VALIDATION OF FAVOR CODE LINEAR ELASTIC FRACTURE SOLUTIONS FOR FINITE-LENGTH FLAW GEOMETRIES

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ABSTRACT

One of the current tasks within the U.S. Nuclear Regulatory Commission (NRC)-funded Heavy Section Steel Technology Program (HSST) at Oak Ridge National Laboratory (ORNL) is the continuing development of the FAVOR (Fracture Analysis of Vessels: Oak Ridge) computer code. FAVOR performs structural integrity analyses of embrittled nuclear reactor pressure vessels (RPVs) with stainless steel cladding, to evaluate compliance with the applicable regulatory criteria.

The initial release of the FAVOR code (release 9401) was made available in February, 1994, to domestic commercial and research organizations active in the field of nuclear RPV integrity assessment. Since the initial release of FAVOR, the HSST program has continued to enhance the capabilities of the FAVOR code. ABAQUS, a nuclear quality assurance certified (NQA-1) general multidimensional finite element code with fracture mechanics capabilities, was used to generate a database of stress-intensity-factor influence coefficients (SIFICs) for a range of axially and circumferentially oriented semielliptical inner-surface flaw geometries applicable to RPVs with an internal radius (RI) to wall thickness (W) ratio of 10. This database of SIFICs has been incorporated into a development version of FAVOR, providing it with the capability to perform deterministic and probabilistic fracture analyses of RPVs subjected to transients, such as pressurized thermal shock (PTS), for various flaw geometries.

For validation purposes, the ABAQUS-generated SIFICs have been compared with those of other investigators. FAVOR linear elastic fracture mechanics (LEFM) solutions for semielliptical flaw geometries have been verified by comparing the FAVOR solutions to ABAQUS direct finite element solutions. A set of benchmark verification problems for various finite-length semielliptical inner-surface flaw geometries has been developed.

INTRODUCTION

The U.S. NRC-funded HSST Program at ORNL is a research activity devoted to developing and extending the technology for assessing the margin of safety against fracture for nuclear RPVs. One of the current tasks within the HSST program is the continuing development of FAVOR [1]. FAVOR performs vessel integrity analyses of embrittled clad RPVs subjected to transient conditions, to evaluate compliance with the applicable regulatory criteria. It is anticipated that FAVOR will continue to mature and evolve, i.e., provide a framework into which future fracture technology developments can be incorporated such that the code will continuously reflect the current state-of-the-art fracture technology and will conform to the applicable regulatory criteria.

The HSST program has an ongoing task that has the objective of developing a comprehensive database of accurate validated SIFICs. This database of SIFICs will be for a range of inner-surface flaw geometries for a range of clad-vessel geometries that will envelope the commercial pressurized water reactor (PWR) and boiling water reactor (BWR) vessel geometries in the United States. The incorporation of this SIFIC database into FAVOR will facilitate the generation of accurate fracture mechanics solutions for a range of flaw geometries as may be required in structural integrity assessments of any U.S. PWR or BWR reactor vessel.

ABAQUS [2], a nuclear quality assurance certified (NQA-1) multidimensional finite element code with fracture mechanics capabilities, was previously used to generate a database of SIFICs for infinite-length axial and continuous 360-degree circumferential inner-surface flaw geometries, applicable to RPVs with an internal radius (RI) to wall thickness (W) ratio of 10. This SIFIC database was previously reported [3] and was incorporated into the initial release (release 9401) of FAVOR. Problems were executed to verify the implementation of the SIFICs into FAVOR. For infinite-flaw geometries, the difference between direct
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ABAQUS finite element solutions and the FAVOR solutions, which utilize SIFICs and superposition, was less than one percent.

Since the initial release of FAVOR, the HSST program has used ABAQUS to generate a database of SIFICs for axially and circumferentially oriented finite-length semielliptical inner-surface flaws with aspect ratios of (ratio of total crack length to crack depth) 2, 6, and 10 that are applicable to RPVs with a (Ri/w) ratio of 10. This ABAQUS-generated database of SIFICs has been incorporated into FAVOR. A future release of FAVOR, currently under development, has the capability to perform deterministic and probabilistic fracture mechanics (PFM) analyses of embrittled clad RPVs that contain axially and/or circumferentially oriented finite-length semielliptical inner-surface flaws with aspect ratios (ratio of total crack length to crack depth) of 2, 6, 10, and infinity.

