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FINAL TECHNICAL PROJECT REPORT

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Granular Flow Advanced Research Objectives

Experiments Using Non-Intrusive Particle Tracing Techniques for Granular Chute Flows

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1. PROJECT DESCRIPTION AND OBJECTIVES

The objective of this contract was to develop a system capable of non-intrusively tracking the motion of an individual particle for the study of granular flows down inclined chutes. The result of the project is a system capable of following the three-dimensional translational and rotational motion of an individual particle embedded with a flowing granular material. The basic system consists of a sphere embedded with three orthogonal transmitters emitting at different frequencies which induce voltages in an antenna array surrounding the flow regime. Analysis of the induced voltage signals within the framework of a derived model yields both the position and orientation of the sphere. Tests were performed in a small scale model chute as well as in a cylindrical vibrated granular bed, which clearly demonstrates the capability of the system.

As a result of discussions at meetings held semi-annually for the GFARO contractors, it was deemed necessary to pursue an additional experimental program as part of this contract related to the measurement of sphere collision properties. The outcome of the work (reported in Appendix C) is the determination of certain properties which are needed for use in computer simulations and theory.

A. Outline of Report

Section II, entitled "Tracking System" discusses the components of the tracking system comprised of the following: A) Theoretical Voltage Model, B) Signal Processing Algorithm and C) the Tracking Sphere and Data Acquisition. The next section ("Radio Transparent Flow Chute System") describes the design and fabrication of the chute systems, which include the model chute on which most of the tracking studies were completed and the full chute on which bulk measurements were performed. A summary of experiments which were carried out with the model and full size chutes appears in Section 4 ("Chute Flow Experiments"). This includes a discussion of additional work required to permit tracking in the full-sized chute system. As the project progressed beyond the second year, it became clear that it was necessary to adapt the tracking system to a smaller granular flow geometry. Because of the interest of vibrations on the behavior of granular flows and its relevance to both chute entrance conditions as well as bumpy chute floors, a vibrating bed system was designed and fabricated. This was then adapted for use with the tracking system. A detailed description of the vibrated bed system and the experimental results obtained are given in Section 5 ("Tracking in a Vibrated Vertical Bed").

An important and major component in the overall GRAFO program was to further develop and extend existing kinetic theories to chute flows (J. T. Jenkins, Cornell University) and also to model the vibrationally energized flows (M. W. Richman, Worcester Polytechnic Institute). For the purpose of validating theories, which were developed for these flows and to provide a broader insight into the important parameters governing behavior, a series of discrete element studies was undertaken. The outcome of this work is included in Appendix B ("Discrete Element Studies of Vibrationally Energized Flows").

In order to make comparisons of theories and simulations with chute flow experiments undertaken at Cornell University and the University of Florida at Gainesville and tracking studies carried out at NJIT, it was necessary to know the collision properties of the flows spheres. Consequently, it was decided that separate experiments would be done at Cornell University and NJIT. A description of the experiments designed and carried out at NJIT appear in Appendix C ("Sphere Collision Properties Experiment").

Section 6 contains supplemental information including a listing of publications, information on the Particle Technology Center at NJIT formed as an outgrowth of this contract, faculty/staff, and a listing of students who were supported and received graduate degrees from this project.

2. TRACKING SYSTEM

The tracking system consists of three main components: 1) the transmitter assembly and associated electronics; and 2) the signal processing codes and 3) the data acquisition system. The fundamental concept is that a sphere which contains three small transmitters, each broadcasting at different frequencies, will induce voltages in an array of receiving loop antennae surround the flow field. The position and orientation of the sphere can then be determined from the measured antenna voltages using a signal processing algorithm based on a derived model.

A. Theoretical Voltage Model

Consider a small loop of wire carrying a sinusoidal current $I = I_0 \sin(\omega t)$, where ω is the frequency (Fig. 2.1). This loop behaves like a small magnetic dipole, and has a magnetic field surrounding it. If another loop is placed anywhere in this magnetic field, there is a corresponding voltage induced in the loop. The strength of the induced voltage depends on the relative position and orientation of the second loop. This is the principle of *electro-magnetic induction*. The strength of the induced voltage depends on the relative position and orientation of the second loop. The current carrying loop is the *transmitter*, and the loop in which voltage is induced is the *receiving antenna*.

Fig. 2.1: Coordinate system

By assuming that the center of the current carrying small loop is located at the origin of a coordinate system (Fig. 2.1), the strength of the magnetic field at point $P(r, \theta, \phi)$ due to the coil at the origin is given by

$$\mathbf{B} = \frac{\mu_0 a^2 I}{4r^3} [2 \cos \theta \mathbf{a}_r + \sin \theta \mathbf{a}_\theta]. \quad (2.1)$$

In the above equation, μ_0 is the permeability of free space, a is the radius of the radiating loop, r is the distance of the point P from the center of the loop, and θ is the angle between the normal to the loop and the line joining P to the loop center. In deriving (2.1), it is assumed that $r \gg a$. By applying Faraday's law, the voltage induced in the arbitrarily placed second loop due to this magnetic field is

$$V_{\text{loop}} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{S} \quad (2.2)$$

Where S is any surface bounded by the second loop. A closed form solution to (2.2) can be obtained if the area of the second loop is very small so that $\mathbf{B}$ does not change significantly over that area. If this is not the case, then a convenient, closed form for (2.2) may not be possible and a numerical solution would be necessary. This is the case in the tracking system since the receiving loops or antenna surround the flow regime are large. Consequently, the principle of reciprocity [10] is applied to avoid a numerical solution to (2.2). Accordingly, the induced voltage in the transmitting coil can be computed by assuming that the current $I = I_0 \sin(\omega t)$ is flowing in the receiving loop (which is much larger than the small transmitting coil inside the tracking sphere). Then, through the reciprocity principle, this can be transformed into the actual induced

voltage in the receiver loop. This procedure is used in deriving the form of the induced voltage used in the signal processing algorithm which follows.

In the tracking system, each receiving loop is planar and rectangular, consisting of four straight wires (Fig. 2.2), two of which have length $2l$ and the other two with a length of $2a$. A local coordinate system (X, Y, Z) is attached to the loop such that its Z axis is normal to the loop's plane. Through the application of the Biot-Savart law and evaluation the necessary integrals, the field intensity at the center of the transmitting loop (x, y, z) due to each straight wire is found. All four components are summed using superposition to yield,

$$\mathbf{B} = \sum_{i=1}^4 \left[\frac{\mu_0 I}{4\pi R_i} (\cos \phi_{i1} - \cos \phi_{i2}) \right] \mathbf{\Theta}_i \quad (2.3)$$

where R_i is the distance from (x, y, z) to wire i , the angles ϕ_{ij} ($j = 1, 2$) are as shown in Fig. 2, and the direction vector $\mathbf{\Theta}_i$ ($i = 1, 2, 3, 4$) for the component i is the unit normal to the triangle formed by connecting the ends of wire i and (x, y, z) . These quantities R_i , ϕ_{ij} ($j = 1, 2$) and $\mathbf{\Theta}_i$ are complicated functions of x, y, z, a and l can be found in the Appendix A, equations (A.1-3). Since all quantities in equation (2.3) are explicitly known, then the components of $\mathbf{B}$ can be written in closed form as,

$$\mathbf{B} = B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k} \quad (2.4)$$

where the components are with reference to the system attached to the receiving loop. Since the integral in (2) is over a small surface of the transmitter coil, $\mathbf{B}$ can be assumed to be constant over this surface and therefore,

$$V_{loop} = -\frac{d}{dt} \mathbf{B} \cdot \mathbf{A} \quad (2.5)$$

where $\mathbf{A}$ is the area vector with direction normal to the transmitter coil and magnitude A equal to the transmitter coil area. From (2.4) and (2.5), the final result for the induced voltage is given by,

$$V_{loop} = -\omega A [B_x \cos \alpha + B_y \cos \beta + B_z \cos \gamma] \quad (2.6)$$

Evaluation of all induced voltages when there are multiple transmitters and receivers (as in the tracking system) requires several coordinate transformations since the above set of equations require the position and orientation of each transmitter in terms of the local coordinate system of each receiver (see Fig. 2.2). This is done through the use of homogeneous transformations [9] which are used to represent position/orientations as well as coordinate frames. A brief description of the homogeneous transformations follows.

Fig. 2.2

A.1 Homogeneous Transformations

Let (x, y, z) be the coordinates of the origin of an arbitrary coordinate system (U, V, W) in a reference coordinate system (X, Y, Z) . Also, let the vector triple $(\mathbf{u}, \mathbf{v}, \mathbf{w})$ represent the orientation or direction cosines of the (U, V, W) coordinate system with respect to the reference frame (X, Y, Z) . That is, $\mathbf{u} = (u_x, u_y, u_z) = (\cos(U, X), \cos(U, Y), \cos(U, Z))$ and similarly for $\mathbf{v}$ and $\mathbf{w}$.

Then the homogeneous transformation can be written as,

$${}^{XYZ}T_{UVW} = \begin{bmatrix} u_x & v_x & w_x & x \\ u_y & v_y & w_y & y \\ u_z & v_z & w_z & z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.7)$$

where the first three columns represent the orientation, and the last column describes the position of an arbitrary coordinate frame in terms of the reference coordinate frame.

In order to specify the location of transmitter i with respect to antenna j , the local coordinate frame (U, V, W) is attached to the transmitter. This frame is defined so that the transmitter lies in the U - V plane of this local system, and the transmitter axis (i.e., the axis of symmetry of the coil) is coincident with the W axis of this local system. Then the transmitter location with respect to the antenna is given by the transformation jT_i , where the pre-superscript j denotes that the transformation is with respect to antenna j , and the post-subscript denotes that the transformation represents the location of transmitter i . The last two columns of this matrix are all that is required to evaluate the induced voltage given by equation (2.6) in conjunction with equations (2.3) and (2.4). Specifically, the third column provides the direction cosines required to find the components of $\mathbf{B}$ in (2.4) while the last column gives the coordinates of the transmitter origin (x, y, z) required for equations (A.1-3).

In the actual tracking system, the sphere contains three transmitters (coils) and there are multiple receiving antenna surrounding the flow region. Consequently, the matrix jT_i must be evaluated for each transmitter-receiver pair using the concatenated transformation,

$${}^jT_i = [{}^jT_g][{}^sT_s][{}^sT_i] \quad (2.8)$$

Here jT_g is the global coordinate system (rigidly attached to the experimental setup) as seen by antenna j , sT_s is the sphere (containing three mounted transmitters, $i = 1, 2, 3$) coordinate system with respect to the global system, sT_i and is the coordinate system of transmitter i with respect to the sphere coordinate system. Since jT_g and sT_i are known for a given experimental setup, then jT_i can be found from (2.8) if the sphere position sT_s is known. (Determination of sT_s from the *forward solution model* is discussed in the next section). Consequently, the voltage induced in antenna j due to transmitter i can be found by using the computed elements of the matrix given in (2.8) as described in the previous paragraph.

A.2 Forward Solution Model

If it is assumed that the position of the sphere is known with respect to the global system, then a complete set of equations for the induced antenna voltages can be obtained which can then be easily computed. This set of equations is called the *forward solution*. Of course it is important to note that in an actual experiment, the position of the sphere is unknown and must be determined from measurements of the antenna voltages.

The location of the tracking sphere at any time instant is specified by six numbers - three for the position of its center with respect to a global Cartesian coordinate system (x, y, z), and three for its orientation (ϕ, θ, ψ). These angles are actually the consecutive rotations of the coordinate system (otherwise known as roll, pitch and yaw) attached to the center of the sphere with respect to the global system. Hence the sphere's location is given by the vector $\mathbf{x} = (x, y, z, \phi, \theta, \psi)$ from which the entries in the transformation sT_s can be calculated, i.e.,

$${}^sT_s(\mathbf{x}) = \begin{bmatrix} \cos \psi \cos \theta & \cos \psi \sin \theta \sin \varphi - \sin \psi \cos \varphi & \cos \psi \sin \theta \cos \varphi + \sin \psi \sin \varphi & x \\ \sin \psi \cos \theta & \sin \psi \sin \theta \sin \varphi + \cos \psi \cos \varphi & \sin \psi \sin \theta \cos \varphi - \cos \psi \sin \varphi & y \\ -\sin \theta & \cos \theta \sin \varphi & \cos \theta \cos \varphi & z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.9)$$

A substitution of the latter into equation (2.8) then yields jT_i for each receiver-transmitter pair $i-j$. As previously noted, the elements from the third and fourth columns of jT_i provide the values needed to compute the terms in equations (A.1-3) and (2.4).

In summary, if the location $\mathbf{x} = (x, y, z, \varphi, \theta, \psi)$ of the tracking sphere with respect to the global coordinate system is known, then the forward solution procedure (by which the voltage induced in receiver i due to transmitter j is found) can be written symbolically written as,

$$V_{ij} = f_{ij}(\mathbf{x}) \quad (2.10)$$

where the notation expresses the functional dependence of V_{ij} on $\mathbf{x}$. The last two columns of jT_i provide all of the required information to compute V_{ij} .

