Investigation of Impulsively Loaded Pressure Vessels

N. Brown, R. Cornwell, D. Hanner, H. Leichter, P. Mohr

October 15, 1963

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Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under Contract W-7405-ENG-48.
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FROM: N. Brown, R. Cornwell, D. Hanner, H. Leichter, P. Mohr

SUBJECT: Investigation of Impulsively Loaded Pressure Vessels

Abstract

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Part II
a) Monolithic Vessel
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October 15, 1963

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ABSTRACT

Explosion containment vessels for containing from 2,000 to 3,000 five ton nuclear explosions are considered. Analysis methods appear adequate and lowest weights using the most advanced materials available in the next five years are:

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Steel</th>
<th>Weight (lbs.)</th>
<th>Thickness (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Room Temp.</td>
<td>Solid</td>
<td>460,000</td>
<td>10</td>
</tr>
<tr>
<td>Strength</td>
<td>&quot;Wallpaper&quot;</td>
<td>400,000</td>
<td>10</td>
</tr>
<tr>
<td>(2) 9.0 ft. radius</td>
<td>Wire Wound</td>
<td>350,000*</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>Titanium</td>
<td>410,000</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>Fiberglass</td>
<td>800,000*</td>
<td>50</td>
</tr>
</tbody>
</table>

None of these can be fabricated today and all require extensive development. Weights are for the sphere alone and in no way account for the "breech" hole, the nozzle or the resulting stress concentrations. No safety factors other than statistical fatigue and notch sensitivity considerations are used.

Fracture toughness concepts lead to the criteria of critical crack size in the finished vessel. To assure quality it is necessary to detect cracks in the vessel as small as .006 inches deep on the surface and .010 inches in the interior, in the case of cryogenic titanium.

A vital question is the exact temperature cycle for the vessel. By operating in the cryogenic range (-320°F) a titanium (Ti-6Al-4V) vessel of 265,000 pounds could be considered. However, use of this same material at 60°F (room temp.) would result in a weight of 410,000 pounds.

Present material technology limits the choice of materials and defines the weight. The addition of safety factors and fixtures (nozzles, etc.) will add to this weight considerably, and may well radically alter the vessel response. Improvements in the strength weight ratios of metals and glasses over * See Text Part III
those considered in this report do not appear reasonable at this time. Winding schemes to utilize the high strength of steel wires and somehow maintain a reasonable thickness appear to offer the most promise. A "ductile" beryllium would of course offer vast improvement, but no indications that this is being developed have appeared and all presently known beryllium is much too brittle.

A very promising approach to weight reduction is attenuation of the input pressure spike, either by controlling the explosion rate, using the propellant as a "foam" to smooth out the pulse, or a spring system between the outer shell and the explosion.

Large pressure vessels, both steel and fiberglass, of high strength weight ratios are currently used in solid rocket motor cases. Aerojet General Corporation is one builder of these and conferences have been held with both research and production personnel to become familiar with current and future problems.

This report is the work of several authors and some duplication exists. A list of references and a bibliography is given at the end of each section. The reader is cautioned not to draw conclusions from any one section but to consider all the factors discussed.
PART I

Dynamic Analysis

Design for explosion containment must be based on an analysis of bottle dynamics. This bottle will almost certainly be a sphere. There are three reasons for this. First, the sphere has a minimum surface to enclosed volume ratio so that all other things being equal it will result in minimum weight. Secondly, to contain a given pressure the stresses in a spherical vessel are less than for any other vessel of the same radius to thickness ratio (being only half those in an equivalent cylinder). Thirdly, and the most important, there are no bending stresses in a sphere even in the dynamic case if the explosion has true spherical symmetry. These bending stresses generally are an order of magnitude higher than the simple tensile stresses over which they are superimposed. A quick look at a high pressure pipe and the associated flanges will quickly bear this out.

A number of people have worked recently on the problem of designing vessels for explosion containment. Most of this work has been A.E.C. sponsored, the motivation being the containment of reactor accidents. W.E. Baker at Aberdeen Proving Ground has analyzed the elastic and elasto-plastic cases (1,2,3) and examined the case of stiffened elastic shells. R. Sankaranarayanan (4) has treated portions of the plastic case and work is currently in progress on this at Brown University.

There are also a series of very general papers on the vibrations of spherical shells principally by P. Naghdi of U. C. Berkeley (5). (Naghdi would be our best source for consultation on shell dynamics.)

Very little has been done in the field of fiber wound spherical vessels. It can easily be shown that all of the simpler winding schemes are basically unstable and comparatively inefficient for the spherical case. These effects can presumably be overcome by modified winding techniques. One of the few papers in this field is that by Read (6).

There has also been a large amount of experimental work done. Baker has included experimental results in his reports.
Considerable work has been done here at the lab under the direction of Bruce Crowley. He has an accumulation of five years worth of unpublished data and has designed vessels for this purpose. Fig. 1 shows summary test results and the resulting empirical design formula. Another interesting set of data is that of Hoffman and Mills of A.P.G. who studied shock effects on walls normal to H. E. blasts\(^7\).

The basic equations for the motion of a spherically symmetric elastic body have been around for a long time. Baker gives an excellent treatment in BRL 1113. This is almost certainly the case in point since the number of shots required restricts us to the elastic range. (Goffin's work above the yield point was for a slightly different case and is probably not applicable here.)

The complexity of the complete elastic solution is drastically reduced if thin shell conditions are assumed. Here the shell thickness is such that the variation in the radial displacements across the shell is very small with respect to the radial displacements themselves. This is a valid assumption in most cases of interest from a propulsion standpoint since a wall thick enough to have a significant variation would be too heavy for our use.
Average Peak Dynamic Stress vs. Weight of Charge

Data by W.B. Crowley from Vortex Program
for "Y" lbs H.E. in 2, 3 or 4 ft. Sphere - Air Medium

Constant Volume Process \( at = K \frac{Y}{r^2} \)

"Crowley's line" \( at = 4 \times 10^4 \left( \frac{Y}{R^2} \right) \)

Legend:
- ○ 4 ft. Dia. 3/8 in. wall
- ▲ 3 ft. Dia. 1/4 in. wall
- □ 2 ft. Dia. 3/16 in. wall

Note: units for each scale
\( \left( \frac{Y}{R^2} \right) \) for Pulse Rocket ~125

Figure 1
The following is the basic thin shell analysis. Assume a spherically symmetric shock system represented by \( F(t) \). Let \( u \) represent the outward displacement of the shell.

\[
\sum F = m \frac{d^2u}{dt^2}
\]

\[
m = \rho h (a d \Theta)(a d \Phi)
\]

\[
\sum F = F(t)(a d \Theta)(a d \Phi) - h(ad \Theta)(2 \sigma \phi \sin \frac{d \phi}{2}) - h(ad \Phi)(2 \sigma \phi \sin \frac{d \Theta}{2})
\]

By symmetry,

\[
\sigma_\theta = \sigma_\phi = \frac{E \epsilon}{1-\nu} = \frac{E}{1-\nu} \frac{u}{a}
\]

Substituting and simplifying,

\[
\frac{d^2u}{dt^2} + \frac{2}{a^2 \rho (1-\nu)} \frac{E}{\rho \nu} \frac{u}{a} = \frac{d^2u}{dt^2} + \sigma^2 u = \frac{F(t)}{\rho h}
\]
Note: A more rigorous approach leads to the following:

\[ \frac{\partial^2 u}{\partial t^2} - \frac{\lambda + 2\mu}{\rho} \left( \frac{\partial^2 u}{\partial r^2} + \frac{\partial}{\partial r} \left( \frac{\partial u}{\partial r} \right) - \frac{2u}{r^2} \right) = \frac{K(t)}{\rho} \]

where \( \lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \), \( \mu = \frac{E}{2(1+\nu)} \)

so \( \lambda + 2\mu = \left( \frac{E}{1-\nu} \right) \left( 1 - \frac{3\nu^2 - 3}{\nu^2 + \nu - 1} \right) \approx \frac{E}{1-\nu} \)

The latter differs from the simpler equation by the underlined terms and the physical constants which for \( \nu = 0.30 \) differ by about 6%.

A brief examination of these yields some interesting features. First, the one dimensional treatment describes a single degree of freedom system. The more rigorous system yields an infinite series of eigenfunctions. However, the nature of these is such that the first mode deviates from that predicted by the simpler approach by but a few percent. Further the amplitude of the higher modes is such that they can be neglected for all practical purposes. This would seem to be borne out by the experimental evidence.

It is an interesting point that it is difficult to excite other modes. Crowley reports one extreme case where he detonated a shaped charge in a spherical vessel. He obtained this first mode excited to an amplitude somewhat greater than by a similar charge symmetrically exploded.
Another interesting feature is the natural frequency expression

\[ \omega = \frac{1}{a} \sqrt{\frac{E}{\rho (1-\nu^2)}} \]

For most materials the ratio of the properties under the radical is a constant. Thus

\[ \omega = \frac{3.42 \times 10^5}{a} \text{ rad/sec} \text{ for steel} \]

\[ \omega = \frac{3.37 \times 10^5}{a} \text{ rad/sec} \text{ for aluminum} \]

\[ \omega = \frac{3.48 \times 10^5}{a} \text{ rad/sec} \text{ for titanium} \]

Consequently the period of the vessel is directly proportional to the radius. Since the period of shock traversal of the vessel cavity is also proportional to the radius, the phase relationships between stimulus and response will be an invariant for these materials.

