Nucleon-Nucleon Interactions

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Abstract. Nucleon-nucleon interactions are at the heart of nuclear physics, bridging the gap between QCD and the effective interactions appropriate for the shell model. We discuss the current status of NN data sets, partial-wave analyses, and some of the issues that go into the construction of potential models. Our remarks are illustrated by reference to the Argonne v_{18} potential, one of a number of new potentials that fit elastic nucleon-nucleon data up to 350 MeV with a χ^2 per datum near 1. We also discuss the related issues of three-nucleon potentials, two-nucleon charge and current operators, and relativistic effects. We give some examples of calculations that can be made using these realistic descriptions of NN interactions. We conclude with some remarks on how our empirical knowledge of NN interactions may help constrain models at the quark level, and hence models of nucleon structure.

1 Introduction

We have accumulated a great deal of empirical knowledge about the nucleon-nucleon (NN) interaction over the last sixty years. Substantial amounts of experimental data have been added in just the last decade. Partial-wave analyses have improved in sophistication to the point where the NN elastic scattering data below $E_{lab} = 350$ MeV can be fit with a $\chi^2 = 1$. There are also now several potential models that fit this data with comparable accuracy.

Realistic NN potentials are very complicated in structure, making nuclear many-body problems the most challenging in all of theoretical physics. Nevertheless, potential representations of the NN interaction can be used to directly study a wide variety of nuclear structure problems, including the ground and excited states of few-body nuclei, light closed-shell nuclei, nuclear matter and neutron stars (Wiringa 1993). In particular, essentially exact calculations of nuclei up to $A=8$ are now possible, using a variety of Faddeev, correlated hyperspherical harmonic, and quantum Monte Carlo methods. Reactions, including response to electron scattering and various electroweak processes of interest in astrophysics, can also be modelled microscopically at this level.

However, there are many phenomena in nuclear physics that are still far too complicated for present many-body theory to handle with bare interactions. Large or mid-shell nuclei, highly excited states, and nuclei at high spin are examples. These problems are best addressed in terms of effective interactions, which should in principle be derivable from the bare NN interaction. Knowledge gained in the exact solution of smaller nuclei should also be useful in this regime.

In turn, the NN interaction is itself an effective interaction between color singlet states, the residual of more complicated QCD effects. At long range, it

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may be described in terms of one-pion exchange, while at intermediate distances the exchange of multiple pions, with the possible excitation of nucleon resonances like the $\Delta(1232)$ becomes important. At short range, the exchange of heavier vector mesons, like the ρ and ω , may start to dominate. However, nucleons, mesons, and nucleon resonances have their own internal quark structure and this must come into play at some scale. Since we know empirically a great deal about NN interactions, we may be able to turn this knowledge to good use in modelling quark-quark interactions.

2 Data and Partial-Wave Analyses

Our first consideration in constructing realistic models for NN interactions is the status of NN data. There are more than 5,000 NN elastic scattering data up to $E_{lab} = 350$ MeV that have been accumulated since 1955. These include total cross sections, differential cross sections, polarizations, asymmetries, and transfer parameters of assorted types. About 40% is pp scattering data and the remainder is np data.

In addition there are bound-state properties of the deuteron, including the binding energy, asymptotic normalizations, radius, and magnetic and quadrupole moments. There are also a variety of reactions involving the deuteron, including the elastic structure functions and tensor polarization measured in electron scattering, photo- and electro-disintegration data, and $p(n, \gamma)d$ radiative capture data.

It is convenient to reduce this mass of data to a more manageable set of ≈ 200 phase shifts by performing a partial wave analysis. That is, the scattering amplitude is expanded in partial waves with phase shifts (and mixing parameters in coupled channels) deduced for each channel, typically up to $L = 5$ or 6. A partial wave analysis can be used to check the data for internal consistency; indeed, it is advisable to "weed out" bad data.

The most comprehensive partial-wave analysis of recent years is that carried out by the Nijmegen group for pp (Bergervoet *et al.* 1990) and np (Stoks *et al.* 1993) elastic data in the range $E_{lab} = 0 - 350$ MeV. As an example of data weeding, their procedure is illustrated in Table 1. They collected all the NN data published in regular journal articles between 1955 and 1992 and performed a global minimization within their chosen parametrization of the phase shifts. They rejected data that was more than three standard deviations off from their fit and then readjusted their parameters to the smaller data set. This resulted in the deletion of 15% of the pp data 28% of the np data. They then added as data to be fit the overall normalizations of many of the data sets, and floated a small number of these where the quoted experimental error added more than nine to the χ^2 . The final set has 1787 pp data and 2514 np data.

This weeding of the data may strike some as rather arbitrary (especially if you are an experimentalist whose data has been rejected!) but it is mechanically unbiased and there is no question that a number of data sets in the literature are

Table 1. Data selection in the Nijmegen PWA

	<i>pp</i>	<i>np</i>
observables	1947	3298
rejected in 3σ test	-291	-932
remainder (groups)	1656 (215)	2366 (211)
normalizations (floated)	+131 (22)	+148 (16)
total data to fit	1787	2514

flawed. In fact there is little difference between the Nijmegen data base and that maintained by the Virginia Polytechnic Institute and State University group in their SAID program.

Partial-wave analyses are typically made in either of two modes: 1) a single-energy analysis, where the data is grouped into a number of bins 25–50 MeV wide and treated as if it were at a single discrete energy, or 2) a multi-energy analysis, in which all the data is treated simultaneously, with some assumptions of underlying smoothness. The Nijmegen group has performed both single- and multi-energy analyses in their studies, achieving the impressive $\chi^2/N_{data} = 0.99$ for the multi-energy analysis. The latest update of the VPI&SU group (Arndt *et al.* 1992) also has both single- and multi-energy PWA's for *NN* data up to 1.6 GeV. Other recent PWA's include a single-energy analysis of *np* data between 142 and 800 MeV (Bugg and Bryan 1992), and a single-energy analysis of *NN* data below 160 MeV (Henneck 1993).

