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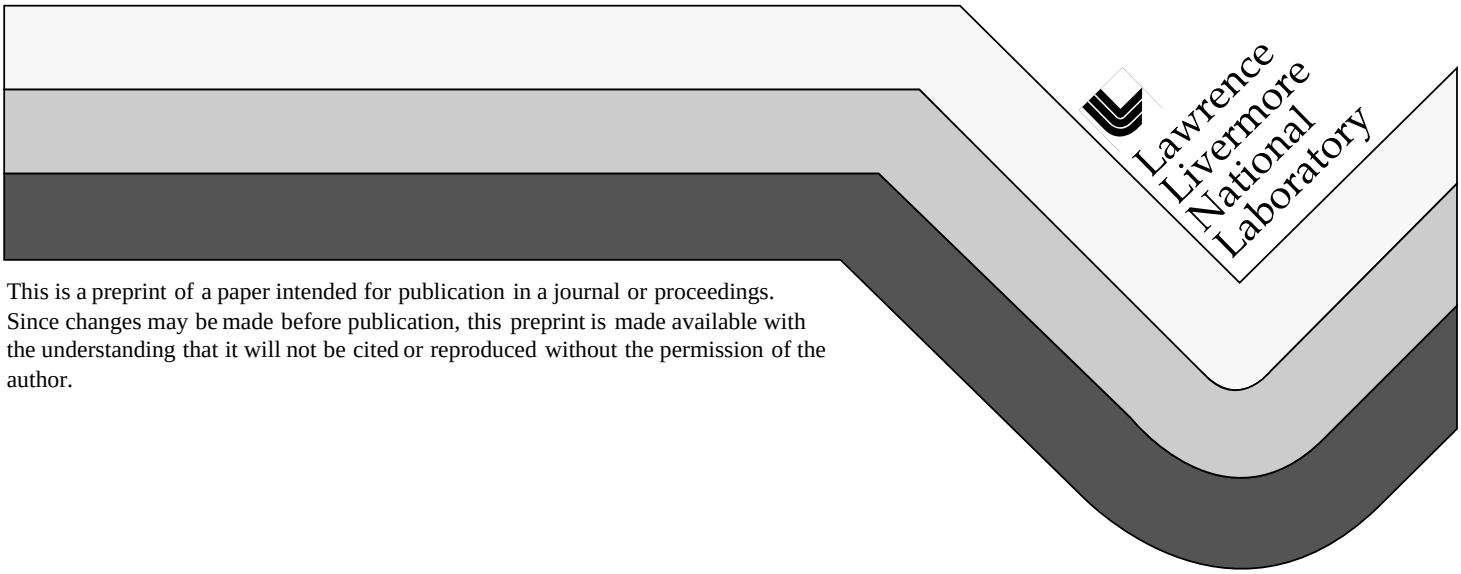

Simple Model for Linear and Nonlinear Mixing at Unstable Fluid Interfaces with Variable Acceleration [1]

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A simple model is described for predicting the time evolution of the half-width h of a planar mixing layer between two immiscible incompressible fluids driven by an arbitrary time-dependent variable acceleration history $a(t)$. The model is based on a heuristic expression for the kinetic energy per unit area of the mixing layer. This expression is based on that for the kinetic energy of a linearly perturbed interface, but with a dynamically renormalized wavelength which becomes proportional to h in the nonlinear regime. An equation of motion for h is then derived by means of Lagrange's equations. This model reproduces the known linear growth rates of the Rayleigh-Taylor (RT) and Richtmyer-Meshkov (RM) instabilities, as well as the quadratic RT and power-law RM growth laws in the nonlinear regime. The time exponent in the RM power law depends on the rate of kinetic energy dissipation. In the case of zero dissipation, this exponent reduces to 2/3 in agreement with elementary scaling arguments. A conservative numerical scheme is proposed to solve the model equations, and is used to perform calculations that agree well with published mixing data from linear electric motor experiments. Considerations involved in implementing the model in hydrodynamics codes are briefly discussed.