If there are p receivers and q transmitters (in the tracking system $q = 3$), then (2.10) represents a set of $m = p \times q$ nonlinear equations in the position variable $\mathbf{x} = (x, y, z, \varphi, \theta, \psi)$. The inverse solution algorithm, discussed in the following section, is concerned with finding the sphere's position given the voltages V_{ij} .

B. Signal Processing Algorithm

The equations represented by (2.10) form a nonlinear algebraic system to which a closed form solution cannot be found. In theory, only six equations are required to find the six unknown components of the sphere's position $\mathbf{x}$. However, due to the highly nonlinear nature of the system, and experimental problems of noise coupled with low voltage signals, an optimization method is used on a system where the number of equations is greater than the unknowns. The numerical procedure is essentially a signal processing technique where signals here refer to the voltage data from the experiment. A "solution" of (2.10) means that value of $\mathbf{x}$ which minimizes the difference or *residual* between the measured voltage values and those predicted by the m model equations. The total least squares residual R is given by,

$$R = \frac{1}{2} \mathbf{r}^T(\mathbf{x}) \mathbf{r}(\mathbf{x}) = \frac{1}{2} \sum_{k=1}^m [r_k(\mathbf{x})]^2 \quad (2.11)$$

where the vector $\mathbf{r}$ is,

$$\mathbf{r} = \{r_k(\mathbf{x}) = [f_k(\mathbf{x}) - V_k], \quad k = 1, \dots, m\} \quad (2.12)$$

The values of the unknowns which minimize R occur when the gradient of R with respect of $\mathbf{x}$ is the zero vector. This leads to applying Newton type methods for finding zeroes of a system of nonlinear equations. This technique produces quadratic convergence and has the form,

$$\mathbf{x}_+ = \mathbf{x}_c - [J(\mathbf{x}_c)^T J(\mathbf{x}_c) + S(\mathbf{x}_c)]^{-1} J(\mathbf{x}_c)^T \mathbf{r}(\mathbf{x}_c) \quad (2.13)$$

where J is the Jacobian of $\mathbf{r}$ and is given by $J_{ij}(\mathbf{x}_c) = \frac{\partial r_i}{\partial x_j}$, $S(\mathbf{x}_c) = \sum_{k=1}^m r_k(\mathbf{x}_c) \cdot \nabla^2 r_k(\mathbf{x}_c)$ and the subscripts

"+" and "c" refer to the next and current iterates, respectively. A Gauss-Newton procedure combined with

the "trust region method" (i.e., the Levenberg-Marquardt algorithm [5]) is employed (S is neglected*) in which a limit δ is set on the distance an iterate may be changed in a single update. The algorithm can be written as,

$$\mathbf{x}_+ = \mathbf{x}_c - \left[J(\mathbf{x}_c)^T J(\mathbf{x}_c) + \mu_c I \right]^{-1} J(\mathbf{x}_c)^T \mathbf{r}(\mathbf{x}_c) \quad (2.14)$$

where I is the 6 x 6 identity matrix and $G = \left[J(\mathbf{x}_c)^T J(\mathbf{x}_c) \right]^{-1} J(\mathbf{x}_c)^T \mathbf{r}(\mathbf{x}_c) \Big|_2$ is the Gauss-Newton step size. If $G \leq \delta$ then $\mu_c = 0$. Otherwise, μ_c is determined by an involved iterative procedure which is performed at each Gauss-Newton iteration. The implementation by More in MINPACK [6] is used. Convergence is achieved in approximately 5 iterations as opposed to the use of a simple Gauss-Newton which requires more than 100 iterations.

In the flowchart (Fig. 2.3), an initial guess $\mathbf{X}$ is provided as input to the voltage model. Considering this input, each transmitter's position and orientation is determined in each antenna's coordinate system through the use of coordinate transformations, from which the theoretical voltage (denoted as V_{model} in the figure) is computed. From the data acquisition system readings⁺, a measured voltage array V_{msd} is determined using a calibration procedure. An error voltage vector EV , calculated as the absolute difference between the theoretical and measured voltages, has magnitude given by $\|EV\| = \sum_{i=1}^m |V_{\text{model}} - V_{\text{msd}}|$.

To compute the solution which minimizes the residual R , *lmdif* (shown as the circular loop in Fig. 2.3) iteratively changes $\mathbf{X}$ to reduce $\|EV\|$ until it falls within an acceptable range. The algorithm also employs a perturbation technique to improve the robustness of the algorithm and prevent the solution from straying to unphysical trajectory points. Details are described in [4].

* This is a nonlinear term involving the product of $\mathbf{r}$ and its Hessian which is rarely available analytically at a reasonable computational cost.

⁺ The forward model equations are given in units of volts, while the experimentally measured signals from the data acquisition system are in arbitrary units of "counts". The process of converting these to voltages is the "calibration procedure" in Fig. 2.3.

Fig. 2.3: Flow chart of solution procedure to obtain sphere trajectory from measured voltages

C. Tracking Sphere and Data Acquisition System

A successful implementation of the tracking system was contingent on the ability to build, integrate and "tune" the electrical, mechanical and computer hardware. Over the course of this project, various arrangements were extensively studied, simulated and evaluated to obtain an optimal configuration. This is described in what follows.

Figure 2.4. Circuitry for 2MHz transmitter.

Fig. 2.5: Schematic of the core of the tracking sphere

Fig. 2.6: Tracking Sphere Assembly

C.1 Tracking Sphere

The tracking particle itself is fabricated from a one-inch high density polyethylene sphere typical of those in the flowing mass. It is constructed by joining two bored halves together via a threaded fitted cylindrical collar. (See Fig. 2.6 for assembly diagram). Packaged within the sphere are three oscillators broadcasting a 2 MHz, 3.65 MHz and 4.4 MHz, as well as associated electronic circuitry and batteries. These frequencies were chosen so as to avoid interference from standard radio broadcast and communication frequencies and also to avoid harmonics. Fig. 2.4 shows the 2 MHz transmitter circuit design.

It is worth noting at this point that during the initial stages of the project, only a single transmitter, consisting of a coil wrapped around a ferrite core, was used. While this did have advantages of ease of fabrication with low battery power consumption, it contained a major disadvantage which prohibited its use. When the transmitter was orthogonal to an antenna, the results were highly inaccurate due to low signal to noise ratio. Furthermore, due to symmetry of the field, if the transmitter was rotated about its own axis, the voltage readings would not change. Such a rotation of the transmitter (and hence the sphere into which it was embedded) could not be detected by the signal processing software. This problem was resolved by developing a three transmitter system schematically shown in Fig. 2.5 and constructed so that the fields are mutually orthogonal. The ferrite cross acts as the core of two of the transmitters while the third is wound around these two using the flat ferrite of the first two transmitters as its core material. The final design of the circuitry to produce the oscillations of the coils was accomplished using integrated circuits.

Power for the oscillators is supplied from two Ni-Cd batteries (EverReady OverTime No. 386) of 1.5 volts each, located on either end of the transmitter assembly. Fig. 2.7 shows that the variation of the battery voltage versus time is such that it is unstable at the start and the end of its life and that there is only an effective life of 30 minutes. Experiments were performed in the stable or flat region of this curve. An optical switch was designed and fabricated so that when the sphere is placed in ambient light, the switch is ON causing the oscillators to transmit; when the sphere is placed in dark, the switch automatically turns OFF after an adjustable 10 to 15 second delay.

Figure 2.7. Variation of Battery Voltage (Count) over its Life ([17])

C.2 Data Acquisition System

The data acquisition system consists of the receiving antennae, multiplexers, amplifiers and A/D converters. On the model-chute there are 7 antennae while the actual chute has 27 antennae. These chutes will be described in Section IV.A.1 of this report. All the antennae consist of uniform hook up wires and

have individual pre-amplifiers physically located within a few inches of its terminals. The antennae's multiplexers and detecting boards are connected to pre-amplifier outputs with twisted pairs of wires to minimize stray pick up and distributed capacity. Current in the antennae is kept to a minimum so that the pre-amplifier input impedance is high, while output impedance is low so that the twisted pair capacity has little or no effect upon the amplitude of the output signal.

Signals picked up by an antenna are filtered through demodulator boards, fed to a pin on the data acquisition card and then stored on a personal computer. A *channel* is defined as the flow path of a signal from a transmitter (emitting at a given frequency) to the equivalent signal (counts) on the hard disk of PC. The signal path is given by: receiving antenna → demodulator boards → input pin on data acquisition card.

Fig. 2.8 shows a schematic of the DAS. The left-dashed block, labeled *Antenna System*, represents sixteen antennae that receive signals from the tracking sphere. Up to sixteen antennae can be read in this setup. The middle dashed block labeled *Antenna Multiplexing and Detection* has three subparts- Multiplexer, Clock circuitry and Demodulator Boards. The multiplexer is used to sequentially select each antenna to be read. The clock generates pulses that are fed to the counter and a variable duty clock. The counter controls the switching of the multiplexer between antennae.

At a given instant, only one antenna is connected to all three demodulator boards, built for each of the 3 frequencies. While the signal induced in an antenna may be positive or negative, only its magnitude is output from the demodulator board. This signal goes to the input pin the of the data acquisition card on the PC. After a delay for transient response of the boards, a *data ready* signal is sent to PC through variable duty clock. This triggers the scanning cycle for scanning sixteen antennae at 2.0, 3.65 and 4.4 MHz frequencies in succession.

A double buffering technique is used to increase the speed of data acquisition. Data in binary form is stored at a very fast rate in the virtual memory of PC during data acquisition. Later, it is transferred in ASCII format to the hard disk. The rate of data acquisition is 463 data sets/second on a 66 MHz Pentium and 208 data sets/second on a 25 MHz 486 PC. Each set consists of all three frequency readings of sixteen antennae. A switching circuit has been developed for handling the 27 antennae on the actual flow chute since the DAS can manage only 16 antennae. A block diagram of the new data acquisition system for the chute is shown in Fig. 2.9.

Fig. 2.8: Data acquisition

Fig. 2.9: Block diagram of new data acquisition system for the chute.

Twenty-seven antennae are used in the chute system with fourteen antennae in the top section and fourteen in the bottom section, one X-antenna being common to both. In the main DAS board there are two binary controlled 16 channel multiplexers. The chute DAS is divided into two sections in order to save switching time and to acquire more relevant information. When the transmitting particle is in the top section, readings are obtained only from antennae 1 to 14. The switching circuit, consisting of a 2 channel

multiplexer, a comparator and a timer, monitors the signal in sixth X-antenna (common to both the top and bottom section) and compares its signal to a preset level. When the transmitter crosses the X-6 antenna, the switching circuit switches to bottom section and the readings are now obtained from antenna 14 to 27. The timer maintains the switched state for 10 to 20 seconds and then resets to the top section.

D. Free Fall Test

In order to check the performance of the data acquisition system with fast data acquisition a controlled trajectory is used where the tracking sphere is allowed to fall freely from rest. The objective of this "free fall test" is to ascertain:

- the real time data acquisition capability of the DAS
- the accuracy of the DAS

In this experiment, the model-chute is oriented with its x-axis vertical and the sphere containing the transmitters is tracked as it falls. Since the data acquisition rate is known, the time between data points is known. From the trajectory $x(t) = 0.5at^2$ predicted by the signal processing backward algorithm, the acceleration a of the sphere can be computed by fitting the trajectory with a quadratic. This yielded $a = 383.97 \text{ inch/s}^2$ which is in very good agreement with the accepted value of $g = 386.4 \text{ inch/s}^2$. Only the typical results in x, y, z are shown in Figs. 2.10 - 2.12. The results of the free fall experiments indicate that the DAS is capable of real time data acquisition.

Fig. 2.10: Free fall test: X plot

Fig. 2.11: Free fall test: Y plot

Fig. 2.12: Free fall test: Z plot

3. RADIO TRANSPARENT FLOW CHUTE SYSTEM

The flow chute system is made of materials which are transparent to the signals emitted from the tracking sphere and consequently do not interfere with voltages measured in the surrounding antennae. The block diagram in Fig. 3.1 shows the system which consists of the flow chute, a hopper feed system, an exit container mounted onto a scale/data acquisition system, and a vacuum conveyor loop to refill the hopper when the experiment is completed. Photographs of the actual system located in *The Particle Technology Center* at NJIT are shown in Fig. 3.2.

Fig. 3.1: Block diagram of radio-transparent flow chute system

Upon opening the hopper gate, flow spheres pass through the sluice entrance gate, down the chute and empty into the container which rests upon a scale to measure mass flow rate exiting the chute. After the hopper is emptied, the spheres are transported back to the hopper using a pneumatic or vacuum conveyor system to prepare for another experiment. The tracking sphere can be inserted into the flowing mass through an injection tube located near the entrance sluice gate. Further visual information on the flow is obtained through the use of a high-speed Kodak Ekta-Pro 1000 system. Below are details on the components of the system.

Inclined Chute: The flow chute is fabricated from acrylic sheets and Extren 500 and 600 of various structural shapes. Components are fastened together by nylon and Extren nuts and bolts. The chute assembly (Fig. 3.2) is 10 feet long with an inclination angle that can be varied from 0 to 25 degrees, an adjustable width from 5 to 14.5 inches and a flexible sluice entrance gate which can open from 0 to 9 inches. The chute floor can be either smooth using a flat acrylic sheet, or bumpy by using an acrylic prismatic sheet. The entire flow region of the chute is tessellated with an array of 27 loop antennae. Through the data acquisition system, voltages can be

measured in each antenna which can be converted into the position of the tracking sphere using the signal processing software previously discussed.

