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M. B. Isichenko and A. J. Wootton

Fusion Research Center
The University of Texas at Austin
Austin, Texas 78712-1068

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On the role of poloidal and toroidal fluctuating electric fields in tokamak transport

M. B. Isichenko and A. J. Wootton

*Fusion Research Center
The University of Texas at Austin
Austin, TX 78712*

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Abstract

The two different expressions for the radial particle flux Γ found in the literature, $c\langle\tilde{n}\tilde{E}_\theta\rangle/B_\varphi$ and $-c\langle\tilde{n}\tilde{E}_\varphi\rangle/B_\theta$, are identical if the parallel electric field is small. The first expression is derivable in a fluid approach, whereas the second follows from the analysis of individual particle orbits. These expressions, without change, are also valid for an arbitrary axisymmetric magnetic geometry. In a situation where the parallel electric field is significant, the more accurate expression for the particle flux is in terms of the standard $\mathbf{E} \times \mathbf{B}$ velocity.

1 Introduction

The usually used expression for the particle cross-field flux caused by the fluctuating electric fields in a tokamak assumes that the transport is dominated by the $\mathbf{E} \times \mathbf{B}$ drift velocity, whose radial component in the standard large-aspect-ratio geometry (r, θ, φ) is

$$v_{r1} = cE_\theta/B_\varphi. \quad (1)$$

Here r is the minor radius, θ the poloidal and φ the toroidal angle. Assuming that the particle density n is governed by the fluid velocity (1), the corresponding fluctuation-average

radial particle flux can be written as

$$\Gamma_{r1} = c \langle \tilde{n} \tilde{E}_\theta \rangle / B_\varphi. \quad (2)$$

A standard assumption here is that the toroidal magnetic field B_φ is much stronger than the poloidal field B_θ and the equilibrium electric field $\langle \mathbf{E} \rangle$ is perpendicular to the magnetic surface. The tilde denotes a fluctuation part. This is the expression routinely used to analyze data from Langmuir probes and heavy ion beam probes.

On the other hand, it has been argued [1, 2, 3] that the combined electric and magnetic drifts result in the average velocity of trapped particles determined by the toroidal electric field E_φ :

$$v_{r2} = -c E_\varphi / B_\theta. \quad (3)$$

Bickerton [4] derives the following radial flux for all particles:

$$\Gamma_{r2} = -c \langle \tilde{n} \tilde{E}_\varphi \rangle / B_\theta. \quad (4)$$

Formulae (1)–(2), on the one hand, and (3)–(4), on the other hand, seem to be at odds both quantitatively and by the different transport physics which they imply. The purpose of this note is to reconcile formulae (2) and (4) for the fluctuation-induced particle flux and also to generalize these results for a general magnetic geometry.

Before getting into the discussion of particle orbits, we first note that the two formulae yield exactly the same result for $E_\theta/B_\varphi = -E_\varphi/B_\theta$; that is, when the parallel electric field is zero:

$$E_\parallel \equiv \frac{E_\theta B_\theta + E_\varphi B_\varphi}{B} = 0. \quad (5)$$

More exactly, the two expressions for the particle flux differ by only a small quantity when the parallel electric field is small: $E_\parallel \ll E_\perp$. As shown in Sec. 4, this condition also reconciles the general-geometry analogs of Eqs. (2) and (4). This settles the quantitative side of the issue. The physical origin of both expressions for the flux is discussed below.

2 Particle motion in an electrostatically turbulent tokamak

In this section we introduce the “toroidal guiding center” of a particle orbit and show that it moves at the velocity proportional to the toroidal electric field.

In a tokamak, especially with a strongly shaped geometry, the contribution of the magnetic drifts is never negligible in comparison with the $\mathbf{E} \times \mathbf{B}$ drift. The reason is in the presence of the substantial magnetic part of the effective electrostatic potential $\phi^* = \phi + (\mu/e)B$. Here $\mu = mv_{\perp}^2/(2B)$ is the conserved magnetic moment. In a geometry with a significant variation of the magnetic field over a flux surface, $\delta B \sim B$, the variable magnetic part of the effective potential ϕ^* is of order the equilibrium temperature, which is typically much larger than the fluctuation potential $\tilde{\phi}$.