Keywords: mixing, instability, Rayleigh-Taylor, Richtmyer-Meshkov, Kelvin-Helmholtz

Introduction

There is considerable current interest in material interpenetration and mixing at unstable fluid interfaces, particularly those driven by the normal acceleration of adjacent fluid layers with different densities. These processes can in principle be computed in detail by direct numerical simulations, and recent advances in computer hardware and numerical methodology now make this feasible in some problems. In most practical applications, however, computer time and storage limitations still preclude a complete simulation of the very wide range of length and time scales that such instabilities produce. It is therefore of interest to develop models which capture the essential physics of such instabilities so that their effects can be simulated with reasonable accuracy on presently available computers. There is a particular need for mix models which are sufficiently simple that they can be retrofitted into existing hydrodynamics codes on a relatively short time scale. In order to be useful in practical problems, it is essential for such models to allow for an arbitrary time-dependent acceleration history $a(t)$. It is also essential for them to reproduce the known growth behavior of the incompressible Rayleigh-Taylor (RT) and Richtmyer-Meshkov (RM) instabilities in both the linear and nonlinear (late-time) regimes as special cases. Some problems with accelerated interfaces also involve tangential velocity discontinuities, so a general mix model should ideally include the Kelvin-Helmholtz (KH) instability as well.

Mix models of this type will necessarily be phenomenological in character, but they should be as fundamentally based as their simplicity allows. Most previous simple mix models have been

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based on bubble rise dynamics, and contain various empirical coefficients (such as drag, buoyancy, and added mass coefficients) which are not directly accessible experimentally. Here we describe a different approach based only on the more general concepts of energy conservation and scale invariance, and we summarize the current status of a family of simple mix models that are being developed using this approach (Ramshaw 1998a,b,c). We also briefly address some of the issues involved in implementing such models in hydrodynamics codes (Ramshaw 1998d). The present discussion is necessarily somewhat abbreviated, and the reader is referred to the papers cited above for further details and references to the original literature.

Review of Planar RT and RM Instabilities

In the linear regime, the growth law for the amplitude $h(t)$ of a small sinusoidal perturbation on an accelerated planar interface between two incompressible fluids is well known and is given by

$$\lambda \ddot{h} = 2\pi A a(t) h \tag{1}$$

where λ is the wavelength of the perturbation, $\dot{q} = dq/dt$ for any q, $A = (\rho_2 - \rho_1)/(\rho_2 + \rho_1)$ is the Atwood number, ρ_i is the density of fluid i, and $a(t)$ is the acceleration, defined as positive when directed from fluid 1 into fluid 2. In the RT case, $a = const$ and Eq. (1) predicts exponential growth when $Aa > 0$. In the RM case, $a(t) = \Delta v \delta(t)$ and Eq. (1) predicts linear growth regardless of the sign of $A\Delta v$.

In the nonlinear late-time regime the mixing layer becomes asymmetrical, and it is conventional to define h as the visual penetration depth of the lighter fluid into the heavier one. The nonlinear RT growth law is $h(t) = \alpha A a t^2$, which follows from a simple dimensional argument. This growth law has been well verified experimentally with values of α clustering in a narrow range about 0.06. The corresponding nonlinear RM growth law is $h(t) \sim t^\theta$, where there is considerable experimental uncertainty in the value of θ. A simple dimensional argument shows that $\theta = 2/3$ in the absence of dissipation (Ramshaw 1998a). Other arguments indicate that this should be an upper bound, which is consistent with most of the values reported in the literature.

Although they are both special cases of instabilities at accelerated interfaces, the RT and RM instabilities are fundamentally different in one respect: the RT instability is generally believed to forget its initial conditions at late times, whereas the RM instability remembers them. The reason for this is that is the implusive acceleration is the only source of energy in the RM problem, so the energy deposited by the impulse is never forgotten. This energy depends on the amplitude and wavelength of the initial perturbation, so it is clear that a useful general model must exhibit a dependence upon these parameters.

In contrast to the linear growth law of Eq. (1), which is valid for an arbitrary acceleration history $a(t)$, the extrapolation of the nonlinear RT and RM growth laws to an arbitrary $a(t)$ is not at all obvious. Our primary objective is to construct a model which captures the essential physics of nonlinear mixing for arbitrary $a(t)$ and properly reduces to the known behavior in the above special cases. This will be done by evaluating the kinetic energy of the mixing layer for a linear perturbation, extrapolating it into the nonlinear regime by means of a wavelength renormalization hypothesis to be described below, and substituting the resulting energy into Lagrange's equations to obtain a nonlinear equation of motion for $h(t)$.