Fig. 3.2: Photographs of flexible radio-transparent chute system

Hopper: The hopper (Fig. 3.3) is fabricated from lumber and has a storage capacity of 36 cubic feet which is three time greater than the maximum chute flow volume. It has a variable discharge area to a maximum of 17" x 17" which is controlled by varying the opening of the hopper gate. The hopper has been designed so that its main flow angle of 26° is larger than the maximum inclination of the chute (25°) and its maximum discharge area is greater than that of the chute sluice entrance gate.

Exit Container and Mass Flow Scale: The lumber rectangular container with a capacity of 45 cubic feet and the scale on which it rests is placed at the chute exit (Fig. 3.4). The scale (Sterling Scale Co.) has a 3,500 lb capacity (accuracy is 0.5 lb) and is connected to a data acquisition system to continuously monitor the container weight during a flow experiment. The indicator (Serling Model 800 Digital Weight Indicator) has 16,000,000 internal counts with a complex digital averaging program to obtain high accuracy and stability. Up to 50,000 counts can be displayed. Keyboard calibration is standard and a special high tech filtering circuit eliminates problems caused by vibrations. The scale has a maximum calibration rate of up to 10 times per second and a sensitivity of 0.5×10^{-6} Volts per graduation. A standard RS232C port is used to interface it with a PC to acquire the data.

Vacuum Recirculation System: This system is used to return the flow spheres from the exit container back to the hopper after an experiment has been completed. It consists of 3 wet/dry vacuums attached to the top of the hopper (Fig. 3.5). Two plastic tubes of 2.25" diameter extend to the exit container to return the spheres back to the hopper. If necessary, an increase of the vacuum can be achieved by plugging up one of these tubes.

Chute Antennae System: This system consists of 27 rectangular loop antennae. It contains 11 X-antennae with their 24" x 24" planes perpendicular to the x-axis, eight Y-antennae with their 24" x 32" loop planes perpendicular to the y-axis, and eight Z-antennae with their 24" x 32" loop planes perpendicular to the z-axis of

the chute. (see Fig. 3.6). The X-antennae are parallel and set at one foot intervals to each other from the entrance to the chute exit along the flow direction. Y-antennae lie in the plane of the chute walls with an overlap of 2.7" between adjacent loops. Z-antennae are arranged in a similar fashion in the upper and lower x-y planes. PVC equal-angle structural materials fastened to the chute with nylon and extren nuts and bolts are used to support the wires (Yel Stranded 22 HWG) constituting each loop.

Flow Spheres: The flow particles are 1 ± 0.002 " diameter solid, high density polyethylene spheres having a mass of 7.57 grams each. For the experiments to be reported, approximately 60,000 such spheres are used having a bulk weight of 454 kg.

Fig. 3.3: Storage Hopper

Fig. 3.4: Chute exit box and scale system

Fig. 3.5: Vacuum recirculation system

Fig. 3.6: Chute antenna array

4. CHUTE FLOW EXPERIMENTS

A. Model Chute Results

In this section, we report on experiments performed using the model chute beginning with a description of the apparatus and a discussion of errors in the system followed by correction schemes to account for these errors.

A.1 Apparatus

The model-chute of dimensions 40 x 20 x 20 cubic inches is made of radio transparent acrylic and is mounted with 7 antennae as shown in Fig. 4.1. The dotted rectangles in the figure represent the antennae. The numbers in parentheses show the sequencing of antennae, while the label (i.e. X, Y or Z) gives the direction of the normal to the plane of the antenna. Antennae dimensions are given in Table 4.1. Since the dimensions and aspect ratio of antennae play an important role in the solution process, a detailed study of the antennae configurations was conducted [8].

Fig. 4.1: Model-chute with Antennae

Table 4.1: Model chute dimensions		
Designation	Number of Antennae	Dimensions (inches)
X - Antennae	3	20 x 20
Y - Antennae	2	20 x 40
Z - Antennae	2	20 x 40

A.2 System Errors

Errors are always present in any physical measurement and can be classified into two main types: random errors (or noise) and systematic errors. Random errors are unpredictable fluctuations that creep into any measurement and are self-normalizing over a period of time. Systematic errors, on the other hand, are introduced due to imperfections in the equipment and/or operator errors in taking measurements. These errors, which are not self-normalizing and tend to accumulate and bias the readings, were studied in detail by Volcy [11] who found that they can be introduced in the tracking system due to: (1) the field of the transmitter is not perfectly symmetrical; (2) the amplifier and data acquisition boards are not perfectly linear; (3) magnetic

coupling exists between antennae; (4) higher order terms are neglected in the derivation of the model; (5) the principle of reciprocity is violated when the transmitter is very near the antenna.

To correct these systematic errors, he proposed various correction schemes consisting of empirical extensions to the voltage model in the form of three-dimensional correction maps. One of these schemes termed the "27-point correction" [1], is explained and demonstrated in what follows.

27-Point Correction Scheme

Recall that the forward model equations are given in units of volts, while the actual measured signals from data acquisition are in arbitrary unit of *counts*. Therefore the counts must be scaled down to the actual voltage in volts. This is done by a process called *calibration*. For the purpose of calibration, voltage readings in units of counts are obtained from the DAS at selected points called the *calibration points*. The forward model is also used to compute the voltage at the calibration points. A *scaling factor* is then obtained by taking a ratio of these two values. Thus,

$$\text{scaling factor} = \frac{\text{voltage obtained from forward model at calibration points (in volts)}}{\text{voltage obtained from D.A.S. at calibration points (in counts)}} \quad (4.1)$$

This factor is obtained for each antenna-transmitter pair. For example, the model chute setup with 7 antennae has 21 factors since there 3 transmitters in the sphere. Although this approach assumes that the scaling factors remain constant throughout the experimental space, it is observed that the scaling factors vary significantly over the experimental space (which can be attributed to the presence of systematic errors as previously noted). Volcy [11] proposed to vary the scaling factors for every antenna-transmitter pair over the experimental space by creating an empirical correction map and interpolating correction to scaling factors.

Fig. 4.2: 27-point correction space and reference nodes.

Fig. 4.3: 27 Nodes in an octant

Consider an antenna shown in Fig. 4.2 with a coordinate system located at its center, and the z-axis pointing outwards from the plane of the antenna. In order to setup the 27-point map, the experimental space surrounding the antenna is divided into octants like (Fig. 4.2), and each of these octants is discretized into 27 points as in Fig. 4.3. These points are referred to by their *relative locations* in the octant $[x_1 y_1 z_1]$, beginning with [000] at the origin of the local coordinate system of the antenna. The second point along the local x-axis is thus [100] and the third is [200] and so on. The point farthest from the origin is [222]. Voltage readings (counts) are taken at all of these 27 points with transmitter axis parallel to antenna axis, so as to induce maximum voltages. Then, scaling factors are calculated at all of these points using equation (4.1).

A correction map is then created by using the scaling factors at the 27 points in the octant. A *correction factor* is applied to the theoretical (forward model) voltage to compensate for the systematic errors in the actual voltage. Interpolation functions are set up between these 27 points to find the correction factor (cf) at any point in the experimental space. The interpolation functions make use of the symmetry of voltages about the antennae planes and can be represented by $cf = \psi(x, y, z, \alpha, \beta, \gamma)$, where ψ is the interpolation function for calculating the correction factor and $(x, y, z, \alpha, \beta, \gamma)$ denotes the position and orientation of transmitter in local antenna coordinates*. The corrective effect of the scheme is demonstrated through the generation of model-reality plots in the following sections.

* Details on the interpolation functions and implementation of the 27-point scheme can be found in Agrawal [1].

A.3 Experimental 27-Point Error Correction Scheme

It is possible to create plots (referred to as *model-reality* plots) comparing the actual voltages obtained from the data acquisition system with those predicted by the forward model. For this purpose, three trajectories (called Type I, Type II and Type III) are chosen along the central planes of the antenna. Readings (in counts) at these points are collected using the data acquisition system. These readings are converted to volts by using each of the calibration points ([000], [001], [002]) thereby resulting in three separate plots corresponding to each of the points. The calibration point which yields the smallest error between the predicted theoretical voltages* and the actual measured voltage is then selected for use in experiments. By carrying out the aforementioned procedure with and without the use of the 27-point scheme, it is also possible to evaluate its effectiveness.

Type I Trajectory

The model-reality plots of Type I, hereby referred to as M-R (I), are generated using a trajectory normal to the plane of the antenna as shown in Fig. 4.4. The z-axis of the antenna (in antenna coordinate system) and the axis of the transmitter are kept coincident and the distance between the transmitter and the antenna is increased from 0" to 20" at 0.5" increments.

The collected data is used to generate the six model-reality plots. The collected data, converted to volts by calibration, gives rise to the "actual" voltage series. The "model" voltages are generated by the forward model with or without 27-point correction, using either of the three points as the reference point. Figs. 4.4-4.9 are representative of the model-reality plots of Type I. Fig. 4.10 is the plot of errors, that is deviation of the model voltage from the actual for various conditions, as a percentage of maximum voltage. Careful observation of the error plots reveals that consistently lower errors are obtained on using 27-point correction with either point [000] or [002] as the reference point.

Fig. 4.4: Trajectory for M-R (I)

Fig. 4.4: M-R (I) without 27-point correction, reference point [000].

* Predicted voltages can be found from the forward model since the sphere positions along the selected trajectories are known.

Fig. 4.5: M-R (I) with 27-point correction, reference point [000]

Fig. 4.6: M-R (I) without 27-point correction, reference point [001]

Fig. 4.7: M-R (I) with 27-point correction, reference point [001]

Fig. 4.8: M-R (I) without 27-point correction, reference point [002]

Fig. 4.9: M-R (I) with 27-point correction, reference point [002]

Fig. 4.10: Error Plots for M-R (I); "0", "1" and "2" are the cases using 27-point correction with [000], [001] and [002] as reference points respectively. "0_no", "1_no" and "2_no" are the corresponding cases without using 27-point correction.

Type II Trajectory

The model-reality plots of Type II, also referred to as M-R (II), are generated using a trajectory in the plane of an antenna as shown in Fig. 4.11. The transmitter is moved in the plane of the antenna with the z-axis of the antenna (in antenna coordinate system) and the axis of the transmitter parallel. The distance between the z-axis of the antenna and the transmitter is varied from -5" to 5" in 0.5" increments.

The collected data is used to generate the six model-reality plots. The collected data, converted to volts by calibration, gives rise to the "actual" voltage series. The "model" voltages are generated by the forward model with or without 27-point correction, using either of the three points as reference point. Figures 4.12-4.17 are representative of the model-reality plots of Type II. Fig. 4.18 is the plot of errors, that is deviation of the model voltage from the actual for various conditions, as a percentage of maximum voltage. Careful observation of error plots shows that use of the 27-point correction with either point [000] or [002] as the reference point results in the smallest errors.

Fig. 4.11: Trajectory for M-R (II)

Fig. 4.12: M-R (II) without 27-point correction, reference point [000]

Fig. 4.13: M-R (II) with 27-point correction, reference point [000]

Fig. 4.14: M-R (II) without 27-point correction, reference point [001]

Fig. 4.15: M-R (II) with 27-point correction, reference point [001]

Fig. 4.16: M-R (II) without 27-point correction, reference point [002]

Fig. 4.17: M-R (II) with 27-point correction, reference point [002]

Fig. 4.18: Error Plots for M-R (II); "0", "1" and "2" are the cases using 27-point correction with [000], [001] and [002] as reference points respectively. "0_no", "1_no" and "2_no" are the corresponding cases without using 27-point correction.

Type III Trajectory

The model-reality plots of Type III, also referred to as M-R (III), are generated by rotating the transmitter 180° about the Y-axis of the antenna as shown in Fig. 4.19. In the antenna system, this is a rotation such that the transmitter axis and antenna Z-axis, which are initially orthogonal, become parallel and coincident and then are orthogonal again as the transmitter completes a rotation of 180 degrees. Thus, the angle between the z-axis of the antenna and the transmitter axis is varied from -90° to 90° in 10° increments.

The collected data is used to generate the six model-reality plots. The collected data, converted to volts by calibration, gives rise to the "actual" voltage series. The "model" voltages are generated by the forward model with or without 27-point correction, using either of the three points as the reference point. Figs. 4.20-4.26 are representative of the model-reality plots of Type III. Fig. 4.27 is the plot of errors, that is deviation of the model voltage from the actual for various conditions, as a percentage of maximum voltage. Careful observation of the error plots shows that consistently lower errors are obtained when 27-point correction is used with either point [000] or [002] as the reference point.

Fig. 4.19: Trajectory for M-R (III)

Fig. 4.21: M-R (III) without 27-point correction, reference point

Fig. 4.22: M-R (III) with 27-point correction, reference point [000]

Fig. 4.23: M-R (III) without 27-point correction, reference point [001]

Fig. 4.24: M-R (III) with 27-point correction, reference point [001]

Fig. 4.25: M-R (III) without 27-point correction, reference point [002]

Fig. 4.26: M-R (III) with 27-point correction, reference point [002]

Fig. 4.27: Error Plots for M-R (III); "0", "1" and "2" are the cases using 27-point correction with [000], [001] and [002] as reference points respectively. "0_no", "1_no" and "2_no" are the corresponding cases without using 27-point correction.

