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Author(s): Joyce A. Guzik (XTA)

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SOLAR STRUCTURE: MODELS AND INFERENCE FROM HELIOSEISMOLOGY

Joyce A. Guzik
Los Alamos National Laboratory, XTA MS B220, Los Alamos, NM 87545-2345
Phone: (505) 667-8927; E-mail: joy@LANL.GOV

ABSTRACT

In this review I summarize results published during approximately the last three years concerning the state of one-dimensional solar interior modeling. I discuss the effects of refinements to the input physics, motivated by improving the agreement between calculated and observed solar oscillation frequencies, or between calculated and inferred solar structure. I have omitted effects of refinements to the input physics, motivated by one-dimensional solar interior modeling. I discuss the recent improvements to the physical input of the observed solar oscillation frequencies, or between observed and calculated oscillation frequencies. For example, calculated low-degree p-mode frequencies agree with observation to within 0.2% (Christensen-Dalsgaard et al. 1996). Although static (as opposed to evolved) solar models can be useful, for example, as test or reference models for helioseismic inversions, most modelers calculate the evolution of the sun from the pre-main sequence or zero-age main sequence to the present, matching the observed solar luminosity (3.846 ± 0.005 x 10³⁵ erg/sec (Willson et al. 1986), mass 1.9891 ± 0.0004 x 10³⁵ g), radius 6.9599 x 10¹⁰ cm (Allen 1973), and photospheric Z/X abundance ratio 0.0245 ± 0.0015 (Grevesse & Noels 1993) at the present solar age 4.52 ± 0.04 Gyr (Guenther 1989). Typically, the modeler adjusts the initial helium mass fraction (Y), initial mass fraction (Z) of elements heavier than H and He, and some parameter in the convection treatment, e.g., the mixing length to pressure scale height ratio (α), to obtain the observed luminosity, radius, and photospheric Z/X abundance. Some have also attempted (successfully) to evolve solar models that take into account the additional constraints of the observed solar Li and Be abundance, which are believed to be reduced by factors of 140 and 2 (±30%), respectively, from the primordial values (Richard et al. 1996). Solar modelers have so far been unsuccessful in creating models that produce the neutrino fluxes observed by the GALLEX, SAGE, Homstake, Kamiokande, and Super Kamiokande neutrino experiments (Bahcall, Basu, & Pinsonneault 1998), while simultaneously matching the constraints of helioseismology.

SOLAR MODEL INGREDIENTS

Recent improvements to the physical input of the standard solar model have resulted in excellent agreement between the calculated and inferred solar structure, or between observed and calculated oscillation frequencies. For example, calculated low-degree p-mode frequencies agree with observation to within several μHz out of 3000 μHz, or a few tenths of a percent (Guzik & Swenson 1997). The inferred sound speed profile agrees with the profiles of standard solar evolution models to within 0.2% (Christensen-Dalsgaard et al. 1996).

Most modern solar models have adopted the OPAL (Iglesias, Rogers & Wilson 1992; Iglesias & Rogers 1996) opacities; the OPAL (Rogers, Swenson & Iglesias 1996) or MHD (Dappen, Mihalas, Hummer, & Mihalas 1988) equation of state tables; and the nuclear reaction rates of Caughlan & Fowler (1988), Bahcall, Pinsonneault, & Wasserburg (1995), or those discussed by Brun et al. (1998). Since the OPAL opacity tables do not extend to temperatures below 6000 K, these have been supplemented by low-temperature opacity tables of, e.g., Kurucz (1992), Alexander & Ferguson (1994, 1995), or Neuforge (1993). While many modelers continue to use the mixing-length treatment of convection (Bohm-Vitense 1958), some have adopted the treatment of Canuto & Mazzitelli (1991, 1992) which allows for a spectrum of eddy length scales, or an attached envelope calibrated by 2-D or 3-D turbulence models (Rosenthal et al. 1998; Demarque, Guenther & Kim 1997). It has also become standard (and essential) in modern solar models to incorporate some treatment of diffusive settling of helium and heavier elements (see, e.g. Cox, Guzik, & Kidman 1989; Thoul, Bahcall, & Loeb 1994).