Single-energy analyses can show a large scatter as a function of energy. In this sense they tend to point out problem areas in the *NN* data where our knowledge is less complete and additional experiments might be valuable. The smooth energy dependence of the phase shifts produced in multi-energy analyses is more physically plausible, and not surprisingly, much easier to fit with a potential model. One of the more poorly-determined phases is the ϵ_1 mixing parameter, which is closely correlated to the 1P_1 phase shift. In Fig.1 we plot these parameters against each other for $E_{lab} = 50$ MeV, where there has been some recent controversy (Machleidt and Slaus 1994, Hammans *et al.* 1994) over the significance of the single-energy PWA. We use this as an illustration of the differences between different single- and multi-energy PWA's and what potential models can fit.

In Fig.1 the single-energy analyses of the Nijmegen and PSI groups are in reasonable agreement with each other, while the SAID analysis is noticeably different due to the retention of some old data that the other groups have discarded. While the single-energy analyses all have large error bars, the Nijmegen multi-energy analysis has a much smaller error as a consequence of considering simultaneously the data over a much larger range of energy. The older potential models that are shown lie in a band that includes the multi-energy point, but are in clear disagreement with the high values of the single-energy analyses. The new high-accuracy potentials which are not shown here all cluster around the

Fig. 1. Partial-wave analyses (crosses with error bars) and potential model fits (x) at 50 MeV

multi-energy point.

An important ingredient in partial wave analyses and NN potentials is the strength of the long-range one-pion exchange coupling. For many years, the accepted $NN\pi$ coupling constant was $f^2 = 0.0790$ (10) as determined from πN scattering data (Bugg, Carter, and Carter 1973). This value was used in many of the potential models of the '70s and '80s. The recent Nijmegen analyses have called this value into question by trying to determine the pion coupling constants and masses solely from the NN scattering data. They succeeded in obtaining a π^0 mass of 135.6 (13) MeV and a $\pi^\pm$ mass of 139.4 (10) MeV, in excellent agreement with accepted values. In the process of fitting these masses, they obtained somewhat weaker $NN\pi$ coupling constants: $f_p^2 = f_{pp\pi^0}^2 = 0.0745$ (6), $f_0^2 = f_{nn\pi^0} f_{pp\pi^0} = 0.0745$ (9), and $f_c^2 = f_{np\pi^-} f_{pn\pi^+} = 0.0748$ (3). This conclusion has caused considerable discussion in the literature. The VPI&SU group has subsequently reexamined πN data (Arndt, Workman, and Pavan 1994) and also obtained a lower value of $f^2 = 0.076$ (1), in agreement with the Nijmegen result. A recent study of NN elastic data from 210 to 800 MeV (Bugg and Machleidt 1995) obtained slightly larger couplings, corresponding to $f^2 = 0.077$ (2), right in between, but with a large enough error bar to touch, both the old and new values. The newest NN potential models, discussed below, which are able to obtain very

good fits to the NN scattering data, all use the new lower value of $f^2 = 0.075$.

3 Potential Modelling

In constructing a potential model for NN interactions there are a number of choices that have to be made. The first decision is what the intended purpose of the model is to be. It might be meant for use in many-body problems, in which case the accuracy with which it reproduces the two-body data and the ease with which it can be used in calculations are the primary considerations. Alternatively, the goal may be to elucidate the role of subnucleonic degrees of freedom in the NN interaction, so the inclusion of known mesons and nucleon resonances may become an important feature. Ideally both aspects would be included, but in practice there are always compromises.

Broadly speaking, potentials can be categorized as either phenomenological or meson-theoretic models, with the former emphasizing the fit to data and use in larger systems, while the latter stress the subnucleon aspects. All models have a one-pion-exchange character at long range, which is absolutely essential to fit high partial-wave data; thus the labels phenomenological or meson-theoretic refer to the treatment of shorter-range interactions. Examples of phenomenological potentials are the Reid soft-core (Reid 1968), Urbana v_{14} (Lagaris and Pandharipande 1981) and Argonne v_{14} (Wiringa, Smith, and Ainsworth 1984) potentials. Meson-theoretic potentials include the Nijmegen (Nagels, Rijken, and de Swart 1978), Paris (Lacombe *et al.* 1980), and Bonn (Machleidt, Holinde, and Elster 1987) models. The recent, highly-accurate potentials include several models from the Nijmegen group (Stoks *et al.* 1994), the Argonne v_{18} potential (Wiringa, Stoks, and Schiavilla 1995), and CD-Bonn (Machleidt, Sammarruca, and Song 1996).

For phenomenological potentials, an important choice is the general structure to be used, which may be either a partial-wave or an operator construct. Operator constructs are based on the demonstration (Okubo and Marshak 1958) that the most general form satisfying translational, rotational, Galilean, and time-reversal invariance, and parity conservation can be written as:

$$V_{NN} = V_c + V_\sigma \sigma_i \cdot \sigma_j + V_t S_{ij} + V_{ls} (\mathbf{L} \cdot \mathbf{S})_{ij} + V_q Q_{ij} + V_{\sigma p} \sigma_i \cdot \mathbf{p} \sigma_j \cdot \mathbf{p} , \quad (1)$$

$$S_{ij} = 3\sigma_i \cdot \hat{\mathbf{r}} \sigma_j \cdot \hat{\mathbf{r}} - \sigma_i \cdot \sigma_j , \quad (2)$$

$$Q_{ij} = \sigma_i \cdot \mathbf{L} \sigma_j \cdot \mathbf{L} + \sigma_j \cdot \mathbf{L} \sigma_i \cdot \mathbf{L} . \quad (3)$$

The individual V_x here can be functions of $\mathbf{r}$, $\mathbf{p}$, or $\mathbf{L}$, and can have a very general isospin dependence:

$$\begin{aligned} V_x = & V_x^0(\mathbf{r}, \mathbf{p}, \mathbf{L}) + V_x^r(\mathbf{r}, \mathbf{p}, \mathbf{L}) \tau_i \cdot \tau_j \\ & + V_x^T(\mathbf{r}, \mathbf{p}, \mathbf{L}) T_{ij} \\ & + V_x^{r^z}(\mathbf{r}, \mathbf{p}, \mathbf{L}) (\tau_i^z + \tau_j^z) \\ & + V_x^M(\mathbf{r}, \mathbf{p}, \mathbf{L}) (\tau_i^z - \tau_j^z) (\sigma_i - \sigma_j) \cdot \mathbf{L} . \end{aligned} \quad (4)$$

The first term above is charge-independent (CI), so that pp , np , nn interactions in the same state of total isospin T are equal. The V_x^T term is charge-dependent (CD) and differentiates np from pp and nn scattering, while the V^{τ^3} term is charge-symmetry-breaking (CSB) and splits pp from nn interactions. These are often referred to as Class II and Class III forces, respectively. The last term is an example of a Class IV CSB force, which mixes isospin in np interactions. The approximate isospin symmetry of nuclear forces makes the V_x^0 and V_x^T terms dominant, but there is clear evidence for small V_x^T and V^{τ^3} contributions to the nuclear force, while electromagnetic forces give contributions of all kinds.