Conclusions from Study

As shown in previous sections, the 27-point correction scheme is effective in correcting the theoretical voltages, and the theoretical voltages modified using the 27-point correction scheme effectively model the actual voltage map. The maximum errors between the theoretical and actual voltages are reduced with this scheme and lie between 2% to 5% of the maximum voltage. The error ranges obtained depend on the choice of reference point. It is observed that the smallest errors are achieved by using either point [000] or point [002]

as the reference. The errors and error profiles obtained for both of these points are almost identical. Therefore the final selection of the reference point is based on practical considerations.

In general, for any antenna, reference point [000] lies in the plane or close to the plane of the antenna and might not be accessible easily in every experimental setup. On the other hand, point [002] lies farthest away from the antenna and is generally in the experimental space and therefore very easily accessible. Also, selection of point [002] will also lead to a reduction in the number of setups to be made for calibrating all of the antennae because the reference points [002] of adjacent antennae are normally common and readings for them can be taken in a single calibration setup. The number of calibration setups required for a chute experiment using point [000] will be 84 (28 antennae x 3 transmitters). These are reduced to just 48 if reference point [002] is used as the reference point. For the vibrated bed experiment described in Section V, the number of calibration setups can be reduced from 18 to 4 and even the jig required for calibration can be made less complicated. Taking all these factors into consideration, point [002] is chosen as the reference point.

One of the main disadvantages to using the 27-point scheme in the chute flow experiments is the necessity of setting up the correction map for every distinct tracking setup. This involves determining the physical limits of the experimental space requiring correction (based on the antenna size, inter-antenna spacing and antenna configuration), and then taking voltage readings at all of the 27 points of an octant in this space*. An additional complication arises if large antennae are needed, in which case 27 nodes may not be enough to model the non-linearity of scaling factors. In these cases, it is necessary to modify the interpolation function accordingly. Also, the 27-point scheme is implemented for a single transmitter and the correction is assumed to be same for all three transmitters.

In order to avoid these disadvantages, an analytical correction method was formulated, which is the same all systems, and eliminates the laborious task of determining the 27 points and taking accurate voltage readings for setting up the map.

A.4 Analytical Error Correction Scheme

A careful study of the model-reality plots indicated that the errors in the voltages depend not only on the position of the transmitter but also on its orientation. This suggests that it could be possible to model the systematic errors by investigating the non-linear nature of the errors and by intuitively adding a small quadratic or higher order term to the voltage equation. Recall from equation (2.6) that the theoretical voltage (V_{th}) induced in a receiving loop antenna by a transmitting coil having N turns is given by,

$$V_{th} = -N\omega A(B_x \cos\alpha + B_y \cos\beta + B_z \cos\gamma) \quad (4.2)$$

This voltage model assumes that the field of the transmitter is perfectly symmetrical, and the amplifier and data acquisition boards are perfectly linear. It also assumes that the principle of reciprocity is valid throughout the experimental space. As a consequence, the theoretical voltage map obtained using this model is symmetrical over the antenna while in practice this is not the case. The systematic errors in voltages can be compensated by adding a correction factor V_c to the theoretical voltage V_{th} yielding a compensated model voltage V given by,

$$V = V_{th} + V_c \quad (4.3)$$

where V_c is the correction added to the theoretical voltage.

In order to find this correction factor V_c , an intuitive approach is used based on the experience and insight obtained from the 27-point experiments previously described. Accordingly, it was observed that the correction equation required quadratic or higher order terms. Therefore, various equations with quadratic, cubic and higher order terms were developed and corresponding model-reality plots were generated. After a

* This is necessary because the effectiveness of the scheme depends on the proper choice of the 27 points and the accuracy with which voltages readings are taken at these points.

careful study of all the model-reality plots, equation (4.4) was selected as the correction equation. Since $N \omega$ and A are constant for a given transmitter, the term $(-N\omega A)$ were combined into a single constant designated as k . Then the correction equation is given by,

$$V_c = -0.15 k \left[\begin{aligned} & \left(B_x^2 \cos^2 \alpha + B_y^2 \cos^2 \beta + B_z^2 \cos^2 \gamma \right) \\ & + 0.01 \left(B_x^3 \cos^3 \alpha + B_y^3 \cos^3 \beta + B_z^3 \cos^3 \gamma \right) \end{aligned} \right] \quad (4.4)$$

Effectively, the above equation adds a 15% quadratic and a 0.15% cubic behavior to the theoretical voltage to give the model voltage. A series of experiments (see typical plots in Fig. 4.28-4.30) carried out to test this clearly indicated that the corrected theoretical voltages agreed quite well with those obtained using the 27-point scheme.

Fig. 4.28: M-R (I) using equation correction
(Using data of Fig. 4.9)

Fig. 4.29: M-R (II) using equation correction
(Using data of Fig. 4.17)

Fig. 4.30: M-R (III) using equation correction
(Using data of Fig. 4.26)

A.5 Multiple Solutions in Orientation

It was often observed that results obtained were correct in position (x, y, z) for the sphere's center, but the orientation was in error. A careful analysis of such solutions revealed that in most of the cases although the

angles were very different, they had nearly the same direction cosines. Consequently, such solutions are called "multiple solutions in orientations".

The forward model predicts theoretical voltages based on the direction cosines of transmitter axis in antenna coordinate system. The direction cosines are functions of α , β and γ , however, a given set of direction cosines does not correspond to a unique set of angles (α , β , γ). This means that in certain cases, different sets of angles (α , β , γ) can result in almost equal direction cosines values and hence almost equal theoretical voltages. Hence, the "multiple solutions in orientations" are the angular solutions that are apparently very different from the actual solution but have almost the same direction cosines as the correct solution. In this section two techniques for minimizing the multiple solutions in orientations are presented: a re-extraction scheme and an "equivalent angle-axis" representation.

Re-Extraction Scheme

A majority of the multiple solutions in orientations are of the type described in [1] and reproduced in Table 4.2. It can be observed that the predicted angles differ by multiples of 90° . For example, the predicted $\alpha = 247^\circ$ is approximately the actual $\alpha = 70^\circ + 2(90^\circ)$, predicted $\beta = 196^\circ$ is approximately the actual $\beta = 6(90^\circ) - 345^\circ$. Though it is possible to correct all the multiple solutions in orientations by examining each solution, no generic equation is found for eliminating these multiple solutions. Hence, the source of the multiple solutions, i.e. the solution algorithm, was closely examined.

Table 4.2: Multiple solutions in orientation ([9])											
Actual °	Predicted °										
X	Y	Z	α	β	γ	X	Y	Z	α	β	γ
6.3	2.8	6.4	0	345	30	6.1	3.2	6.5	174	193	211
12.3	6.0	8.1	70	345	30	12.1	6.1	7.9	247	196	213
14.0	7.0	8.6	90	345	30	13.8	6.7	8.2	267	197	212
14.8	7.5	8.8	100	345	30	15.3	7.8	8.9	260	16	208

The backward solution algorithm employs a perturbation technique for calculating the positions and orientations from the voltages. This involves individually perturbing each of the unknowns x , y , z , α , β and γ about initial guesses to converge to a solution where the residuals are minimum. Due to the random nature of the perturbation technique and periodicity of trigonometric functions, often the algorithm converges to an angular solution that is different from the expected solution but has the same direction cosines. The solution re-extraction scheme maps the multiple angle solutions into the interval $[-180^\circ, 180^\circ]$ for post-processing and better visualization of results. To do this, the solution given by the backward algorithm is used to calculate the global-to-transmitter transformation matrix, T given by equation (4.5). A simple representation of the transformation matrix T is given by equation (4.6).

$$T = \begin{bmatrix} \cos\beta \cos\gamma & \sin\alpha \sin\beta \cos\gamma - \cos\alpha \sin\gamma & \cos\alpha \sin\beta \cos\gamma + \sin\alpha \sin\gamma \\ \cos\beta \sin\gamma & \sin\alpha \sin\beta \sin\gamma + \cos\alpha \cos\gamma & \sin\gamma \\ -\sin\beta & \sin\alpha \cos\beta & \cos\alpha \cos\beta \end{bmatrix} \quad (4.5)$$

$$T = \begin{bmatrix} n_x & o_x & a_x \\ n_y & o_y & a_y \\ n_z & o_z & a_z \end{bmatrix} \quad (4.6)$$

The angles can be re-extracted from this matrix using the $\text{atan2}(\sin, \cos)$ function as shown in equations (4.7), (4.8) and (4.9). Since the atan2 function takes both the sine and the cosine of the angle as its arguments, information about the sign of the angle is preserved and a unique value in the interval $[-180^\circ, 180^\circ]$ is found.

$$\alpha = \text{atan2}(o_z, a_z) \quad (4.7)$$

$$\beta = \text{atan2}\left(-n_z, \sqrt{n_x^2 + n_y^2}\right) \quad (4.8)$$

$$\gamma = \text{atan2}(n_y, n_x) \quad (4.9)$$

Results obtained after re-extracting the actual and the predicted angles shown in Table 4.2 are presented in Table 4.3. The results, except for the last one, no longer exhibit multiple solutions in orientations.

Table 4.2: Solution after re-extraction of results of Table 4.2.

Actual (degree)						Predicted (degree)					
X	Y	Z	α	β	γ	X	Y	Z	α	β	γ
6.3	2.8	6.4	0	-15	30	6.1	3.2	6.5	-6	-13	31
12.3	6.0	8.1	70	-15	30	12.1	6.1	7.9	67	-16	33
14.0	7.0	8.6	90	-15	30	13.8	6.7	8.2	87	-17	32
14.8	7.5	8.8	100	-15	30	15.3	7.8	8.9	-100	16	-152

The solution re-extraction process results in a marked improvement in visualization of results. This can be seen from the results of an experiment (Fig. 4.31 and 4.32) in which the transmitting sphere was allowed to roll down an inclined plane with the rotation (β) taking place only about the y-axis.

Fig. 4.31: Rotation β before solution re-extraction

Fig. 4.32: Rotation β after solution re-extraction

B. Full Scale Chute Experiments

In the following sections, we present results on flow experiments using the full scale chute described in Section 3. In the first part, there is a discussion of bulk measurements without the tracking sphere, which were carried out to characterize the chute. This is followed by results using controlled trajectories with the tracking sphere.

B.1 Bulk Measurements

A flow experiment consists of releasing spheres from the hopper which flow through the sluice gate, down the inclined chute and into the exit container. Depending on the opening of the gate and the floor conditions, an experiment can run for approximately one minute, although a typical time was about 35 to 40 seconds. In an experiment, there are five factors that can be altered:

1. The size of the hopper gate opening which varies from completely closed to an area of 17" x 17"
2. The sluice gate opening (height from 0 to 9")
3. Chute width (from 5" to 14.5")
4. Chute floor conditions (smooth or bumpy)
5. Chute inclination (from 0° to 25°)

Characterization of the mass flow rate exiting the chute was done using the scale and attached data acquisition system. Preliminary experiments were carried out using a smooth floor, a 9" high sluice opening, and a chute width of 14.5". At a rate of 5 to 7 times per second, the scale data acquisition system captured the weight of the sphere in the exit container. Figure 4.33 shows a typical result when the chute inclination angle was 13.7°. The curve designated as "Series 1" is the mass flow rate in lb./sec while "Series 2" refers to the total weight (lb.). A summary of the flow rate experiments for inclination angles of 8.8°, 10.8° and 15° are presented in Fig. 4.34 which shows the maximum mass floor rates for smooth and bumpy floor conditions. For the angles considered, the rate increases with angle of inclination while the smooth floor produces a larger flow rate.

Figure 4.33: Scale measurements, 13.7° chute inclination and smooth floor conditions.

Fig. 4.34: Maximum mass flow rates for smooth and bumpy floor conditions

A characterization of the effect of the floor conditions is obtained by computing a "condition flow ratio" (CFR) defined by,

$$CFR = \frac{\text{Hopper empty time for the bumpy floor}}{\text{Hopper empty time for the smooth floor}}$$

Results from the experiments for the 3 inclination angles are summarized in Fig. 4.35 which shows an inverse relationship between the *CFR* and inclination angle.

Fig. 4.35: Condition flow ratio versus chute inclination angle

B.2 Controlled Tracking Experiments

Figs. 4.36 - 4.41 are typical results obtained from controlled trajectory experiments shown here for the bottom half of inclined chute. The transmitting sphere is held in the calibration jig, which is then moved along the global x-axis from $x = 59''$ to $x = 107''$ in 2'' increments*. The y and z coordinates are kept fixed at 12'' and rotations α , β and γ are zero degrees. Very good results are obtained with mean errors of only 0.34'', 0.10'' and 0.06'' in x, y and z coordinates and 1.47°, 0.70° and 0.56° in α , β and γ rotations, respectively.

