in B/B mode. It is premature to predict whether the reduction in measurement error standard deviation can be sufficiently reduced to meet the detection goal for 8 kg of Pu over one year. The total variance reduction achieved will depend on the relative sizes of the random and systematic errors and on the number of “known to have zero volume or mass change” transfers that we have available. This partial cancellation in systematic errors is achieved via a “poorman’s calibration,” in which we gather enough SRD data between each pair of communicating tanks that we can do an empirical recalibration.

Also recall that the effective false alarm rate must consider the total number of tanks in the system and systems with more tanks will require smaller false alarm rate per tank. And, we have not yet treated B/C or C/B tanks. We believe that those tanks can be treated in the same manner as B/B tanks, but with larger measurement errors in both the wait mode and the transfer mode. However, there is no direct communication between the main process tanks that ship to and receive from any set of pulsed columns. That means we cannot do bias corrections between, say, Tanks 4 and 5 in Fig. 4 because Tank 4 never ships directly to Tank 5. Methods to deal with tanks such as Tanks 4 and 5 in Fig. 4 are currently under investigation.

Also, unless we have an in-line concentration measurement for Pu, then we cannot formally achieve the loss detection ability we hope for, because a diverter could replace solution with proper-density solution and remain undetected. If we do have an in-line Pu concentration measurement then we can perform a Pu Mass balance example, as in Section 3.2, and the results will be very encouraging. Even without an in-line Pu concentration measurement there is considerable advantage in using solution monitoring, as we have described.

Concerning other quantitative benefits, we also recognize the utility of using solution-monitoring data to partially validate measurement-error models and of maintaining an ability to compare operator declarations to solution-monitoring software results for the number of shipments, sampling events, etc.

We have given an initial list of some qualitative benefits and take the opportunity to add one more here: traditional decision theory that specifies a target-detection probability for a given false alarm rate is well established. However, in practice, if we declare that we think material has been lost
because a decision threshold has been exceeded, then the next step is **anomaly resolution**. Solution monitoring data will be the first thing to request in cases where apparent anomalies must be resolved. We therefore strongly believe that there is a practical utility to the IAEA having access to solution-monitoring data, and we believe that during routine use, the demand on the inspector’s time can be acceptably low provided the software is well conceived.

### 5. References


Appendix A

Simplified Example Calculation to Demonstrate that a Reprocessing Facility with an 8280-kg Pu/year Throughput Cannot Meet the IAEA Detection Goal for Protracted Diversion

Consider the chemical separations area material balance area, with inputs measured in the input accountability tank and outputs measured in the output accountability tank. In a sense that was quantified in Ref. 4, the most powerful test for loss over one year is a test that ignores the monthly MBs, and simply compares the annual output $T_{out}$ to the annual input $T_{in}$. (This assumes that there is an annual cleanout of the entire plant and that the beginning and ending inventories are zero.) As an aside, there is no such thing as a “best test” because the best test depends on both the particular loss scenario (abrupt, protracted, or something in between) and on the 12-by-12 covariance matrix, $\Sigma$, which holds the variances of the monthly MBs on the diagonal and the covariances between each MB on the off-diagonals. However, the Ref. 4 definition of “best” is one reasonable definition and serves as a quick check for whether the one-year protracted diversion goal can be met. The quick check is if the annual $MB = T_{in} - T_{out}$ test cannot detect an 8 kg loss with probability at least 0.95 for a 0.05 false alarm probability, then no other test will be able to either. To apply the quick check here, assume that the measurement control checks were all satisfactory during the year, so that no recalibration of either the volume-to-level relation or the analytical procedures for the Pu assay are performed. Also assume that all measurement-error models are multiplicative (measurement-error standard deviation is proportional to the true value). Then a simple formula often used by the IAEA is applicable

$$\sigma^2_{inputs}= Inputs^2 \times (\sigma^2_S + \sigma^2_R/n_{inputs}) \quad (A-1)$$

where $Inputs^2\sigma^2_S$ is the total systematic error variance (sum of systematic error variances for volume-to-level calibration and the analytical procedure for Pu concentration), and similarly for the random error variance (sum of random error variances for volume measurement, for analytical procedure for Pu concentration, and for sampling error). In most error analyses, the systematic error variances contribute more to the final variance than do the random error variances. In Eq. (A-1), that tendency is reflected in the fact that the random error variance of the measured amount of input is divided by the number of inputs. Current international standards for volume and Pu mass measurements cannot do better than about 0.1% relative standard deviation, for each of the five (two systematic and three random) individual error types so we let $\sigma^2_S = 2 \times 0.001^2$, 

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Appendix A

\( \sigma_R^2 = 3 \times 0.001^2 \), Inputs = 8280 kg, and \( n_{\text{Inputs}} = 150 \) in Eq. (1) to get \( \sigma_{\text{Inputs}} = 11.8 \) kg (of which 11.4 kg is due to the systematic errors). Applying Eq. (1) to the outputs with Outputs = 8280 kg and \( n_{\text{Outputs}} = 40 \), we get \( \sigma_{\text{Outputs}} = 12.1 \) kg. Combining the results, we get

\[
\sigma_{MB} = 16.9 \text{ kg}.
\] (A-2)

To meet the IAEA detection goal, we need \( \sigma_{MB} = \frac{8}{3.3} = 2.42 \) kg, so we cannot meet the IAEA detection goal. Stated another way, given a value of 16.9 kg for \( \sigma_{MB} \), the IAEA detection goal can be met only if the loss is \( 16.9 \times 3.3 = 55.8 \) kg.