The equations of the guiding center motion are as follows:

$$\dot{\mathbf{x}} = v_{\parallel} \mathbf{b}^* - (c/B) \nabla \phi^* \times \mathbf{b}, \quad (6)$$

$$\dot{v}_{\parallel} = -(e/m) \mathbf{b}^* \cdot \nabla \phi^*, \quad (7)$$

where $v_{\parallel}$ is the parallel velocity, $\mathbf{b} = \mathbf{B}/B$ is the unit vector in the direction of the magnetic field, $\mathbf{b}^* = \mathbf{b} + (v_{\parallel}/\omega_B) \mathbf{c}$, $\omega_B = eB/(mc)$ is the gyrofrequency, and $\mathbf{c} = \mathbf{b} \times (\mathbf{b} \cdot \nabla) \mathbf{b}$ is the magnetic curvature. In a toroidally symmetrical system, Eqs. (6) and (7) conserve the canonical toroidal momentum

$$P_{\varphi} = R \left(mv_{\varphi} + \frac{e}{c} A_{\varphi} \right) = R \left(mv_{\parallel} \frac{B_{\varphi}}{B} - \frac{e}{c} \frac{\psi}{2\pi R} \right) \equiv -\frac{e\psi^*}{2\pi c}, \quad (8)$$

where ψ is the poloidal magnetic flux specifying a flux surface, and

$$\psi^* = \psi - \frac{2\pi R B_{\varphi} v_{\parallel}}{\omega_B} \quad (9)$$

is a conserved quantity. By analogy with the gyro guiding center, we will call the point with the radial coordinate ψ^* “the toroidal guiding center.” The convenience of this concept

is that the toroidal guiding center does not move radially in the absence of fluctuations, whereas the gyro guiding center does. For a trapped particle, ψ^* is the radial coordinate of the reflection point on the banana; for passing particles, ψ^* lies either inside or outside the orbit at the distance of about the poloidal Larmor radius. If this radius is small compared with the radial scale of interest (which may be not true at the very plasma edge or in the scrape-off layer), for the purpose of transport study it suffices to follow the motion of the toroidal guiding centers.

When the toroidal symmetry is broken, as in the presence of a toroidal electric field, ψ^* is no longer conserved. The equation of motion for ψ^* can be derived from Eqs. (6)–(7), but it is more straightforward to use the canonical momentum balance $\dot{P}_\varphi = -e\partial_\varphi\phi$. We thus obtain

$$\dot{\psi}^* = 2\pi c \frac{\partial\phi(\psi, \theta, \varphi, t)}{\partial\varphi}. \quad (10)$$

The toroidal coordinates (ψ, θ, φ) used in Eq. (10) are strictly defined for a general geometry by the standard flux representation of the magnetic field:

$$\mathbf{B} = \frac{\nabla\chi \times \nabla\theta - \nabla\psi \times \nabla\varphi}{2\pi} \equiv \frac{\nabla(\varphi - q\theta) \times \nabla\psi}{2\pi}, \quad (11)$$

where $\chi(\psi)$ is the toroidal magnetic flux, $q(\psi) = \chi'(\psi)$ is the safety factor, and φ is the plain angle about the axis of symmetry.

Equation (10) is true for both trapped and passing particles. In the presence of only the inductive electric field $E_\varphi = V_{loop}/(2\pi R)$, the toroidal guiding center is moving inward as follows:

$$\dot{\psi}^* = -cV_{loop}. \quad (12)$$

For trapped particles, Eq. (12) presents the velocity of the Ware-Galeev drift, which is the basis of the neoclassical pinch effect [1, 2, 5]. The effect of the loop voltage on the gyro guiding center's ψ is smaller for passing particles in proportion to $(B_\theta/B)^2$ [6], because the change in ψ^* is mostly due to the parallel acceleration [change in the second term in Eq. (9)],

whereas for trapped particles the velocity term is uniformly bounded and ψ changes itself:

$$\langle \dot{\psi} \rangle_{trapped} = \dot{\psi}^*.$$

This distinction is no longer true for a mostly turbulent electrostatic field with a negligible loop voltage, because no secular growth of $v_{\parallel}$ is expected. Thus Eq. (10) does describe the radial displacement of each particle due to the electrostatic fluctuations. However, passing electrons can undergo a smaller radial transport than trapped electrons, because the electron bounce frequency is typically larger than the turbulence frequency, and the orbit (i.e., surface) averaging of the toroidal force (10) takes place for the passing electrons [6].