Now consider the shock system. This may be described mathematically as the sum of a series of delta functions and step functions.
Now, if the input is assumed to be

\[ F(t) = A \delta + B \]

where \( \delta \) represents the Dirac delta function, \( A \) represents the area under the impulse curve, and \( B \) the height of the step input, the solution will be of the form

\[ h u(t) = \frac{A}{\omega} \sin \omega t \left[ \begin{array}{c} \frac{B}{\omega^2} (1 - \cos \omega t) \\ \end{array} \right] \]

The computer solution of T. Stubbs indicates there are three significant pulses. This would seem to agree with the experimental data, the shock system generally decaying into hash by the time the third shock arrives.

Thus using the Heaviside gating operator \( H(t) \)

\[ H(t) = 0, \quad t < t_1 \]
\[ H(t) = 1, \quad t \geq t_1 \]

\[ \rho h u(t) = \frac{A}{\omega} \sin \omega t \left[ \begin{array}{c} \frac{B}{\omega^2} (1 - \cos \omega t) + H(t) \left[ \frac{C}{\omega} \sin \omega (t - t_1) \\ \frac{D}{\omega^2} (1 - \cos \omega (t - t_1)) \right] + H(t_2) \left[ \frac{E}{\omega} \sin \omega (t - t_2) + \frac{F}{\omega^2} (1 - \cos \omega (t - t_2)) \right] \end{array} \right] \]

\[ = \sin \omega t \left[ \frac{A}{\omega} + H_1 \frac{C}{\omega} \cos \omega t - H_2 \frac{D}{\omega^2} \sin \omega t + H_3 \frac{E}{\omega} \cos \omega t e^{-\frac{t}{\omega^2}} \right] + \cos \omega t \left[ -\frac{B}{\omega^2} - H_1 \frac{C}{\omega} \sin \omega t - H_2 \frac{D}{\omega^2} \cos \omega t - H_3 \frac{E}{\omega} \sin \omega t e^{-\frac{t}{\omega^2}} \right] \]
\[ + \left[ \frac{B}{\omega^2} + H_1 \frac{D}{\omega^2} + H_2 \frac{E}{\omega^2} \right] \]
This is of the form

\[ \phi u(t) = \alpha \sin \omega t + \beta \cos \omega t + \gamma \]

Consequently extrema appear at

\[ \tan \omega t = \alpha / \beta \]

For purposes of illustration we have used the input calculated by T. Stubbs, as follows:

- \( A = 334 \times 10^{-6} \) Kb sec
- \( C = 138 \times 10^{-6} \) Kb sec
- \( E = 215 \times 10^{-6} \) Kb sec
- \( B = 0.30 \) Kb at \( t = 0 \) sec.
- \( D = 0.35 \) Kb at \( t_1 = 400 \times 10^{-6} \) sec
- \( F = 0.15 \) Kb at \( t_2 = 870 \times 10^{-6} \) sec

Now assuming a nine foot radius vessel with 14 cm walls to conform to a computer calculation by Stubbs we find a natural frequency of \( 3.09 \times 10^3 \) rad/sec. The phase angles for the second and third pulses are \( 71^\circ \) and \( 154^\circ \) respectively. Thus the first maximum occurs after the second pulse and is a displacement of 1.73 cm corresponding to 264 KSI. The third pulse comes at \( 154^\circ \) while the vessel is returning and retards this motion so that further displacements are at 1.54 cm and \(-0.585\) cm, a half amplitude of \( 0.06 \) cm oscillating about a mean of \( 0.48 \) cm. Thus after the first maximum we have an alternating tangential stress of 162 KSI alternating about a mean 72.4 KSI to give maximum and minimum of 234 KSI and -89 KSI respectively.

It is interesting to note that an empirical formula developed by Crowley

\[ \sigma_{max} = \frac{4 \times 10^4}{h (in)} \left( \frac{W (lb)}{r^2 (ft^2)} \right)^{0.75} \]
yields a maximum stress of 259 KSI, only 2% different although it has been extrapolated by two orders of magnitude. The latter is also somewhat disconcerting because the model laws on the basis of which people are drawing conclusions do not yield such a formula!! There are some other significant conclusions that can be drawn from these results. For example, consider the case where the impulse is completely attenuated by some shock absorbent material or mechanism prior to striking the wall so that only the step functions remain. We then have an alternating stress of 72.4 KSI cycling about an identical mean to yield a stress varying sinusoidally from zero to 145 KSI.

Both the proportionate magnitude and the form are important. In the second case we do not have the very large first pulse. Furthermore, here the alternating stress component is less than half the corresponding component in the first case (72 vs. 162). Further, this component alternates about an identical mean in the second case so that the average stress cycles between zero and twice the mean. This is similar to the thermal stress case studied by Coffin. There are a number of regions where there is a concentration factor of two. In these regions yielding would occur on the first cycle so that here there would be a built in compressive stress at the zero or equilibrium position. Neglecting fatigue factors, etc., a vessel of the given dimensions built for the second system would require a yield strength of about 150 KSI. Built for the first case it would require a yield of about 360 KSI using the same type of argument.

An analytical study is being made here to determine the important shock absorber parameters of a mechanical system. Los Alamos reports considerable success with aerosol foams. On the other hand, one laboratory experience with a solid plastic foam showed that it was detrimental, apparently forming load concentration points.

There are two materials for which the ratio of Young's modulus to density vary considerably from that quoted above. In the case of beryllium the natural frequency is higher. Since this appears as a square in the terms
pertaining to step function response, the coupling here is poor. However, it is so slight that the impulse couples to an extent which just compensates for this. In the case of lead on the other hand the effect is just the opposite. Only the step functions couple appreciably yielding a factor of two rather than a factor of four. This property together with its fabricability makes lead reinforced with steel wire an interesting contender, but it is out of the question for a flight vessel.

Now there are several basic problems that have not yet been treated analytically. The first of these is the nozzle. This should probably not be rigidly attached to the vessel at the point of attachment since this will set up an additional mode of vibration, this time resulting in high bending stresses.

Another problem is the hole. For a circular opening in a sheet in biaxial tension the theory of elasticity predicts a stress concentration factor of two. For a case where the impulse load is completely eliminated we could probably live with this, since it would yield to a point where it would be cycling between equal compression and tension stresses rather than between zero and a tensile stress. For the impulsive case this is not possible. The common technique is to increase the thickness in this region. To do so here for a hole of large diameter results in a significant increase in stiffness locally and results in bending stresses close by. Crowley reports that he was able to reduce this factor of 2 to only $1\frac{1}{2}$ for this reason.

Another problem is the magnitude of the stresses involved. No one has to our knowledge investigated the fatigue characteristics of materials due to impulsive loading of this magnitude. We are not at all certain that the traditional elastic analysis is valid for the radial compressive wave. Evidences of energy dissipation from equation of state work may well be evidences of internal damage which will show up after repeated loadings.
A sacrificial goat may well be the answer, but it is going to take a fantastic amount of testing to determine this.

Another unknown in vessel design is the so called "scabbing" or spalling due to high impulse loads. Davids (7) develops some criteria for stress waves penetrating axially into plates. According to his results we are operating in a very bad region. The effects of reflected waves from the back surface can accentuate cracking and lead to spalling, especially if tensile forces are already trapped in the material.

2. W. E. Baker - The Elastic-Plastic Response of Thin Spherical Shells to Internal Blast Loading. BRL 1194

9. A. L. Austin - Thermally Induced Vibrations of Thin Concentric Spherical Shells - U.C.R.L. 7402
PART II  a)

**MONOLITHIC VESSEL**

**Purpose:**

The purpose of this section is to examine the feasibility of constructing the spherical pressure vessel as a metallic monolith, capable of operating through a temperature range of cryogenic to about $200^\circ F$ or higher.

**Material Selection**

Some of the more important properties which must be considered in evaluating materials for this purpose are listed below:

1. Yield-strength to density ratio throughout operating temperature range
2. Notch-strength to tensile strength ratio
3. Fracture-toughness as a function of section thickness and temperature
4. Endurance or fatigue limits
5. Damping Capacity
6. Metallurgical size effects
7. Reaction of material to shock loading
8. Formability
9. Weldability and weld efficiencies
10. Embrittlement due to large neutron doses

Fig. 2 shows yield strength to density ratios for a number of alloys over a wide temperature range. It will be noted that at cryogenic temperatures heat treated 6 Al-4V titanium alloy has the highest ratio. Note that beryllium sheet (extrapolated to cryogenic temperatures) also indicates a good strength to density ratio. Beryllium at the present, however, must be eliminated from consideration due to its lack of development, and its extreme brittleness. Other metals showing acceptable strength to density ratios in
YIELD STRENGTH TO DENSITY RATIOS FOR VARIOUS ALLOYS

18% Ni (250) MARAGING STEEL

301 SS (60% COLD REDUCED)

TITANIUM 6AL-4V (AGED BAR)

6AL-4V (ANNHELED)

4340

301 SS

RENE 41

5 Ti, Mo

900 1200 1400 1600 1800 2000 2200

E = [psi]
this range are some steels including the 18% Ni Maraging variety (1),
some aluminum alloys, and annealed 6 Al-4V and AlloMAT Titanium Alloys.

