Appendix to Theory of Seismic Coupling (HAB-59-4)

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APPENDIX TO THEORY OF SEISMIC COUPLING (FAB-52-1)

The Letter Method of Concealment

A. Letter has proposed to reduce the seismic coupling by placing the bomb in the center of a cavity in hard rock. He has pointed out that it is possible to have the energy propagate in the cavity by radiation rather than by shock so that the ultimate situation is a certain statical pressure in the cavity which is given by

\[ p_1 = (\gamma - 1) \frac{W_1}{\frac{4}{3} \pi R^3} \]  

(A-1)

where \( R \) is the cavity radius and \( \gamma - 1 \) can be as small as .1 to .15 for a hot gas.

The pressure \( p_1 \) is applied suddenly to the wall of the cavity. Care must be taken to make \( p_1 \) small enough so that the surrounding rock behaves elastically. How small \( p_1 \) has to be is hard to tell; in particular, fissures in the rock may open under the influence of the hoop stress induced by \( p_1 \). Letter has suggested that 50 atmospheres may be tolerable. If the rock behaves elastically then (33) may be used with \(-\sigma_2 = p_1\).

Asymptotically at large \( t \) this gives

\[ \gamma = \gamma_1 = \frac{R^3 p_1}{4 \mu} = (\gamma - 1) R_0^3 \]  

(4-2)

where we have set \( n = 1 \) and used (70). It is interesting to note that \( \gamma_1 \) is independent of the cavity radius.
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Classification (Delegation/Review Date) Changed to:

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We now have to solve (45) with the initial condition \( \gamma = \gamma = 0 \). The solution is of the form (45) and may be written explicitly

\[
\frac{\gamma}{\gamma_1} = 1 + \frac{\alpha_2}{\alpha_1 - \alpha_2} e^{-2\alpha_1 x} - \frac{\alpha_1}{\alpha_1 - \alpha_2} e^{-2\alpha_2 x}
\]

(A-3)

where we have defined

\[
x = c \tau /2R
\]

(A-4)

If we set \( n = 1 \) then (46) gives

\[
2 \alpha_{1,2} = -1 \pm i
\]

(A-5)

so that explicitly

\[
\frac{\gamma}{\gamma_1} = 1 + e^{-x} (\sin x - \cos x)
\]

(A-6)

The Fourier transform (3), using (33), is given by

\[
\hat{\phi} (\omega) = -\pi \int \sigma_\star (\tau, t) e^{i\omega \tau} d\tau = \hat{\sigma} \int \gamma (\tau) e^{i\omega \tau} d\tau = -i\omega \hat{\rho} \int \gamma (\tau) e^{i\omega \tau} d\tau
\]

(A-7)

This can easily be calculated for (A-6). For low frequencies such that

\[
2 \omega R/c \ll 1
\]

(A-8)
which are the only ones of interest to us, we obtain (except for the factor $-i$)

$$\phi(\omega) = \phi \gamma, \quad \omega = (\gamma_g - 1) \frac{\rho_0}{c^2} \omega$$

(A-9)

This may now be compared with the expression for hard rock without a cavity, (20). Denoting quantities referring to hard rock by a subscript $h$, to the cavity by a subscript $c$ and to the gas in the cavity by $g$, we find

$$\frac{\phi_c(\omega)}{\phi_h(\omega)} = (\gamma_g - 1) (\gamma_h + 1) \frac{R_0}{c} \omega$$

(A-10)

Taking $\gamma_g = 1.15$, $\gamma_h = 2.5$, $\omega = 10 \text{ sec}^{-1}$, $R_0 = 2.15$ meters and $c = 5 \text{ km/sec}$, we find

$$\frac{\phi_c(\omega)}{\phi_h(\omega)} = 2.2 \cdot 10^{-3}$$

(A-11)

This is a substantial reduction. If the result (20) for tuff is correct, the reduction relative to tuff is still a factor $0.04$. However, the reduction is less great at higher yield.
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