Thus, in the approach of the toroidal guiding centers, it is the *toroidal* electric field which makes the particles drift across the magnetic flux surfaces in tokamak, and Eq. (10) presents the basic framework for studying the radial transport in a general tokamak geometry.

In a large-aspect-ratio tokamak, we have $\psi = \int_0^r B_{\theta} 2\pi R dr$, and Eq. (10) reduces to Eq. (3), implying the Bickerton expression (4) for the radial flux. Interestingly and less transparently, formula (4) is also exactly valid for an arbitrary magnetic geometry. To show this, we rewrite the ψ -velocity (10) in terms of the real-space velocity perpendicular to the magnetic flux surface $\psi = \text{const}$:

$$u_{\perp} = \dot{\psi}^* / |\nabla \psi|. \quad (13)$$

From Eq. (11) it follows that the gradient of ψ is simply related to the poloidal magnetic field $B_{\theta} = |\nabla \psi| / (2\pi R)$, where $R = |\nabla \varphi|^{-1}$ is the local major radius. Upon eliminating $|\nabla \psi|$ from the flux $\Gamma^* = n u_{\perp}$, we arrive at the usefully simple result for the instantaneous local cross-surface flux of toroidal guiding centers in a general axisymmetric magnetic field:

$$\Gamma^* = -cn E_{\varphi} / B_{\theta}. \quad (14)$$

For a time average flux, an averaging over the fluctuations of n and E_{φ} must be performed. We thus find that the Bickerton formula (4) expresses the particle (more exactly, the toroidal guiding center) flux due to an arbitrary electrostatic turbulence in an arbitrary shaped

geometry. As a consequence, for $E_{\parallel} \ll E_{\perp}$, formula (2) is as good in a general geometry, despite its appearance as an approximation for a large-aspect-ratio, circular-cross-section tokamak.

3 Particle flux in a fluid approach

The results of the previous section appear to indicate that formulae (1) and (2) result from a popular misperception, which is only accidentally remedied by the outside fact that the parallel electric field is small. Here we show that this is not true; namely, the poloidal-field-dominated particle flux (2) naturally arises from a fluid description, whereby one looks at all particles at a given spatial location, rather than follows the extended orbits of chosen particles.

By denoting $f(\mathbf{x}, v_{\parallel}, \mu, t)$ the particle density in the phase space of $(\mathbf{x}, v_{\parallel}, \mu)$, the collisionless drift kinetic equation reads:

$$\partial_t f + \partial_{\mathbf{x}}(\dot{\mathbf{x}}f) + \partial_{v_{\parallel}}(\dot{v}_{\parallel}f) = 0, \quad (15)$$

where the phase-space velocities are given by Eqs. (6)–(7). Upon integrating Eq. (15) over $v_{\parallel}$ and μ , we obtain the continuity equation for the particle density $n(\mathbf{x}, t) = \int f dv_{\parallel} d\mu$: $\partial_t n + \partial_{\mathbf{x}} \Gamma = 0$, where the instantaneous flux is

$$\Gamma = nu_{\parallel} \mathbf{b} + nc \frac{\mathbf{E} \times \mathbf{b}}{B} + \frac{nc}{eB} \left[(T_{\parallel} + mu_{\parallel}^2) \mathbf{c} - T_{\perp} \frac{\nabla B}{B} \times \mathbf{b} \right]. \quad (16)$$

Here $u_{\parallel}$ is the parallel fluid velocity defined by $nu_{\parallel} = \int f v_{\parallel} dv_{\parallel} d\mu$, and the temperatures are introduced according to $nT_{\perp} = \int f \mu B dv_{\parallel} d\mu$ and $nT_{\parallel} = \int f m(v_{\parallel} - u_{\parallel})^2 dv_{\parallel} d\mu$.

