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Martin A. Lopez de Bertodano
Stephen G. Beus
Jian-Feng Shi

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BETTIS ATOMIC POWER LABORATORY
WEST MIFFLIN, PENNSYLVANIA 15122-0079

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The Effect of Pressure on Annular Flow Pressure Drop in a Small Pipe

Martin A. Lopez de Bertodano and Jian-Feng Shi
School of Nuclear Engineering
Purdue University
West Lafayette, IN 47907-1290
bertodan@ecn.purdue.edu

Stephen G. Beus
Bettis Atomic Power Laboratory
Westinghouse Electric Corporation
West Mifflin, PA 15122-0079

ABSTRACT

New experimental data was obtained for pressure drop and entrainment for annular up-flow in a vertical pipe. The 9.5 mm. pipe has an L/D ratio of 440 to insure fully developed annular flow. The pressure ranged from 140 kPa to 660 kPa. Therefore the density ratio was varied by a factor of four approximately. This allows the investigation of the effect of pressure on the interfacial shear models. Gas superficial velocities between 25 and 126 m/s were tested. This extends the range of previous data to higher gas velocities.

The data were compared with well known models for interfacial shear that represent the state of the art. Good results were obtained when the model by Assali, Hanratty and Andreussi (1985) was modified for the effect of pressure. Furthermore an equivalent model was obtained based on the mixing length theory for rough pipes. It correlates the equivalent roughness to the film thickness.

INTRODUCTION

The two fluid model application to two phase flows depends upon the extent of the experimental data base available to support it. There are many engineering applications in the chemical and the power industries under high pressure conditions. However most of the data compatible with the two fluid model was obtained at atmospheric pressure. One good example is the pressure drop in annular flow.

An air-water experiment was performed to measure pressure drop, entrainment and film thickness in annular flow at various pressures from atmospheric to 660 kPa. The purpose of these data is to assess the effect of pressure on the models for pressure drop available in the open literature.

The models of Wallis (1969), Henstock and Hanratty (1976) and Assali, Hanratty and Andreussi (1985) represent the state of the art in annular interfacial shear models that are compatible with the two fluid model. The model by Wallis is based on the assumption that the effect of the interfacial waves may be modeled as a function of the ratio of the film thickness to the diameter, h/D. However in order to implement this model it is necessary to have a model for the film thickness. Henstock and Hanratty developed one model based on single phase mixing length theory in the liquid film. This model correlates h^* = h u^*/V_f to the Reynolds number of the liquid film. They made the same assumption as Wallis for the interfacial shear and then defined a new factor F which was shown to be equivalent to the Martinelli-Nelson parameters, X_n and X_u. This model includes the effect of pressure since the Martinelli-Nelson model -which is a mixture model- was validated for a wide range of pressures. The model by Assali, Hanratty and Andreussi for film thickness is similar to the model by Henstock and Hanratty. However their model for interfacial friction factor includes the properties of the gas flow.

REVIEW

Wallis (1969) proposed the first correlation for interfacial shear based on the analogy with surface roughness. He based his correlation on atmospheric pressure data from various experiments. He found that he could fit these data with the correlation:
\[ f_i = 0.02 \left( 1 + 300 \frac{h}{D} \right) \]  

(1)

where \( \tau_i = 1/2 f_i/4 \rho_g (u_g - u_p)^2 \). Equation (1) is equivalent to:

\[ f = 0.02 \left( 1 + 75 \frac{k_s}{D} \right) \]  

(2)

and \( k_s = 4h \). Equation (2) is an approximation to Moody's chart for large \( Re \). The key assumption in this model is that \( k_s / D \) is proportional to \( h / D \).

A similar approach was taken by Henstock and Hanratty (1985) who also proposed the following model for film thickness:

\[ h_f^+ = \left[ 0.707 \left( \frac{0.5 Re_f^{0.5}}{0.9 Re_f^{0.5}} \right) + \left( 0.037 Re_f^{0.5} \right)^{0.4} \right] \]  

(3)

where \( Re_f = 4 \dot{m}_f / \pi D \mu_f \) and \( \dot{m}_f = \dot{m}_f(1-E) \). This model based on single phase mixing length theory has two extremes which may be obtained by integration of Newton's law of shear for laminar flows or Prandtl's logarithmic law for turbulent flows. Henstock and Hanratty defined a parameter:

\[ F = \frac{h_f^+ v_f}{Re_g^{0.9} v_g \sqrt{\rho_f / \rho_g}} \]  

(4)

and then proposed a correlation for the interface friction factor:

\[ \frac{f_i}{f_g} - 1 = 1400 F \left[ 1 - e^{-k_s \rho_g g \tilde{h}} \right] \]  

(5)

where the term in the square brackets accounts for the effect of gravity. This term has been neglected in the calculations performed for this paper. It can be shown that \( F \) is proportional to \( X_m \) at low \( Re \) and it is proportional to \( X_m \) at high \( Re \) if the Martinelli-Nelson parameters are corrected by replacing the total liquid flow by the flow of the liquid film. Therefore this correlation includes the correct pressure dependence.