FORWARD AND INVERSE METHODS

The two approaches to helioseismic comparisons can be categorized as forward and inverse methods. The objective of the forward method is to test the physical input of solar and stellar models, and not necessarily to derive the absolute structure of the solar interior. In the forward approach, one generally uses the best available physical input to calculate solar evolution as described above, and directly compares the observed and calculated oscillation frequencies. The differences between predictions and observation are used to evaluate and suggest further refinements in physical input. For an excellent overview, see “Testing a Solar Model: The Forward Problem” (Christensen-Dalsgaard 1995).
The objective of the inverse approach is to derive the internal structure of the sun independent of the physical input of the models. One uses a reference model to derive weighting functions, or kernels, that relate a specific change in model structure to a change in predicted frequency. The differences between observed frequencies and those of the calculated reference model (incorporating as well the observational errors), are used as input to back out differences between the reference model structure and the sun for such quantities as sound speed, density, adiabatic index, or rotation rate as a function of radius. In the case of the solar rotation, the observed rotationally-produced frequency splittings are used to determine the internal rotation profile as a function of latitude as well as of radius. The two commonly used methods of solving the inversion integral are known as Regularized Least Squares (RLS) and Optimally Localized Averages (OLA). The OLA averaging kernels give better localization, as the name implies; however, the RLS method is more sensitive to internal inconsistencies in the data, and is computationally less expensive than the OLA method (Sekii 1996). For introductory reviews of inversion methods in helioseismology, see Basu (1997), Gough & Thompson (1992), and Kosovichev et al. (1992).

The location of discontinuities in the inferred sound speed profile has been used to derive the convection zone depth, 0.7135 ± 0.0005 R⊙, and the extent of convective overshoot below the convection zone, 0.05 pressure scale heights (Basu 1998). The deviation from the perfect gas value of ρ1/3 due to partial ionization of elements in the solar envelope has been used to derive the convection zone helium abundance, Y = 0.248 ± 0.001, and convection zone Z/X = 0.0245 ± 0.0006 (Basu 1998). Note that these composition determinations are affected by the adopted equation of state and opacity; in this case Basu used the 1996 OPAL values. Basu’s results also assume a proposed small reduction in the solar radius discussed below. The sound speed, density, and Γ1 profiles, as well as the convection zone depth and composition are now being used as “constraints” to test solar models with modified input physics, which is a simpler alternative to calculating the model frequencies as in the forward method.

**IMPROVEMENTS TO SOLAR INTERIOR MODELING**

1. OPACITIES

A major focus for refining the input physics of solar models has been the radiative opacities in the solar interior. The OPAL opacities have recently been updated (Iglesias & Rogers 1996) from the 1992 tables (Iglesias, Rogers & Wilson 1992) to take into account additional elements and improvements in the EOS treatment. The 1996 tables show an opacity decrease of 5%, compared to the 1992 values, for conditions below the solar convection zone. The effect of these lower opacities is to slightly decrease the convection zone depth, which worsens the agreement with the inferred helioseismic depth (Guzik & Swenson 1997; Bahcall, Basu, & Pinsonneault 1998). Figure 1 from Guzik & Swenson (1997) shows the observed minus calculated (O-C) versus calculated low-degree p-mode frequencies for two solar models using the old and new opacities; the new opacities reduce the frequencies of these modes by about 1 μHz.

![Figure 1](image-url)

**Figure 1:** Observed minus calculated nonadiabatic frequencies of two solar models of Guzik & Swenson (1997) using the 1992 and 1996 OPAL opacity tables. Both models use the OPAL 1996 EOS tables. Observations are from Chaplin et al. (1996) and Schou & Tomczyk (1997).