Fig. 4.36: X plot
(maximum error = 0.61'', mean error = 0.34'', standard deviation = 0.15°)

Fig. 4.37: Y plot
(maximum error = 0.29'', mean error = 0.10'', standard deviation = 0.08'')

* Exception at data point 15, where the increment is 3 inches, and point 17, where the increment is 1 inch.

Fig. 4.38: Z plot
(maximum error = 0.21", mean error = 0.06", standard deviation = 0.05")

Fig. 4.39: α plot
(maximum error = 2.07°, mean error = 1.47°, standard deviation = 0.47°)

Fig. 4.40: β plot
(maximum error = 1.92° , mean error = 0.70° , standard deviation = 0.37°)

Fig. 4.41: γ plot
(maximum error = 1.09° , mean error = 0.56° , standard deviation = 0.25°)

5. TRACKING IN THE VIBRATED VERTICAL BED

This section describes results of the first application of the tracking technique to an experimental investigation. A description of the apparatus and experimental procedures is given, followed by a panorama of the possible trajectories and detailed descriptions of the three-dimensional motion of the sphere. All trajectory figures appear at the end of this section.

A. Experimental Apparatus and Procedures

The non-intrusive system previously described is used to find the three-dimensional position and rotation of the sphere in a vibrated bed composed of acrylic spheres of diameter d having a normal restitution coefficient e . The bed is contained in an acrylic cylindrical container of height H whose floor is a piston mounted onto a vibration exciter consisting of a B&K* general purpose head (type 4812) and exciter body (type 4801). The vibration exciter is driven by a B&K power amplifier (type 2707). An accelerometer is fixed onto the piston base and connected to the vibration exciter controller (type 1050). This is a device containing a digitally controlled generator which provides an input signal to the power amplifier, a vibration meter enabling accurate regulation of any vibration parameter, and a compressor for regulation of the vibration exciter excitation. During an experiment, the vibration exciter control permits an easy variation of the vibration frequency at fixed amplitude, velocity or acceleration. The mounted cylinder is surrounded by an array of six orthogonal receiving antenna (i.e., antenna cage) whose voltages are induced by the tracking sphere transmitters. Fig. 5.1a-c shows actual photos of the experimental set-up and a sketch of the system antenna cage surrounding the cylinder. Relevant parameters are listed in Table 5.1 while the dimensions of the antenna are given in Table 5.2.

Table 5.1: Physical parameters for the vibrated bed experiment

Parameter	D_{cyl}	d	H/D_{cyl}	Γ	e
Value/Defn.	4.5"	0.125"	0.44	$a\omega^2/g$	~ 0.9

Table 5.2: Antenna dimensions

Antenna	Quantity	Dimensions (inch x inch)
X	2	6 x 6
Y	2	6 x 10
Z	2	6 x 10

Before beginning an actual experiment, the tracking sphere ($D = 1$ inch) must go through calibration procedures. The ball is placed in the middle of the antenna cage, oriented such that each transmitter is successively perpendicular to each receiving antennae, and then the corresponding voltages measured. Because the transmitters in the sphere are mutually perpendicular, only four positions of the sphere (also called *correlation points*) are required. When this has been completed, the bed is vibrated at amplitude a and frequency f which causes the tracking sphere, initially placed in the center of the piston floor, to rise to the bed surface.

Acquisition of the induced voltages begins when the piston oscillations are initiated. Each sampling point through the data acquisition systems consists of 18 voltages, i.e., 6 antennae x 3 frequencies/antennae. Due to the lifetime of the battery and the sampling rate used, experiments longer than 20 minutes are not possible without uncontrollable loss of accuracy. Once the acquisition is completed, the voltage data is processed (see

* Bruel & Kjaer

Section IIA,B) to compute the coordinates of the sphere. Since the computation time required for processing is long, it was possible on the average to obtain one complete position per second. Furthermore, the acquisition rate used (i.e. approximately 900 Hz) meant that one real second of the experiment could be processed in about 15 minutes. When the time required for the large sphere to rise to the surface required several minutes or even hours, it was not possible to use the tracking system without modification of the data acquisition rate **.

Fig. 5.1(a),(b): Photographs of experimental set-up showing the acrylic cylinder, partially filled with 1/4" diameter acrylic spheres, surrounded by 6 orthogonal receiving antenna loops. This is mounted onto the B&K general purpose head and exciter body; **(c)** Schematic of the cylinder and surrounding antennae

B. Phenomenology of the Sphere Trajectories

With the experimental setup used, surface heaping is observed either for relative accelerations $\Gamma < 1$ or $\Gamma > 1$. When $\Gamma = 1.29$ and $f = 7.5$ Hz, the trajectory of the sphere is composed of single flights with a rise which appears essentially linear in time, except at the very beginning and end of its rise (See Fig. 5.2). After reaching the top of the surface heap, the sphere falls along the slope towards the walls and consequently its altitude decreases as seen in the figure. When $\Gamma = 2.06$ and $f = 12$ Hz in Fig. 5.3, the first part of the trajectory before the sphere actually begins to rise, is very similar to the previous case simply because it corresponds to the establishment of the permanent vibration regime. The last part of the trajectory in Fig. 5.3 is qualitatively different. The sphere slows down earlier and this results in a more pronounced curvature of its trajectory. In addition, the rise time is slower despite the large relative acceleration because the normalized amplitude in Fig. 5.3 ($a/d = 1.12$) is smaller than in Fig. 5.2 ($a/d = 1.8$). Thus, it appears that the amplitude has an effect on the shape of the sphere's trajectory, especially far from the vibration source (i.e., the floor piston)

When $f > 15$ Hz, surface heaping no longer occurs although there is a bulk convective flow in the bed whereby particles in central core stream upwards while a downward flow occurs in an annular region adjacent to the cylinder wall. The apparently very linear time trajectory for the case when $\Gamma = 7.45$ and $f = 18$ Hz ($a/d = 1.8$) is presented in Fig. 5.4. Although the motion consists of single flights, before it reaches the surface a period doubling* motion occurs. We hypothesize that this is due to the preferential occurrence of period doubling in regions of smaller energy dissipation (i.e., in regions of lower density) which may be the case for the applied level of agitation here. Fig. 5.5 shows the sphere's trajectory when $f = 18$ Hz ($a/d = 1.12$) and $\Gamma = 4.64$ where the same observations can be made as in the previous case. However, here the period doubling is much less important and the trajectory displays a more pronounced curvature when the sphere approaches the top surface. The difference in the amplitudes between the two case ($a/d = 1.8$ versus $a/d = 1.12$) may be responsible for this since a smaller amplitude results in a denser upper layer which increases the dissipative properties of the medium there and thus hinders period doubling.

In some cases, an inverse convective flow is observed which seems to occur only for high relative accelerations. Fig. 5.6 shows a trajectory sinking to the bottom of the container when $f = 27$ Hz and $\Gamma = 16.7$ ($a/d = 1.8$). An analysis of the Fourier transform of the trajectory reveals a very noisy spectrum with three dominant frequencies at f , $f/2$ and $f/4$ (see Fig. 5.10). When the amplitude level is reduced ($a/d = 1.12$, $f = 27$ Hz, $\Gamma = 10.4$), Fig. 5.7 shows that the ball sinks more slowly and the trajectory is slightly curved at the top which is perhaps a consequence of the change in amplitude. Here the downward motion is dominated only by the vibration frequency f as shown in the Fourier analysis of Fig. 5.11.

Several trajectories were found which corresponded to a "whale" effect (whereby the sphere goes up and down), but the sphere never reaches the bed surface and it does not seem to follow the convective flow. Figs. 5.8 and 5.9 show examples of this trajectory where, in both cases, the amplitude of the oscillation is approximately 0.6 inches and the sphere finds it easier to move down than up. This phenomenon is not understood and it is difficult to predict the required conditions for this to occur.

* This is characteristic of a nonlinear and dissipative system.

C. Analysis of the Sphere's Three Dimensional Motion

The trajectories described here are obtained in a cylindrical coordinate system as shown in Fig. 5.12. The principal displacement occurs in the vertical direction while rotational components of the motion are insignificant. The effect of initial radial position is illustrated using two examples where $a/d = 1$ and $f = 15$ Hz.

In the first example, the ball is initially positioned at the center of the piston surface. The vertical z -coordinate is plotted in Fig. 5.13 where the rise appears very regular and the oscillatory motion has a frequency $f/2$. In Fig. 5.14, the radial displacement r during the first 20 seconds (when the sphere rises) is approximately one bed particle diameter d .

Just before it reaches its maximum altitude, r increases significantly because the sphere is moving towards the cylinder walls. There is also a curious periodic oscillation of the radial displacement at $f/2$, which can also be seen on all of the other coordinates. In Fig. 5.15, angle θ varies slightly which implies that there is almost no orthoradial motion. In addition, rotational motions appear, but they are small in comparison with a complete 360° rotation as shown in Figs. 5.16, 5.17 and 5.18.

In the second example, the ball is initially positioned on the piston surface, but it is now touching the cylinder walls. The vertical z motion shown in Fig. 5.19 is qualitatively similar to that in Fig. 5.13. All of the rotations and the orthoradial displacement are very small. The only important difference is the radial displacement (Fig. 5.20) r which decreases, reaching a minimum with a slight increase at the end. In order to rise, the ball must avoid the downward flow along the walls. Consequently, it is pushed inside the bed while rising; close to the surface, it starts moving back towards the walls. The maximum distance from the wall that it attains is approximately $5d$, a value larger than the width $3d$ of the downward flow observed on the bed surface. It is certainly possible that the width of the downward flow at certain locations within the bed can be $5d$; however, there must also be a horizontal flow to allow circulation of the mass of spheres to change directions. Most likely, it is this horizontal component of the flow that is carrying the tracking sphere further than $3d$ away from the walls.

D. Smoothing of Results

Errors present in the tracking system are of two types: systematic errors and random errors. Sources of the systematic errors and techniques employed to correct these have been discussed in Section 4A. In this section we discuss the random errors and a smoothing technique for minimizing their effect applied to the vibrated bed experiments.

Random errors are unpredictable fluctuations that may be introduced by various factors such as radio noise, electromagnetic reflection etc. In order to check the nature of these random errors in the tracking system, an experiment was performed in the vibrated bed setup. The transmitting sphere was kept stationary at one position and data was collected at 463 Hz for two seconds. This data was then used to generate histograms, one of which is presented in Fig. 5.22.

Fig. 5.22: Histogram for voltage (counts)

After processing the voltage data, histograms were also generated for the calculated trajectory $(x, y, z, \alpha, \beta, \gamma)$. The histogram for α rotation is shown in Fig. 5.23 which appear to be Gaussian in nature.

Fig. 5.23: Histogram for α rotation

It is known that the effect of random errors which are generally Gaussian in nature, can be minimized by using Gaussian smoothing techniques [2] which can be expressed by the following equation:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad (5.1)$$

where, σ is the standard deviation of the data and μ is the mean value of data. The smoothing technique is implemented in the form of equation (7.2) [2],

$$smo_pt(n_p) = \sum_{P=0}^{P_n} \sum_{n=0}^{n_p} \sum_{i=i_{min}}^{i_{max}} \left(pt(n) - smo_pt(n_{p-1}) \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{i-n}{\sigma}\right)^2} \right) \quad (5.2)$$

where $smo_pt(n_p)$ refers to the smoothed value of data point n after P passes, $pt(n)$ is the unsmoothed value of data point n , P_n is the total number of passes, n_p is the total number of points, and i_{min} and i_{max} define the moving range of points over which Gaussian weights are applied. The smoothing is done in steps, determined by the number of passes P_n . The number of points in the range i_{min} to i_{max} used to apply Gaussian weights is determined by the standard deviation σ as an input. In the first pass, smoothing is done directly on the raw data and in the subsequent passes, Gaussian weights are applied to the difference between raw data and smoothed values of the points over the interval i_{min} to i_{max} . The degree of smoothing is thus affected by the number of successive passes and standard deviation. Thus there is a complete control over the smoothing process.

Fig. 5.24 shows the effect of smoothing applied to the results. The plot depicts the result of a "rigid vibration" experiment, where the transmitting sphere is rigidly mounted on the vibration exciter of the vibrated bed setup and vibrated at a frequency of 25 Hz. The figure shows both the raw (unsmoothed) and smoothed results. Note that since the vibration is in x direction, only the X plot is shown.

Fig. 5.24: Effect of smoothing on rigid vibration result: X plot

Fig. 5.25: Effect of smoothing on rigid vibration result: FFT

Fig. 5.25 shows Fast Fourier Transforms of the raw and smoothed data of Fig. 5.24. From (a), it can be seen that besides the dominant 25 Hz frequency, noise of higher frequencies is also present, while in (b), high frequency noise no longer appears.

E. Velocity Computation Technique

We emphasize here that the only direct measurement available is that of the trajectory of the sphere. Velocity and acceleration are derived quantities. Since calculation of velocity and acceleration is the process of computing derivatives of the positions and orientations, it is very sensitive to noise. Therefore, the accuracy with which velocity and acceleration can be computed is dependent on the accuracy of calculation of the positions and orientations and thus very much depends on success in filtering out the random errors and noise from the solution. Caution should be exercised in directly interpreting the calculated velocity and acceleration values.