We now see that, locally, the main cross-field flux is given by the $\mathbf{E} \times \mathbf{B}$ contribution, specifically due to the perpendicular electric field lying in the magnetic flux surface. There also are contributions due to the fluctuating parallel flow $\tilde{u}_{\parallel}$ and temperatures $\tilde{T}_{\perp}$ and $\tilde{T}_{\parallel}$,

but these are typically small in proportion to

$$\frac{\Gamma_{\tilde{T}}}{\Gamma_{\tilde{\phi}}} \sim \frac{\tilde{T}}{e\tilde{\phi}} \frac{1}{k_{\theta}R} \ll 1. \quad (17)$$

Thus the usual $\mathbf{E} \times \mathbf{B}$ flux (2) is a natural outcome of the fluid approach.

4 Reconciliation of fluid and particle fluxes in general geometry

In this section we show that the difference between the “particle” flux (14) and the “fluid” flux (16) is due to the difference between the gyro and the toroidal guiding centers, and this difference is small when the parallel electric field is small.

In principle, the fluxes of the gyro guiding centers and the toroidal guiding surfaces need not be the same. For example, the difference between the particle flux and the gyro guiding center flux is known as the diamagnetic current [7]. The similar difference for the gyro guiding centers and the toroidal guiding centers is related to the toroidal bootstrap current [8, 9]. As this current has no radial component, we anticipate that the radial flux of gyro guiding centers equals that of the toroidal guiding centers.

Equation (16) is the local flux of gyro guiding centers. We now wish to compare its $\mathbf{E} \times \mathbf{B}$ part with the flux of toroidal guiding centers (14):

$$\Gamma_{\mathbf{E} \times \mathbf{B}} - \Gamma^* = nc \left(\frac{E_{\theta} b_{\varphi} - E_{\varphi} b_{\theta}}{B} + \frac{E_{\varphi}}{B_{\theta}} \right) = nc \frac{B_{\varphi} E_{\parallel}}{B B_{\theta}} \quad (18)$$

In addition to this, the fluid flux (16) also differs from Eq. (14) by additional thermal terms. To reconcile the “particle” and the “fluid” fluxes, we note that the derivation of formula (14) was not quite consistent, because, in the equation of motion of the toroidal guiding center ψ^* (10), the right-hand side is taken at the point of the gyro guiding center ψ , rather than at ψ^* . The problem can be fixed by writing the radial velocity of the gyro

guiding center using Eq. (9):

$$\dot{\psi} = \dot{\psi}^* + 2\pi \frac{d}{dt} \frac{RB_\varphi v_{\parallel}}{\omega_B} = -2\pi R c E_\varphi + v_{\parallel}^2 \nabla_{\parallel} \frac{RB_\varphi}{\omega_b} + \frac{RB_\varphi}{\omega_b} \dot{v}_{\parallel}. \quad (19)$$

The distribution-function-average flux normal to the magnetic surface can be written

$$\Gamma = \int dv_{\parallel} d\mu f(\mathbf{x}, v_{\parallel}, \mu) \frac{\dot{\psi}}{|\nabla\psi|}. \quad (20)$$

Using Eq. (7), the integration yields:

$$\Gamma = nc \left(-\frac{E_\varphi}{B_\theta} + \frac{B_\varphi E_{\parallel}}{BB_\theta} + \frac{T_{\parallel} + mu_{\parallel}^2}{e} \frac{\nabla_{\parallel} \frac{RB_\varphi}{B}}{RB_\theta} - \frac{T_{\perp}}{e} \frac{B_\varphi \nabla_{\parallel} B}{B^2 B_\theta} \right). \quad (21)$$

Upon some manipulations with magnetic flux coordinates, we find that Eq. (21) is identical to Eq. (16).