Since portions of the vessel will likely operate at times at
cryogenic temperatures and must withstand impulsive shock loading within
this temperature range, the most important consideration is probably
notch sensitivity and fracture toughness of the material.

Table 1 lists for comparison the notch tensile ratio and impact
toughness for a group of sheet materials selected for a high or relatively
high strength to density ratio, Ref. (2) through (7). Based on notch
sensitivity it may be seen that all of the materials are acceptable with
the exception of 7178-T6 and 4340 steel, both of which seem to exhibit
high notch sensitivity at low temperatures. The 301 and 310 cold re-
duced stainless steels both appear to have excellent low temperatures
toughness, but unfortunately they derive their high strength-to-density
ratio from cold work and therefore will be eliminated from consideration
for thick sections. Similarly, a metallurgical size effect exists for
the aluminum alloys, in that quench delay times must be kept to a minimum
and, thus, they cannot be fully hardened in sections over 5 to 6 inches
thick. This is also true for titanium alloys such as 6Al-4V. These al-
loys are shallow hardening and where heat treated in large sections exhibit
strengths only slightly higher than in the annealed condition.

From the foregoing considerations it is apparent that the material used
in a thick wall monolith must not require quenching or rapid cooling in
order to attain a high strength to density ratio. Among the materials con-
sidered, only extra low intersitral (E.L.I.) vacuum melted AlloMAT and 6 Al-4V
titanium alloys and the recently developed 18% Ni Maraging Steel heat treat-
ed to 250,000 psi yield, seem to fulfill these requirements.

The AlloMAT is used in the annealed condition, the 6Al-4V can be used
annealed, and the maraging steel requires only slow air cooling in its heat
treatment cycle. Only these materials will be considered for a monolithic
vessel.
<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>TEMP.</th>
<th>σ_y/σ</th>
<th>NOTCHED-UNNOTCHED Tensile Ratio</th>
<th>*Charpy V-notch Ft-lb.</th>
</tr>
</thead>
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<td>Kt = 6.3</td>
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</tr>
<tr>
<td>6Al-4V (aged)</td>
<td>+78</td>
<td>1.01</td>
<td>1.13</td>
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<td></td>
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<tr>
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<td>5</td>
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<td></td>
<td>-423</td>
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<td>.60</td>
<td>-</td>
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<tr>
<td>18% Ni Maraging Steel (250) (Vac Melt)</td>
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<td>1.4</td>
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<tr>
<td></td>
<td>-320</td>
<td>1.09</td>
<td>.74</td>
<td>20</td>
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*Bar
PART II  b)  

FRACTURE TOUGHNESS CRITERIA

It has recently become generally recognized that one of the primary causes for failure of high-strength materials in structural components (welded or unwelded) is that they contain flaws or other defects which act to trigger crack propagation. The size of the flaw that will cause catastrophic failure of the structure depends upon the "fracture toughness" of the material (the larger the flaw and the lower the fracture toughness, the higher the probability of catastrophic failure). The smallest size defect in a material of a given fracture toughness which will become self-propagating at a given stress is called the "critical crack length".

The original hypothesis proposed by Griffith about 40 years ago stated that the stress necessary to propagate a surface crack in a thick plate of brittle material is

$$\sigma = \left[ \frac{2 \cdot E \cdot \gamma}{(1 - \nu)^2 \cdot \pi \cdot a} \right]$$

where  
$E$ = Young's Modulus  
$\gamma$ = surface energy  
$\nu$ = Poisson's ratio  
$a$ = crack length

The Griffith Theory has been modified by Irwin (11) and his associates into the currently most widely accepted criterion for fracture toughness in metals which takes into account a small amount of plastic deformation on a microscopic scale. It relates a parameter called the strain energy release rate "$G$" or crack extension force (in units of in-lb/in$^2$) to the critical crack size. When this quantity reaches a critical value $G_c$ the
crack will propagate rapidly. For a finite plate of width $W$ with a critical crack length of $2a$ or a surface crack of length $a$, the crack extension force for tensile loading is given by

$$G_c = \frac{\sigma^2 W}{E} \tan \left( \frac{\pi a}{W} \right) \quad (12)$$

For a crack of length $2a$ in an infinitely wide plate the relationship between stress and $G_c$ is given by

$$G_c = \frac{\pi \sigma^2 a}{E}$$

$G_c$ is usually determined by precracked center-notch tensile tests, precracked slow bend tests, or precracked Charpy impact tests. Fair correlation has been obtained with all of these methods; however, the precracked Charpy impact tests tend to give higher values of $G_c$ in the more plastic regions. Fig. 3 shows $G_c$ (Charpy) values for 4340 steel as a function of sheet or plate thickness at three temperatures (13). With increase in sheet thickness, the fracture toughness value (at 100°C) increases to a maximum and then decreases. The initial increase is associated with an increase in the volume of metal deformed per unit cross-sectional area, as a result of lateral contraction. The toughness then decreases due to an increase in the area subject to flat fracture (plane strain) at the center, relative to the area undergoing shear fracture at the surface. This maximum toughness usually occurs between 0.1 and 0.2 inches thickness for most materials at room temperature.

The fracture toughness $G_c$ approached asymptotically a value $G_{ic}$ with increasing plate thickness (14). This limiting value of toughness is associated with a mode of fracture which is primarily of the flat or plain strain type, and represents the fracture toughness of a relatively thick plate (one inch or more for most materials). At very low temperatures the fracture toughness, $G_c$, may also equal $G_{ic}$ and be the same for all plate thickness as indicated in Fig. 2 at -196°C.
Fig. 3. Fracture toughness vs. specimen thickness for specimens tempered at 800°F (taken from ASD-TDR-62-868, June 1963).
It is obvious that the material property $G_{IC}$ is of primary importance in the design of thick-walled pressure vessels. It indicates the size of a flaw or crack that may be tolerated. Unfortunately, however, materials may contain flaws too small to be detected by practical inspection techniques, and yet such imperfections can cause catastrophic failures in high strength materials of low fracture toughness.

Only recently has the fracture toughness concept been applied quantitatively to the design of rocket motor cases and cryogenic pressure vessels. At present, much research is being done in determining fracture toughness values for high strength sheet materials, but to date, only a small amount of data has been published. Table 2 is a compilation of fracture toughness values for high strength sheet materials and their corresponding $G_{IC}$ values. From this and other data, $G_{IC}$ values for 18% Ni Maraging Steel, Ti 6 Al-4V, and A 110AT (Ti 5Al-2.5Sn-) have been estimated for welded and unwelded plate at both room temperature and -320°F. Using these values, the critical crack size ($2a$) was calculated over a wide stress range, and are presented in Figures 4 through 9.

For various reasons (stress level uncertainty and the possibility of unequal stress distribution throughout the structure) it seems unwise, for crack propagation considerations, to count the applied stress as less than the yield strength of the material. From this we see that for a welded pressure vessel of 18% Ni Maraging Steel operating at room temperature, an internal crack ($2a$) of .09" or a surface crack ($a$) of .045" would likely cause catastrophic failure if the stress reached the yield point of the material. At -320°F an internal crack of .04" or a surface crack of .02" would cause failure at the yield stress. Since cracks smaller than critical size may slowly grow, a growth factor of 2 is customarily allowed. Based on this factor, it would be necessary to inspect out all cracks in the cryogenic steel vessel greater than .01 inches on the surface and .02 inches interior. Table 3 summarizes the permissible crack size for the three materials under consideration.
Permissible Crack Sizes for Heavy Welded Sections

<table>
<thead>
<tr>
<th>Material</th>
<th>78°F Permissible</th>
<th>-320°F Permissible</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Surface Crack</td>
<td>Interior Crack</td>
</tr>
<tr>
<td>18% Ni Maraging</td>
<td>.02</td>
<td>.045</td>
</tr>
<tr>
<td>Steel</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AlloMAT</td>
<td>.03</td>
<td>.05</td>
</tr>
<tr>
<td>Ti 6Al-4V</td>
<td>.025</td>
<td>.05</td>
</tr>
</tbody>
</table>

Examination of the permissible crack size for cryogenic vessels in Table 3 indicates that interior cracks from .01 to .02 inches must be detected and eliminated in welded areas. In practical production inspection techniques, a flaw of about 100 mils may be detected and located with a high degree of confidence, using ultrasonic methods (15). With much greater care and less certainty this size may be extended down to possibly 20 mils. It may be seen, therefore, that detection and location of 10 mil flaws in a material which is 10 to 18 inches thick may prove quite difficult. Flaws as small as a few thousandths of an inch have been detected and located ultrasonically in thick sections of metal (16).