Since the estimated uncertainties in the OPAL opacity calculations are ~5%, a few groups (e.g. Gabriel & Carlier 1997; Brun et al. 1998) have considered opacity “ramps” of 1 to 5% to improve the agreement between inferred and predicted sound speed below the convection zone. Inversion techniques are also under development (Elliott 1995; Tripathy & Christensen-Dalsgaard) to derive the opacity in the solar interior; it would be a significant achievement if helioseismology could constrain radiative interior opacities to a range that is smaller than the estimated uncertainties of the calculated tables.

2. EQUATION OF STATE

Both inversions and direct frequency comparisons demonstrate much-improved agreement with inferred solar structure for models using modern equation of state treatments with additional physical detail, compared to the Eggleton, Faulkner & Flannery (1973, EFF) analytical EOS (Basu & Christensen-Dalsgaard 1997; Guenther 1994). The largest improvement comes from including the Debye-Hückel term of the Coulomb interaction, which is neglected in the EFF equation of state. Helioseismic comparisons also seem to favor the OPAL (Rogers, Swenson, & Iglesias 1996) tables over the MHD (1988) equation of state (Basu & Christensen-Dalsgaard 1997; Guzik & Swenson 1997).
Helioseismic comparisons are so sensitive that even subtle improvements to the equation of state treatment can be detected, and arguably must be included in solar models. Some improvements investigated recently are: 1) including the Z variation with radius due to element diffusion in the EOS, which has up to a 0.13% effect on pressure and 0.05% effect on sound speed in the solar interior (Guzik & Swenson 1997); 2) electron exchange corrections, which are included in OPAL but not in MHD, and which lower the central pressure enough to require an increase in initial hydrogen abundance of about 0.001 (Guzik & Swenson 1997); 3) relativistic effects, omitted in both the OPAL and MHD treatments, which decrease \( \Gamma_1 \) by about 0.2% in the solar core, in the direction of improving agreement with inferred \( \Gamma_1 \) (Elliot & Kosovichev 1998); 4) including excited states in the hydrogen internal partition function (Nayfonov & Dappen 1998).

3. ABUNDANCES AND DIFFUSIVE SETTLING

There is strong evidence along several lines of inquiry for diffusive settling of helium by about 0.03 in mass fraction from the solar convection zone during the sun's lifetime. Inversions for the hydrogen abundance profile \( X(r) \) (Kosovichev 1995) show a discontinuity in \( X \) at the convection zone base, expected from helium draining out of the convection zone and piling up near the base where the diffusion velocities become slower. Helium diffusion also eliminates a discrepancy amounting to several percent between the calculated sound speed of solar models without diffusion and the inferred sound speed (Christensen-Dalsgaard et al. 1996). Finally, the inferred convection zone helium mass fraction \( Y \approx 0.248 \) is less by \(-0.03\) than the initial \( Y = 0.272-0.275 \) required to generate the observed luminosity in solar evolution models, and is in accord with the amount of settling expected due to diffusion.

The evidence for, and constraints on the amount of diffusive settling of elements heavier than hydrogen and helium are not as well established due to uncertainties in the solar element abundance mixture, uncertainties in the diffusion coefficients for various elements, and also due to the smaller sensitivity of solar models to variations in element abundances. Some helioseismic comparisons favor models with less element diffusion than predicted, or even with no Z diffusion (Gabriel 1997). Most modelers now include the additional constraint of normalizing their surface abundances after diffusion to the observed Z/X, which does seem to improve the overall agreement (e.g. Richard et al. 1996). Recent attempts to include radiative levitation of individual elements as well as a more detailed settling treatment (Turbtou et al. 1998, these proceedings) appear to worsen the calculated and inferred sound speed agreement. This may indicate that other compensating changes to input physics, such as in the opacities, may be needed to restore agreement.