After having obtained the position and orientation of the tracking sphere, the results are post-processed (see Sections 4.A.5 and 5.B) to smooth the data and remove any multiple orientations*. The velocity of the transmitting sphere, both linear and angular, is calculated numerically using the "Five Point Formula" discussed by Burden et al. [3].

$$V_{ij} = \frac{(X_{i,j-2} - 8X_{i,j-1} + 8X_{i,j+1} - X_{i,j+2})}{12t} \quad (5.3)$$

where $X = \{x, y, z, \alpha, \beta, \gamma\}$, $V = \{V_x, V_y, V_z, \omega_x, \omega_y, \omega_z\}$, t is the time between consecutive readings, X_i is X_i at data point j , and V_j is V_i at data point j .

Velocities computed for the rigid vibration experiment (see previous section) are presented in Figs. 5.26-5.31. Fig. 5.26 - 5.28 are the components of the linear velocity V (m/s), while Figs. 5.29 - 5.31 are the components of the angular velocity ω (radians/sec). As expected, V_x (in the direction of vibration) is the major component of linear velocity with small fluctuations in V_y and V_z .

* Random errors and multiple orientation solutions can results in large unrealistic fluctuation in the computed velocities.

Fig. 5.26: Linear velocity of rigid vibration: V_x

Fig. 5.27: Linear velocity of rigid vibration: V_y

Fig. 5.28: Linear velocity of rigid vibration: V_z

Fig. 5.29: Angular velocity of rigid vibration: ω_x

Fig. 5.30: Angular velocity of rigid vibration: ω_y

Fig. 5.31: Angular velocity of rigid vibration: ω_z

F. Critical Issues and Discussion

All of the trajectories analyzed via the Fourier transform show a peak at high frequency, usually near 250Hz. An example is provided in Fig. 5.21. This peak cannot be considered as noise since the amplitude of noise usually decreases with frequency. An explanation for the existence of this peak is important since it introduces fluctuations in the actual motion of the sphere and affects the precision of the tracking technique.

Numerous computations of the sphere's velocity and acceleration have been tested during the course of this project and it appeared that there was a large sensibility to noise. A time derivative in

physical space results in a multiplication by the frequency in Fourier space, and consequently the noise at high frequencies is amplified. To address this problem, various smoothing techniques were employed. One of these is the convolution of the trajectory by a Gaussian which has the major disadvantage of a loss of energy in the signal. A possible solution to this loss is to apply the Gaussian convolution several times to the difference between the original signal and the previously smoothed signal to retrieve some of the lost energy. However this results in a loss of trajectory details.

Analysis of the trajectory is itself a formidable task. For instance, it is very difficult to obtain a phase "portrait", (i.e., the evolution of the sphere's position at a fixed vibration phase), information which is needed to understand the behavior of such nonlinear systems. The main problem is that there is no reference between the trajectory and the piston motion. Consequently, a true time reference is missing, and the motion of the sphere relative to the piston is unknown. It is therefore necessary to include the piston motion in the data acquisition system by collecting the accelerometer signal as a reference. To establish a phase "portrait", precise control of the frequency would be helpful since the data acquisition rate is unrelated with the vibration frequency, and therefore the possibility of having a point at the same phase each period becomes fortuitous. Thus it would be interesting to lock the acquisition onto the vibration frequency with a variable waiting time between each data point. It is not possible to scan the 18 voltages readings required for each point with a controllable waiting time between captures since the current acquisition system board only has 16 channels.

A major issue to be addressed is related to the calibration procedures which consist of taking voltages induced when the sphere is placed in the center of the antenna cage and then rotated to obtain four different physical positions. The size of the antenna cage introduces errors in the determination of the actual positions when the sphere is close to an antennae and not on a symmetry axis. One possible explanation is that the dipolar approximation is no longer valid because the current loops (i.e., transmitters in the sphere) are too close to the antennae. Furthermore, one of the loops in the cage is quite large in comparison to the others, which makes it more difficult to fulfill the conditions for the dipolar approximation. It should be recalled that the magnetic field, created by the loop itself, may have some non-negligible components in the loop plane and there are clearly not taken into account in the model used for the determination of position from induced voltages. The additional limitation for the study of segregation is that the amount of rising from one period to another is often very small compared with the amplitude of the oscillatory motion of the sphere. This means that the precision of the recorded trajectory must be extremely good to localize very tiny variations in the trajectory.

Figure 5.2 Vertical position z as a function of time in the case of surface heaping at large amplitude ($a/d = 1.8$) and $f = 7.5$ Hz.

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Fig. 5.3: Vertical position z as a function of time in the case of surface heaping at small amplitude ($a/d = 1.12$) and $f = 12$ Hz.

Fig. 5.4: Vertical position z versus time in the case of convection without surface heaping at high amplitude ($a/d = 1.8$) and $f = 18$ Hz. Period doubling is observed above $z \approx 3.5$ ".

Fig. 5.5: Vertical position z versus time in case of convection without heaping at small amplitude $a/d = 1.12$, $f = 18$ Hz.

Fig. 5.6: Position z versus time for inverse convection at large amplitude $a/d = 1.8$ and $f = 27$ Hz. Oscillatory motion is dominated by three frequencies: $f, f/2, f/4$.

Fig. 5.7: Position z versus time for inverse convection case at small amplitude $a/d = 1.12$ and $f = 27$ Hz. Oscillatory motion is associated with only one frequency, f .

Fig. 5.8: Plot of vertical position z as a function of time for $a/d = 1$ and $f = 25$ Hz. The motion qualitatively corresponds to the "whale" effect.

Fig. 5.9: Plot of the vertical position z as a function of time for $a/d = 1$ and $f = 30$ Hz. The motion qualitatively corresponds to the "whale" effect.

Fig 5.10: Amplitude spectrum, corresponding to the trajectory of Fig. 5.6, shows three dominant frequencies f , $f/2$, and $f/4$. Frequencies higher than the vibration frequency are not physically significant.

Fig. 5.11: Amplitude spectrum corresponding to the trajectory of Fig. 5.7. There is only one dominant frequency $f = 27$ Hz. Frequencies higher than the vibration frequency are not physically significant.

Fig. 5.12: Sphere position (r, θ, z) in cylindrical coordinate system (u_r, u_θ, u_z) with the corresponding rotations (ψ, κ, α) .

Fig. 5.13: Vertical position z versus time for $a/d = 1$, $f = 15$ Hz. Surface heaping is observed, rise is linear in time and the sphere oscillates at $f/2$.

INTRODUCTION

Bulk solids or granular materials are assemblies of discrete solid particles bathed in a fluid. They exhibit very complex and diverse static or dynamic behaviors. In recent years granular materials have motivated significant research efforts: because bulk solids occupy such a preponderant place in human activities and environment, and have so many economical consequences, engineering design solutions cannot afford to rely on error and trial development methods. In the past 15 years attention has been focused on microstructural level studies that in turn allow for quantitative predictions of large scale flows. Hence, more fascinating phenomena have been intensively studied. Typical examples [1] are size-segregation [2, 3, 4], convection [5, 6], surface waves [7, 8] and heap formation [9, 10]. Understanding of the collision process between two granules plays a crucial role in the knowledge of the stress tensor and transport coefficients in rapid granular flows [11].

Advances in computers have made simulations to study flows feasible. Owing to the complex nature of any granular flow, numerical simulations of bulk solids have been turned into a very powerful and widely used tool [12], trying to make up for the difficulty to perform any direct, noninvasive physical measurements in real experiments, to complement existing experimental results and test theoretical ones [13, 14, 15]. They provide a means whereby the sensitivity of theoretical models may be tested and validated in the absence of experimental data. However, interpretation of physical experiments and validation of computer simulations and theoretical models require a knowledge of the properties of the flow materials. A distinctive property of grains is that their collisions are inherently inelastic. Mechanisms generating energy losses and/or transfers during collisions are extremely complicated but have to be somehow included in a proper approximation of collisions because they constitute a major factor in the complexity of granular flows.

Even though the problem of the impact of two elastic spheres has not been completely solved, the advances that have been made since the early work of Hertz have brought much insight in the processes involved so they can now be modeled

reasonably well. Yet, any model requires experimental validation to insure that realistic values of material properties are used. Unfortunately this data is still scarce although some efforts have been made using video analysis techniques to track the motion of colliding/flowing spheres. For many flow phenomena, the behavior of impacting particles can be described in terms of a collision operator, in which the post-collisional particle kinematics are a function of the values before collision and three parameters. The primary tasks to accomplish are on the one hand to measure collisional properties for various materials and on the other hand to study their sensitivity to variations of impact velocities and sizes. In this paper we perform experimental measurements of collisional properties of spheres and investigate the dependance of the coefficient of restitution on the impact velocity as well as on the size of the spheres. Finally, these results are compared with existing theoretical predictions found in the literature.

EXPERIMENTAL TECHNIQUES

Experimental measurements of collision properties of spheres depend entirely on the ability to determine as accurately as possible the kinematics of the two colliding bodies before and after collision. Previous experiments were designed in which either the available velocity range was very narrow [16] or the geometry of the collision [17], i.e. the angle of relative approach, was maintained to zero. Our aim is to increase flexibility for both parameters. The experimental setup we use is shown in Fig. 1. It consists of two main units: The "Collision unit" is designed to produce collisions of two spheres of arbitrary diameters with adjustable collision geometry: two steel tubes are mounted on micro-positionning slides; one of them is allowed to move horizontally and the other vertically thus allowing for changes of the relative incidence of the two spheres. One sphere is inserted in each tube and held at the end of the latter by a void pump. A manual trigger switches from the void pump to the pressurized air supply thus releasing the two spheres. After emergence from the tubes the spheres follow a ballistic trajectory until and after collision. The initial motion takes place in the vertical plane containing the axes of the tubes. The "Recording Unit" is a Ektapro 1000 High-Speed Video System consisting of the camera itself whose focal plane is parallel to the above pre-collisional plane, an image processing unit, a video monitor and a image intensifying unit. The collision scene is lightened from the front with two 750 Watt Lowell DP lights positionned symmetrically on both sides of the camera. Collisions are recorded on a video tape at a rate of 1000 images per second. A varying number of the resulting gray level images, depending on the velocities and sizes of the spheres, are saved and downloaded on a Unix platform for processing. In order to keep the processing time within reasonable limits, this number is usually of the order of 8 to 12 images (but maybe as low as 2 for high velocities) with approximately as many images before as after collision. Fig. 3 shows a

sequence of such images. The time is indicated in milliseconds at the lower left corner of each frame. In this particular example images from the left column are taken before collision and those from the right column after collision. The collision takes place between time $t=8$ ms and $t=9$ ms. Image processing techniques have been implemented to determine the trajectory of the spheres: this includes edge detection [18] and clustering techniques [19]. Once the positions of the centers of the spheres have been determined, we perform a second degree polynomial fit of those for each sphere, before and after the collision independantly. We use only the pre-collisional fits to find the "collision time", i.e. the time when the two trajectories intersect. The velocity components of each sphere before or after the collision are then computed by differentiating the best fits at the "collision time". The individual spins of the falling spheres are determined by following the motion, about the sphere centers, of black dots imprinted on their surfaces as can be seen on Fig. 3 The detection of these markers in the gray level image is performed with the same method as in the case of the sphere boundaries. Once the correspondence between markers from one image to the next is known, the position of the markers are shifted to a reference frame connected to the center of the sphere so that only the rotational motion of the markers subsists. The rotation vector from one frame to the next may then be determined by a least squared method. We assume that this rotation does not vary significantly during the recording so that an average rotation vector is computed before and after collision for each sphere. After collision, the velocity components of the spheres perpendicular to the focal plane are then determined from conservation of angular momentum. The ability of this experiment to provide results accurate enough relies entirely on the resolution of the camera. This is why before extracting the features of the images, i.e. the sphere and marker boundaries, a *subpixel* expansion [18] of the image is performed, which is part of the above mentioned "edge detection technique". This amounts to inserting interpolation points of the intensity of the gray level images. therefore increasing the apparent resolution of the camera. The final images have dimensions 4 times larger than the camera resolution, i.e. 768 by 960. We may expect that this expansion method will improve the accuracy. Assuming that the accuracy on the position of center is of s subpixel units, that we may estimate the error made on the measurement of the component of relative velocity $\vec{v}$ in a given direction ξ as:

$$\Delta v_{\xi} = \frac{s d}{n_s \delta t \sqrt{N}} \quad (1)$$

where δt is the average time between two consecutive frames, n_s is the number of subpixel per diameter and the factor $\sqrt{N}$ (N is the average number of frames before or after collision) is included in a statistical sense to account for the decrease of the standard deviation of our measurements with the increasing number of frames used for a fit. In a series of test collision experiments with 25.4mm nylon spheres, we measured the Δv_{ξ} from the difference in total momentum of the two

spheres, before and after the collision in both directions. If $v_{1\xi}$ and $v_{2\xi}$ are the velocity components of two colliding spheres in either x - or y -direction before collision with a prime after collision, we computed Δv as: $\Delta v = \frac{1}{2}(v'_{1\xi} + v'_{2\xi} - v_{1\xi} - v_{2\xi})$. Using equation 1 where s is now considered as a parameter, we find that a value of s of 2 subpixel units gives an upper bound for Δv_ξ : for velocities of the order of 1 m/s, this corresponds to an error of 4cm/s.