A partial-wave potential like Reid has the potential specified separately in each $^{2S+1}L_J$ channel, but different channels are not necessarily connected by a consistent operator structure. For example, in the Reid soft-core potential, the 3P_2 – 3F_2 coupled channels are specified by a $V_c + V_t S_{ij} + V_{ls} (\mathbf{L} \cdot \mathbf{S})_{ij}$ combination, but these same V_c , V_t , and V_{ls} do not produce the potential in 3P_0 and 3P_1 channels. This kind of channel-by-channel formulation implies some nonlocality in the potential which is not easily characterized.

In many-body applications like Faddeev calculations of the three- and four-body nuclei, partial-wave expansions for the wave function are common, and this kind of potential is straightforward to use. However, for a number of variational and quantum Monte Carlo methods in configuration space that do not use partial-wave expansions, potentials based on a consistent operator structure are far easier to use. Of course, operator-based potentials can easily be projected into partial waves as needed, and all channels are specified, whereas partial-wave potentials are truncated at some J . Both operator and partial-wave constructs typically have ≈ 40 parameters to get a reasonable fit to the data.

In meson-theoretic potentials, one has several options for treating the shorter-range parts of the interaction. The simplest is a one-boson-exchange (OBE) format such as the original Nijmegen potential and some of the Bonn models. The Nijmegen potential is constructed using the exchange of π , η , η' , ρ , ω , ϕ , ϵ , and S^* mesons, plus Pomeron, f , f' , and A_2 contributions. Its parametrization includes the various meson couplings (the masses are taken from particle data), the Pomeron mass, and a cutoff factor for a total of about 12 parameters to be fit, which is typical of meson-theoretic models.

A common problem among meson-theoretic potentials is how to represent the observed scalar attraction at intermediate ranges. The Pomeron provides this attraction in the Nijmegen model. A more common treatment in OBE models is to introduce a fictitious σ meson, a scalar of mass ≈ 500 MeV, as is done in some of the Bonn models. The microscopic source of the intermediate-range attraction is believed to be due to two-pion-exchange (TPE) with the possible excitation of the intermediate nucleons to $\Delta(1232)$ resonance states. Other multiple-meson exchanges and nucleon resonances can also contribute. The full Bonn model treats these terms in a perturbative manner, drawing the leading-order Feynman diagrams and attempting to integrate over the intermediate states. An alternative followed by the Paris group is to use information on $\pi\pi$ and πN scattering and dispersion techniques. Both methods inevitably lead to a certain amount of

Table 2. Potential χ^2/N_{data} comparison for 5–350 MeV

Potential	pp	np	Comments
Reid soft-core (1968)	2.5	(10.7)	Partial-wave ($J \leq 2$) local r-space
Nijmegen (1978)	2.0	(?)	OBE non-local r- & k-space
Paris (1980)	2.2	(3.8)	OBE+TPE $\Rightarrow$ Operator non-local r- & k-space
Argonne v_{14} (1984)	(6.9)	2.1	Operator local r-space
Bonn (1987)	(13.0)	3.0	OBE+TPE E-dependent non-local k-space
Nijmegen (1993)	1.9	1.9	OBE CD non-local r- & k-space
Nijm I (1994)	1.00	1.05	Partial-wave CD non-local r- & k-space
Nijm II (1994)	1.00	1.04	Partial-wave CD local r- & k-space
Argonne v_{18} (1995)	1.10	1.06	Operator CD-CSB local r-space
CD Bonn (1996)	1.03	1.03	OBE $\Rightarrow$ Partial-wave CD-CSB non-local k-space

energy-dependence and non-locality not present in the OBE models. To make a tractable potential for use in many-body calculations, the energy-dependence may be replaced by a momentum-dependence that is fit to the original model, as was done in the parametrization of the Paris potential.

A final decision in constructing a potential model, whether phenomenological or meson-theoretic, is what data to fit. Through the 1980's the charge-dependence of the nuclear force was neglected and different groups chose to fit either pp or np data in $T = 1$ channels. The pp data is easier to obtain experimentally and has smaller error bars. However there has generally been more np data available and not having to treat the long-range Coulomb interaction makes life a little easier for the potential builder. With the growing size of the NN database and the increasing accuracy of many-body calculations, it has become more important to fit both kinds of data, which necessarily introduces CD into the nuclear potential. The Nijmegen group emphasized this point (Stoks and de Swart 1993, 1995) by comparing a number of potentials to their database and PWA. We show a selection of their results in Table 2, updated to include the new high-accuracy potentials of the last few years.

In looking at the table we see that through the 1980's, the best potentials achieved a χ^2 of ≈ 2 for the data, either pp or np , that the builders chose to fit. However, potentials fit to pp data did not reproduce np data well and *vice versa*. Even when CD is added to a OBE model, as in the case of the updated Nijmegen (1993) model, the χ^2 could not be improved significantly beyond 2 within the limited set (≈ 12) parameters available. The highly accurate potentials of the last few years incorporate a significantly larger number of parameters, not so different from the number required in the Nijmegen PWA itself.

A final consideration is what amount of deuteron data to fit. The binding energy, E_d , is fit in all models, but there are other static properties, such as the asymptotic normalizations A_S and η_d , and the electromagnetic radii, r_d , magnetic moment, μ_d , and quadrupole moment, Q_d , which could be fit. The electromagnetic terms suffer from having two-body charge and current operator

contributions from meson-exchange and relativistic effects that are not uniquely specified by the potential. Ideally one would construct such operators simultaneously with the potential model, but this has not been done yet as part of the data fitting procedure.