4. MIXING VS. MASS LOSS AND THE SOLAR LITHIUM/BERYLLIUM PROBLEM

Two explanations have been proposed to account for the depletion of the light elements Li and Be from their presumed protosolar abundance to the present photospheric abundance: Mixing below the solar convection zone, induced by either differential rotation or gravity waves; and mass loss of about 0.1 solar masses early in the sun's main-sequence lifetime. Both mechanisms also have the advantage of smoothing out the steep composition gradient at the base of the convection zone resulting from helium and element diffusion, and somewhat reducing the small remaining discrepancy between the inferred and calculated sound speed profiles.

Regarding the mass-loss scenario, Guzik & Cox (1995) and Morel, Provost, & Berthomieu (1997) conclude that the mass-loss phase must end quite early, no later than about 0.2-0.3 Gyr after the sun arrives on the main sequence, to avoid ruining the good agreement with the sound speed in the core of the models. The diffusion-produced composition gradient at the convection zone base is smoothed by the changing location of the convection zone boundary as the sun loses mass. For example, Anderson et al. (1996) show for the Guzik & Cox model that a 0.3% difference in sound speed at the convection zone base is decreased to about 0.2%. However, in the early higher-mass phase, the increased nuclear energy generation rate steepens the resultant composition gradient in the core. Anderson et al. and Morel et al. show that if the mass loss phase does not end early enough, the core sound speed discrepancy is increased from 0-0.2% to about 0.4%.

![Figure 2: Sound speed profile differences between models of Gabriel (1997) and the seismically inferred profile of Basu (1997). Two models with helium and element diffusion, plus mixing below the convection zone (thick solid and dashed line) deplete Li and Be while removing the discrepancy between calculated and inferred sound speed near the convection zone base. Note also the large disagreement in sound speed between the model without helium diffusion and the seismic model (thin solid line).](image-url)
Mixing induced by rotation or gravity waves, depending on the parametrization of the mixing, can do very well in improving the agreement in sound speed below the convection zone base (Gabriel 1997; Richard et al. 1996; Guenther & Demarque 1997). However, the mixing must not extend too deep to avoid ruining the good sound speed agreement in the solar core (Richard & Vauclair 1997). Figure 2 shows Gabriel's result for the sound speed difference between several solar models and that of Basu's (1997) seismic solar model; the discrepancy between the calculated and inferred sound speed of a few tenths of a percent near the convection zone base (at \( R = 0.6-0.7 \, R_\odot \)) is removed by including rotationally-induced mixing.

### 5. THE SOLAR RADIUS

Several groups have reported a disagreement between the "seismic" solar radius as measured by the f modes (surface gravity waves whose frequencies depend almost exclusively on the solar radius), and the generally adopted "photospheric" radius 695.99 Mm listed in Allen's Astrophysical Quantities (1973). Schou et al. (1997) determine a "seismic" solar radius of 695.68 Mm from the f modes, a decrease of 310 km, while Antia (1998) finds 695.78 Mm, a decrease of 210 km.

Brown and Christensen-Dalsgaard (1998) reassess the photospheric solar radius. Rather than simply identifying the inflection point in solar intensity decrease at the solar limb with the photosphere, as they speculated was done for the value reported by Allen, they use model atmospheres to determine the distance between this inflection point and the true photosphere at optical depth \( \tau = 2/3 \). They find a photospheric radius of 695.508 \( \pm 0.026 \) Mm, a decrease of 482 km, and even smaller than the proposed "seismic" radii of Schou et al. and Antia.

Such a decrease in radius will not only affect the f modes, but also other p-mode frequencies, because the overall mean solar density increases with the smaller radius. Based on the period–mean density relationship, \( \Delta R/R = -(2/3) \Delta \rho/\rho \). Figure 3 compares the observed minus calculated (O-C) frequencies of two models of Guzik & Despain (1998), evolved to different final radii, with the photospheric radius of the low-radius model smaller by 484 km. The smaller radius tends to flatten the otherwise unexplained upward trend in O-C for frequencies less than 2000 \( \mu \)Hz, and in this sense appears to improve agreement. The remaining downward trend in O-C for higher frequencies can be removed by changes in the solar surface structure (see section 6 below).