The foregoing discussion has assumed that the vessel material would exhibit crack propagation properties similar to those thus far experienced in structures such as missiles, cryogenic pressure vessels, and high performance aircraft. The vessel under consideration, must withstand thousands of impulsive or shock loadings, giving rise to high alternating stresses superimposed upon a decaying static stress. The effect of this environment may considerably change the concept of fracture criteria for this vessel.
### Table 2

Fracture Toughness Values for Various Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Thickness</th>
<th>Yield Str. #/in²</th>
<th>Fractures Toughness in-lbs/in²</th>
</tr>
</thead>
<tbody>
<tr>
<td>18% Ni Maraging Steel</td>
<td>.06</td>
<td>250,000</td>
<td>G&lt;sub&gt;c&lt;/sub&gt; 500* 270*</td>
</tr>
<tr>
<td>TIG Welded</td>
<td>.08</td>
<td>227,000</td>
<td>G&lt;sub&gt;c&lt;/sub&gt; 1200 1600* 300* 200*</td>
</tr>
<tr>
<td>7075-T6</td>
<td>.06</td>
<td>72,000</td>
<td>G&lt;sub&gt;lc&lt;/sub&gt; 700 - 115 -</td>
</tr>
<tr>
<td>2024-T6</td>
<td>.125</td>
<td>50,000</td>
<td>G&lt;sub&gt;lc&lt;/sub&gt; 2740 - 300 -</td>
</tr>
<tr>
<td>SAE 4340</td>
<td>.25</td>
<td>230,000</td>
<td>G&lt;sub&gt;c&lt;/sub&gt; 140 90 125 80</td>
</tr>
<tr>
<td>Ti B-210-VCA</td>
<td>.09</td>
<td>165,000</td>
<td>G&lt;sub&gt;c&lt;/sub&gt; 700 - 218 -</td>
</tr>
<tr>
<td>Ti 6Al 4V (annealed)</td>
<td>.063</td>
<td>129,000</td>
<td>G&lt;sub&gt;lc&lt;/sub&gt; - - 275* 120*</td>
</tr>
<tr>
<td>Welded</td>
<td>.063</td>
<td>129,000</td>
<td>G&lt;sub&gt;c&lt;/sub&gt; - - 170* 110*</td>
</tr>
<tr>
<td>Ti Allo AT (annealed)</td>
<td>.063</td>
<td>135,000</td>
<td>G&lt;sub&gt;c&lt;/sub&gt; - - 300* 130*</td>
</tr>
<tr>
<td>Welded</td>
<td>.063</td>
<td>130,000</td>
<td>G&lt;sub&gt;c&lt;/sub&gt; - - 180* 120*</td>
</tr>
</tbody>
</table>

*Estimated from best data available.*
<table>
<thead>
<tr>
<th>WORKING STRESSES X 10^5 PSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25</td>
</tr>
</tbody>
</table>

* VNELO STRNEHT *

WORKING STRESSES AT 10^6 CYCLES

WORKING STRESSES AT 1.66 X 10^6 CYCLES

\[
\frac{170 - 0.2}{2.6} = 2.66^2
\]

Estimated values of CID

Crack size calculated from

WELOED

UNWELOED

Temp = 185

Thickess: 2.5

Critical crack size for 18% Ni Martekine Steel (E50)

1.6
FIG. 5

CRITICAL CRACK SIZE* FOR 18% Ni, MARAGING STEEL (250)

THICKNESS > 5"

TEMP. = -320°F

- UNWELDED
- WELDED (TIG)

* CRACK SIZE CALCULATED FROM ESTIMATED VALUES OF GIC

2a = \frac{2 \cdot GIC \cdot E}{\pi \cdot \sigma^2}

WORKING STRESS AT 1.66 \times 10^5 CYCLES

WORKING STRESS AT 10^6 CYCLES

WORKING STRESS \times 10^{-5} PSI
FIG. 6

CRITICAL CRACK SIZE* FOR TI 6Al-4V (ANNEALED)

THICKNESS > 5"
TEMP. = 78°F
-- UNWELDED
--- WELDED

* CRACK SIZE CALCULATED FROM ESTIMATED VALUES OF GIC:

\[ 2a = \frac{2.6Gic \cdot E}{\pi - \sigma^2} \]

WORKING STRESS AT 10^6 CYCLES

WORKING STRESS AT 10^6 CYCLES

YIELD STRENGTH

WORKING STRESS X 10^-5 PSI
FIG. 8

CRITICAL CRACK SIZE* FOR A110 AT (T2-5A1-2.5Sn)

THICKNESS > 5"
TEMP. = 75°F

UNWELDED
WELDED

*CRACK SIZE CALCULATED FROM ESTIMATED VALUES OF GIC

$$2a = \frac{2 \cdot GIC \cdot E}{\pi \cdot \sigma^2}$$

WORKING STRESS AT $10^6$ CYCLES

WORKING STRESS AT $10^4$ CYCLES

YIELD STRENGTH

WORKING STRESS X 10^-5 PSI
**Fig. 9**

**Critical Crack Size**

For A110 at (T = 51F - 2.55h)

**Thickness** = 5"  
**Temp** = 320°F

Crack size calculated from:

\[ a = \frac{2GcE}{\sigma_y^2} \]

*Crack size at 10^4 cycles

Yield strength

Working stress at 10^4 cycles

<table>
<thead>
<tr>
<th>Working Stress x 10^4 PSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.0</td>
</tr>
<tr>
<td>15.0</td>
</tr>
<tr>
<td>16.1</td>
</tr>
<tr>
<td>17.0</td>
</tr>
<tr>
<td>18.0</td>
</tr>
<tr>
<td>19.0</td>
</tr>
<tr>
<td>20.0</td>
</tr>
</tbody>
</table>

Critical Crack Size (inches)
PART II  c)

THICKNESS EFFECTS

From the dynamic response equation for a spherical vessel one finds the frequency of the vessel is determined by the following equation

$$\omega = \frac{L}{a} \sqrt{\frac{2E}{\rho(1-\nu)}}$$

where
- $E$ = Modulus of elasticity
- $\rho$ = Density
- $\nu$ = Poisson's ratio
- $a$ = Mean radius of vessel

If the internal radius of the vessel is held constant and Poisson's ratio is assumed to be the same for materials under consideration the frequency becomes a function of $\frac{E}{\rho}$ only. This is only approximately true as the thickness will vary for each material, therefore varying the mean radius. For aluminum, titanium and steel, $\frac{E}{\rho}$ is constant for all practical purposes.

Again if one goes through the dynamic equation the circumferential stress becomes a function of $\frac{E}{\rho}$ when $a$ and $\nu$ are constant. Therefore, the circumferential stress times thickness will be approximately the same for the three above materials for any given input. The solid line in Figure 10 shows this stress as a function of thickness for the dynamic response reported by T. Stubbs for a nine foot vessel.

For the above case the vessel contains an internal gas pressure of 10,000 psi at the instant the maximum circumferential stress is reached. Taking this into account the vessel material is under tri-axial stress and the yield criterion should become something other than circumferential stress alone. Among the many tri-axial yield criteria proposed, the Von Mises or
Distortion Energy Theory fits the experimental data best and will be used in this case. The equation proposed by Von Mises is

\[ \sigma_0 = \frac{1}{V^2} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2 \right]^{\frac{1}{2}} \]

- \( \sigma_0 \) = allowable uniaxial working stress of material
- \( \sigma_1, \sigma_2 \) = circumferential stress
- \( \sigma_3 \) = normal stress

For our case \( \sigma_1 = \sigma_2 \) and \( \sigma_3 \) equals the internal pressure. For these conditions the above equation reduces to

\[ \sigma_0 = \sigma_1 = \sigma_2 - \sigma_3 \]

As \( \sigma_1 \) and \( \sigma_2 \) are tensile stresses and \( \sigma_3 \) is a compressive stress,

\[ \sigma_3 = -10,000 \text{ psi} \]

\[ \sigma_0 = \sigma_1 = \sigma_2 + 10,000 \text{ psi} \]

The dashed line in Figure 10 gives \( \sigma_0 \) as a function of thickness. However, this stress should not be confused with the maximum stress within the material and gives only a yield criterion for the material. It can be seen from Figure 10 that for a vessel thickness larger than 10 inches that the normal stress becomes an increasingly important factor. It also becomes apparent that the strength to weight ratio would not be the only factor in determining final weight of a vessel made from a material with a yield strength of less than 150,000 psi. Vessel weights for various materials as a function of thickness are shown in Figure 11.
Stress vs Thickness for Internal Radius Sphere \( (E = 10,600 \times 10^7 \text{ in}) \)

Distortion Energy Theory (Von Mises)

\[ \sigma_{0s} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \]
PART II  d)

Fatigue Failure Criteria

As the allowable working stress for all materials is reduced under cyclic stress, a fatigue failure criterion must be used. The spherical vessel under consideration is displaced mostly by impulsive loading which causes large amplitude vibrations of the vessel. The vessel frequency of materials with $\frac{E}{\rho} = 10.6 \times 10^7$ in. is approximately 500 cycles for a 9 ft. radius sphere. If no damping occurred this would account for $1.25 \times 10^7$ cycles for 10 seconds between detonations. This would reach the endurance limit for most materials and should represent the maximum number of cycles. On the other hand, it would seem that a minimum number of four cycles per detonation would occur which would give a total of $10^4$ cycles. As the endurance limit can be as low as 40% of the yield strength it becomes apparent that the total number of cycles should be kept as low as possible. Data collected by Crowley at LRL indicates that spherical vessels shocked by H. E. produce only a few large amplitude cycles before becoming low amplitude hash. If this held true in the case under discussion only $10^4$ cycles would need be considered, assuming there would be no side effects. However, if this is not true, then it becomes necessary to consider the self damping capacity of the metal of the vessel.