The vessel dimensions used in the ABAQUS models were 86.0 inches for the inner radius and a wall thickness of 8.5 inches. This vessel geometry corresponds to a large percentage of the commercial pressurized water reactor vessels in the United States [4]. The SIFICs were calculated in a nondimensional form; therefore, FAVOR is capable of applying them to any vessel that has an (Ri/w) ratio of 10.

The basis for the FAVOR quality assurance plan [5] has the following validation and verification requirements: (1) validation shall be accomplished by comparing the code results to either physical data or a validated code designed to perform the same type of analysis, and (2) verification shall be performed to ensure that the mathematical models have been properly coded. The HSST database of SIFICs was validated by comparing the results to those of other investigators where possible. Verification that the SIFIC database was correctly implemented into FAVOR was accomplished by comparing FAVOR KI solutions to direct ABAQUS three-dimensional finite element solutions.

**GENERATION AND APPLICATION OF STRESS-INTENSITY-FACTOR INFLUENCE COEFFICIENTS**

An important aspect of performing fracture analyses of RPVs is the calculation of accurate stress intensity factors (KIs) for inner-surface cracks in cylinders. Calculation of KI by superposition is a technique proposed by Bueckner [6]. Several investigators [7-11] have focused on a superposition technique that requires calculation of SIFICs for a given 3-D crack model. In this technique, the flaw surface of a specified geometry is subjected to four stress distributions: uniform, linear, quadratic, and cubic, versus distance from the RPV inner-surface. SIFICs obtained from these solutions can be superposed to obtain mode-I stress-intensity factors for arbitrary stress distributions. Considerable economy can be achieved in this approach because the only direct 3-D solutions required are those used to calculate the SIFICs. This approach also provides considerable generality in that KIs may be obtained for arbitrary stress distributions such as those produced by complex time-varying thermal-hydraulic boundary conditions associated with PTS.

The presence of a thin layer of stainless steel cladding on the inner surface of RPVs can have a significant effect on the KI values for shallow inner-surface flaws because of high thermal stresses generated in the cladding during a thermal transient. Therefore, to accommodate the stress discontinuity associated with the cladding, SIFICs were calculated separately for stresses in cladding and base material. For an arbitrary continuous stresses distribution in the base material and the case of a 3-D semielliptical flaw, the truncated stress distribution is approximately by a third-order polynomial as indicated in Eq.(1):

$$\sigma(a') = C_0 + C_1(a/a) + C_2(a/a)^2 + C_3(a/a)^3,$$

where \(\sigma(a')\) is the stress normal to the crack plane at radial position \(a'\), where \(a'\) and \(a\) are defined in Figure 1. The KI values are calculated for each of the individual stress terms in Eq. (1) and are then added to obtain the total KI value due to the base material as indicated by the following equation:

$$K_I(a/w, \theta) = \sum_{j=0}^{3} C_j \sqrt{a} a K^*_j(a/w, \theta),$$

where \(a\) is the crack depth and \(\theta\) is the elliptical angle denoting the angular position on the crack front. \(K^*_0(a/w, \theta), K^*_1(a/w, \theta), K^*_2(a/w, \theta), \text{ and } K^*_3(a/w, \theta)\) are the SIFICs for the uniform, linear, quadratic, and cubic stress distributions, respectively. Since the stress distribution in the cladding is essentially linear, a first-order polynomial is used with the cladding SIFICs to calculate the KI value due to the cladding. The KI due to the stresses in the cladding is superimposed on the KI due to stresses in the base material to obtain the total KI.

ABAQUS was used to generate SIFICs at various angular positions around the crack front for flaw depths in the range of 0.01 ≤ \(a/t\) ≤ 0.5, with particular emphasis on shallow flaws (\(a/t < 0.1\)). SIFICs were also calculated for two cladding thicknesses \(t_{clad} = 3.96 \text{ mm (0.156 in.) and } t_{clad} = 6.35 \text{ mm (0.25 in.)}\). A 3-D finite element model was generated for each crack depth and aspect ratio [12-13].

![Fig. 1 Axially oriented semielliptical flaw on inner-surface of cylinder.](image)
The 3-D finite element models of semielliptical inner-surface flaw geometries were generated with the ORMGEN [14] automatic mesh generating program. In the process of calculating the SIFICs, careful attention was paid to using adequately converged finite element meshes and an appropriate cylinder length. The number of elements in the circumferential and axial directions and around the crack front was increased, one at a time, to the point where the addition of one element changed the value of \( K_I \) by less than one percent [12–13].

The complete SIFIC databases and further details regarding the generation and application of the SIFICs can be found in references 12 and 13. The technique implemented into FAVOR for calculating \( K_I \) due to cladding is described in references 7 and 12.