Basu (1998) and Antia (1998) demonstrate that it is important to invert oscillation frequencies using reference models with the same radius as the sun in order to derive the correct sound speed profile. The differences in derived sound speed are of the same order as the discrepancy between the inferred sound speed and sound speed of the best standard models, so this small radius change is non-negligible.

![Figure 3: Observed minus calculated nonadiabatic frequencies of standard and low-radius solar models calculated by Guzik and Despain (1998). Both models use the analytical SIREFF EOS (Guzik & Swenson 1997). The photospheric radius of the "lowrad" model is 484 km smaller. The lower solar radius removes most of the trend of increasing O-C with increasing frequency for \( \nu < 2000 \mu \)Hz. The remaining downward trend in O-C for higher frequencies can be removed by changes in the solar surface structure (see section 6).](image)

The top panel of Fig. 4, reproduced from Basu (1998), shows that the exact sound speed difference between two models (solid line) is recovered by the inversion technique if the reference model has the same radius as the test model. The lower panel shows the sound speed difference between two models with identical input physics, but different radii (695.99 and 695.78 Mm). When the inversion is carried out using a reference model with different radius than the test model, the sound speed difference is not recovered (dotted line), with the discrepancy amounting to nearly 0.1% in squared sound speed over the entire solar interior.

Figure 5, also reproduced from Basu (1998), shows the relative sound speed difference between the sun and two reference models with radii 695.99 and 695.78 Mm. Note that this radius decrease (210 km) is at the smaller end of the proposed decrease. If the solar radius should actually be decreased by 482 km, as proposed by Brown & Christensen-Dalsgaard, sound speed inferences will need to be revised by as much as 0.2-0.3%.
6. SOLAR SURFACE STRUCTURE

A nearly universal feature of comparisons between observed and calculated frequencies of standard solar models is the downward trend in observed minus calculated (O-C) frequency with increasing frequency for frequencies greater than about 2000 \( \mu \text{Hz} \) (see, e.g., Fig. 1). This trend can be removed by modifying the structure of the superadiabatic regions of the model (e.g., Guzik, Cox, & Swenson 1996; Demarque, Guenther, & Kim 1997). Altering the location and shape of the superadiabatic temperature gradient changes the upper turning points of the \( p \)-modes, which become higher with increasing \( p \)-mode order and degree.

To illustrate the effect, Figure 6 (from Guzik, Cox, & Swenson 1996) shows the run of temperature versus density near the surface of two solar models. Both models are evolved using identical input physics, except for an increase of 50\% in the opacities for temperatures less than 15,000 K in the model represented by the dashed line, compared to that of the solid line. Figure 7 compares the observed minus calculated non-adiabatic low-degree frequencies of these two models, showing the significant flattening of the slope in the O-C frequencies for the model with the higher surface opacities (thin lines). Note that this example is intended only to be illustrative, and is not necessarily intended to suggest an increase of 50\% in opacities for \( T < 15,000 \) K. Any change to the input physics of the model that produces a similar effect on the temperature gradient will improve the O-C frequency agreement.

Figure 4: Sound speed inversion tests from Basu (1998). When the reference and test models have the same radius, the inversion technique recovers the sound speed difference between models (upper panel). When the models have different radii (695.99 and 695.78 Mm), inversion techniques do not recover the actual sound speed difference (solid line versus dotted line, lower panel).

Figure 5: Relative sound speed differences between the sun and two reference models with radii 695.99 Mm (upper curve, triangles) and 695.78 Mm (lower curve, circles), versus fixed fractional radius (from Fig. 2 of Basu 1998).