DEFINITION OF COLLISIONAL PROPERTIES

Subsequently to the work of Mindlin [20, 21] on the oblique contact of frictional spheres, Maw et al. [22, 23] modeled the collision of two elastic spheres by subdividing their contact patches into a series of concentric annuli, each of them being either in sticking or in sliding motion relatively to the same annulus belonging to the other sphere. Solving numerically the equations of elasticity with mixed boundary conditions they evidenced the possibility during a collision of storing and partly retrieving elastic energy stored as tangential deformation of the spheres in the contact region. These results are schematized by a simple "collision operator" proposed by Walton [24]: the collision is considered as an instantaneous event and three collisional properties are defined relating the pre- and post-collision kinematics. These properties are phenomenological constants describing the inelastic and frictional interactions of colliding spheres. Let us consider two homogeneous (see Fig. 2) spheres with masses m_1 and m_2 , diameters σ_1 and σ_2 , moments of inertia about their center I_1 and I_2 ($I_i = m_i \sigma_i^2 / 10$) and colliding when their centers lie at $\vec{r}_1$ and $\vec{r}_2$. Prior to the collision the centers of the spheres have velocities $\vec{v}_1$ and $\vec{v}_2$ and the spheres are spinning with rotation vectors $\vec{\omega}_1$ and $\vec{\omega}_2$. The new values of velocities and rotation, hereafter denoted with a prime, are obtained from the conservation of linear and angular momenta and prescribed relations using the collisional properties. $\vec{n} = (\vec{r}_1 - \vec{r}_2) / |\vec{r}_1 - \vec{r}_2|$ is the unit vector joining the centers of the two spheres. The relative velocity of the spheres at their contact point, or sliding velocity, before collision is (see Fig. 4):

$$\vec{v}_c = \vec{v}_1 - \vec{v}_2 - \left(\frac{\sigma_1}{2} \vec{\omega}_1 + \frac{\sigma_2}{2} \vec{\omega}_2 \right) \times \vec{n} \quad (2)$$

This velocity has a normal component $\vec{v}_n = (\vec{v}_c \cdot \vec{n}) \vec{n}$ and a component lying in the tangential plane $\vec{v}_s = \vec{v}_c - \vec{v}_n$. The normal coefficient of restitution is defined as:

$$(\vec{v}'_1 - \vec{v}'_2) \cdot \vec{n} \stackrel{\text{def}}{=} -e (\vec{v}_1 - \vec{v}_2) \cdot \vec{n} \quad (3)$$

while the coefficient of tangential or rotational restitution is:

$$\vec{v}'_s \stackrel{\text{def}}{=} -\beta \vec{v}_s \quad (4)$$

approximately constant. The direction of approach of the spheres is varied by moving the tubes on their respective slides. Fig. shows the coefficient of tangential restitution β versus the cotangent of the angle of incidence, $-\frac{v_n}{v_a}$. On this plot we observe the qualitative behavior predicted by Walton's model. For low values of $-\frac{v_n}{v_a}$, β first increases linearly until it reaches a maximum positive value $\beta_0 = 0.44 \pm 0.4$. The average coefficient of normal restitution is found to be over all these collisions $e = 0.97 \pm 0.1$. From the slope of the linear part we can extract $\mu_0 = 0.175 \pm 0.1$. Interestingly enough, the only two other measurements of β_0 we found in the literature [16] were very close to the value just obtained: 0.43 for soda lime glass spheres and 0.44 for cellulose acetate spheres.

In another series of experiment we now investigate the mass and velocity dependence of the coefficient of restitution e . In this case the tubes are kept at the same height so that at high velocities the angle of incidence of the spheres will be close to 0. The pressure is varied to obtain different impact velocity magnitudes. The experiment is performed first on Nylon spheres with diameters 25.4, 12.7 and 6.35 mm. Some errorbars are shown which are estimated from the change of total momentum as explained earlier, according to:

$$\frac{\Delta e}{e} \simeq 2 \frac{\Delta v_n}{v_n}$$

Results are shown on Fig. 7. It can be observed that the general trend is that of a decrease of the coefficient of restitution from values close to one as the velocity increases. On the other hand although the results appear quite scattered, there is an aggregation of points according to the size, indicating larger values of e , i.e. less dissipation, for larger spheres at fixed velocity. This result is very encouraging and allows us to compare with existing theoretical models of dissipation during collisions. At relatively low speeds, a few meters per second, as used here, energy dissipation appears due either to viscoelasticity of the materials or plastic deformation if in some region of the contact surface the local stress exceeds a typical yield stress or even fracture. It is very likely that in real collisions both viscoelastic and plastic behaviors will come into play and that viscoelastic effects will prevail for small deformations, i.e. small impact velocities. Both behaviors have been modeled theoretically: Kuwabara et al. [17] proposed a model of viscoelastic effects based on Hertz's theory [25]. It was compared with experimental measurements made on spheres hung by a bifilar suspension and showed good agreement for materials with the highest restitution. Johnson [26] proposed a model of plastic dissipation in which the coefficient of restitution does not depend on the size of the object and decreases as the power $-1/4$ of the velocity at high velocities. We compare our measurements of e with both predictions. In the viscoelastic model, the force P acting between two grains, is the sum of an elastic interaction given by Hertz's theory and of a viscoelastic

force describing the internal friction of the material:

$$P = \underbrace{2\eta d \dot{\delta} \left(\frac{\delta}{d}\right)^{\frac{1}{2}}}_{\text{viscoelastic damping force}} + \underbrace{Ed\left(\frac{\delta}{d}\right)^{\frac{3}{2}}}_{\text{Hertzian elastic force}} \quad (7)$$

so that the time evolution of the interparticle "penetration" δ ($\delta = (\vec{r}_1 - \vec{r}_2 - 2R\vec{n}) \cdot \vec{n}$) is solution of the following differential equation:

$$m_{red} \ddot{\delta} = P \quad (8)$$

$E = Y/[3(1 - \nu^2)]$ (Y is modulus of elasticity and ν Poisson's ratio) comes from Hertz's theory and η is a phenomenological constant with the units of a viscosity. While the value of E is imposed by mechanical properties of the material (Y and ν), we have no information on what η -values should be, it may therefore be considered as a fitting parameter. This equation was shown [27] to yield coefficient of restitution e obeying approximately:

$$1 - e \propto \frac{\eta \rho^{2/5}}{E^{-3/5} R} V_i^{1/5} \quad (9)$$

The size dependence in this model is therefore very strong. η was chosen so as to grossly center three curves obtained from equation 8 at a location satisfactory to the eye. We used approximate available values of Y and ν : $Y \simeq 10^9 \text{ N.m}^{-2}$, $\nu \simeq 0.3$, $\eta \simeq 33.2 \text{ kg.s}^{-1}.\text{m}^{-1}$. The mass density for nylon is $\rho \simeq 1.14 \cdot 10^3 \text{ kg.m}^{-3}$. We solve this equation numerically over the time interval where $\delta > 0$, i.e. when the spheres are in contact, to compute the coefficient of restitution e . The lines labelled "VM" in Fig. 7 are the results thus obtained from equation 8 for the three diameters employed: this comparison tells us that the size dependance ($1 - e \propto 1/d$) predicted by equation 8 appears to be overestimated; the rate of decrease of e is qualitatively compatible with our measurements.

For higher velocities, the rate of decrease of e with the normal velocity is higher than that predicted by equation 8 as can be seen on Fig. 7 for the 6.35 mm balls data for $V_i \geq 30 \text{ m.s}^{-1}$ which falls abruptly below the viscoelastic curve. In this velocity range we do not have any data for the larger sizes. However this deviation incites us to compare our data with a model for plastic deformation since we expect that plasticity has to come into play. In order to obtain a full prediction of e starting at velocities just above the yield velocity, where the energy dissipation is very low and e close to one, up to the point where the power law decay predicted by Johnson is to take place, we used the formalism proposed by Ning et al. [28]. In their model, the stress distribution is Hertzian, across the contact surface which is a disk of radius a such that $2a^2 = R\delta$, with a cut-off at a specified value of yield stress σ_y . The force-displacement relationship after yield has been reached becomes linear. We did not take into account any variation of the contact curvature, as was done in [28], in this region during the

elastic-plastic loading but instead kept it constant equal to the initial radius. The force-displacement curve is shown in Fig. 6. During loading the repulsive force between the two spheres is therefore:

$$P = Ed\left(\frac{\delta}{d}\right)^{\frac{1}{2}} \delta \quad \text{for } \delta \leq \delta_y$$

$$P = \frac{1}{2}\pi\sigma_y R(\delta - \delta_y) + Ed\left(\frac{\delta_y}{d}\right)^{\frac{1}{2}} \delta_y \quad \text{for } \delta \geq \delta_y \quad (10)$$

Unloading is done elastically with a new contact curvature R' so that the elastic force comes to zero for an elastic displacement $(\delta^* - \delta_0)$ smaller than δ^* , that achieved during loading. Both the new curvature and the "permanent indentation", δ_0 , stem directly from the continuity of the maximum force P^* and the radius of the contact area at the end of the loading phase. A discontinuity of the curvature thus arises that may be regarded as a discontinuity of the average curvature. Yield first occurs when the stress as predicted by Hertz's theory at the center of the contact patch reaches σ_y . This happens if the initial impact velocity is larger than the yield velocity V_Y given by:

$$V_Y = \left(\frac{\pi^4 \sigma_y^5}{10\rho E^4}\right)^{\frac{1}{2}} \quad (11)$$

For $V_i \geq V_Y$, the pressure at the center of the contact area will first reach σ_y when $\delta = \delta_y = \frac{1}{2}\pi^2 R\left(\frac{\sigma_y}{E}\right)^2$. The work done until the instant of maximal compression is:

$$W_i = \frac{1}{2}m_{red}V_i^2 = \int_0^{\delta^*} P d\delta \quad (12)$$

During rebound, the work done by the elastic force is:

$$W_r = \frac{1}{2}m_{red}V_r^2 = \frac{3P^{*2}}{5Ea^*} \quad (13)$$

From equations 12 and 13, an exact solution may be found for the coefficient of restitution $e = V_r/V_i$. Using $\sigma_y \simeq 1.15 \cdot 10^8 \text{ N/m}^2$, $V_Y \simeq 13 \text{ m.s}^{-1}$. When V_i becomes very large, e behaves according to:

$$e \simeq 1.18\left(\frac{V_i}{V_Y}\right)^{-\frac{1}{4}} \quad (14)$$

The solution is plotted on Fig. with the data for 6.35 mm spheres and gives a quite satisfactory agreement, showing the existence of a crossover region between viscoelastic and plastic behaviors for velocities around 25 cm/s. This plastic model defines a "universal curve" with no size dependance. More data for the larger sizes is needed to confirm the predicted scaling of e .

Conclusion

An experimental apparatus was designed to perform measurements of sphere collision properties. This kind of measurements is of crucial importance to scientists dealing with modelling of bulk solids. These properties are extracted by using a collision operator as was previously done in [22, 16]. In this model the possibility of retrieving part of the elastic energy stored in the early moments of a collision where sticking is involved, responsible for the reversal of the relative surface velocity, is measured by the coefficient β_0 which was found to be very close to previous values obtained for other materials [16]. With this experimental setup we were able to provide results on the size and mass dependance of the coefficient of restitution for the collision of spheres in free flight and for the first time to test the relevant dissipation hypotheses. It appears first that it is still not possible to support precisely a functional form of the velocity dependance of e . However our results showed two essential features: a noticeable size dependance of the coefficient of restitution occurs for moderate impact velocities and a crossover between to dissipation regimes. Although the viscoelastic model is not satisfactory from a quantitative point of view, it agrees qualitatively as far as size- and velocity-dependance of e are concerned. For high velocities, which were reached mostly for the smallest spheres, a crossover to the plastic deformation regime is in good agreement with our data. Scaling for the velocity dependance is also in very good agreement with the measurements. Unfortunately more data is needed to substantiate the scaling predicted by the plastic theory. This experimental device proved to be a suitable apparatus to study present issues concerning the collision of inelastic frictional spheres and should be used further to answer the still open questions just mentioned.

Figure 1: Experimental Setup

Figure 2: Colliding spheres with spins.

No. 1

No. 4

No. 2

No. 5

No. 3

No. 6

Figure 3: Sequence of grey level images from a typical collision experiment

Figure 4: Relative motion of the two spheres of Fig. 2 and impulse $\Delta \vec{P}$ exerted by sphere 2 on sphere 1.

Figure 5: Coefficient of tangential restitution β versus the cotangent of the angle of incidence.

Figure 6: Force displacement curve.

Figure 7: Coefficient of restitution versus normal impact velocity for nylon spheres for different diameters. The diameter of the spheres is shown in the figure. (VM)=viscoelastic model. (PM)=plastic model.

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Fig. 5.14: Radial displacement r versus time for $a/d = 1$, $f = 15$ Hz. A residual oscillation is observed at $f/2$.

Fig. 5.15: Plot of the angular motion θ as a function of time for $a/d = 1$ and $f = 15$ Hz. A residual oscillation at $f/2$ is observed.