4 The Argonne v_{18} potential

As an example of some of the considerations that go into constructing an NN potential, I will discuss in more detail the structure of the new Argonne v_{18} model and some of the "philosophy" behind it. Our primary goal was to produce an accurate potential fit to the latest data that could be easily used in configuration-space variational and quantum Monte Carlo calculations of finite nuclei and nucleon matter. We want to use the two-body potential as part of a many-body Hamiltonian:

$$H = \sum_i K_i + \sum_{i < j} v_{ij} + \sum_{i < j < k} V_{ijk} . \quad (5)$$

To fit both pp and np data it was clear that we would have to introduce some CD into the force. We are also interested in looking at CSB in finite nuclei (the Nolen-Schiffer anomaly) so we decided to additionally fit the limited nn data – the singlet scattering length and effective range. To pull the strong-interaction CSB component out of the data, it is essential to first take care of all the electromagnetic contributions, including not just Coulomb, but Darwin-Foldy, vacuum polarization, and magnetic moment terms. This had been done in the Nijmegen PWA, where it was essential for fitting low-energy data accurately, but it had not been done in NN potential models before.

After the electromagnetic interaction, the primary source of CD is the effect of $\pi^\pm - \pi^0$ mass differences on the OPE potential. We decided to treat these terms carefully, and to adopt the weaker $f_{NN\pi}^2$ coupling constant indicated by the Nijmegen PWA. However, we were not particularly interested in further microscopic details of the NN force, so we did not attempt to impose a one-boson-exchange or other meson-theoretic model for the short-range structure. We did opt for an operator structure that could immediately be used in our many-body applications where we already have a substantial computational investment. For simplicity, we adopted the general operator structure of the earlier Urbana and Argonne v_{14} models, but found we had to increase the number of functions and parameters to get a good fit to the data, in addition to adding extra CD and CSB operators. One requirement we imposed was that the potential be finite with zero slope at the origin, to represent the finite size of the nucleons. This necessitated form factors on all terms including the electromagnetic ones. We also required that the tensor potential go to zero at the origin.

A final consideration is that we also want to develop the three-body force and two-body charge and current operators in a consistent fashion. This means that the long-range two-pion-exchange three-nucleon force, $V_{ijk}^{2\pi}$, is constructed with the same radial functions and cutoffs as the v_{ij}^π . We also investigated the

Fig.2. Phase shifts for np , nn , and pp scattering in 1S_0 channel from Argonne v_{18} potential compared to various PWA

corresponding CD of $V_{ij}^{2\pi}$, although it turns out to be quite weak. The charge and current operators are also constructed in a manner consistent with the v_{ij} following a prescription (Schiavilla, Pandharipande, and Riska 1989,1990) that preserves the continuity equation and their contributions to the deuteron static moments and structure functions calculated. However, we did not proceed far enough to incorporate these calculations in the fitting procedure, so only the deuteron binding energy was used as a constraint.

The final potential is written as a sum of electromagnetic, OPE, and remaining short-range terms:

$$V_{NN} = v^\gamma + v^\pi + v^R. \quad (6)$$

The electromagnetic terms include one- and two-photon exchange, Darwin-Foldy, vacuum polarization, and magnetic moment contributions:

$$v^\gamma(pp) = v_{C1}(pp) + v_{C2} + v_{DF} + v_{VP} + v_{MM}(pp), \quad (7)$$

$$v^\gamma(np) = v_{C1}(np) + v_{MM}(np), \quad (8)$$

$$v^\gamma(nn) = v_{MM}(nn), \quad (9)$$

where all terms have form factors for finite size effects, derived from the standard

Fig. 3. Phase shifts for np , nn , and pp scattering in 3P_0 channel from Argonne v_{18} potential compared to various PWA

nucleon dipole forms:

$$G_E^p = \frac{G_M^p}{\mu_p} = \frac{G_M^n}{\mu_n} = (1 + q^2/b^2)^{-2}, \quad (10)$$

$$G_E^n = \beta_n q^2 (1 + q^2/b^2)^{-3}. \quad (11)$$

The value of b in the form factors is taken from the proton charge radius, and β_n in G_E^n is the experimentally measured slope at $q = 0$. The np Coulomb term $v_{C1}(np)$ arises from the overlap of the nucleon form factors, and while small, it is comparable to the v_{C2} , v_{DF} , and v_{VP} terms. Details of the electromagnetic terms can be found in the paper (Wiringa, Stoks, and Schiavilla 1995).

The OPE terms are given by:

$$v^\pi(pp) = f_{pp}^2 v_\pi(m_{\pi^0}), \quad (12)$$

$$v^\pi(np) = f_{pp} f_{nn} v_\pi(m_{\pi^0}) + (-)^{T+1} 2 f_c^2 v_\pi(m_{\pi^\pm}), \quad (13)$$

$$v^\pi(nn) = f_{nn}^2 v_\pi(m_{\pi^0}), \quad (14)$$

where T is the isospin and

$$v_\pi(m) = \left(\frac{m}{m_s}\right)^2 \frac{1}{3} mc^2 [Y_\mu(r) \sigma_i \cdot \sigma_j + T_\mu(r) S_{ij}]. \quad (15)$$

Fig.4. Phase shifts for np , nn , and pp scattering in ϵ_1 channel from Argonne v_{18} potential compared to various PWA

Here $Y_\mu(r)$ and $T_\mu(r)$ are the usual Yukawa and tensor functions with the exponential cutoff of the Urbana and Argonne v_{14} models

$$Y_\mu(r) = \frac{e^{-\mu r}}{\mu r} (1 - e^{-cr^2}) , \quad (16)$$

$$T_\mu(r) = \left(1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2}\right) \frac{e^{-\mu r}}{\mu r} (1 - e^{-cr^2})^2 , \quad (17)$$

where $\mu = mc/\hbar$. The scaling mass m_s makes the coupling constant dimensionless, and is taken to be the charged-pion mass, $m_{\pi^\pm}$. As mentioned above, the Nijmegen PWA finds very little difference between the different coupling constants, so we take $f_{pp} = -f_{nn} = f_c \equiv f$, and adopt their value $f^2 = 0.075$ (although we did some searching on this value to confirm it was optimal for this model). v^R term, and was neglected in this model.