Kinetic Energy in the Linear Regime

The kinetic energy per unit area of an accelerated mixing layer can readily be evaluated from the linear potential flow solution, with the result (Ramshaw 1998a)

$$T = \frac{\bar{\rho}\lambda}{4\pi}\dot{h}^2 - \frac{1}{2}\Delta\rho h\dot{h}\dot{Z} + \frac{1}{2}M\dot{Z}^2 \tag{2}$$

where $Z(t)$ is the location of the original unperturbed interface, $2\bar{\rho} = \rho_1 + \rho_2$, $\Delta\rho = \rho_2 - \rho_1 = 2\bar{\rho}A$, and M is the total mass per unit area. The acceleration of the layer is simply $a = \ddot{Z}$, which is produced by external forces acting on the boundaries of the mixing layer far from the interface. These forces are independent of h, so they may be omitted from Lagrange's equation for h. The Lagrangian L can therefore simply be identified with T, and it is then easy to verify that the resulting Lagrange equation of motion for h is simply Eq. (1) above. The use of Lagrange's equations therefore reproduces the correct linearized dynamics of the interface. The advantage of this approach is that it permits a straightforward extension into the nonlinear regime as shown below.

The Wavelength Renormalization Hypothesis

The present class of models is based on the fundamental but apparently drastic hypothesis that the linear expression for T remains valid in the nonlinear regime with λ replaced by $\lambda(t) = b|h(t)|$, where b is a dimensionless coefficient of order unity. We refer to this hypothesis as the wavelength renormalization hypothesis (WRH). The WRH may be motivated heuristically by observing that in the nonlinear regime, the mixing layer contains no intrinsic length scale other than its own width, so there is no objective basis for describing it as either thick or thin. Viewed from sufficiently far away (i.e, if you back up or "zoom out" enough), the mixing layer always looks thin. But this means the interface always looks like it is only slightly perturbed, with a perturbation amplitude of order h. Of course this perturbation is irregular rather than sinusoidal, but the characteristic length scale of these irregularities is also of order h and may be expected to play the role of the effective wavelength of the perturbation. These considerations suggest that, at least in some rough scaling sense, the system is always in the linear regime! It is then not unreasonable to expect the mixing layer to behave in an essentially linear manner but with a time-dependent λ which is continuously dynamically renormalized to a value of order h.

Construction of the Nonlinear Model

The nonlinear model may now be constructed simply by replacing λ by $\lambda = \lambda(h)$ in Eq. (2) for T and substituting the result into Lagrange's equation of motion for h, with an additional generalized force introduced to represent the dissipation of kinetic energy due to viscosity at small length scales. This gives (Ramshaw 1998a)

$$\lambda\ddot{h} + \frac{1}{2}\dot{\lambda}\dot{h} + 2\pi c|\dot{h}|\dot{h} - 2\pi Aah = 0 \tag{3}$$

where $c \sim 1$ is the dissipation coefficient, which is set to zero in the linear regime. Equation (3) is the basic dynamical evolution equation of the model. In the linear regime, $\dot{\lambda} = 0$ and we regain the correct linear growth law of Eq. (1).

Since the nonlinear model was derived by means of Lagrange's equations, it automatically conserves energy when dissipation is absent. This essential property takes its simplest form in the

pure RT case, where $a = const.$ It is then easy to verify that when $c = 0$, the total perturbation energy

$$E = \frac{\bar{\rho}\lambda}{4\pi}\dot{h}^2 - \frac{1}{2}\bar{\rho}Aah^2 \tag{4}$$

is conserved by Eq. (3) in both the linear and nonlinear regimes.

We have not yet verified that the model correctly reproduces the known growth behavior of the nonlinear RT and RM instabilities. This will be done in the next section, and in the process we shall find that the parameters b and c are uniquely determined by the measured experimental growth rates for these instabilities.