Figure 6: Temperature versus density near the surface of two solar models of Guzik, Cox, & Swenson (1996). The models use identical input physics, except that the model represented by the dashed line was evolved with opacities increased by 50\% for \( T < 15,000 \) K.
Many refinements to the physics of modeling the solar surface regions have been considered. These can broadly be grouped into categories involving improved radiation and convective transport modeling. Some proposed improvements in the radiation modeling are: 1) including the effect of nonadiabaticity in the oscillation frequency calculation (e.g., Cox, Guzik & Kidman 1989; Kosovichev 1995); 2) adopting the Eddington instead of the diffusion approximation to radiation transport (Guenther 1994); 3) improving the surface T-r relation (Schlattl, Weiss, & Ludwig 1997); 4) including the chromospheric layers (Ulrich & Rhodes 1983); 5) modified low-temperature opacities (Guzik, Cox, & Swenson 1996; Neuforge 1993); 6) Planck vs. Rosseland mean opacities in the outer layers (A.N. Cox 1995, private communication).

In the convection category are such refinements as: 1) including turbulent pressure and energy (Kosovichev 1995; Rosenthal et al. 1998); 2) variable mixing length (Gabriel 1995; Demarque, Guenther, & Kim 1997); 3) improved mixing length treatment that takes into account the spectrum of turbulent eddies (Canuto & Mazzitelli 1991, 1992; Canuto, Goldman, & Mazzitelli 1996); 4) “patched” model with attached atmosphere calibrated to two- or three-dimensional turbulence models (Rosenthal et al. 1998; Demarque et al., these proceedings).

Most of the uncertainty in the radiative opacities occurs at too low a temperature (less than 6000 K) to have much effect on the superadiabatic layers at ~9000 K. The most important refinements appear to be including nonadiabatic effects and turbulent pressure. However, because all of the proposed improvements have a significant effect, it will be very difficult to use solar oscillations to unequivocally “calibrate”, for example, surface opacities, or free parameters in a convection treatment.

7. THE SOLAR CORE AND THE NEUTRINO PROBLEM

A comparison of sound speed inversions using different data sets, e.g., the GONG (Anderson et al. 1996), GOLF (Turck-Chieze et al. 1998), or BISON + Lowl (Basu et al. 1997) data sets show disagreements of several tenths of a percent for the inner ~20% of the solar radius. Therefore it is perhaps premature to evaluate the advantages of proposed improvements to the physics of the structure of the solar core. Nevertheless, some modifications to the core structure that have been proposed to solve the neutrino problem can be ruled out by helioseismology. Some recently investigated modifications include lowering the Z abundance or the opacity in the solar core (Guenther & Demarque 1997), and mixing "He from larger radii where it is burned to higher equilibrium concentration into the core (Cumming & Haxton 1996; Bahcall et al. 1997). The latter proposal has the advantage of substantially reducing the "Be neutrino flux, which seems to be absent in the neutrino detection experiments, as well as reducing the "B neutrino flux. Burning "He at small radii causes the "He+"He reactions to be favored over "He + "He, thereby reducing the "Be and "B neutrino fluxes. In addition, when "Be is produced, it is preferentially at small radii, where the temperature is high, favoring the "B neutrino production of the ppII chain over the ppIII chain produced "Be neutrinos. However, Bahcall et al. show that mixing the inner 25% of the solar radius according to the prescription of Cumming & Haxton results in a discrepancy of the sound speed in the core of 8%, compared to a discrepancy of only 0.2% for the standard (unmixed) solar model.

Richard & Vauclair (1997) investigate a proposed mixing scenario suggested by Morel & Schatzman (1996) involving stochastic internal waves, which alleviates, but does not completely solve the solar neutrino problem. For three parameterizations of the proposed mixing, Richard & Vauclair find rather large discrepancies of 2 to 6% between the calculated and helioseismically inferred Δυ/υ, where υ=η/p, which argue against even this relatively small amount of mixing.