An additional term that could have been added to this OPE expression is the short-range δ -function (spread out with a form factor) that is generated when a $(\sigma_i \cdot k)(\sigma_j \cdot k)$ term is Fourier transformed. Such a term would get mixed up with the short-range phenomenology of the

The remaining part of the potential is simply written as a phenomenological

Fig. 5. Phase shifts for np , nn , and pp scattering in 1P_1 channel from Argonne v_{18} potential compared to various PWA

operator structure:

$$v_{ST}^R(NN) = v_{ST,NN}^e(r) + v_{ST,NN}^t(r)S_{12} + v_{ST,NN}^{ls}(r)L \cdot S \\ + v_{ST,NN}^{l2}(r)L^2 + v_{ST,NN}^{ls2}(r)(L \cdot S)^2. \quad (18)$$

Each of these terms is given the general form

$$v_{ST,NN}^i(r) = I_{ST,NN}^i T_\mu^2(r) + [P_{ST,NN}^i + \mu r Q_{ST,NN}^i + (\mu r)^2 R_{ST,NN}^i] W(r), \quad (19)$$

with μ the average pion mass and $T_\mu(r)$ given by (17). Thus the $T_\mu^2(r)$ term has the range of a two-pion-exchange force. The $W(r)$ is a Woods-Saxon function which provides the short-range core:

$$W(r) = [1 + e^{(r-r_0)/a}]^{-1}. \quad (20)$$

The four sets of constants $I_{ST,NN}^i$, $P_{ST,NN}^i$, $Q_{ST,NN}^i$, and $R_{ST,NN}^i$ are parameters to be fit to data, subject to the regularization condition mentioned above:

$$v_{ST,NN}^t(r=0) = 0, \quad (21)$$

$$\frac{\partial v_{ST,NN}^{i \neq t}}{\partial r} \Big|_{r=0} = 0, \quad (22)$$

Table 3. Scattering lengths and effective ranges for Argonne v_{18} , with and without the electromagnetic terms

	experiment	v_{18}	w/o v^7
$^1a_{pp}$	-7.8063 (26)	-7.806	-17.164
$^1r_{pp}$	2.794 (14)	2.788	2.865
$^1a_{nn}$	-18.5 (5)	-18.487	-18.818
$^1r_{nn}$	2.8 (1)	2.840	2.834
$^1a_{np}$	-23.749 (8)	-23.732	-23.084
$^1r_{np}$	2.81 (5)	2.697	2.703
$^3a_{np}$	5.424 (3)	5.419	5.402
$^3r_{np}$	1.760 (5)	1.753	1.752

which effectively constrains one of the parameters.

We explored a number of different parametrizations fitting to the phase shifts of the Nijmegen PWA. When we settled on the final form, we fixed the f^2 , the cutoff factor, c , in the OPE $Y_\mu(r)$ and $T_\mu(r)$ functions, and the r_0 and a parameters in $W(r)$, and then adjusted some forty $I_{ST,NN}^i$, $P_{ST,NN}^i$, and $R_{ST,NN}^i$ parameters by making a direct fit to the Nijmegen NN database, rather than the phase shifts. The 1787 pp and 2514 np elastic scattering data from 0–350 MeV were fit with a $\chi^2/N_{data} = 1.09$. In addition we fit the deuteron binding energy $E_d = 2.224575$ MeV, and, by a slight alteration of the 1S_0 pp potential, the nn scattering length and effective range. As a check, we also compared our elastic scattering to the VPI&SU SAID database, and got the somewhat larger, but still very respectable $\chi^2/N_{data} = 1.34$.

Some of the more interesting phase shifts are shown in Figs. 2-5. The greatest amount of CD and CSB occurs in the 1S_0 channel, as seen in Fig.2, where we plot the pp , np , and nn phases of the potential and compare to the recent PWA analyses. The bulk of the difference between the pp and nn phases is due to the electromagnetic interaction, whereas the difference between the nn and np phases is primarily a measure of the CD interaction, with roughly equal amounts coming from the OPE and the short-range part of the interaction. The next greatest amount of CD comes in the 3P_0 channel, as seen in Fig.3; here the data are not good enough to identify more CD than just that coming from OPE. The ϵ_1 mixing parameter, which is a sensitive indicator of the strength of the tensor force, is given in Fig.4. Here the various single-energy PWA's show a wide variation, but the Nijmegen multi-energy PWA has a quite smooth behavior, and all the new high-accuracy potentials are fairly close to the latter. Finally in Fig.5, we show the 1P_1 phases, which are closely correlated with ϵ_1 as discussed earlier.

Scattering lengths and effective ranges produced by Argonne v_{18} are given in Table 3. The experimental values are reproduced fairly well. An interesting point to note is the contribution of the v^7 terms; the importance of the Coulomb terms to $^1a_{pp}$ are well known, but there is also a significant contribution to $^1a_{np}$, coming mostly from the magnetic moment interaction, with some part from the $v_{C1}(np)$ term. Deuteron properties are displayed in Table 4, including meson-

Table 4. Deuteron properties for Argonne v_{18} , including meson-exchange contributions

	experiment	v_{18} + MEC	
E_d (MeV)	-2.224575 (9)	-2.224575	
A_S (fm ^{1/2})	0.8781 (44)	0.8850	
η_d	0.0256 (4)	0.0250	
r_d (fm)	1.953 (3)	1.967	
μ_d (μ_0)	0.857406 (1)	0.847	0.871
Q_d (fm ²)	0.2859 (3)	0.270	0.275
P_D (%)		5.76	

exchange contributions (MEC) to the electromagnetic moment operators. The electromagnetic interaction contributes +18 keV to E_d , 14 keV from $v_{MM}(np)$ and 4 keV from $v_{C1}(np)$.