**COMPARISON OF SIFICs WITH OTHER INVESTIGATORS**

**Axially oriented flaws**

Some of the ABAQUS-generated 3-D SIFICs were compared with similar SIFICs obtained by other investigators. To compare with other investigators, the ABAQUS-generated SIFICs were multiplied by \( \sqrt{Q} \), where the shape factor \( Q \) is the square of the complete elliptic integral of the second kind. The shape factor for an elliptical crack is approximated by \( Q = 1 + 1.464 \left( \frac{a}{c} \right)^{1.65} \), where \( a \) is the crack depth and \( 2c \) is the crack length.

Figure 2 compares the HSST-generated SIFICs to those generated by Raju-Newman [8] at various positions around the crack front of an axially oriented 2:1 semielliptical flaw of depth \( (a/w) = 0.2 \).

**Fig. 2** Comparison of HSST-generated SIFICs with those of Raju-Newman for an axially oriented semielliptical 2:1 flaw of depth \( (a/w) = 0.2 \)

Figure 3 compares the HSST-generated SIFICs to those generated by French investigators at Framatome [9] at various positions around the crack front of an axially oriented 6:1 semielliptical flaw of depth \( (a/w) = 0.5 \).

**Fig. 3** Comparison of HSST-generated SIFICs with those of Framatome for an axially oriented semielliptical 6:1 flaw of depth \( (a/w) = 0.5 \).

Figure 4 compares the HSST-generated SIFICs to those generated by Raju-Newman [8] at various positions around the crack front of an axially oriented 10:1 semielliptical flaw of depth \( (a/w) = 0.2 \).

**Fig. 4** Comparison of HSST-generated SIFICs with those of Raju-Newman for an axially oriented semielliptical 10:1 flaw of depth \( (a/w) = 0.2 \).

As illustrated in Figs. 2, 3, and 4, the HSST-generated SIFICs for axially oriented semielliptical flaw geometries are in good agreement with those previously generated by other investigators, thus providing mutual validation.
Circumferentially oriented flaws

References 10 and 11 provide normalized $K^*$ solutions for inner-surface circumferential surface flaws in cylindrical vessels subjected to remote tension loading. Direct comparisons of these solutions with the HSST-generated SIFICs can be obtained by multiplying the HSST values for uniform pressure loading by $V/Q$ where $Q$ is as defined above.

Figure 5 compares the HSST-generated SIFICs to those generated by Raju-Newman [10] at various positions around the crack front of circumferentially oriented 2:1 semielliptical flaws of depth $(a/w) = 0.2$ and 0.5. Raju-Newman provide SIFICs for remote tension loading only at the free surface and maximum depth locations, i.e., angular positions of 0 and 90 degrees, respectively.

![Fig. 5 Comparison of HSST-generated SIFICs with those of Raju-Newman for circumferentially oriented semielliptical 2:1 flaws of depth $(a/w) = 0.2$ and 0.5.](image)

Figure 6 compares the HSST-generated SIFICs to those generated by Kumar, et al., in Ref. 11, which employed a line spring model implemented in the ADINA [15] finite element code. The results shown are for a circumferentially oriented 1.7 inch-deep flaw with an aspect ratio of 6:1.

As illustrated in Figs. 5 and 6, the HSST-generated SIFICs for circumferentially oriented semielliptical flaw geometries are in good agreement with those previously generated by other investigators, thus providing mutual validation.

As mentioned earlier, the HSST SIFIC database placed emphasis on shallow flaws $(a/w) < 0.1$. The motivation for this was that shallow flaws are usually the main contributors to vessel failure in PTS probabilistic fracture mechanics analyses. The other investigators did not publish SIFICs for flaw depths of $(a/w)$ ratios less than 0.2; therefore, a comparison could not be made. Verification problems were run to check results for these shallower flaws (see Problems 1 and 2 below).

![Fig. 6 Comparison of HSST-generated SIFICs with those of an ADINA line-spring model for a circumferentially oriented semielliptical 6:1 flaw of depth $(a/w) = 0.2$.](image)

IMPLEMENTATION OF SIFICs INTO FAVOR

The development version of FAVOR calculates $K_I$ at 10 angular locations around the semielliptical crack front at each transient time step for a range of flaw depths. $K_I$ is calculated at 0 degrees, which corresponds to the inner-surface of the vessel, and at 90 degrees, which corresponds to the deepest point of the flaw. $K_I$ is also calculated at each 10 degree elliptical angle increment between the inner-surface and the deepest point of the flaw.