Available literature report the damping capacity will vary with material, temperature, frequency, applied stress and the ratio of applied stress to static stress. Damping capacity for various materials varies over two orders of magnitude. Also the damping capacity will vary over one order of magnitude for a given material under various conditions.

The specific damping capacity is defined as

$$\psi = \frac{2(A_1 - A_m)}{(A_1 + A_m)}$$

where

$\psi' = $ specific damping capacity

$A_1 =$ amplitude of first cycle

$A_m =$ amplitude of mth cycle

$M =$ number of cycles from $A_1$ to $A_m$
The damping properties of materials are frequently expressed in terms of the logarithmic decrement and

\[ 2\delta = \nu \]

The literature on titanium damping capacity shows it to be extremely low with no room temperature values above \( \nu = 0.0015 \) \((8)\). This would give a total number of cycles of \( 1.1 \times 10^6 \) above 50\% of the allowable working stress. No data is available on the damping capacity or fatigue life of 18\% Ni Maraging Steel \((250)\). However, a room temperature endurance limit of 95,000 psi has been reported \((10)\) with no mention of the number of cycles where it occurred. If a damping capacity of .01 is assumed for maraging steel the total number of cycles becomes \( 1.66 \times 10^5 \).

The fatigue life and endurance limit of various materials varies with temperature, the ratio of the variable stress to the mean stress, the type of stress (bending, tension, compression or torsion), with size and surface effect. In general, both the fatigue life and endurance limit are increased with decreasing temperature down to cryogenic temperatures.

In all cases, the endurance limit is increased with a decreasing ratio of the applied stress to the mean stress. One conservative design criterion is this Soderberg Line which states

\[
\frac{1}{N} = \frac{S_m}{S_y} + \frac{S_v}{S_m}
\]

where
- \( S_m \) = mean stress
- \( S_v \) = variable stress
- \( S_y \) = yield strength
- \( S_m \) = endurance strength with no mean stress
- \( N \) = safety factor

Fig. 11 shows the \% increase in the endurance limit as a function of the ratio of the endurance limit to the yield strength for the 9' radius
vessel where \( \frac{E}{P} = 10.6 \times 10^7 \) in. Most fatigue data is for bending stress but our case is more analogous to a tension-compression fatigue. For steels where some tension-compression data is available the reduction in endurance limit from bending to tension-compression is approximately 15\% (9). Data shows very little size effect and no surface effect for tension-compression fatigue.

For a large number of materials, \( \frac{S_m}{S_y} \) is between 0.4 to 0.6. Fig. 11 shows a gain of 12\% to 18\% and would approximately offset the loss incurred in going from bending to tension-compression endurance limit. Until fatigue data is available for materials under consideration it would seem reasonable to use uncorrected bending fatigue data for our design. No mention thus far has been made to the increase in fatigue life due to damped vibrations except for ignoring all cycles at a stress level less than half the working stress. However, the fact that no safety factor has been set for fatigue failure criterion and only approximate damping capacities are being used, this gain should probably be ignored for the present.

Fig. 12 shows an estimated S-N curve for 18\% Ni Maraging Steel using the following data.

1. Yield strength at room temp. - 245,000 psi
2. Yield strength at -321°F - 290,000 psi
3. Endurance limit - 95,000 psi

For this same curve the following assumptions were made.

1. Endurance limit occurs at \( 10^7 \) cycles
2. Fatigue life for cycles less than \( 10^7 \) is linear on a semi-log plot
3. Endurance limit at -321°F - 110,000 psi
Using $1.66 \times 10^5$ cycles (estimated damping capacity) the allowable working stress is 133,000 psi (Fig. 13). From Fig. 10 the thickness for a 9 ft. radius sphere is 12 inches and the weight is 560,000 pounds. For $10^4$ cycles the allowable working stress is 158,000 psi, the thickness is 10 inches and the weight is 460,000 pounds. An estimated curve for cryogenic temperatures is also plotted in Fig. 12 and is based on the increase in yield strength at the lower temperature. These values are shown in Table 4.

Fatigue values for titanium alloys are also shown in Table 5. The Ti-4Al-4Mn alloy would be eliminated from a fabrication standpoint, but values are given to show the increase in fatigue life of titanium alloys at cryogenic temperatures. Final design values for a titanium alloy vessel are shown in Table 4. The weights shown in this table are for a spherical vessel without openings and exclude the weight of the nozzle.
<table>
<thead>
<tr>
<th>Material</th>
<th>Temp. °F</th>
<th>No. Cycles</th>
<th>Allowable Working Stress KPSI</th>
<th>Thickness Inches</th>
<th>Weight 1000 pounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1-6Al-4V (Annealed)</td>
<td>RT</td>
<td>$10^6$</td>
<td>93</td>
<td>18</td>
<td>470</td>
</tr>
<tr>
<td>T1-6Al-4V (Annealed)</td>
<td>RT</td>
<td>$10^4$</td>
<td>110*</td>
<td>15</td>
<td>410</td>
</tr>
<tr>
<td>T1-6Al-4V (Annealed)</td>
<td>-321</td>
<td>$10^6$</td>
<td>120*</td>
<td>13.5</td>
<td>365</td>
</tr>
<tr>
<td>T1-6Al-4V (Annealed)</td>
<td>-321</td>
<td>$10^4$</td>
<td>155*</td>
<td>10.2</td>
<td>265</td>
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<tr>
<td>T1-5Al-2.5 Sn &quot;</td>
<td>RT</td>
<td>$10^6$</td>
<td>78</td>
<td>19</td>
<td>530</td>
</tr>
<tr>
<td>T1-5Al-2.5 Sn &quot;</td>
<td>RT</td>
<td>$10^4$</td>
<td>100*</td>
<td>16.5</td>
<td>450</td>
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<tr>
<td>T1-5Al-2.5 Sn &quot;</td>
<td>-321</td>
<td>$10^6$</td>
<td>115*</td>
<td>14.5</td>
<td>390</td>
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<tr>
<td>T1-5Al-2.5 Sn &quot;</td>
<td>-321</td>
<td>$10^4$</td>
<td>145*</td>
<td>11</td>
<td>290</td>
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<tr>
<td>18% Ni Maraging Steel (250)</td>
<td>RT</td>
<td>$1.66 \times 10^5$</td>
<td>133*</td>
<td>12</td>
<td>560</td>
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<tr>
<td>18% Ni Maraging Steel (250)</td>
<td>RT</td>
<td>$10^4$</td>
<td>158*</td>
<td>10</td>
<td>460</td>
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<td>$1.66 \times 10^5$</td>
<td>152*</td>
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<td>18% Ni Maraging Steel (250)</td>
<td>-321</td>
<td>$10^4$</td>
<td>183*</td>
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*Estimated Values
<table>
<thead>
<tr>
<th>5 x 10^6</th>
<th>10^7</th>
<th>Material Conditions</th>
<th>Remarks</th>
<th>Refs.</th>
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</thead>
<tbody>
<tr>
<td>78</td>
<td></td>
<td>Ann Bar</td>
<td>Butt fusion welded joints</td>
<td>10</td>
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<tr>
<td>62</td>
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<td>Ann Sheet</td>
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<td>74</td>
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<td>Notched K=3.3</td>
<td>7</td>
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<td>62</td>
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<td>Ann Bar</td>
<td>Notched K=3.3</td>
<td>7</td>
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<tr>
<td>130</td>
<td>130</td>
<td>Ann 1 hr. at 1300°F air cooled</td>
<td>.25&quot; dia. unnotched rod</td>
<td>9</td>
</tr>
<tr>
<td>84</td>
<td>84</td>
<td>Hot Swaged</td>
<td>Unnotched</td>
<td>9</td>
</tr>
</tbody>
</table>
FABRICATION

The fabrication of a very thick wall monolithic spherical vessel nine feet in diameter would certainly present many new problems not heretofore encountered. This would require an order of magnitude extension in present technology. There is no fundamental assurance that these developments can be accomplished. It would undoubtedly require the development of new fabrication techniques, very large equipment, and meticulous workmanship, inspection and quality control.

The preformed plates would be made from vacuum melted material with the composition carefully controlled. The dissolved gas content would necessarily be kept at an absolute minimum. Each plate would require thorough surface inspection by Zyglo or magnetic-particle methods (which ever was applicable) followed by a highly accurate and sensitive ultrasonic inspection for interior flaws.

Highly efficient butt welding of these thick plates would again require an advancement in the state of the art. At present titanium alloys have been efficiently welded in sections up to 3 inches thick and maraging steel up to about 4 inches thick. Inert gas fusion techniques would be used with some method of preheating the joint before welding. The filler metal would probably be of the same composition and purity as the base metal. After each pass it would be necessary to clean and grind the weldment and employ rigid inspections such as ultrasonics, zyglo, magnetic particle, and possibly x-ray or gamma ray examination of the weld and heat affected zone. All unacceptable flaws and defects would be eliminated before further welding.

A very large furnace would be necessary in order to stress, relieve, and/or anneal the entire structure at various stages of fabrication. The furnace would be of the inert gas or vacuum type to prevent oxidation and scaling and would probably be electrically heated.

The titanium alloys would require intermittent stress relieving treatments during fabrication at 1150°F followed by a final anneal at the com-
pletion of fabrication at 1400°F with a slow furnace cool to 1100°F.

