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Spin Dependent Drell-Yan beyond Leading Order: Non-Singlet Virtual corrections to $O(\alpha_s^2)$

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Abstract

We present parton-level analytical results for the next-to-leading order non-singlet virtual corrections to the Drell-Yan differential cross-section. The dependence of the differential cross section on the helicity of the initial state partons is shown explicitly (the spins of the final state partons are summed). The calculation is implemented in dimensional regularization within the $\overline{MS}$ scheme and with the t'Hooft Veltman prescriptions for the n-dimensional γ_5 . Both the polarized initial state and the unpolarized cross sections can be obtained from our result. Our unpolarized cross section agrees with the previous result of Ellis, Martinelli and Petronzio in the non-singlet sector.

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1 Introduction

Spin physics has emerged in the last few years as one of the most active fields of perturbative QCD. Given the planned experiments at RHIC and HERA, it is likely that such interest will continue to grow. Among the interesting processes which are planned to be investigated at hadron colliders such as RHIC are the Drell-Yan production of muon pairs and two-jet production in polarized proton-proton collisions [1]. The difficulty of calculating spin dependence in next-to-leading order QCD is notorious and therefore only $O(\alpha_s)$ results have been presented in the literature.

The difficulty in the calculations comes from the presence of chiral fermions or gluons of fixed helicity in the initial state which renders the study of the spin dependencies extremely time consuming. The analysis of the infrared structure of the cross section in perturbation theory is also much more complex than the unpolarized case. If dimensional regularization is used to control both the infrared and ultraviolet singularities of the amplitudes, then the presence of chiral projector for the fermions requires a suitable definition of γ_5 in n -dimension. In this work we present results for the $O(\alpha_s^2)$ virtual corrections to the $q\bar{q}$ annihilation channel for the classical Drell-Yan process. This is an important ingredient to the overall non-singlet differential cross section, $d\sigma/dQ^2 dtd u$, and for the total cross section $\sigma(Q^2)$, where Q^2 is the invariant mass of the photon. The differential cross section is used to predict the transverse momentum distribution of the photon in the process $p p \rightarrow \gamma^* + X$ with polarized beams.

It has been observed [8] that Drell-Yan production of charged lepton pairs is a case where transverse and longitudinal asymmetries are comparable. To leading order, the ratio between the transverse and the longitudinal asymmetries $R \equiv A_{TT}/A_{LL}$ is estimated to be of order 1. This result is due to the fact that the gluons don't couple to transverse asymmetries and to order α_s only the quark-antiquark channel contribute. Furthermore, Drell-Yan is one of the possible avenues to obtain information about the twist-2 transversity distribution, which can be related to the ratio between transverse and longitudinal asymmetries.

It is important to study the polarized Drell-Yan process, including its factorization and renormalization scale dependence by going to next-to-leading order. The Drell-Yan process has been always considered a possible way to disentangle possible new gauge interactions. The inclusion of polarization in the initial states can be a useful tool to investigate such effects, and other effects, such as compositeness and supersymmetry (see [2] and references therein). The presence of a massive photon in the final state makes an analytic calculation much more involved than, for instance, two other recent similar studies [4]. Although the non-singlet contribution to the process can only give information about the polarized quark distributions, it is the most lengthy Drell-Yan subprocess. We present the order α_s^2 virtual corrections in this paper. Work on the initial state helicity dependence of the real corrections is close to completion [12] and will follow shortly.

2 General Structure

The lowest order (*non-singlet*) contributions to the large transverse momentum production of virtual photons (see Fig. 1) with invariant mass $q^2 = Q^2$ arise from the two, $q + \bar{q} \rightarrow \gamma^* + g$, “Born” amplitudes shown in Fig. 3. We refer to these two diagrams as the *direct* and the *exchange* (or crossed) amplitudes, respectively. In Fig. 2 we have generically illustrated the expansion of the amplitude which appears in $d\sigma/dQ^2$ up to order α_s . Notice that the quark form factor contributions represented in Fig. 3 (and the related $O(\alpha_s^2)$ corrections, not included in the picture) do not appear in the study of the cross section, $d\sigma/d^2q_t dy$, for transverse momentum, q_t , greater than zero. The two-loop $O(\alpha_s^2)$ corrections to the quark form factor can be added in the study of the total cross section, $d\sigma/dQ^2$, by interfering the 2-loop on-shell quark form factor of ref. [11] with the lowest order $q\bar{q}\gamma$ annihilation channel and by using helicity projectors for the initial quark states.