The final step in constructing the v_{18} is the projection from specific ST, NN channels to an operator format. The strong interaction part can be written as a sum of 18 operators:

$$v_{ij} = \sum_{p=1,18} v_p(r_{ij}) O_{ij}^p. \quad (23)$$

(It is for this reason that the potential is called a “ v_{18} ” model, although the electromagnetic interaction adds an additional four operators, so it might have been called a “ v_{22} ”.) Here the first fourteen operators are the same charge-independent ones used in the Argonne v_{14} potential and are given by

$$O_{ij}^{p=1,14} = 1, \tau_i \cdot \tau_j, \sigma_i \cdot \sigma_j, (\sigma_i \cdot \sigma_j)(\tau_i \cdot \tau_j), S_{ij}, S_{ij}(\tau_i \cdot \tau_j), \mathbf{L} \cdot \mathbf{S}, \mathbf{L} \cdot \mathbf{S}(\tau_i \cdot \tau_j), \\ L^2, L^2(\tau_i \cdot \tau_j), L^2(\sigma_i \cdot \sigma_j), L^2(\sigma_i \cdot \sigma_j)(\tau_i \cdot \tau_j), (\mathbf{L} \cdot \mathbf{S})^2, (\mathbf{L} \cdot \mathbf{S})^2(\tau_i \cdot \tau_j) \quad (24)$$

The four additional operators break charge independence and are given by

$$O_{ij}^{p=15,18} = T_{ij}, (\sigma_i \cdot \sigma_j) T_{ij}, S_{ij} T_{ij}, (\tau_{zi} + \tau_{zj}), \quad (25)$$

where $T_{ij} = 3\tau_{zi}\tau_{zj} - \tau_i \cdot \tau_j$, is the isotensor operator, defined analogous to the S_{ij} operator. The first three of these operators provide charge-dependence, while the latter provides charge-symmetry-breaking. The projections can be made from the appropriate CI, CD, and CSB combinations:

$$v_{S1} = v_{S1}^{CI} + v_{S1}^{CD} T_{ij} + v_{S1}^{CSB} (\tau_{zi} + \tau_{zj}) \quad (26)$$

where

$$v_{S1}^{CI} = \frac{1}{3}(v_{S1,pp} + v_{S1,nn} + v_{S1,np}), \quad (27)$$

$$v_{S1}^{CD} = \frac{1}{6} \left[\frac{1}{2}(v_{S1,pp} + v_{S1,nn}) - v_{S1,np} \right], \quad (28)$$

$$v_{S1}^{CSB} = \frac{1}{2}(v_{S1,pp} - v_{S1,nn}). \quad (29)$$

Overall, the resulting Argonne v_{18} gives a pretty good fit to both pp and np data. The $\chi^2/N_{data} = 1.09$ is not quite as good as the several new Nijmegen or CD-Bonn models, which are different in each partial wave, but the latter cannot be expressed in the closed operator form that our quantum Monte Carlo many-body methods (discussed below) require. There remain several interesting questions that cannot be resolved with the present NN data. The CD of the interaction in S-waves is well established, but poorly constrained in P-waves; consequently the relative strength of T_{ij} and $(\sigma_i \cdot \sigma_j)T_{ij}$ terms is not pinned down. By fitting the nn scattering length we have also introduced CSB into the strong interaction, but again, this is only an S-wave quantity, and there is no P-wave data. We chose to use only a $(\tau_{zi} + \tau_{zj})$ CSB term, but one may reasonably expect $(\sigma_i \cdot \sigma_j)(\tau_{zi} + \tau_{zj})$ and $S_{ij}(\tau_{zi} + \tau_{zj})$ terms in addition on the basis of meson-theoretic effects such as ρ - ω mixing. However, any careful treatment of CD and CSB effects should only be made after the well-known electromagnetic contributions are included.

Perhaps the least satisfactory feature of this model is the low value for the deuteron quadrupole moment, Q_d . The MEC provide some correction in the right direction, but are too small. This problem is also present in the new Nijmegen potentials and CD-Bonn, and may be attributable to the small value of f^2 that all these models use. The error in μ_d is less disturbing because there are two-body current contributions to the electromagnetic operators that are not constrained by v_{ij} through the continuity equation, and these terms can probably be adjusted to fit the data. In future work, it would be nice to develop the two-body potential and charge and current operators simultaneously and fit both the NN scattering data and the considerable amount of deuteron data together.

5 Many-Body Calculations

A major reason for constructing new highly-accurate NN potentials is the great progress that has been made in many-body calculations in recent years. In the last ten years several groups have developed Faddeev calculations for the trinucleons with an accuracy of 10 keV or better for the binding energy (Chen *et al.* 1985, Glöckle, Witala, and Cornelius 1990). These methods have since been extended to Faddeev-Yakubovsky studies of the alpha-particle ground state (Glöckle and Kamada 1993). Another recent development is the correlated hyperspherical harmonic (CHH) method (Kievsky, Viviani, and Rosati 1993; Viviani, Kievsky, and Rosati 1995) which also gives three- and four-nucleon ground state energies with very high precision. The Faddeev and CHH methods have also made significant progress in describing problems such as nd and pd elastic scattering (Hüber *et al.* 1995).

Another line of attack has been the development of variational Monte Carlo (VMC) methods with ever-more sophisticated trial functions (Wiringa 1991) and the Green's function Monte Carlo method (GFMC) which can stochastically evolve these trial functions to the exact ground or lowest excited state of a given

quantum number (Carlson 1988). The advantage of these quantum Monte Carlo methods is that extension to larger nuclei is relatively straightforward, and has been made successfully for $A = 5-8$ nuclei (Pudliner *et al.* 1995, 1996). A cluster-expansion VMC has also been developed for larger nuclei such as ^{16}O (Pieper, Wiringa, and Pandharipande 1992) and it is hoped that a cluster-expansion adaption of GFMC will be forthcoming.

All these calculations, which have been made possible by the tremendous advances in computer technology in the last decade, give us an unprecedented chance to understand nuclear structure at the level of the NN interaction. The accuracy of these methods demands accurate NN potentials. It also allows us to examine small effects such as CD and CSB in many-body systems at a far more meaningful level than heretofore possible.

One challenge is immediately obvious when we calculate the binding energy of ^3H : the realistic NN potentials do not reproduce the observed value of 8.48 MeV when used in the nonrelativistic Schrödinger equation. The local potentials Argonne v_{18} , Nijm-II, and Reid'93 all give a binding energy of 7.62 ± 0.01 MeV (Friar *et al.* 1993). The nonlocal Nijm-I gives a little more binding at 7.72 MeV, while CD-Bonn, with a more extensive nonlocality, gives a reported 8.00 MeV (Machleidt, Sammarruca, and Song 1996).