This correlation is based on the assumption that \( k_s / D \) is proportional to \( h / D \). However Assali et al. (1985) found that this assumption did not scale their data properly. They then proposed a new correlation:

\[ \frac{f_i}{f_g} - 1 = 0.45 \left( h_g^+ - 4 \right) Re_g^{-0.2} \]  

(6)

where \( h_g^+ = h u_g^+ / v_g \). This dimensionless group scales a broad atmospheric pressure database better than \( h / D \). Equation (36) in Assali et al. has been used to calculate this term in the present work.

A link with the well established mixing length theory for single phase turbulent flow in rough pipes would be desirable. According to Tennekes and Lumley (1972):

\[ \frac{u}{u^*} = \frac{1}{\kappa} \ln y^* + \phi_0(k_s^+) \]  

(7)

where \( k_s^+ = k_s u^* / v \). Then averaging this equation across the pipe it may be shown that:

\[ \sqrt{\frac{8}{f}} = \frac{1}{\kappa} \ln Re \sqrt{f - \frac{1}{\kappa} \left( \frac{3}{2} + \ln 2 \right) + \phi_0(k_s^+)} \]  

(8)

so that

\[ f = \phi_1(Re, k_s^+) = \phi_2(Re, k_s^+ / D) \]  

(9)

because \( \frac{k_s^+}{Re} = \sqrt{\frac{f}{8 D}} \). Then equation (8) is equivalent to the more familiar equations for rough surfaces. However equation (8) is implicit so a more practical approximate relationship derived from White's formula (1974) is:

\[ f = 1.02 \left[ \log_{10} \left( \frac{2.51 Re}{\sqrt{8 k_s^+ + 2.51}} \right) \right]^{2.5} \]  

(10)

As will be shown there is a simple empirical relationship between \( h_g^+ \) and \( k_s^+ \) so a direct comparison between (6) and (10) is possible.
EXPERIMENT

Experimental Loop

The experiment was carried out in a pipe with adiabatic upwards air-water flow. The loop schematic is shown in Figure 1. The test section is made of a 9.5 mm acrylic tube. The length between the top of the mixer and the inlet of the first extraction unit is 4.2 m, so the L/D ratio is 440. A second extraction unit is located 300 mm above the first one.

![Figure 1: Test Section Schematic](image)

Deionized water was injected into the test section at the mixer through a 76 mm long porous wall with 100 μm porosity. The water formed a liquid film around the periphery of the tube, and the air flow was injected to the bottom of the mixer. The mixture of air and entrained liquid droplets downstream of the extraction unit was discharged to a separator tank. The water collected in the separator tank and the extraction flow meters were drained to the water reservoir.

The comprehensive measurements conducted included: air and water flow rates at the injection, and film flow, pressure drop, and film thickness near the exit. The blueprint of the assembly where pressure drop and film thickness were measured is shown in Figure 2.

![Figure 2: Assembly of conductivity probes and pressure taps.](image)

Air and Water Flow Rate Measurement

The range of the experimental data compared to a previous similar experiment by Cousins and Hewitt was extended on the ranges of pressure, air flow and water flow. The compressor available provided very high air flow rates at 930 kPa. A rotameter, with a range of $2.53 \times 10^{-7}$ to $1.14 \times 10^{-5} \text{ m}^3/\text{s}$ with ±2% accuracy, was used to measure the inlet air flow rate. The water flow rate was measured using two calibrated rotameters for different flow ranges, the rotameter for low water flows has a range of $7.58 \times 10^{-6}$ to $1.14 \times 10^{-4} \text{ m}^3/\text{s}$ with ±2% accuracy, and the other one for high water flows has a range of $9.44 \times 10^{-4}$ to $9.44 \times 10^{-3} \text{ m}^3/\text{s}$ with ±3% accuracy.