In an alternative approach, Takata & Shibahashi (1997) solve for the temperature and hydrogen and helium composition profiles of the sun, given the inferred

![Low Degree p-Modes](image-url)
sound speed profile, luminosity, convection zone depth, and photospheric Z/X abundance. These profiles do not show any signature of core mixing, and the neutrino fluxes predicted for this seismic model are significantly larger than the observed rates. These results are independent of reasonable errors in the opacities, EOS or nuclear reaction rates.

To make further progress in deriving the core structure, a few g-modes would provide a very powerful discriminant. Figures 8 and 9 show the differences between calculated g-mode periods for some recent solar models, relative to the g-mode predictions for model 4 of Guzik & Swenson (1997) that uses the analytical SIREFF equation of state. All of these models use relatively modern input physics. The Guenther et al. (1992) model is the oldest, and uses 1992 OPAL opacities, Bahcall (1989) nuclear reaction rates, an analytical equation of state including Debye-Huckel corrections, but does not include helium or element diffusion. The Gabriel (1997) model includes 1996 OPAL opacities, a modern EOS similar to MHD, helium and element diffusion, and also includes rotationally-induced mixing below the convection zone which improves the sound speed and P/p agreement. The Brun et al. (1998) model includes 1996 OPAL opacities and EOS, helium and element diffusion, and their own nuclear reaction rate compilation. The three Guzik & Swenson (1997) models include 1996 OPAL opacities, helium and element diffusion, the Caughlan & Fowler (1988) nuclear reaction rates, but different equations of state—either MHD, 1996 OPAL, or the analytical SIREFF EOS. As can be seen, the g-mode periods can vary by several percent between models, which is a much larger percentage variation than for the p-mode predictions for these models (under 1%).

The two very tentative g-mode detections reported at this conference at 251.85 and 220.65 µHz, or 66.2 and 75.5 minutes, were identified as l=2. The models are unfortunately least sensitive to these short-period g-modes, but knowing the periods to within one or two percent would be sufficient to begin to discriminate between models.

CONCLUSIONS

We can consider whether we are finally reaching the limit of the usefulness of helioseismic comparisons to reveal deficiencies in the physics treatments of our solar interior models. As several of the papers in this conference have shown, some recent improvements to the input assumptions or physics, such as more detailed diffusion treatments, new opacities, solar radius determinations, or heavy element mixtures, have in fact made the agreement with helioseismology worse. But this criterion should by no means be the reason to reject these improvements. Helioseismic comparisons have been used to calibrate free parameters in physical models, for example, in proposed mixing scenarios or convection treatments, but there is a limit to the usefulness of this approach when different sets of input assumptions or parameters produce the same result, as is the case for the solar surface structure.

It is always desirable to compare predictions with the data in as many ways as possible. For example, modifications in the solar model structure that improve the overall sound speed agreement with the seismically inferred value may not improve the absolute frequency.
agreement for all ranges of p-mode order or degree. And it is imperative to test physical models and data (e.g., opacities, convection, diffusion coefficients, parametrized mixing scenarios, and nuclear reaction rates) for other types of stars where they may have larger observable consequences. Laboratory data for opacities, the equation of state, or nuclear reaction rates is continuing to improve, and should be assessed and included in the modeling.

We next consider future directions for solar interior modeling research. Our current one-dimensional standard solar models agree with helioseismic data remarkably well, but the extreme accuracy of observations leaves room for further refinements. Perhaps the most progress could be made in modeling explicitly in two and/or three dimensions the hydrodynamics of rotation, convection, the solar dynamo, or proposed rotational shear-induced or wave-driven mixing processes. These processes have usually been introduced in a simplified, parametrized manner to explain the solar surface stratification, Li or Be depletion, and sound speed discrepancies in the solar interior. We have yet to calculate solar evolution models that reproduce the inferred internal rotation profile, or to model in detail possible magnetic field variations that reproduce changes in p-mode frequencies during the solar cycle.

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