Nonlinear RT and RM Instabilities

According to the WRH, $\lambda = b|h|$ in the nonlinear regime. It is then easy to verify that the RT scaling law $h = \alpha Aat^2$ satisfies Eq. (3) for constant a provided that

$$\alpha = \frac{\pi}{2b + 4\pi c} \tag{5}$$

Similarly, in the RM case, $a(t) = \Delta v \delta(t)$ and Eq. (3) predicts that $h \sim t^\theta$ with

$$\theta = \frac{2b}{3b + 4\pi c} \tag{6}$$

These relations can be inverted to obtain

$$b = \frac{\pi\theta}{\alpha(2-\theta)} \tag{7}$$

$$c = \frac{2 - 3\theta}{4\alpha(2-\theta)} \tag{8}$$

When $c = 0$, Eq. (6) reduces to $\theta = 2/3$ as it should. Equations (7) and (8) determine the model parameters b and c in terms of the experimentally accessible parameters α and θ, which may be obtained from pure RT and RM experiments, respectively.

Transition Between the Linear and Nonlinear Regimes

In the linear regime the perturbation wavelength λ is constant with its initial value λ_0, whereas in the nonlinear regime we have $\lambda = b|h|$ in accordance with the WRH. It is necessary to prescribe a transition between these two limits so that an initially small perturbation can grow and evolve into a fully developed mixing layer. A simple way of effecting this transition is to set

$$\lambda = \max(\lambda_0, b|h| + (1 - mb)\lambda_0) \tag{9}$$

where $m \sim 1$ is a parameter which makes the transition occur at $|h| = m\lambda_0$. However, this is clearly an abrupt and highly oversimplified transition rule, and more gradual and realistic alternatives should also be explored.

Relation to Previous ODE Models

When $c = 0$ and the linear-to-nonlinear transition is replaced by $\lambda = \lambda_0 + b|h|$, the model reduces to an earlier unpublished model proposed by Pete Stry. The relation to other simple mix

models may be seen by letting $v = \dot{h}$. In the nonlinear regime, the present model then takes the form

$$\dot{v} = \frac{2\pi h}{b|h|} Aa - \left(\frac{v}{2h} + \frac{2\pi c}{b} \left| \frac{v}{h} \right| \right) v \tag{10}$$

Equation (10) strongly resembles earlier bubble-dynamics models used by Youngs, Dimonte, et al. However, these models frequently omit the linear regime and the absolute value signs. The former is essential to obtain the correct dependence on initial conditions in RM problems, while the latter are essential to obtain the correct oscillatory and demixing behavior in RT stable cases with $Aa < 0$ (Ramshaw 1998a). Note in particular that the term quadratic in v is not a pure drag term as has previously been presumed, as it does not always oppose v and does not vanish when $c = 0$.

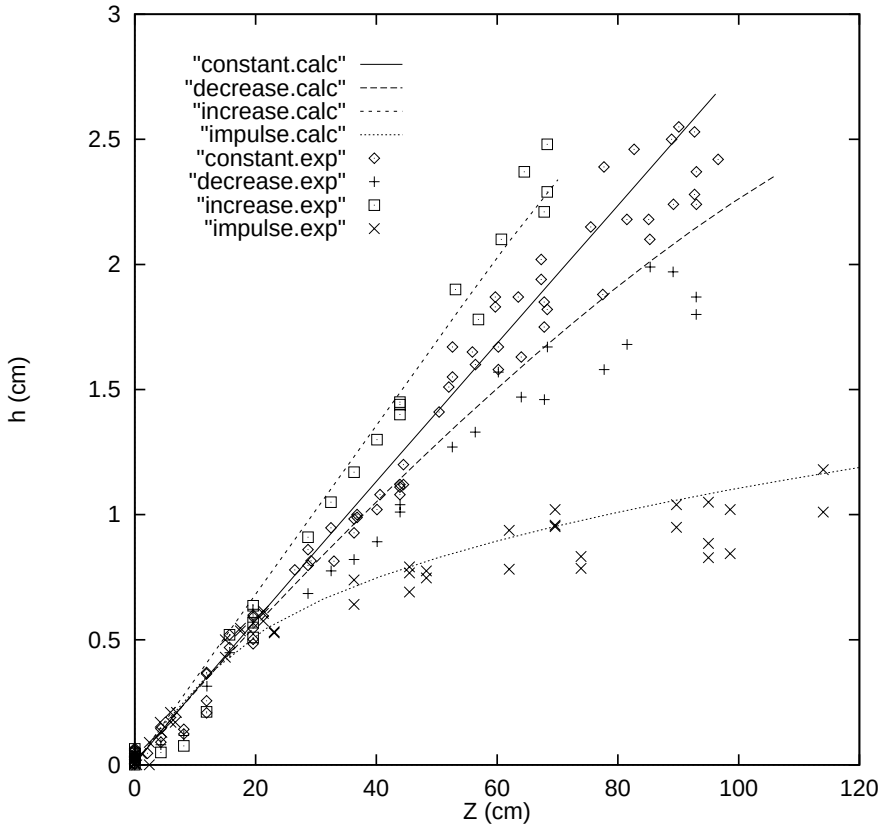

Figure 1: Comparison of model calculations with experimental data for four different acceleration histories.