The 18% Ni Maraging Steel would also probably require intermittent stress relieving and a final heat treatment of the entire vessel at the end of fabrication. The heat treat cycle would consist of an anneal at 1500°F for 10 to 20 hrs. (depending on thickness) followed by air cooling. Maraging at 900°F for about 4 hours and air cooling would then produce the required properties (18).
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PART III
COMPOSITE PRESSURE VESSELS

In the past few years the use of composite pressure vessels has increased considerably within the aerospace industry. Composites allow the engineer to take advantage of the greater apparent strengths characteristics of small material sections, or of materials for which fabrication technology in large sections does not exist. Secondly, in a composite, one can sometimes design with less structural material placed in a more efficient orientation with respect to the predicted loads. Finally, equipment and fabrication costs have proven to be much less for the composite, especially where vessel geometry may be complex.

Neglecting for a moment dynamic considerations, a principal objective in the pulse rocket is to contain a volume of fluid under pressure, in a vessel of minimum weight. The principal material selection criterion is working strength compared to weight (density). The strongest material forms available based on this criterion ($\sigma/\rho$) as measured in uniaxial tension fall into two general categories, filament and thin sheet. However, the strengthening mechanisms for various specific materials may be quite different.

Some General Relationships

In the case of the filament structure vessel, the optimum configuration can be shown to be one in which the structural material is distributed so as to be uniformly stressed in a continuous "isotensoid" manner(1). Satisfying a general biaxial or triaxial stress condition thus requires placement of material (filaments) in the actual direction and proportion of the stresses to be supported. A thin sheet on the other hand is capable

---

1 Schuerch, Astro Research Corp. AIAA paper 2914-63
of withstanding a biaxial stress condition of various proportions. Thus, a sheet can be applied directly to shell like membrane structures where the normal (radial) stresses are negligible. Notice that if for a moment we consider a "real" dynamic vessel vs. our temporarily assumed static case; it is this ability to withstand impulsive normal stresses in an essentially biaxial composite which will prove to be perhaps the most significant impediment to proposing a reasonable solution at this time.

For the typical static case of a thin wall pressure vessel where the principal stresses are essentially simple biaxial tension a few generalizations are in order. First, the optimum shape for a pressure vessel made of sheet isotropic in two directions is the sphere. The weight of such a vessel of volume \( V \) under pressure \( P \) is simply

\[
W_m = 3/2 \frac{V P \rho}{\sigma_b}
\]

Where \( \rho \) is the wall density and \( \sigma_b \) the biaxial limiting stress, which by a yielding distortion energy criterion (Von Mises) is equal to the uniaxial tensile yield strength.

Conversely, in the case of the ideal filament vessel, there is no optimum isotensoid geometry, although only a few interesting shapes can be wound in a continuous, stable, isotensoid manner. These shapes are basically the cylinder, toroid and spheroid, but not in a strict sense the true sphere. The weight of the isotensoid vessel depends only on the contained volume, pressure and filament strength but not on the particular geometry. Using a strain energy approach (See below), and equating the work done in straining the vessel to the energy stored in the wall yields the following weight relationship for the isotensoid.
\[ PE = \frac{1}{2} P \Delta V \quad \text{and} \quad SE = \frac{1}{2} E \epsilon^2 \]

for small expansions

\[ \Delta V \approx 3 \epsilon V \]

i.e., \[ \Delta V \approx V_0 (1+\epsilon)^3 - V_0 \approx 3 \epsilon V_0 \]

\[ Q_{PV} = \frac{1}{2} PV \approx \frac{3 \epsilon V P}{2} \]

which can be equated to the strain energy

i.e., \[ Q_{s} = \frac{1}{2} \epsilon \frac{W_f}{E_f} \]

\[ = \frac{1}{2} \epsilon \frac{E_f}{E_f} W_f \]

\[ \therefore \quad \frac{3}{2} \epsilon V P = \frac{1}{2} \epsilon \frac{E_f}{E_f} W_f \]

\[ \text{and} \quad 3 V P = \frac{E_f}{E_f} W_f \]

or \[ W_f = 3 \frac{V P E_f}{E_f} \] (neglects binder)
Thus, the uniaxial strength of the vessel filament material must be twice that of the sphere membrane for equal vessel weight. This in effect assumes that the filaments are present as uniformly distributed orthogonal pairs which are independent. In practice however, the windings are applied as nearly as possible in coherent layers of a helical trajectory. This is generally done to preserve filament continuity and to minimize inter-layer shear stresses or flexural fatigue damage between filaments of differing orientation.

**Simplified Analysis of Filamentary Structure**

A complete design analysis of a filamentary vessel would require exact consideration of the load carrying ability, orientation and interaction of each element, combined with the effect of any binding material. This approach is inevitably laborious and has only been attempted for the most elementary geometries \(^{(2)}\). In many instances inability to adequately represent in detail the complexities of the composite yields results which are inexact or worse yet, misleading. Physical discontinuities within the winding such as the crossing or bridging of individual strands for instance are unavoidable in some geometries. The resulting section buildup, local bending, filament interaction, etc., represent considerations which general analysis cannot account for and are not likely to be easily resolved.

Fortunately, a reasonable approximate analysis can be obtained by resorting to the substitution of properties of a reasonable but hypothetical anisotropic composite. While the latter approach does allow some consideration of actual composite properties, its application to analysis is justifiable only where the following restrictions apply.

1. Stress is proportional to strain in all elements
2. Strain is continuous between phases
3. Filament portions are continuous and available in directions appropriate to all loads
4. All phases are homogeneous and elastically isotropic.

\(^{(2)}\) Brown, Aerojet, General
The assumption of elastic behavior is particularly questionable where binders such as glues, resins or work hardenable brazes are involved.

Consider as the basic element of a unidirectional system, a single resin encased filament under uniaxial load (shown below).

![Diagram of uniaxial and biaxial composites](image)

The strain in both elements will be equal and the following relationships can be derived.

\[
\varepsilon_c = \varepsilon_f = \varepsilon_b
\]

\[
\frac{Z_c}{E_c} = \frac{\varepsilon_f}{E_f} = \frac{\varepsilon_b}{E_b}
\]

and,

\[
\frac{Z_c A_c}{E_c} = \frac{\varepsilon_f A_f}{E_f} + \frac{\varepsilon_b A_b}{E_b}
\]

or,

\[
\frac{Z_c}{E_c} \varepsilon_c A_c = \frac{\varepsilon_f}{E_f} A_f \varepsilon_f + \frac{\varepsilon_b}{E_b} \varepsilon_b A_b
\]

from which

\[
E_c = \frac{E_f A_f + E_b A_b}{A_c}
\]

and

\[
\frac{Z_c A_c}{E_c} = \frac{\varepsilon_f}{E_f} \left( A_f + \frac{E_b}{E_f} A_b \right)
\]

which for \(E_b \ll E_f\) gives

\[
\varepsilon_c \ll \frac{\varepsilon_f A_f}{A_c}
\]
Thus, in the direction of the filament axis, the composite properties are essentially those of the filament. Similarly, in the direction normal to the filament, the properties of the composite more nearly approximate those of the binder. Also, since the binder generally represents very little load carrying capacity, we can see that it is desirable to have the highest possible filament density in the direction of the applied stress.

For circular section filaments of equal diameter, the optimum packing density (C) in uniaxial array is about 0.9 for a hexagonal cluster. This array is seldom used however, because it is unstable under normal (transverse) load and difficult to wind when in the form of pre-bound roving. The more commonly used square array has a filament density of 0.78, which is also the density of crossed layers in square array. Densities of less coherent arrays vary widely. A typical value for a convergent (at the pole) opposed helix winding on a spheroid would vary from 0.35 to 0.70. Random fiber or mat arrays seldom exceed a density of more than 0.25.

\[ \text{Fig. III-2} \]

- **SQUARE**
- **HEXAGONAL**
- **CROSSED SQUARE**

We need also consider the efficiency with which structural loads are distributed among the individual filaments. Without discussing mechanisms for the moment, it will suffice to say that glass-epoxy composites behave as if the average filament were capable of being stressed to only 60 or 70% of its single fiber breaking strength. This value will be termed by the fiber efficiency \( \eta_f \) and taken as the ratio of effective filament stress to uniaxial ultimate filament strength.
The Filament Reinforced Sphere

In the special case of a sphere, where the principal stresses are equal, the ideal filament distribution would be one which resulted in a composite with equal orthogonal properties. This results in a simple filament to composite strength relationship.

\[ 2 \zeta_c = \zeta_f \frac{A_c}{A_f} \]

which for a composite of filament density "c" and efficiency \( \eta_f \) becomes:

\[ \zeta_c = \frac{1}{2} C \zeta_f \eta_f \]

where \( \zeta_c \) is now the allowable biaxial hoop stress in the composite shell, and \( \zeta_f \) some allowable single filament working stress such as an ultimate strength or endurance limit.

Aside from the annoying fact that there is no reasonable continuous isotensoid of a true sphere which is stable, we are otherwise ready to compute composite material properties for application to an appropriate vessel stress or strain expression. Great circle windings must be discontinuous, or if skewed are unstable. The only stable spherical winding geometry of which the writer is aware, results in an oblate spheroid with polar openings and axial length about 0.6 the equatorial diameter (4). The dynamic response of the pulse rocket vessel will likely force the use of the regular sphere in order to avoid excessive shell bending moments.