We calculate the spin dependence of the cross section by using the helicity projectors, $P_\pm = \frac{1}{2}(1 \pm \gamma_5)$, which project out the helicity states of an initial state quark and antiquark, respectively, as follows:

$$u(p_1, h_1) = \frac{1}{2}(1 + h_1 \gamma_5)u(p), \quad \bar{v}(p_2, h_2) = \frac{1}{2}\bar{v}(p_2)(1 - h_2 \gamma_5), \quad (1)$$

where $h_1 = \pm 1$ corresponds to quark helicity $\pm \frac{1}{2}$, and $h_2 = \pm 1$ corresponds to antiquark helicity $\mp \frac{1}{2}$.

The squares of the direct amplitude L_1 and exchange amplitude L_2 in Fig. 3 in $N = 4 - 2\epsilon$ dimensions are given by

$$\begin{aligned} M_{dd}(h_1, h_2) &= e_f^2 g^2 g_s^2 \frac{C_F}{N_c} \frac{2u}{t} \left((1 - \epsilon)^2 + h_1 h_2 (1 + \epsilon)^2 \right), \\ M_{cc}(h_1, h_2) &= e_f^2 g^2 g_s^2 \frac{C_F}{N_c} \frac{2t}{u} \left((1 - \epsilon)^2 + h_1 h_2 (1 + \epsilon)^2 \right). \end{aligned} \quad (2)$$

where $\alpha = g^2/4\pi$ is the fine structure constant, and e_f the charge of the quark. The quantity C_F/N_c is the color factor, and $\alpha_s = g_s^2/4\pi$ is the QCD strong coupling constant. The interference term is more complicated and is given by,

$$\begin{aligned} 2M_{dc}(h_1, h_2) &= \\ e_f^2 g^2 g_s^2 \frac{C_F}{N_c} \frac{4}{tu} &\left[(1 - \epsilon)(Q^2 s - \epsilon tu) + h_1 h_2 (1 + \epsilon)(Q^2 s + \epsilon tu) + 2h_1 h_2 \epsilon tu \right]. \end{aligned} \quad (3)$$

The sum of the two Born amplitudes squared is

$$\begin{aligned} |M_B(h_1, h_2)|^2 &= |M_B(h)|^2 = M_{dd}(h) + 2M_{dc}(h) + M_{cc}(h) \\ &= e_f^2 g^2 g_s^2 \frac{C_F}{N_c} \frac{2}{tu} \left[(1 - \epsilon) \left(2Q^2 s + (1 - \epsilon)(t^2 + u^2) - 2\epsilon tu \right) \right. \\ &\quad \left. + h(1 + \epsilon) \left(2Q^2 s + (1 + \epsilon)(t^2 + u^2) + 2\epsilon tu \right) + 4h\epsilon tu \right], \end{aligned} \quad (4)$$

and depends only on the product $h = h_1 h_2$.

To any order in perturbation theory we can write,

$$|M(h_1, h_2)|^2 = |M(h)|^2 = (1 + h)|\bar{M}|^2 - h|M_{++}|^2 = |\bar{M}|^2 - h\Delta|M|^2, \quad (5)$$

where $h = h_1 h_2$ and where

$$|\bar{M}|^2 = \frac{1}{4} \sum_{h_1, h_2} |M(h_1, h_2)|^2 = \frac{1}{4} \sum_{h_1, h_2} |M(h_1, h_2)|^2, \quad (6)$$

is the spin averaged (*unpolarized*) amplitude squared. Furthermore,

$$|M_{--}|^2 = |M_{++}|^2 \text{ and } |M_{-+}|^2 = |M_{+-}|^2, \quad (7)$$

so that

$$|\bar{M}|^2 = \frac{1}{2} (|M_{++}|^2 + |M_{+-}|^2). \quad (8)$$

The spin asymmetry, $\Delta|M|^2$, is defined according to

$$|M_\Delta|^2 = \frac{1}{2} (|M_{++}|^2 - |M_{+-}|^2) = |M_{++}|^2 - |\bar{M}|^2. \quad (9)$$