Comparison with Dimonte-Schneider Experimental Data

Dimonte and Schneider (1996) (DS) have reported experimental measurements of the growth of a mixing layer between two incompressible fluids subjected to four different acceleration histories. Figure 1 shows a comparison between the experimental data and the model predictions. The latter were obtained by solving Eq. (3) numerically using a conservative numerical scheme (Ramshaw

1998a) and the DS parameter values of $\alpha = 0.061$ and $\theta = 0.37$. The agreement is quite satisfactory, but these four experiments do not of course explore all the conditions of interest, and further comparisons of model predictions with experiments and/or direct numerical simulations will be required to obtain a better assessment of the overall accuracy and utility of the model.

Inclusion of Kelvin-Helmholtz Instability

We now consider the case in which the two fluids in the accelerated mixing layer also have nonzero tangential velocities u_1 and u_2 far from the interface. The potential flow solution for this case is easily obtained, and the kinetic energy T can be evaluated from it just as before. Unfortunately, Lagrange's equations no longer apply, because h is no longer a proper generalized coordinate when $\Delta u \equiv u_2 - u_1 \neq 0$ (Ramshaw 1998c). However, energy is of course still conserved, and this may be invoked to derive the equation of motion for h from T. The result is

$$\lambda\ddot{h} + \frac{1}{2}\dot{\lambda}\dot{h} - 2\pi Aah - \frac{2\pi^2\rho_1\rho_2\Delta u^2}{(\rho_1+\rho_2)^2}\frac{h}{\lambda}\left(2 - \frac{h}{\lambda}\frac{d\lambda}{dh}\right) = -\,2\pi c|\dot{h}|\dot{h} \tag{11}$$

Note that there are no new parameters associated with the KH instability. In the linear regime where $\lambda = \lambda_0 = const$ and $c = 0$, Eq. (11) reduces to the known linear stability result for that case. It also reduces to Eq. (3) when $\Delta u = 0$ as it should. Moreover, for a pure KH instability in the nonlinear regime, where $\lambda = b|h|$, we find

$$\dot{h} = \frac{\alpha(2-\theta)}{\sqrt{\theta(1-\theta)}}\,\frac{\sqrt{2\rho_1\rho_2}}{\rho_1+\rho_2}\,|\Delta u| \tag{12}$$

In the special case of equal densities and using the Dimonte-Schneider values of α and θ, Eq. (12) reduces to $\dot{h} = \eta|\Delta u|$ with $\eta = 0.15$. This is reasonably close to values of η reported in the literature for this case, which are subject to considerable uncertainty but tend to lie between 0.06 and 0.12.

Spherical Geometry

Accelerated material interfaces are of course not always planar. It is therefore of interest to examine the effects of interface curvature, the simplest cases of which are cylindrical and spherical interfaces. Here we consider the spherical case (Ramshaw 1998b), which occurs in inertial confinement fusion and in certain astrophysical problems. In this case we can again follow the same procedure used to obtain Eq. (3). The kinetic energy of the linearized potential flow resulting from a spherical harmonic perturbation of order ℓ is readily found to be

$$T = T_0 + 2\pi(2\rho_\ell - \Delta\rho)R\dot{R}h(R\dot{h} + \dot{R}h) + \pi\rho_\ell R^3\dot{h}^2 \tag{13}$$

where $R(t)$ is the radius of the interface between adjacent concentric spherical shells of inner and outer radii R_1 and R_2 respectively, $\rho_\ell = \rho_1/\ell + \rho_2/(\ell+1)$, and

$$T_0 = 2\pi R^4\dot{R}^2\left[\rho_1\left(\frac{1}{R_1} - \frac{1}{R}\right) + \rho_2\left(\frac{1}{R} - \frac{1}{R_2}\right)\right] \tag{14}$$