In this example calculation, the key assumptions we have made are

1) All measurement errors are proportional to the true value.
2) There is no recalibration of either the volume-to-level relation or the analytical procedure to assay Pu.
3) The mass of Pu in a tank is measured with an error that follows this model:

\[
M = VC = V_T \times (1 + \varepsilon_{VR} + \varepsilon_{VS}) \times C_T \times (1 + \varepsilon_{CR} + \varepsilon_{CS} + \varepsilon_{SR}).
\] (A-3)

where \( V_T \) denotes the true volume, \( C_T \) denotes the true Pu concentration, and the error \( \varepsilon_{VR} \) is the random error of the volume measurement (and similarly for the other errors). Note that the concentration measurement includes two random errors: one for the assay itself and one for the sampling error reflecting that the sample selected for assay might not have the same concentration as the average concentration of the tank.

4) All errors such as \( \varepsilon_{VR} \) in Eq. A-3 are assumed to have an approximate normal distribution with mean 0 and variance \( 0.001^2 \). This was an arbitrary but reasonable choice.

Note that after combining all the above assumptions, the result in Eq. (2) in Section 3.1 represents a standard deviation of about 0.2%.
Appendix B

A Bias-Correction Example

The type of bias correction we suggest in Section 3 is identical to what we have seen in safeguards in a different setting. In safeguards, bias corrections are usually intended to be applied to measurements on a collection of process items. The typical situation is that the process items were measured during a period in which measurement control data that was taken during that period suggests (statistically) that a bias correction would reduce the average squared distance between the measurement and the true value. The main source of controversy over the use of bias corrections in safeguards seems to be that bias corrections must place considerable faith in our rather simple error models such as

\[
V_i = V_{Ti} \times (\varepsilon_{VS} + \varepsilon_{VR_i}).
\]  

(B-1)

where the systematic volume error is assumed to be a constant times the true volume for all measurements made during the calibration period, so there is no \(i\) index on \(\varepsilon_{VS}\) in Eq. (B-1). Note that Eq. (B-1) can be written separately for both Tank 1 and Tank 2, which gives Eq. (3) in Section 3

\[
SRD_i = V_{1,i} - V_{2,i} = V_{1T} \times ((\varepsilon_{VS1} - \varepsilon_{VS2}) + (\varepsilon_{VR1,i} - \varepsilon_{VR2,i})).
\]

(B-2)

Throughout this paper we make the simplifying assumption that the true volume shipped is the same for each shipment, so in Eq. (B-2) there is no \(i\) index on \(V_{1T}\).

The situation we consider is as follows. Assume we have \(n_{previous}\) shipments (at least one) from Tank 1 to Tank 2 that are known to have zero true volume loss, that is, all of the liquid shipped from Tank 1 is received by Tank 2 without any hold-up, losses, etc. Those previous shipments can be used to bias-correct all future shipments during the calibration period because \(\varepsilon_{VS1}\) and \(\varepsilon_{VS2}\) are assumed to be constant during the calibration period.

Denote the bias-corrected volume difference between \(\tilde{V}_{1,i}\) and \(\tilde{V}_{2,i}\) as \(S\tilde{R}D_i\). The obvious bias correction is the average SRD during the \(n_{previous}\) shipments, say \(\overline{SRD}_{prev}\). The “trick” is to write an error model for \(\overline{SRD}_{prev}\):

\[
\overline{SRD}_{prev} = V_{1T} \times ((\varepsilon_{VS1} - \varepsilon_{VS2}) + (\varepsilon_{VR1} - \varepsilon_{VR2})).
\]

(B-3)
where we see that if we let $\tilde{SRD}_i = SRD_i - SRD_{prev}$, then the systematic errors would exactly cancel if over a few shipments $\tilde{\epsilon}_{VR1} - \tilde{\epsilon}_{VR2} = 0$. Unfortunately, we can only expect that $\tilde{\epsilon}_{VR1} - \tilde{\epsilon}_{VR2} = 0$, because $\tilde{\epsilon}_{VR1} - \tilde{\epsilon}_{VR2} \sim N(0, (\sigma^2_{VR1} + \sigma^2_{VR2})/n_{prev})$ (normal with mean 0 and specified variance) so the systematic errors only approximately cancel, leaving us with a new estimate of the systematic error variance equal to $(\sigma^2_{VR1} + \sigma^2_{VR2})/n_{prev}$. We used this new systematic error variance estimate in the first bias-correction example given in Section 3. In conclusion, if we let $\tilde{SRD}_i = SRD_i - SRD_{prev}$, then the random error variance will be the same as that for the non-bias-corrected $SRD_i$, and the systematic error variance will (perhaps surprisingly) be determined by the random error variance and the number of previous shipments that are known to have zero volume change.