The spin averaged (*unpolarized*) amplitude squared is determined from $|M(h)|^2$ by setting $h = 0$ and $-\Delta|M|^2$ is the coefficient of h . For the Born term, this results in

$$|\bar{M}|^2 = e_f^2 g^2 g_s^2 \frac{C_F}{N_c} \frac{2}{tu} [(1 - \epsilon)(2Q^2 s + (1 - \epsilon)(t^2 + u^2) - 2\epsilon tu)], \quad (10)$$

and

$$\Delta|M|^2 = -e_f^2 g^2 g_s^2 \frac{C_F}{N_c} \frac{2}{tu} [(1 + \epsilon)(2Q^2 s + (1 + \epsilon)(t^2 + u^2) + 2\epsilon tu) + 4\epsilon tu]. \quad (11)$$

Adding these two terms yields

$$|M_{++}|^2 = -e_f^2 g^2 g_s^2 \frac{C_F}{N_c} \frac{8\epsilon}{tu} [Q^2 s + (t + u)^2], \quad (12)$$

which is proportional to ϵ and vanishes in the limit $\epsilon \rightarrow 0$. Since at the Born level there are no $\frac{1}{\epsilon}$ singularities that might combine with this term to yield a finite contribution, in the limit $\epsilon \rightarrow 0$,

$$|M_{++}|^2 = 0 \text{ and } \Delta|M|^2 = -|\bar{M}|^2. \quad (13)$$

For the $q + \bar{q} \rightarrow \gamma^* + g$ subprocess the condition that $|M_{++}|^2 = 0$ means that the quark helicity is maintained (does not flip) in the collision. The incoming quark line with helicity $\pm \frac{1}{2}$ turns around and becomes an incoming antiquark with helicity $\mp \frac{1}{2}$.

We refer to this as “helicity conservation”. We see that at the Born level helicity is conserved in the limit $\epsilon \rightarrow 0$.

In $N = 4 - 2\epsilon$ dimensions the differential cross section is related to the $2 \rightarrow 2$ invariant amplitude according to

$$s \frac{d\sigma}{dt}(s, t, h) = \frac{1}{16\pi s} \left(\frac{4\pi s}{tu} \right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} |M(h)|^2, \quad (14)$$

which can be written as

$$s \frac{d\sigma}{dt}(s, t, h) = (1+h) s \frac{d\bar{\sigma}}{dt}(s, t) - h s \frac{d\sigma_{++}}{dt}(s, t), \quad (15)$$

or as

$$s \frac{d\sigma}{dt}(s, t, h) = s \frac{d\bar{\sigma}}{dt}(s, t) - h s \frac{d\sigma_{LL}}{dt}(s, t), \quad (16)$$

where $s d\bar{\sigma}/dt$ is the unpolarized cross section and

$$s \frac{d\sigma_{LL}}{dt} = \frac{1}{2} \left(s \frac{d\sigma_{++}}{dt} - s \frac{d\sigma_{+-}}{dt} \right). \quad (17)$$

At the Born level we have,

$$\begin{aligned} s \frac{d\bar{\sigma}}{dt}(s, t) &= e_f^2 K_2 \frac{\alpha_s}{s} T_B(Q^2, u, t), \\ s \frac{d\sigma_{++}}{dt}(s, t) &= -e_f^2 K_2 \frac{\alpha_s}{s} \epsilon A_B(Q^2, u, t), \end{aligned} \quad (18)$$

where

$$\begin{aligned} T_B(Q^2, u, t) &= \frac{2}{tu} \left[(1-\epsilon) (2Q^2 s + (1-\epsilon)(t^2 + u^2) - 2\epsilon tu) \right] \\ &= 2(1-\epsilon) \left[(1-\epsilon) \left(\frac{u}{t} + \frac{t}{u} \right) + \frac{2Q^2(Q^2 - u - t)}{ut} - 2\epsilon \right], \end{aligned} \quad (19)$$

and

$$A_B(Q^2, u, t) = \frac{8}{tu} [Q^2 s + (t+u)^2], \quad (20)$$

where K_2 is defined by

$$K_2 = \pi \alpha \frac{C_F}{N_c} \frac{1}{\Gamma(1-\epsilon)} \left(\frac{4\pi\mu^2}{Q^2} \right)^\epsilon \left(\frac{sQ^2}{tu} \right)^\epsilon, \quad (21)$$

where we have rescaled, $\alpha_s \rightarrow \alpha_s(\mu^2)^\epsilon$, so that it remains dimensionless in $N = 4 - 2\epsilon$ dimensions. At this order of perturbation theory, we have

$$s \frac{d\sigma}{dt}(s, t, h) = e_f^2 K_2 \frac{\alpha_s}{s} \left[(1+h) T_B(Q^2, u, t) + h \epsilon A_B(Q^2, u, t) \right]. \quad (22)$$