FAVOR performs a cubic-spline interpolation to calculate $K_{IS}$ for any crack depth that does not correspond to one of the discrete flaw depths for which SIFICs were calculated.

VERIFICATION OF FAVOR

A set of benchmark verification problems for finite-length semil elliptical inner-surface flaw geometries has been developed. A problem definition and solution for each of the finite-flaw geometries is presented below.

The transient definition used in these benchmark problems is the one utilized in the NRC / Electric Power Research Institute (EPRI) co-sponsored PTS benchmarking exercise [16]. The thermal transient is characterized by a stylized exponentially decaying coolant temperature. The following formulation is used for the exponentially decaying thermal transient:

$$T(t) = T_f + (T_i - T_f) \exp(-\beta \times \text{time}),$$  \hspace{1cm} (3)

where:

$T(t)$ = coolant temperature at time $t$,
$T_i$ = coolant temperature at time $= 0$,
$T_f$ = final coolant temperature,
$\beta$ = exponential decay constant (min$^{-1}$).
The final coolant temperature was 150°F. Beta, the exponential decay constant, was 0.15 min⁻¹. The initial temperature was 850°F.

The vessel has an internal radius and wall thickness of 86 and 8.5 inches, respectively. The thermoelastic properties used in these analyses are tabulated in Table 1.

**TABLE 1  Thermoelastic Properties of Base and Cladding**

<table>
<thead>
<tr>
<th>Property</th>
<th>Base</th>
<th>Clad</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal conductivity (Btu/hr-ft-OF)</td>
<td>24.0</td>
<td>10.0</td>
</tr>
<tr>
<td>Specific heat (Btu/lb-°F)</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>Density (lb/ft³)</td>
<td>489</td>
<td>489</td>
</tr>
<tr>
<td>Modulus of elasticity (ksi)</td>
<td>28000</td>
<td>27000</td>
</tr>
<tr>
<td>Thermal expansion coefficient (°F)</td>
<td>8.05 E-6</td>
<td>9.1 E-6</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.3</td>
<td>0.3</td>
</tr>
</tbody>
</table>

**PROBLEM 1:** Axially oriented semieliptical finite-length inner-surface flaw with an aspect ratio of 2:1.

(a) Determine the $K_I$ time history at angular positions of 0, 30, and 90 degrees around the crack front of a 0.425 inch ($a/w=0.05$) deep axially oriented flaw with an aspect ratio of 2 to 1 located on the inner-surface of a clad pressure vessel that is subjected to the PTS benchmarking thermal transient and a constant pressure of 1.0 ksi. The clad thickness is 0.156 in. The heat transfer coefficient is constant and is equal to 500 BTU/hr - ft²-°F.

Figure 7 illustrates the FAVOR and ABAQUS direct 3D finite element solutions.

(b) Determine $K_I$ at various angular positions around the crack front at transient times of 20, 40, and 100 minutes.

Figure 8 illustrates the FAVOR and ABAQUS direct 3D finite element solutions.

**PROBLEM 2:** Axially oriented semieliptical finite-length inner-surface flaw with an aspect ratio of 6:1.

Determine $K_I$ at a transient time of 20 minutes at various angular positions around the crack front of a 0.6375 inch ($a/w=0.075$)-deep semieliptical flaw with an aspect ratio of 6:1 located on the inner-surface of a clad pressure vessel that is subjected to the PTS benchmarking thermal transient. The cladding thickness is 0.25 in. The heat transfer coefficient is constant and is equal to 320 BTU/hr - ft²-°F.
Figure 9 illustrates the FAVOR and ABAQUS direct 3-D finite-element solutions for $K_I$ at various angular positions at the transient time specified.

![Graph](image)

Fig. 9 Comparison of FAVOR and ABAQUS direct 3-D finite-element solution of $K_I$ for axially oriented 0.6375 inch-deep flaw with an aspect ratio of 6:1 at various angular positions around the crack front at a transient time of 20 minutes.

**PROBLEM 3: Axially oriented semielliptical finite-length inner-surface flaw with an aspect ratio of 10:1.**

Determine $K_I$ at a transient time of 20 minutes at various angular positions around the crack front of a 4.25 inch (a/w=0.50)-deep flaw with an aspect ratio of 10:1 located on the inner-surface of a clad pressure vessel that is subjected to the PTS benchmarking thermal transient. The cladding thickness is 0.25 in. The heat transfer coefficient is constant and is equal to 320 BTU/hr - ft$^2$°F.