The equation of motion for h is then obtained by substituting T into Lagrange's equations as before. In the linear regime where $\ell = \ell_0 = const$, this yields the well known Plesset stability equation for this case. In the nonlinear regime we again adopt the WRH, with the effective wavelength defined

as twice the mean distance between nodes; i.e., $\lambda = 2\pi R/\ell$. The resulting equation of motion for $s = R^2 h$ is

$$2R^2 \frac{d}{dt}\left(\frac{\rho_\ell \dot{s}}{R}\right) - \left(\frac{\partial \rho_\ell}{\partial s}\right) R\dot{s}^2 - 2\Delta\rho \ddot{R} s + 4c\bar{\rho}\frac{|\dot{s}|\dot{s}}{R^2} = 0 \tag{15}$$

The volume of fluid exchanged across the interface is proportional to s, so s is a better measure of the degree of mixing than h. Indeed, in problems with large changes in R, impressions of mixing inferred from h alone can be highly misleading (Ramshaw 1998b).

Implementation in Hydrodynamics Codes

Simple models of the present type can be used in a stand-alone manner for a given interface acceleration $a(t)$ and slip velocity $\Delta u(t)$. In practical problems, however, these quantities are not known *a priori* but are influenced by the mixing itself. That is to say, the mixing dynamically interacts with the hydrodynamic processes that produce the acceleration and slip velocity, and it is necessary to allow for this interaction in order to obtain self-consistent results. This requires the mix model to be dynamically coupled to the hydrodynamics code that is used to compute the fluid flow field in which the unstable interface is embedded. It is by no means clear how best to effect this coupling. In fact, the answer to this question will depend on the particular hydrodynamics code in question, including the numerical scheme and the other physical submodels (such as turbulence models and diffusion fluxes) with which the mix model must communicate. These issues are fraught with subtleties and ambiguities and are not yet resolved. We shall therefore not attempt a comprehensive discussion of the many possible approaches, but will confine our attention to one particularly simple approach which is currently being explored (Ramshaw 1998d).

The approach in question consists in essence of using the mix model to determine the outer boundaries of the growing mixing layer; i.e., the region within which the two fluid materials have been mixed together by the instability, and outside of which they remain pure. Once these boundaries have been located, an auxiliary procedure is used to compute the transport of materials in the interior of the mixing layer. This approach is particularly well suited for use in one-dimensional Lagrangian hydrodynamics codes, but can be adapted to other codes as well.

If the original interface is horizontal and located at $z = Z(t)$, with fluid 1 below and fluid 2 above, then the boundaries of the mixing layer are clearly located at $z = Z(t) - h_1(t)$ and $z = Z(t) + h_2(t)$, where $h_i(t) = |h(t)|$ if i is the heavier fluid and $h_i(t) = |h_s(t)|$ if i is the lighter fluid, where $h_s(t)$ is the spike penetration depth. The present class of models computes the bubble penetration depth h but not the spike penetration h_s. However, empirical correlations are available which express h_s in terms of h and A, so h_s may be assumed known for present purposes.

One might at first think that $Z(t)$ should be identified with the Lagrangian surface which coincides with the original unperturbed interface at $t = 0$ and moves with the normal fluid velocity w. This would be wrong, however, because the surface $z = Z(t)$ is a surface across which no net *volume* flows, whereas a Lagrangian surface is one across which no net *mass* flows. Since the two fluids have different densities, the mixing carries heavier fluid into lighter fluid (and vice versa), which results in a net mass flux across the surface $z = Z(t)$ in the direction of the lighter fluid. The surface $z = Z(t)$ is therefore not Lagrangian, and consequently moves with respect to a Lagrangian mesh. It is essential to compute this relative motion in order to determine $Z(t)$ and hence the boundaries of the mixing layer.