As we saw earlier, in the limit $\epsilon \rightarrow 0$ helicity is conserved so that

$$s \frac{d\sigma}{dt}(s, t, h) = (1 + h) s \frac{d\bar{\sigma}}{dt}(s, t), \quad (23)$$

which implies that

$$s \frac{d\sigma_{++}}{dt}(s, t) = 0 \text{ and } s \frac{d\sigma_{LL}}{dt}(s, t) = -s \frac{d\bar{\sigma}}{dt}(s, t). \quad (24)$$

To connect with the notation of ref. [3], we note that

$$K_2 T_B(Q^2, u, t) = K T_0(Q^2, u, t), \quad (25)$$

where K and T_0 are define in ref. [3] as

$$K = 2\pi\alpha \frac{C_F}{N_c} \frac{(1-\epsilon)}{\Gamma(1-\epsilon)} \left(\frac{4\pi\mu^2}{Q^2} \right)^\epsilon \left(\frac{sQ^2}{tu} \right)^\epsilon, \quad (26)$$

and

$$T_0(Q^2, u, t) = \left[(1-\epsilon) \left(\frac{u}{t} + \frac{t}{u} \right) + \frac{2Q^2(Q^2 - u - t)}{ut} - 2\epsilon \right]. \quad (27)$$

3 Virtual diagram contribution

The list of the diagrams with virtual corrections contributing to the non-singlet sector of Drell-Yan is given in Fig. 4. We have omitted all the self-energy insertions of quarks and gluons and the ghost contributions.

Our calculations are performed in the $\overline{MS}$ scheme using dimensional regularization to regulate both the ultraviolet and the infrared singularities. We remove the ultraviolet singularities in the relevant subdiagrams by off shell regularization. Then by sending on shell the initial state quarks and the final state gluon, we encounter singularities in the form of double poles and single poles in $\epsilon = 2 - n/2$. The one-loop order result for the cross section for the process $q + \bar{q} \rightarrow \gamma^* + g$ is given by

$$\begin{aligned} s \frac{d\sigma^{\text{virtual}}}{dt du}(s, t, u, h) = & e_f^2 K_2 \frac{\alpha_s}{s} \delta(s + t + u - Q^2) \left\{ ((1+h)T_B + h\epsilon A_B) \left[1 - \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu^2}{Q^2} \right)^\epsilon \right. \right. \\ & \times \left. \left(\frac{2C_F + N_C}{\epsilon^2} + \frac{1}{\epsilon} \left(3C_F - 2C_F \ln \frac{s}{Q^2} + \frac{11}{6} N_C + N_C \ln \frac{sQ^2}{ut} - \frac{1}{3} N_F \right) \right) \right] \\ & \left. + \frac{\alpha}{2\pi} (1+h) \left[\pi^2 (4C_F + N_C) \frac{2Q^2 s + t^2 + u^2}{3tu} - 2(2C_F - N_C) \frac{Q^2(t^2 + u^2)}{tu(t+u)} \right] \right\} \end{aligned}$$