Film Flow and Entrainment Rate Measurements

As the liquid film moves up the test section, a portion of liquid film is entrained into the gas core. In the first extraction unit, the liquid film was extracted by suction through a porous wall section with porosity of 100 μm. The extracted liquid flow was collected by using one of two calibrated extraction flow meters. To extract the liquid it was necessary to extract a small amount of air as well. The same procedure was carried out on the second extraction unit. A more detailed description of these measurements is given by Lopez de Bertodano and Jan (1996).

Pressure Drop Measurement

Two manometers with different manometric fluids, i.e. mercury and Meriam 175 Blue Fluid (Specific
gravity 1.75) were used for pressure drop measurements. The pressure difference was measured between two pressure taps at 3.850 m and 4.158 m from the top of mixer. The tubes connecting the pressure taps to the manometers were filled with water to prevent bubbles in the lines. The liquid purge technique described by Hewitt (1978) was adopted. The injected water flow rate was less than 1% of the total water flow so that it would not affect the pressure drop measurement or the entrainment measurement.

**Film Measurements**

The film thickness and wave velocity were measured with two conductivity probes shown in Figure 2. Each conductivity probe consists of two 0.8 mm thick ring electrodes separated with a 1.6 mm thick layer of Teflon insulator mounted on the surface of the tube. A 50 kHz sinusoidal signal is applied across the electrodes. Sodium chloride was added to the deionized circulating water to increase the electrical conductivity so that the film thickness may be measured by measuring conductance between two electrodes. For very thin water films like those found in our experiments, the output signal varies linearly with the film thickness.

Prior to the tests, the conductivity probe was calibrated for film thickness. An acrylic conical rod which could produce films of various thickness was inserted into the test section and it was moved vertically with a micrometer. A plot of voltage vs. film thickness is shown in Figure 3 together with the theoretical prediction by Coney (1973). A plot of the probe conductance G, which is normalized by that for the tube full of water G_\infty, shows that a linear relationship exists when the relative thickness H=h/a is less than 0.3 (see Figure 3).

The output responses of the probes is sent to a solid state circuit, similar to that used by Koskie et. al.(1989), where they are demodulated and converted to a DC voltage. The voltage is then acquired by an A/D converter at 10 kHz. The analog voltage signals are stored in a PC. A wave trace measured by one of the probes is shown in Figure 4.

The wave profiles in the time domain were converted to the spatial domain by multiplying the time by the average roll wave velocity. Roll wave velocities were calculated two different ways. A FORTRAN program was developed to calculate the time delay between the signals of the two probes by direct comparison. In addition the Xmgr software was used to perform a cross correlation of the traces of the two signals. Both methods gave similar results. Figure 10 shows the wave profiles in the spatial domain for four different flow conditions.

![Figure 3: Conductivity probe calibration](image)

**RESULTS**

Interfacial friction factor
First we compare the lower pressure data (i.e., 20 and 35 psia). Since the pressure tap lines are full of water the measured pressure drop represents the wall shear plus the hydrostatic difference between the column of water in the pressure tap line and the column of two phase flow in the test section. In the following calculations the void fraction in the test section is assumed to be 1. Then:

\[
\left( \frac{dp}{dz} \right)_{\text{frictional}} = \left( \frac{dp}{dz} \right)_{\text{measured}} + \rho_f g \tag{11}
\]

This introduces some error, however since the void fraction is at least 0.9 the maximum error is only 1 kPa/m. The wall shear is given by the relation:

\[
\tau_i = -\frac{D}{4} \left( \frac{dp}{dz} \right)_{\text{frictional}} - \bar{d}(u_g - u_i) \tag{12}
\]

where it is assumed that the velocity of the droplets impinging on the film is \( u_i = u_f \). For our data the first term on the RHS is much bigger than the second term.

Figure 5 shows the comparison with Wallis’ correlation. The predicted interfacial shear is \( \tau_i = 1/2 \frac{f_f}{4} \rho_f (u_g - u_i)^2 \). For the interfacial velocity it was assumed that the film resembles Couette flow such that \( u_i = 2u_f \) and \( u_{if} = \frac{m_{if}}{\rho_f \pi D_h} \). Henstock’s correlation, equation (3), was used for the film thickness. The comparison is not very good. Evidently the effect of the film thickness on interfacial friction is not as simple as proposed by Wallis. The Figure shows that the predictions become more inaccurate as the liquid flow increases.

Figure 6 shows the comparison with Henstock’s correlation, equation (5). The agreement with the data is reasonable at high gas flows but at low gas flows it is poor.