The relation between the interface velocity $\dot{Z}$ and the fluid velocity w normal to the interface may be determined by evaluating the mass flux across the interface due to the mixing. This in turn requires knowledge of the volume per unit area $V(t)$ transported (exchanged) across the original interface. Clearly $V(t)$ is of order $|h(t)|$, so we may write $V = \gamma|h|$, where γ is a dimensionless coefficient of order unity. It is easy to show that $\gamma = 1/\pi$ for sine waves and $\gamma = 0.5$ for square waves, so it is probably reasonable to set $\gamma \approx 0.35$–0.4 for general use. The net mass flux across the interface is then given by

$$\dot{M} = (\rho_1 - \rho_2)\dot{V} = -\gamma\Delta\rho \frac{d|h|}{dt} \tag{16}$$

But in a hydrodynamics code, this same mass flux is given by $\dot{M} = \bar{\rho}_0[w(Z(t),t) - \dot{Z}]$, where $\bar{\rho}_0$ is the mean fluid density at $z = Z$. Equating these two expressions for $\dot{M}$, we obtain

$$\dot{Z} = w + \gamma\frac{\Delta\rho}{\bar{\rho}_0}\frac{d|h|}{dt} \tag{17}$$

This determines the motion of the interface, and together with h thereby determines the outer boundaries of the mixing layer.

In order to calculate the nonuniform distribution of material masses within the mixing region, we must compute the local mass fluxes $J_1(z)$ and $J_2(z)$ of materials 1 and 2 relative to w. The values of these fluxes at $z = Z(t)$ can be evaluated in the same way as $\dot{M}$, with the result

$$J_1(Z) = -J_2(Z) = \gamma\frac{\rho_1\rho_2}{\bar{\rho}_0}\frac{d|h|}{dt} \tag{18}$$

To obtain the fluxes $J_i(z)$ at intermediate points, it is then necessary to interpolate between $J_i(Z)$ and the edges of the mixing layer, where $J_i = 0$. There is no unique way to do this, but it is easy to come up with reasonable prescriptions (Ramshaw 1998d). Further experimental and/or direct numerical simulation data on the profiles of these fluxes and material distributions would be most welcome.

Unresolved Issues and Missing Physics

These models are still evolving, and do not yet contain all of the ingredients that are expected to be needed for realistic simulations of practical problems. There are several unresolved issues and areas where further improvement is needed, including:

- The sudden transition between the linear and nonlinear regimes provided by Eq. (9) should be replaced by a more realistic and gradual transition, possibly including a weakly nonlinear transitional regime.

- When $Aa(t) < 0$ the model properly predicts demixing (reseparation) of the two fluids, but at a rate which is expected to be too fast (Ramshaw 1998a).

- The models described here are incompressible, and hence neglect compressibility effects which are often important in practical applications. In particular, RM instability studies have shown that it is important to distinguish between and correct for differences between the preshock and postshock values of h and A. Previous simple mix models have sometimes

employed an ad hoc compression correction which consists in essence of introducing a term $h\partial w/\partial z$ into $\dot{h}$. This can be done in the present models as well, but it would be preferable to rebuild the models allowing for compression effects from the outset. This is currently in progress and will be reported elsewhere in due course (Ramshaw 1998c).

- The present models are limited to single-mode perturbations in the linear regime, and should be generalized to allow for multimode perturbations and mode coupling. Unfortunately, it is not clear how to accomplish this without a substantial increase in complexity.

- The treatment of shocks (impulsive accelerations) in RT stable cases presents some problems. In such cases $h(t)$ exhibits oscillations about $h = 0$, and if the impulse arrives when h is very small or zero, it will have little or no effect. This problem should be ameliorated by a multimode capability with incommensurate frequencies. In addition, many hydrocodes treat shocks by shock smearing or capturing techniques, which artificially thicken the shock and prolong its duration. The numerical shock may then become gradual rather than impulsive in comparison to the time scale of the interface oscillations, and it will not then deposit the correct energy. This problem may be dealt with by monitoring the artificial viscosity to detect the presence of a shock, storing the accumulated velocity jump until the shock is just past the interface, and then impulsively depositing this accumulated velocity jump into the mix model.

- In their present form, these models neglect surface tension and do not compute the transition from chunk to atomic mix. Removal of this restriction will probably require additional variables to represent information about the spectrum of length scales and the rate of atomic mixing.

- In some problems the accelerating interface is simultaneously ablating, and this will affect the instability growth in ways that the models cannot currently represent.

- Some problems involve multiple interacting interfaces, and the present models do not allow for this.

- Finally, the consistent coupling between simple mix models of this type and the various turbulence models currently used in hydrodynamics codes presents a variety of unresolved issues.

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