In principle one might easily expect the NN potential, as an effective force between nucleons with a composite quark substructure, to be nonlocal. Unfortunately, NN elastic scattering data does not tell us anything about how much nonlocality the NN potential should have. In addition, relativistic effects can be expected to contribute at the level of a few hundred keV to the trinucleon binding, but at present there is no consensus on the sign of the relativistic contribution. Using a relativistic three-dimensional Blankenbecler-Sugar formalism in the trinucleon, the ^3H binding increases to 8.19 MeV for CD-Bonn, a result consistent with earlier work (Rupp and Tjon 1992). However, studies based on relativistic quantum mechanics have consistently found repulsive contributions of order 0.3 MeV in the triton (Glöckle, Lee, and Coester 1986, Forest *et al.* 1995). Much work remains to be done on these issues.

Because of the composite nature of the nucleon and the corresponding nucleon excitation spectrum we also expect there to be nontrivial three-nucleon ($3N$) forces. Models for the $3N$ potential have been discussed for nearly forty years. It was realized early on that they could contribute to the effective spin-orbit splitting in nuclei, and the first models made use of the idea of excitation of an intermediate $\Delta(1232)$ state (Fujita and Miyazawa 1957). A more sophisticated treatment using knowledge of πN scattering and PCAC was made in the Tucson-Melbourne model (Coon *et al.* 1979).

For the current VMC and GFMC calculations we use one in a series of Urbana $3N$ potentials called model IX, which is constructed with long-range two-pion-exchange and a shorter-range phenomenological piece:

$$V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^R, \quad (30)$$

Table 5. Energy of $A = 3, 4$ nuclei for Argonne v_{18} + Urbana IX Hamiltonian

	${}^3\text{H}$	${}^3\text{He}$	${}^4\text{He}$
Experiment	-8.482	0.764	-28.296
Argonne v_{18}	-7.61 (3)	0.739	-24.09 (10)
($npT = 1$)	-7.79		-24.69
($ppT = 1$)	-7.44		-23.69
+ Urbana IX	-8.47 (2)	0.757	-28.32 (12)
K	51. (1)	0.014	118. (2)
v_{ij}	-58. (1)	0.743	-142. (2)
(v^γ)	0.04	0.677	0.88
(v^π)	-44.1 (5)	0.	-107. (1)
(v^R)	-14. (1)	0.066	-36.
V_{ijk}	-1.20 (3)	0.	-6.6 (3)
($V^{2\pi}$)	-2.25 (6)	0.	-12.6 (3)
(V^R)	1.05 (3)	0.	6.0 (2)

where

$$V_{ijk}^{2\pi} = A_{2\pi} \sum_{cyclic} \{X_{ij}, X_{jk}\} \{\tau_i \cdot \tau_j, \tau_j \cdot \tau_k\} + \frac{1}{4} [X_{ij}, X_{jk}] [\tau_i \cdot \tau_j, \tau_j \cdot \tau_k], \quad (31)$$

$$X_{ij} = f^2 \frac{1}{3} mc^2 [Y_\mu(r) \sigma_i \cdot \sigma_j + T_\mu(r) S_{ij}], \quad (32)$$

and

$$V_{ijk}^R = \sum_{cyclic} U_0 T_\mu^2(r_{ij}) T_\mu^2(r_{jk}). \quad (33)$$

The $Y_\mu(r)$ and $T_\mu(r)$ are the same functions given in (16)-(17). The overall strengths $A_{2\pi}$ and U_0 are adjusted to reproduce the ${}^3\text{H}$ binding energy and to give a reasonable saturation density in variational calculations of nuclear matter. In general, the $V_{ijk}^{2\pi}$ is attractive at all densities, and gives the extra binding needed in light nuclei, while the V_{ijk}^R is made repulsive and has a more rapid density dependence, helping to lower the saturation density in nuclear matter which is too large with NN potentials only. The shorter-range piece could in principle have extensive spin- and isospin-dependence, but we can no more derive the short-range character of $3N$ forces from first principles than we can for the NN force.

The binding energy of $A = 3, 4$ nuclei using the Argonne v_{18} with or without Urbana IX is shown in Table 4. The second line gives the energy (with MC statistical error) calculated for Argonne v_{18} alone; the difference with experiment of 0.9 MeV in ${}^3\text{H}$ and 4.2 MeV in ${}^4\text{He}$ must be made up by some combination of nonlocality, relativistic effects, and many-nucleon forces. In the present work, we assume all the difference is due to a V_{ijk} , with the hope that the short-range part may be mocking up some of the nonlocal and relativistic effects. The third (fourth) line gives the energy obtained if we use only the np (pp) potential for $T = 1$ states. This variation of 0.35 MeV in ${}^3\text{H}$ and 1.0 MeV in ${}^4\text{He}$ is a reflection

Fig. 6. Spectrum for $A \leq 7$ nuclei calculated using variational and Green's function Monte Carlo methods

of the amount of CD in the NN force, and is a sizeable fraction of the amount we attribute to V_{ijk} , hence the necessity of getting the CD correct.

The total energy for the full Hamiltonian and the breakdown into kinetic, NN and $3N$ potential terms is given on subsequent lines. It is interesting to note that a very large fraction of the total v_{ij} comes from OPE, and so to the dominant part of V_{ijk} is from TPE. The total V_{ijk} contribution is a significant fraction of the net binding, but it is small compared to the total v_{ij} , so we may hope that any four-nucleon force, which should exist in principle, will be negligible. The second column of Table 5 gives the ${}^3\text{He}$ – ${}^3\text{H}$ energy difference, which is a measure of the CSB in the NN force. This difference is fairly well explained within the present force model, especially when considering that the experimental uncertainty in ${}^1a_{nn}$ is large enough to cover a 20% correction to the strong-interaction CSB term. Basically, we can say the ${}^3\text{He}$ – ${}^3\text{H}$ energy difference is completely consistent with the difference in low-energy pp and nn scattering lengths.