In this instance it will ultimately be necessary to attempt an exact analysis since the simple isotensoid (or netting) analysis for which the present discussion is valid is simply not applicable to a true sphere. An actual spherical design will likely require auxiliary windings in the equator region and will be stable only under the restraint of the binder.

Filament Properties

The best current example of an applied filament-resin system is the previous mention glass-epoxy solid propellant rocket motor case material. This material is based on windings of pre-bound tapes of about 20
strands of .0004 in. (10^-2 mm) diameter. Most commercial forms have a single filament strength on the order of 400,000 psi. Design working stresses of the epoxy bound composite in static cylindrical vessels are generally on the order of 100,000 psi (5). As will be seen, the low density of the composite results in vessel wall thickness for the pulse rocket of from 3 to 5 feet. Considering winding discontinuities and the high fraction of low modulus binder present, such a thickness is probably intolerable from a strain coupling standpoint. Present practice is limited to vessel sections of 1/2 in. thickness (1000 layers) or 1 to 2% of the radius, resulting in static burst pressures of about 2000 psi. An isotropic shell analysis is needed to establish the reasonable limit for filament strain coupling as a function of low modulus fraction and wall thickness to radius ratio:

Some Approximate Material Properties

<table>
<thead>
<tr>
<th>Filament Mat'l.</th>
<th>Density ρ</th>
<th>Modulus E</th>
<th>Strength Es</th>
<th>Str./Wt $\frac{E}{\rho}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass (present)</td>
<td>.09 lb/in³</td>
<td>10x10⁶ psi</td>
<td>400x10⁰ psi</td>
<td>4.4x10⁶ in.</td>
</tr>
<tr>
<td>Dacron</td>
<td>.05</td>
<td>1.9</td>
<td>140</td>
<td>2.8</td>
</tr>
<tr>
<td>Carbon Steel</td>
<td>.29</td>
<td>29</td>
<td>600</td>
<td>2 +</td>
</tr>
<tr>
<td>Boron</td>
<td>.09</td>
<td>55</td>
<td>500</td>
<td>5.5</td>
</tr>
<tr>
<td>Beryllium</td>
<td>.067</td>
<td>45</td>
<td>125</td>
<td>1.85</td>
</tr>
<tr>
<td>Composites</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uniaxial Glass-Epoxy</td>
<td>.08</td>
<td>7</td>
<td>180-220</td>
<td>2.5</td>
</tr>
<tr>
<td>Crossed Glass-Epoxy</td>
<td>.07</td>
<td>3</td>
<td>75</td>
<td>1.1</td>
</tr>
<tr>
<td>Mat Glass-Epoxy</td>
<td>.05-.07</td>
<td>3</td>
<td>15-50</td>
<td>.7</td>
</tr>
</tbody>
</table>
Some Approximate Material Properties - Cont'd.

<table>
<thead>
<tr>
<th>Filament Mat'l.</th>
<th>Density</th>
<th>Modulus</th>
<th>Strength</th>
<th>Str./wt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass Filament</td>
<td>.11</td>
<td>14-17</td>
<td>700</td>
<td>6</td>
</tr>
<tr>
<td>Steel Wire</td>
<td>.29</td>
<td>29</td>
<td>1000</td>
<td>3 +</td>
</tr>
<tr>
<td>Beryllium Wire</td>
<td>.067</td>
<td>45</td>
<td>.500 ?</td>
<td>7.5 ?</td>
</tr>
</tbody>
</table>

While higher filament strengths may help the apparent section thickness and all up weight to an appreciable degree, there is not much prospect of improving the winding uniformity and density so long as we are constrained to the sphere.

Other Filament Properties Considerations

In addition to the problem of load distribution among the filaments which was previously mentioned, there are questions relating to the behavior of composites under dynamic conditions about which very little is known. One of these would be the effect of repeated or cyclic loading (fatigue). The data which is available (eg. Ref. 6) suggests a rather rapid decrease in the strength of epoxy bound glass composites with number of cycles and possibly a cyclic rate effect. Endurance strengths for 100,000 cycles are deduced from incomplete data to be about 35% of the single cycle ultimate strength. Perhaps even more significant, the same reference suggests that a cumulative damage mechanism is operating and that rupture occurs by brittle fracture propagation according to Weibull's statistical probability theory. Further, creep strength is said to vary inversely as section dimension. All of this suggests that the individual filaments are not as generally assumed structurally independent. Should these data be verified, design of composites for cyclic loading in large sections would look extremely unattractive, especially if the fracture propagating advantages sought in separated structural elements is lost.
There is no known data on the properties of composites under impulsive (shock) loadings and virtually no data on damping properties. One can make dire suggestions as to the effect of normal shock waves on the inter-laminar bond, but what is needed are some well thought out tests. The damping properties may offer an area for optimism and ingenuity, but again a few tests would be invaluable.

A Rough Cut Design

Using the principal case of ADN 36 and the peak dynamic strain of the preceding dynamic response analysis, a rough estimate of vessel size and weight can be made. The allowable working strength of an assumed glass epoxy composite would be: \( \sigma_{cc} = \frac{1}{2} C \sigma_{\text{e}} \nu_{\alpha} \)

Using (optimistically) \( C = .7; \nu_{\alpha} = .6, \sigma_{\text{tu}} = 400 \text{ kpsi} \) and \( \sigma_{\text{e}} \) (endurance strength at \( 10^5 \) cycles) = 35 \( \sigma_{\text{tu}} \)

\[
\sigma_{cc} = \frac{.6 (.7) 35 (400)}{2} \times 10^3
\]
\[
\sigma_{cc} \approx 29,400 \text{ psi}
\]

This value can then be applied to the peak dynamic stress expression derived in the previous section or to the empirical form developed by B. Crowley et.al in the Vortex program. The result in either instance is a seemingly unreasonable first order wall thickness approximation of more than 50 inches and weight of about 800,000 lbs. While the numbers used are the best applicable to glass at this time, there is some hope for improvement.

First, attenuation of the input function could lower the peak dynamic strain to the neighborhood of twice the steady state. Even more important to the composite might be attendant elimination of the reflex (compressive) portion of the vessel strain response. Secondly, a more ductile filament and binder system (eg. wire plus braze) might allow a better average fiber stress distribution efficiency (\( \nu_{\text{f}} \)) of perhaps 90% (1). The reported
fatigue strengths of wires are also much better (~ 70% of the ultimate) than those of the glass. As was said above we are probably stuck with a low filament density (C) for the sphere. Applying these projected improvements to a 1/2 million psi wire would yield a composite working stress of nearly 100,000 psi. The resultant wall thickness would drop to 16 inches and the all up weight would be on the order of 350,000 lb. to 500,000 lb. (for steel wire) depending on the density of the binder. The ultimate development of a useful (i.e. ductile) beryllium wire system could reduce these weights by a factor of two.

Laminated Shells - "Wall Paper"

The low biaxial strengths efficiency characteristic of uniaxial filament composites might be overcome while maintaining high specific strength (\(\sigma_p\)) through the use of laminated shell wall sections. In the ideal spherical vessel, transverse shear is absent, such that the division of the wall into lamina should have no effect on the structural use efficiency of the material. From a structural efficiency standpoint however, an obvious difficulty lies in achieving a system of shells which are in effect circumferentially continuous. The implied necessity is that we either form the lamina in situ, or succeed in making joints of 100% efficiency. Since the shell materials we would consider would be chosen from those with the highest strength attainable, self bonding is implied. The alternate use of flanges, rivets, lap shear joints on glue will however need to be considered since the possibility of attaining a self bond (weld) without affecting the properties of the adjacent material seems remote.

These other joining schemes will however, invariably reduce the average working stress (structural efficiency) of the section as compared to the ideal constant thickness lamina. A reduction by a factor of two is likely for a simple lap shear joint with a factor of three being more likely for most mechanical connectors. At first glance, the scarf (beveled) joint would appear to offer most promise for either bonding or welding, although the complexity of fitting and joining many (say 100) layers seems staggering.
Welding as a means of joining would be consistent with short time joint strengths of up to say 200,000 psi. Fusion welding materials which depend on elaborate heat treatment or cold work for strength however seems unprofitable. For such materials (e.g. ausformed steel) diffusion bonding or low temperature (less than 1000°F) brazing would have to be developed.

As an exercise, we might consider an approach which would involve the prospective use of a hypothetical high strength alloy in sheet form. First, the trade off between ultimate strength and fracture toughness (see section on monoliths) would place the optimum sheet thickness in the range of 0.1 inches.

Next, we can consider as likely candidates for the lamina a number of metals in sheet form.

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (ρ)</th>
<th>Modulus (E)</th>
<th>Ultimate Strength (σtu)</th>
<th>Str./Wt. (σf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>.10</td>
<td>10 x 10^6 E</td>
<td>80 (ksi)</td>
<td>.8 x 10^6 ksi</td>
</tr>
<tr>
<td>Maraged Steel</td>
<td>.29</td>
<td>29</td>
<td>290</td>
<td>1.0</td>
</tr>
<tr>
<td>Ausformed Steel</td>
<td>.29</td>
<td>29</td>
<td>480</td>
<td>1.65</td>
</tr>
<tr>
<td>Steel (projected)</td>
<td>.29</td>
<td>29</td>
<td>(700)</td>
<td>(2.4)</td>
</tr>
<tr>
<td>Titanium</td>
<td>.16</td>
<td>16</td>
<td>195</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Notes: All data for room temperature. Fatigue strength in flexure at 10^6 cycles approx. 50% of short time ultimate. Tensile fatigue strength approx. 70% flexure value.