Thus the difference between the simply derived flux of the toroidal guiding centers (14) from the complete fluid expression (16) is due to the finite distance, of order the poloidal Larmor radius, between the toroidal and the gyro guiding centers. This difference results in several additional, but typically small, terms in the flux. The relative contribution of the parallel electric field is of order $E_{\parallel}/E_{\perp}$. The effect of temperature fluctuations is of order $\tilde{T}/(e\tilde{\phi})(k_{\theta}R)^{-1}$, which is always small; the effect of the parallel flow is even smaller.

5 Conclusion

The apparent contradiction between the two expressions, (2) and (4), for the fluctuation-induced particle flux in a tokamak is settled. We find that in most situations the standard formula (2) and the Bickerton formula (4) are identical possibly except near a limiter or divertor target, where the parallel electric field can be strong [10, 11]. The more general expression for the flux involves the perpendicular component of the electric field tangent to the magnetic flux surface:

$$\Gamma = nc \frac{E_{\theta} b_{\varphi} - E_{\varphi} b_{\theta}}{B}. \quad (22)$$

Here the φ components of the vectors are in the direction of toroidal symmetry, the poloidal (θ) components are in the plane tangent to the flux surface, and the frame of (ψ, θ, φ) is assumed right-handed.

For the case when the parallel electric field is much less than the perpendicular one, Eq. (22) is equivalent to either formulae (2) or (4), which thereby are more general than originally intended and valid in an arbitrary axisymmetric magnetic geometry.

As the condition of the small parallel electric field appears to be universally valid (possibly except near the edge in some limiter or divertor geometries), the question of whether the self-consistent tokamak transport is determined by poloidal or toroidal electric fields is somewhat academic. In the test-particle approach, the role of the toroidal electric field is highlighted by the fact that the confinement is determined by the change of the toroidal momentum due to the toroidal force. In the fluid approach, the usual $\mathbf{E} \times \mathbf{B}$ transport is sensitive to the electric field perpendicular to the magnetic field and parallel to the magnetic flux surface.

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References

- [1] A. A. Ware. Pinch effect for trapped particles in a tokamak. *Phys. Rev. Lett.*, 25(1):15-17, 1970.
- [2] A. A. Galeev. Diffusion-electrical phenomena in a plasma confined in a tokamak machine. *Zh. Eksp. Teor. Fiz.*, 59:1378-1389, 1970. [English transl. *Sov. Phys. JETP* **32**, 752 (1971)].
- [3] C. S. Liu, D. C. Baxter, and W. B. Thompson. Anomalous diffusion in axisymmetric toroidal systems. *Phys. Rev. Lett.*, 26(11):621-624, 1971.
- [4] R. J. Bickerton. Particle diffusion due to fluctuations in toroidal systems. *Nucl. Fusion*, 13:290-292, 1973.
- [5] R. D. Hazeltine and J. D. Meiss. *Plasma Confinement*. Addison-Wesley, New York, 1992.
- [6] M. B. Isichenko, A. V. Gruzinov, and P. H. Diamond. Invariant measure and turbulent pinch in tokamaks. Report FRCR-452, Fusion Research Center, The University of Texas at Austin, October 1994. Submitted to *Phys. Rev. Lett.*
- [7] L. Spitzer. *Physics of fully ionized gases*. Interscience, New York, 1956.
- [8] R. J. Bickerton, J. W. Connor, and J. B. Taylor. Diffusion driven plasma current and bootstrap tokamak. *Nature*, 229(4):110-112, 1971.
- [9] B. B. Kadomtsev and V. D. Shafranov. Stationary tokamak. In *International Conference on Plasma Physics and Contr. Fusion*, pages 479-89, Vienna, 1971. IAEA. IAEA-CN-28/F-10.
- [10] A. Grossman, L. Schmitz, F. Najmabadi, and R. W. Conn. RF ponderomotive forces and electrostatic barriers for scrape-off layer control: phenomenological model. *J. Nucl. Materials*, 196-198:775-784, 1992.
- [11] L. Schmitz, L. Blush, G. Chevalier, R. Lehmer, Y. Hirooka, P. Chia, G. Tynan, and R. W. Conn. Impurity transport and retention in a gas target divertor: simulation experiments in PISCES-A and modeling results. *J. Nucl. Materials*, 196-198:841-847, 1992.