$$\begin{aligned}
& -2C_F \left(\frac{8(2Q^2s + t^2 + u^2)}{tu} - \frac{Q^4s(t+u)}{tu(Q^2-u)(Q^2-t)} - \frac{t^2+u^2}{(Q^2-u)(Q^2-t)} \right) \\
& -2 \left(Li_2 \left(\frac{t}{t-Q^2} \right) + \frac{1}{2} \ln^2 \left(1 - \frac{Q^2}{t} \right) \right) \left(N_C \frac{2s+t}{u} + 2C_F \frac{s^2 + (s+u)^2}{tu} \right) \\
& -2 \left(Li_2 \left(\frac{u}{u-Q^2} \right) + \frac{1}{2} \ln^2 \left(1 - \frac{Q^2}{u} \right) \right) \left(N_C \frac{2s+u}{t} + 2C_F \frac{s^2 + (s+t)^2}{tu} \right) \\
& +2(2C_F - N_C) \left(Li_2 \left(-\frac{t+u}{s} \right) \frac{2Q^2s + u^2 + t^2 + 2s^2}{tu} \right. \\
& + \left(2 \ln \left(\frac{s}{Q^2} \right) \frac{Q^4 - (t+u)^2}{(t+u)^2} + \ln^2 \left(\frac{s}{Q^2} \right) \frac{s^2}{tu} \right) \\
& - \left(\ln \left(\frac{|t|}{Q^2} \right) \ln \left(\frac{s}{Q^2} \right) - \frac{1}{2} \ln^2 \left(\frac{|t|}{Q^2} \right) \right) \frac{s^2 + (s+u)^2}{tu} \\
& - \left(\ln \left(\frac{s}{Q^2} \right) \ln \left(\frac{|u|}{Q^2} \right) - \frac{1}{2} \ln^2 \left(\frac{|u|}{Q^2} \right) \right) \frac{s^2 + (s+t)^2}{tu} \\
& +2 \ln \left(\frac{|u|}{Q^2} \right) \left(C_F \frac{4Q^2s - 2su + tu}{(Q^2-u)^2} + N_C \frac{u}{Q^2-u} \right) \\
& +2 \ln \left(\frac{|t|}{Q^2} \right) \left(C_F \frac{4Q^2s - 2st + tu}{(Q^2-t)^2} + N_C \frac{t}{Q^2-t} \right) \\
& \left. -2 \ln \left(\frac{|t|}{Q^2} \right) \ln \left(\frac{|u|}{Q^2} \right) N_C \frac{2Q^2s + t^2 + u^2}{tu} \right\}, \tag{28}
\end{aligned}$$

where $h = h_1 h_2$. The quantities T_B and A_B appear in at the Born level (22) and are given by equations (19) and (20), respectively. Notice that the double poles and single pole structure (in the $h \rightarrow 0$ limit) automatically reproduces the singularities of the virtual contributions of ref. [3]. In particular, the double poles in ϵ correctly multiply the two-to-two Born contribution T_B , which is the Born level unpolarized result. Furthermore, it is possible to show [12] that the structure of the virtual contributions presented here correctly factorize, after adding the real contributions.

The presence of the $h\epsilon A_B$ term in (28) implies that in the t'Hooft-Veltman [9, 10] scheme the virtual corrections by themselves do not conserve helicity (*i.e.* they do not satisfy the equation (23)). In particular, the finite part of $d\sigma_{++}/dt$ in the limit $\epsilon \rightarrow 0$ for the virtual corrections is given by

$$\begin{aligned}
& s \frac{d\sigma_{++}^{\text{virtual}}}{dt}(s, t) = \\
& -e_f^2 K_2 \frac{\alpha_s^2}{2\pi s} A_B(s, t) \left(3C_F - 2C_F \ln \frac{s}{Q^2} + \frac{11}{6} N_C + N_C \ln \frac{sQ^2}{ut} - \frac{1}{3} N_F \right), \tag{29}
\end{aligned}$$

where $s + t + u = Q^2$. In some regularization schemes, such as those enforcing an anticommuting γ_5 in n dimensions, helicity would be conserved and equation

(13) maintained throughout. In the t'Hooft-Veltman [9, 10] scheme helicity is not manifestly conserved. However, we expect that the non-singlet real contributions, $s \, d\sigma_{++}^{\text{real}}/dt$, will cancel the virtual contribution in (29) leaving a final result that conserves helicity and maintains equation (23). Work on the helicity dependent real corrections to order α_s^2 is in progress [12]. A study of the transversity distribution to $O(\alpha_s^2)$ is also forthcoming [13].

4 Conclusions

We have presented the parton-level analytical results for the next-to-leading order non-singlet virtual corrections to the Drell-Yan differential cross-section. The dependence of the differential cross section on the helicity of the initial state partons is shown explicitly (the spins of the final state partons are summed). Although the calculation is very involved due to the presence of chiral projectors in the initial state, the result is quite compact and has been presented in a form from which cancelation of the infrared, collinear and infrared plus collinear singularities is evident. Both the polarized initial state and the unpolarized cross sections can be obtained from our result. Our unpolarized cross section agrees with the previous result of Ellis, Martinelli and Petronzio [3] in the non-singlet sector. Our calculation is a first step toward the extension of the classical $O(\alpha_s^2)$ Drell-Yan result to the case of longitudinal polarized beams. A more detailed discussion of our results and of the methods developed by us in the analysis of the virtual corrections will be presented elsewhere [12], where we will provide a complete analysis of the initial state helicity dependence in the non-singlet sector.

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Figures

Figure 1: The Drell-Yan process

Figure 2: Radiative corrections of order α_s

Figure 3: The Born diagrams

Figure 4: One-Loop contributions