Figure 10 illustrates the FAVOR and ABAQUS direct 3-D finite-element solutions for $K_I$ at various angular positions at the transient time specified.

![Graph](image)

Fig. 10 Comparison of FAVOR and ABAQUS direct 3-D finite-element solution for $K_I$ of axially oriented 4.25 inch-deep flaw with an aspect ratio of 10:1 at various angular positions around the crack front at a transient time of 20 minutes.

**PROBLEM 4: Circumferentially oriented semielliptical finite-length inner-surface flaw with an aspect ratio of 2:1.**

(a) Determine the $K_I$ time history at the angular positions of 0 and 30 degrees around the crack front of a 1.7-inch (a/w=0.20) deep circumferentially oriented flaw with an aspect ratio of 2 to 1 located on the inner-surface of a clad pressure vessel that is subjected to the PTS benchmarking thermal transient. The cladding thickness is 0.25 in. The heat transfer coefficient is constant and is equal to 500 BTU/hr - ft$^2$°F.

Figure 11 illustrates the FAVOR and ABAQUS direct 3-D finite-element solutions.

Determine the maximum thermally induced $K_I$ at the deepest point ($\theta=90$ degrees) of a 6:1 circumferentially oriented semieliptical inner-surface flaw during a linear cooldown of 100 degrees F/hour. The clad thickness is 0.0 in. The heat transfer coefficient is constant and is equal to 500 BTU/hr-ft$^2$-OF. Figure 12 illustrates the FAVOR and ABAQUS direct 3D finite element solutions and the solution by Zahoor [17]. The ABAQUS and FAVOR solutions utilized identical material properties. It is unknown what material properties were utilized by Zahoor.

Figure 11 Comparison of FAVOR and ABAQUS direct 3-D finite-element solutions for $K_I$ time history of a circumferentially oriented 1.7 inch-deep flaw with an aspect ratio of 2:1 at angular positions around the crack front of 0 and 30 degrees.

Fig. 12 Comparison of FAVOR, ABAQUS direct 3-D finite-element, and Zahoor solutions of maximum thermally induced $K_I$ at the deepest point of a circumferentially oriented semieliptical flaw with an aspect ratio of 6:1 during cooldown rate of 100°F/hr.


Determine the stress intensity factor at a transient time of 20 minutes at various angular positions around the crack front of a 4.25 inch ($a/w=0.50$) deep circumferentially oriented semieliptical finite length flaw with an aspect ratio of 10:1 located on the inner-surface of a clad pressure vessel that is subjected to the PTS benchmarking thermal transient and a constant pressure of 1.0 ksi. The cladding thickness is 0.156 in. The heat transfer coefficient is constant and is equal to 500 BTU/hr-ft$^2$-OF.

Figure 13 illustrates the FAVOR and ABAQUS direct 3-D finite-element solutions. The above six problems provide verification that FAVOR produces $K_I$ solutions that are within 1–2% of those obtained by direct 3-D ABAQUS finite-element solutions. FAVOR $K_I$ solutions for semieliptical flaws have also been compared with results of other methodologies in Ref. 18.
Fig. 13 Comparison of FAVOR and ABAQUS direct 3-D finite-element solution for $K_I$ of circumferentially oriented 4.25 inch-deep flaw with an aspect ratio of 10:1 at various angular positions around the crack front at a transient time of 20 minutes.

SUMMARY
A database of SIFICs has been generated for a range of axially and circumferentially oriented finite length semielliptical flaws residing on the inner-surface of a clad RPV. This database of SIFICs was generated by the NQA-1 certified general finite element multidimensional ABAQUS code. The SIFICs are applicable to RPVs with an internal radius (Ri) to wall thickness (w) ratio of 10. This vessel geometry corresponds to a high percentage of the commercial pressurized water reactor vessels in the United States. This database of SIFICs has been compared with those of other investigators and found to be in good agreement, thus providing mutual validation. The ABAQUS-generated database of SIFICs has been implemented into the FAVOR code. FAVOR utilizes this SIFIC database and superposition to perform deterministic and probabilistic fracture analyses of clad embrittled RPVs. The implementation of this database of SIFICs into FAVOR has been verified to produce $K_I$ solutions that are within approximately 1.2% of those obtained by direct ABAQUS 3-D finite element solutions.

REFERENCES
(4) ASME Section XI, Task Group on Reactor Vessel Integrity Requirements, White Paper on Reactor Vessel Integrity Requirements for Level A and B Conditions, EPRI Project 2975-13, January 1993.


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