Figure 7 shows the comparison with Asali et. al.’s correlation, equation (6). Asali et al.’s correlation for the film thickness is also used. The agreement with the data is very good at the lowest pressure and clearly better than Henstock’s correlation, but it varies systematically as the pressure is increased. A modification to Asali’s correlation is proposed:

\[
\frac{f_i}{f_g} = 0.6 \left( h_s + \frac{\rho_f / \rho_g}{(\rho_f / \rho_g)_{\text{atm}}} - 4 \right) Re_g^{-0.2} \tag{13}
\]

Figure 8 shows the comparison of equation (13) with the data is very good.
**DISCUSSION**

In order to match correlation (13) with mixing length theory the following relationship was empirically obtained:

\[
k_{x}^{*} = 0.035 \left( \frac{\rho_g}{\rho_g} \right) \left( \frac{\rho_f}{\rho_g} \right)^{\frac{1}{2}} \left( \frac{\rho_f}{\rho_g} \right)_{atm} \tag{14}\]

Figure 9 shows a comparison of the data with equations (10) and (14). Comparing Figures 8 and 9 it appears that both correlation methods are equivalent. Furthermore equation (14) may be rewritten as

\[
\frac{k_{x}^{*}}{D} = 0.035 \left( \frac{\rho_g}{\rho_g} \right) \left( \frac{\rho_f}{\rho_g} \right) \left( \frac{\rho_f}{\rho_g} \right)_{atm} \left( \frac{h}{D} \right) \tag{15}\]

which shows the relationship between the present model and the simpler one by Wallis. The agreement obtained shows the relationship between equation (13) and the mixing length turbulence theory for single phase flow.

**Interfacial structure:**

The surface roughness model represented by \( k_{x}^{*} \) may be used to calculate the effect of the ripples on the film surface. However as the liquid flow rate increases roll waves appear. The effect of these may be
different from the effect of the ripples (i.e.: form drag vs. skin friction) if the roll waves are not steep a theory by Beicher et. al. (1993) for the form drag over an undulating surface may be applicable. This theory applies to turbulent flows provided that there is no flow separation behind the waves. The drag force occurs because the boundary layer becomes thicker in the leeward side of the roll waves so the pressure is smaller there. For sinusoidal roll waves the coefficient of drag is given by:

\[ C_D = 2 \left( \frac{a}{\lambda} \right)^2 \frac{k_s}{\lambda} f_i \]  

(16)

where \( a \) is the amplitude of the roll wave, \( \lambda \) is the half height wavelength and \( \psi \) is a function defined by the authors. Calculations performed with this model indicate that for all of our data:

\[ \frac{dp}{dx}_{\text{form drag}} \leq 0.025 \frac{dp}{dx}_{\text{skin friction}} \]

Figure 10 shows how the roll waves look for the four worst cases from our data set. The ordinate scale corresponds to the distance from the wall to the centerline of the tube. The sides of these roll waves have relatively low gradients which justify the assumption that the gas flow does not separate. Probably the waves are more steep near the inlet of the test section but because the test section is so long they flatten out. Therefore the results obtained correspond to the ripple regime only and should not be extended to the roll wave regime.

CONCLUSION

The effect of pressure on the interfacial shear was identified and correlated for the ripple-annular regime based on new experimental data. The proposed empirical correlations, either equations (13) or (14), should not be used beyond the experimental data that support them (i.e.: the data used to develop the original correlation by Asali et. al. and the present data).

In order to remove the empiricism in equation (14) it would be necessary to obtain an analytical result for the amplitude and distribution of the interfacial ripples.

REFERENCES


Lopez de Bertodano, M. A., C. S. Jan and Beus, S.  


NOMENCLATURE

\( \dot{d} \) = droplet deposition rate  
\( D \) = diameter  
\( dp/dz \) = pressure gradient  
\( E \) = droplet entrainment  
\( f \) = friction factor  
\( f_s \) = \( 1.02 \left( \log_{10} Re \right)^{2.5} \)  
\( g \) = gravitational constant  
\( h \) = film thickness  
\( Re \) = Reynolds number  
\( u \) = velocity  
\( u^* \) = friction velocity  
\( y \) = distance from the wall

Greek:

\( \kappa \) = Von Karman's constant \((0.41)\)  
\( \nu \) = kinematic viscosity  
\( \rho \) = density  
\( \tau \) = shear stress

Subscripts:

\( e \) = entrained droplets  
\( f \) = liquid  
\( g \) = gas  
\( i \) = interfacial  
\( lf \) = liquid film