Fig 5.16: Plot of the rotational motion α as a function of time for $a/d = 1$ and $f = 15$ Hz. A residual oscillation at $f/2$ is observed.

Fig 5.17: Rotational motion ψ as a function of time for $a/d = 1$ and $f = 15$ Hz. A residual oscillation at $f/2$ is observed.

Fig. 5.18: Rotational motion κ as a function of time for $a/d = 1$ and $f = 15$ Hz. A residual oscillation at $f/2$ is observed.

Fig. 5.19: Plot of vertical position z as a function of time for $a/d = 1$ and $f = 15$ Hz. The ball is initially touching the side walls. The rise is linear with time, and the ball oscillates at $f/2$. No major differences with Fig. 5.13 are observed.

Fig. 5.20: Plot of radial displacement r as a function of time, for $a/d = 1$ and $f = 15$ Hz. First the ball is observed to move from the walls towards the center of the cylinder. Before reaching the bed surface, it starts to move back towards the walls, apparently following the bulk convective flow.

Fig. 5.21: Amplitude spectrum corresponding to the trajectory of Fig. 5.6, where $a/d = 1.8$ and $f = 27$ Hz, showing a high frequency peak around 250 Hz, whose origin is undetermined. Otherwise, there are 3 dominant peaks at f , $f/2$ and $f/4$, while there is a profile in $1/f$ corresponding to the FFT of the almost linear average motion.

6. SUPPLEMENTAL INFORMATION

A. Publications

1. R. N. Dave, J. Volcy, J. Agarwal, V. Gupta, A. Troiano, A. Rosato, I. Fischer, W. Carr, "Non-Intrusive Rigid Body Tracking for Dry Particulate Flows, Part II: Practical Aspects", in preparation (1997).
2. Y. Lan and A. D. Rosato, "Convection Related Phenomena in Vibrated Granular Beds," under revision for *Physics of Fluids* (1997).
3. R. N. Dave and Bukiet, B. G., "Non-Intrusive Rigid Body Tracking Technique for Dry Particulate Flows, Part I: Theoretical Aspects," under review, *Review of Scientific Instruments* (1997).
4. L. Vanel, A. D. Rosato, R. N. Dave, "Rise Regimes of a Single Large Sphere in a Vibrated Bed," *Phys. Rev. Lett.*, Feb. 17, 1997.
5. L. Vanel, R. N. Dave and A. D. Rosato, "Experimental Observations of a Rise of a Single Large Sphere in a Bed of Granular Material Subjected to Vertical Vibration," *Proceedings of the 5th World Congress of Chemical Engineering*, Vol. VI, pp. 645-50, July 14-18, 1996, San Diego, CA.
6. Y. Lan and A. D. Rosato, "Macroscopic Behavior of Granular Vibrating Beds of Smooth Inelastic Spheres," *Phys. Fluids* **7** [8], 1818-1831 (1995).
7. R. N. Dave, A. D. Rosato, K. Bhaswan, "Fractal Characterization of Clustered Microstructure in Uniform Shear Flows of Smooth, Inelastic Particles," *Mech. Res. Comm.* **22** [4], 335-342 (1995).
8. R. N. Dave and T. Fu, "Robust Shape Detection Using Fuzzy Clustering: Practical Applications," *Fuzzy Sets and Systems* **65**, 161-185 (1994).
9. A. D. Rosato and Y. Lan, "Granular Dynamics Modeling of Vibration-Induced Convection of Rough Inelastic Spheres," *Proceedings of the First International Particle Technology Forum, Symposium on Solids Transport, Separation and Classification*, p. 446-453, Aug. 17-19, 1994, Denver, AIChE, NY (1994).
10. A. D. Rosato, R. N. Dave, I. S. Fischer, Y. Lan, "Experiments and Simulations in Particle Technology," *Proc. NSF/DOE Workshop, Flow of Particulates and Fluids*, Oct. 6-7, 1994, Philadelphia, PA.
11. A. D. Rosato, R. N. Dave, and E. Mosch, "Experimental Study of Vibrational Size Segregation," *Proceedings of the AIChE Particle Technology Forum*, pp. 325-330, Denver, CO, Aug. 17-19, 1994.
12. R. N. Dave, J. Yu, A. D. Rosato, "Measurement of Collisional Properties of Spheres Using High-Speed Video Analysis," *1993 ASME WAM: Symposium on Mechatronics*, DSC-Vol 50, pp. 217-222, Nov. 28 - Dec. 3, 1993.
13. R. Dave, B. Bukiet., A. Rosato., I. Fischer and J. Volcy, "Non-Invasive Rigid Body Tracking," *Proceedings of the DOE/NSF Workshop, Flow of Particulates and Fluids*, Sept. 29 - Oct. 30, 1993, Ithaca, New York.

14. R. N. Dave, A.D. Rosato, I. S. Fischer, B. G. Bukiet, and W. N. Carr, "Non-Intrusive Three-Dimensional Particle Tracking System," *Joint NSF/DOE Workshop on Flow of Particulates and Fluids*, Gaithersburg, MD, Sept. 17-19, 1992.
15. A. D. Rosato and Y. Lan, "Discrete Element Modeling of Vibrating Granular Beds," *Powders & Grains 93*, A. A. Balkema, Publ. Netherlands (1993).
16. R. N. Dave and K. Bhaswan, "Adaptive Fuzzy c-Shells Clustering and Detection of Ellipses," *IEEE Transaction of Neural Networks* **3** (5), 643-663 (1992).
17. R. N. Dave, "Generalized Fuzzy c-Shells Clustering and Detection of Circular and Elliptical Boundaries," *Pattern Recognition* **25** (7), 713-721 (1992).
18. R. N. Dave, A. S. Ashok, and B. G. Bukiet, "On Development of a Three Dimensional Particle Motion Tracking System," *Proceedings Of the 1992 ASME Winter Annual Meeting*, Paper No. 92-WA/DSC-5, Nov. 8-13, 1992, Anaheim, CA.
19. A. D. Rosato, R. Dave, I. Fischer, W. Carr, "Methodology of a Non-Intrusive Particle Tracing Technique for Inclined Chute Flows", *Proceedings of the Third DOE/NSF Workshop, Flow of Particulates and Fluids*, Oct. 23-25, 1991, Worcester, MA.

B. Particle Technology Center* at NJIT

As a result of this contract, the Mechanical Engineering Department established the *Particle Technology Laboratory (PTL)* in 1991 which maintained this status within the department until 1995. At this point, a proposal was submitted to NJIT by A. D. Rosato and R. N. Dave to designate the laboratory as an Institute Center. With approval granted by the Board of Trustees in June 1995, the *Particle Technology Center (PTC)* was established with its first year officially beginning in September of 1995. Since its establishment, the Center has seen a substantial growth in its activities as well as level of funding, both from federal and industrial sponsors.

The *PTC* is located on the third floor of the Mechanical Engineering Building which houses its facilities for experimental and computational research as well as a fully-equipped instructional laboratory. Involved Center faculty members from the Departments of Mathematics and Mechanical, Civil, Chemical and Electrical Engineering, encompass a broad range of expertise in particle technology and they are at the forefront of researchers in their respective fields. The mission of the *PTC* is three-fold:

- To conduct basic experimental research and mathematical modeling at the microlevel to gain an understanding of the macroscopic behavior of bulk solids in dry and slurry form
- To educate undergraduate and graduate students and provide training to other professionals in the engineering practice of particle technology
- To develop cost effective flow, handling, and processing technology of particulate systems relevant to existing and emerging industries, and transfer this technology to industrial companies working in partnership with the Center

Thus the long term goals of the *PTC* include enhanced scholarly activities, encompassing presentations, publications and intellectual property, submission of multi-disciplinary federal and industrial proposals, broadening education and training, continuing laboratory development and modernization of experimental equipment, solicitation of corporate memberships, and expansion of the center participation to enhance expertise in particle technology.

* A comprehensive overview of the activities of the PTC can be found on its web site at:
http://www-ec.njit.edu/ec_info/image2/PTC

C. Faculty / Staff

Dr. Rajesh N. Dave: Associate Professor of Mechanical Engineering, NJIT

Dr. Ian S. Fischer: Associate Professor of Mechanical Engineering, NJIT

Dr. Anthony D. Rosato: Associate Professor of Mechanical Engineering, NJIT

Mr. Songyao Ren: Electrical Engineer

Mr. Anthony Troiano: Electrical Engineer

D. Students / Degrees Awarded

1. Zhang, Yahong, "Chute Flow Experiments, M.S. Thesis, NJIT, October 1996.
2. Gupta, Vivek, "Implementation of Three-Dimensional Particle Tracking System," M.S. Thesis, NJIT, January 1995.
3. Agrawal, Jitesh, H., "The Use of Multiple Transmitters for 3-D Non-Intrusive Particle Tracking," M.S. Thesis, NJIT, January 1995.
4. Patel, Prashant, "Simulation of a Nonintrusive Particle Tracking System," M.S. Thesis, NJIT, January 1995.
5. Volcy, Jerry, "Development of a Non-Intrusive Particle Motion Tracking Technique for Granular Flow Experiments," M.S. Thesis, January 1994.
6. Lan, Yidan, "Particle Dynamics Modeling of Vibrating Granular Beds", Ph.D. Dissertation, NJIT, May 1994.
7. La Rosa, Anthony, "Experimental Studies of Size Segregation," M.S. Thesis, NJIT, August 1993.
8. Sen, Sumit, "Application of Computer Vision for Particle Collision Parameter Analysis, " M.S. Thesis, Mechanical Engineering NJIT, May 1992.
9. Ashok, Avadhani, "Computational Aspects of a 3-Dimensional Non-Intrusive Particle Motion Tracking System, " M.S. Thesis, NJIT, May 1992.
10. Caesar, J., "Automatic Motion Analysis of Colliding Spheres," M.S. Thesis, NJIT, May 1992.

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APPENDIX A: Equations for the Parameters of Equation (2.3)

$$R_1 = \sqrt{(y+a)^2 + z^2} \quad (\text{A.1.1})$$

$$R_2 = \sqrt{(x-l)^2 + z^2} \quad (\text{A.1.2})$$

$$R_3 = \sqrt{(y-a)^2 + z^2} \quad (\text{A.1.3})$$

$$R_4 = \sqrt{(x+l)^2 + z^2} \quad (\text{A.1.4})$$

$$\cos\phi_{11} = \frac{x+l}{\sqrt{x^2 + y^2 + z^2 + l^2 + a^2 + 2lx + 2ay}} \quad (\text{A.1.5})$$

$$\cos\phi_{12} = \frac{x-l}{\sqrt{x^2 + y^2 + z^2 + l^2 + a^2 - 2lx + 2ay}} \quad (\text{A.2.1})$$

$$\cos\phi_{21} = \frac{y+a}{\sqrt{x^2 + y^2 + z^2 + l^2 + a^2 - 2lx + 2ay}} \quad (\text{A.2.2})$$

$$\cos\phi_{22} = \frac{y-a}{\sqrt{x^2 + y^2 + z^2 + l^2 + a^2 - 2lx - 2ay}} \quad (\text{A.2.3})$$

$$\cos\phi_{31} = \frac{-x+l}{\sqrt{x^2 + y^2 + z^2 + l^2 + a^2 - 2lx - 2ay}} \quad (\text{A.2.4})$$

$$\cos\phi_{32} = \frac{-x-l}{\sqrt{x^2 + y^2 + z^2 + l^2 + a^2 + 2lx - 2ay}} \quad (\text{A.2.5})$$

$$\cos\phi_{41} = \frac{-y+a}{\sqrt{x^2 + y^2 + z^2 + l^2 + a^2 + 2lx - 2ay}} \quad (\text{A.2.6})$$

$$\cos\phi_{42} = \frac{-y-a}{\sqrt{x^2 + y^2 + z^2 + l^2 + a^2 + 2lx + 2ay}} \quad (\text{A.2.7})$$

$$\Theta_1 = \frac{z\mathbf{j} - (y+a)\mathbf{k}}{\sqrt{y^2 + z^2 + a^2 + 2ay}} \quad (\text{A.3.1})$$

$$\Theta_2 = \frac{-z\mathbf{i} + (x-l)\mathbf{k}}{\sqrt{x^2 + z^2 + l^2 - 2lx}} \quad (\text{A.3.2})$$

$$\Theta_3 = \frac{-z\mathbf{i} + (y-a)\mathbf{k}}{\sqrt{y^2 + z^2 + a^2 - 2ay}} \quad (\text{A.3.3})$$

$$\Theta_4 = \frac{z\mathbf{i} - (x+l)\mathbf{k}}{\sqrt{y^2 + z^2 + l^2 + 2lx}} \quad (\text{A.3.4})$$

APPENDIX B: Discrete Element Calculations of Vibrationally Energized Flows

This section contains a reprint "Macroscopic Behavior of Vibrating Beds of Smooth Inelastic Spheres," *Phys. Fluids* 7 (8), 1818-1831 (1995).

Reprint - removed for separate
processing

APPENDIX C: Sphere Collision Properties Experiment

This section contains a preprint of the results of our experiments to measure the collision properties of impacting spheres using the three-parameter model derived by O. R. Walton.

Preprint - removed for separate processing