Beryllium would rate interest except for a complete lack of low temperature ductility. Aluminum on the other hand would look more attractive if working temperatures were in the cryogenic range.

Finally, we will imagine the mode of fabrication to be the lamination of preformed gores or cusps into layers much as was typical of the historic hot air balloon. Joints will be staggered so as not to coincide either in location or alignment. All joining will be inter-laminar by means of a fusible metal braze. A single layer preformed inner shell will serve as
the mandrel. The entire assembly will be in an oven at slightly below braze melting temperature. Each segment of the laminar structure will be placed over the preceding layer, having been previously coated with the braze metal. The sheet being attached will be held in place by means of a stretched sheet (drum head) pulled firmly over it. The surface layers will then be raised locally to the braze melting temperature by means of induction or direct r.f. heating. Thus, after a few thousand operations one would expect to have a completed vessel.
The properties of the resultant vessel might reasonably exceed those of the equivalent monolith by an appreciable amount. The principal question in establishing an exact structural efficiency relates to the mode of load-transfer between ends of segments and from layer to layer. If for instance, we assume a monotonic transfer model as is often cited in the classic glue bonded lap joint,

![Diagram](attachment:fig_001)

we lower the average local stress to $1/2$ the optimum value. If however, the braze joint is truly thin compared to the sheet, we should have continuity of strain and by implication a strain discontinuity at each end of the joint.

![Diagram](attachment:fig_002)

In this latter instance, the model attributable to St. Venant would predict that the shear due to the strain discontinuity would disappear in a distance of a few (3-5) thicknesses and except for a stress concentration at the discontinuity the average stress in the lamina return to the
undisturbed average value. By this argument, providing we can in effect ignore the implied yielding of the braze at the discontinuity, application of a stress concentration criteria (and generous fillet radii) can reduce the probable stress concentration to a value of about 1.3 \( f_t \approx t/2 \). There are however, two obvious potential flaws in the scheme which need be investigated. First, the implied average shear stress in the braze is 20 to 30\% of the sheet stress. This requires a braze metal appreciably better than those now available. Secondly, with repeated cycling, the implied yielding of the braze at the stress concentration will result in failure unless the allowable elastic strain in the braze is at least \( 1/2 \) the local total strain and the reversed stress portion of the vessel strain cycle can be eliminated.

The result of all this would seem to be a reasonable prospect for a laminar design of lower all up weight than is attainable by other methods. If we presume as Zackay suggested that ausformed sheet can ultimately be produced with an endurance limit strength of 250 kpsi or better, a peak working stress on the other of 180 kpsi is conceivable. This would result in a vessel thickness on the order of 10 inches and an all up vessel weight of about 400,000 lbs.

If however, the required brazing temperatures exceed the maximum ausform or titanium tempering temperature (c.a. 1000°F), the next best choice would be maraged steel which would allow brazing at temperatures up to 1500 or 1800°F. The maraged vessel would weigh about 750,000 lb. (vs. 500,000 lb. for titanium). Note that these values are for room temperature. Aluminum, Titanium and maraged steel will increase in usable strength at cryogenic temperatures. (Fig. 6). Ausformed steel suffers severely in ductility and toughness below room temperature and is probably unusable below -100°F.
LAWRENCE RADIATION LABORATORY - UNIVERSITY OF CALIFORNIA

ENGINEERING NOTE

NAME: P. B. Mohr
DATE: Oct. 15, 1963

Fig. III-6

-320°F

-423°F

0.2% YIELD STRENGTH TO DENSITY RATIO
NOTCH TO 0.2% YIELD STRENGTH RATIO

FILAMENT WOUND REINFORCED
PLASTIC 0.025 IN. THICK
TI-6Al-4V ANNEALED
0.055 IN. THICK
TI-5Al-2Sn 6% WELT 0.025 IN. THICK
K-36 0.025 IN. THICK
COLD REDUCED
0.125 IN. THICK
ALUMINUM 7075-T6
0.25 IN. THICK
ALUMINUM 7075-T6 6% COLD REDUCTION
0.033 IN. THICK
STAINLESS STEEL AISI 301 6% COLD REDUCTION
0.033 IN. THICK
STAINLESS STEEL AISI 301 7% COLD REDUCTION
0.033 IN. THICK
STAINLESS STEEL AISI 301 10% COLD REDUCTION
0.033 IN. THICK
STAINLESS STEEL AISI 301 15% COLD REDUCTION
0.033 IN. THICK
STAINLESS STEEL AISI 301 20% COLD REDUCTION
0.033 IN. THICK
STAINLESS STEEL AISI 301 25% COLD REDUCTION
0.033 IN. THICK
STAINLESS STEEL AISI 301 30% COLD REDUCTION
0.033 IN. THICK
STAINLESS STEEL AISI 301 40% COLD REDUCTION
0.033 IN. THICK
STAINLESS STEEL AISI 301 50% COLD REDUCTION
0.033 IN. THICK
STAINLESS STEEL AISI 301 60% COLD REDUCTION
0.033 IN. THICK
STAINLESS STEEL AISI 301 70% COLD REDUCTION
0.033 IN. THICK
STAINLESS STEEL AISI 301 80% COLD REDUCTION
0.033 IN. THICK
STAINLESS STEEL AISI 301 90% COLD REDUCTION
0.033 IN. THICK
STAINLESS STEEL AISI 301 100% COLD REDUCTION
0.033 IN. THICK

FROM: HICKEL, JOHNOVÉ KEMP - LEWIS RESEARCH CENTER
R. S. W. METALS ENGINEERING QUARTERLY MAY 1963
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PART IV

TEST PROGRAM

The more obvious test programs involve setting off charges in bottles. Before we scare anybody let it be pointed out that we are considering crude, massive, reusable bottles that can be produced at the local steel foundries for $1.00 per lb. These would be two to four feet in diameter and several inches thick. The charges might be sufficiently small that much of the preliminary work could be done here rather than at Site 300.

First we need input-response data. We have an abundance of response data available, principally from Crowley. However, no one has recorded the input system. The reason for this was that transducers were so heavy that they would interfere with the vessel response. Further, they were dealing with a system so massively overdesigned that they did not need such a close study of the actual coupling. A few of their experiments could be repeated in massive bottles to determine what the input was. This should be done at a very early date.

Fabrication of large thick monolithic structures will require a considerable advance in the state of the art of welding and inspecting thick sections. An early start on this program is necessary.

Of secondary importance are studies concerning the dynamics of vessels containing nozzles. These would have to be precision workpieces unlike the crude vessels above.

There is also a great lack of heat transfer information (both for the vessel and nozzle). There is a slight amount of analogous information available from shock tube workers, but no systematic study has been made of the significant parameters. The approximations made so far have completely ignored turbulence and radiation. This may or may not be justified. Crowley has tried to obtain such data however, and his results which may or may not be applicable would indicate that present estimates are at least the correct order of magnitude. This study should be made in a massive reusable
vessel using pulse heating instrumentation of the type used for shock tube work. Another more exotic heating problem concerns the heat generated in fissile materials that will preferentially deposit out on the cold vessel walls.

We need to make photoelastic and pull tests on laminated structures and to examine the impulsive fatigue characteristics of the bonds in laminates and composites. The large area of bond material is such that the usual assumptions made about bond stress distribution are invalid. Furthermore, it would appear that notch toughness criteria are associated with the manner of crack propagation. We have talked to people in the missile industry who report orders of magnitude difference in the notch toughness of plates slabbed to different thickness from the same block. These effects are claimed to be real, and to correlate well with vessel burst tests. Now if there is a critical thickness of plate yielding maximum toughness, all other things being equal, simply because of the manner in which cracks propagate in the material, there is a question as to whether brazing or otherwise laminating these plates into heavier sections will affect this apparent property. A small but careful program in this direction is an early necessity.

Since fatigue is a problem, the material damping characteristics which will determine the number of cycles, should receive early attention. Information is available for only a few materials under limited frequency and stress conditions. We must study damping capacities of materials of interest at appropriate stress levels, frequencies, and temperatures.

Due to the high levels of impulsive shock near the Hugoniot elastic limit, there is serious question of fatigue characteristics of both isotropic and composite materials. The equation of state experiments in this region are almost without exception, single shot tests. We need to study the effects of thousands of shots on a single specimen.
Further, we need to study the effects of these large, high rate loads on laminated materials, paying particular attention to reflections from the laminar surfaces. These will have no effect on localizing possible damage and may form part of the criteria for braze strength.

A difficult problem we will face in materials testing will be dynamic stress simulation. The impulse portions of the loading are much too fast for mechanical simulation. We need to find a way to impose a general in-plane variable biaxial stress while simultaneously coupling a normal shock.

Obtaining a statistically meaningful number of data points probably rules out the use of expensive vessels. We should make some attempt to devise a test which could use a relatively simple reproducible specimen. One possibility would be an internal combustion autoclave with test panels inserted in the wall.
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