We conclude this section with two results from VMC and GFMC studies of

Fig. 7. Calculated elastic and transition form factors in ${}^6\text{Li}$

light p-shell nuclei. We have calculated the ground and low-lying excited states for $A=5-7$ nuclei with the same Argonne v_{18} + Urbana IX Hamiltonian (Pudliner *et al.* 1995, 1996). The results of these calculations are summarized in the spectrum shown in Fig. 6. The VMC and GFMC calculations have an upper-bound property (within their statistical error bars, which are shown by shaded boxes) for the lowest state of given $J^\pi; T$ quantum numbers. The VMC calculations for $A=3,4$ nuclei are 2–3% above the exact results given by Faddeev, CHH, or GFMC, but as A increases, they seem to get progressively worse. However,

they do get the correct ordering and a reasonable magnitude for the splitting of states for $A=5-7$. The GFMC calculations, which agree very well with Faddeev-Yakubovsky and CHH methods for $A \leq 4$, give a significant improvement over the VMC calculations in $A=5-7$. It appears that the $A = 6$ nuclei are underbound by about 1 MeV, and the $A = 7$ nuclei by about 2 MeV with the present Hamiltonian. This may be attributable our neglect of nonlocal and/or relativistic effects, but most likely to the overly simple form we have used for V_{ijk}^R . We are now investigating whether simple generalizations of this term can improve the spectrum without introducing too many new force parameters. In any event, one can truly see the beginnings of shell structure in these nuclei, arising from a realistic bare two- and three-nucleon interactions.

Our second result is a VMC calculation of elastic and transition electromagnetic form factors in ${}^6\text{Li}$ shown in Fig. 7 (Schiavilla and Wiringa 1996). These have been made in both impulse approximation (IA) and with MEC. The comparison with data for the longitudinal elastic is excellent, as is the E2 transition to the 3^+ first excited state. The MEC corrections are small, but stand out noticeably in the first minimum where they significantly improve the fit to data. The M1 transition to the 0^+ isobaric analog state is also reproduced reasonably well. The elastic transverse form factor is good up to the first zero, but then is too large in the region of the second maximum. We plan to repeat these studies with the GFMC wave function to see if the present deficiencies can be removed. Nevertheless, these calculations, which have been done without effective charges as commonly needed in shell-model studies, give a remarkably good explanation of the electron scattering data.

6 Influence of NN Interactions on qq Models

Our empirical knowledge of NN interactions is substantial, and our theoretical understanding of the long-range part is long-established. One of the key features we know is that there is a strong tensor force, arising from pion exchange at long range and perhaps modified at short distances by heavier meson-exchange. This leads to a large angular dependence in $S, T = 1, 0$ states and induces a toroidal (dumbbell) shape in the deuteron $M_d = 0$ ($M_d = \pm 1$) at a femtometer scale (Forest *et al.* 1996). A graphic illustration is the set of constant-density surfaces shown in Fig. 8 for a polarized deuteron. These angular-dependent pair correlations persist in larger nuclei, and are independent of the particular NN potential chosen.

If one wanted to construct a constituent quark model for the deuteron, using six valence quarks (and GFMC methods may make such a calculation as feasible as ${}^6\text{Li}$ in the near future) then the quark-quark (qq) interaction V_{qq} must reproduce this feature of the deuteron. Plausible models would have to incorporate a tensor force *between quarks*. While there is a tensor component in one-gluon exchange (OGE) it is not nearly strong enough to provide the correlations observed in the deuteron. The answer may be to introduce a direct $qq\pi$ coupling,

Fig. 8. Surfaces of constant density $\rho = 0.24 \text{ fm}^{-3}$ (A & B) or $\rho = 0.08 \text{ fm}^{-3}$ (C & D) for polarized deuteron

as has been suggested on the basis of systematics in hadronic spectra (Glozman and Riska 1996), leading to a OPE between constituent quarks.

It is also interesting to note that these toroidal structures are predicted by classical Skyrme effective Lagrangians, which have been related to QCD in the limit of infinite colors (Verbaarschot *et al.* 1987). The Skyrme models obtain a toroidal deuteron of radius $\approx 1 \text{ fm}$ and binding of $\approx 150 \text{ MeV}$ is obtained. Realistic NN potentials are most attractive in the $S, T = 1, 0$ channel for $M_S = 0$ in a toroid of radius just under 1 fm , where the potential combination $v^c + 2v^t$

has a depth of 100–200 MeV, depending on the model. In the classical limit, this would lead to exactly the same kind of deuteron as predicted by the Skyrme field theory.

7 Future Developments

The field of NN interactions and potential modelling remains an active and dynamic area of research. The major accomplishments of the last few years are the superb partial-wave analysis of the Nijmegen group, and the construction of a number of new NN potentials, all with a $\chi^2/N_{data} \sim 1$. Coupled with the rapid progress in many-body calculations, we are now able to answer some interesting questions with unparalleled accuracy. The fact that local potentials that are phase-equivalent give the same binding energy for the triton, even though the details of the potentials are quite different, is a significant discovery.

Nevertheless, there remain many unsettled questions dealing with nonlocality, relativity, and many-nucleon forces. Most importantly, the microscopic origin of the NN force still needs to be connected to hadronic structure and QCD at a more intimate level. Two recently developed potentials are making some interesting steps in that direction. One is a meson-exchange model with intrinsic nucleon structure called Ruhrpot (Plümper, Flender, and Gari 1994). This model combines long-range meson exchange with direct NN interactions at short distance and pays particular attention to calculating meson-baryon vertices (form factors) in a self-consistent manner. The χ^2 fit to data is not quite as good as the new high-accuracy models discussed above, but is still quite respectable.

The second model is a potential built on an effective chiral Lagrangian (Ordóñez, Ray, and van Kolck 1994) that makes use of an expansion in chiral perturbation theory. One interesting feature of the χ PT treatment is a power counting argument that leads to the statements

$$V_{NN} > V_{3N} > V_{4N} \dots, V_{CD} > V_{CSB}(T=1) > V_{CSB}(n-p), \quad (34)$$

where we differentiate between class III and IV CSB in the latter relation. Both of these features are empirically known facts, but it is nice to see them come out of the same theoretical source. Interestingly both of these potentials are being extended to make consistent predictions for $3N$ forces (Eden and Gari 1996, van Kolck 1994).

For applications to nuclear structure and effective interactions, there also remain many challenging problems. I have not attempted at all to discuss how one goes about making an effective interaction for use in shell model studies from the realistic NN potentials. Assuming that the shell model will continue to be treated as a nonrelativistic Schrödinger equation, one of the big challenges will be how to incorporate $3N$ forces, which seem to be a necessary ingredient in the microscopic calculations of light nuclei. In the meantime, we can expect that these direct calculations using realistic potentials will continue to progress to larger and larger nuclei.

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