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**ANALYSIS OF THE ANOMALOUS SCALE-DEPENDENT BEHAVIOR OF
DISPERSIVITY USING STRAIGHTFORWARD ANALYTICAL EQUATIONS:
FLOW VARIANCE VS. DISPERSION**

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ABSTRACT

Recent field and laboratory data have confirmed that apparent dispersivity is a function of the flow distance of the measurement. This scale effect is not consistent with classical advection dispersion modeling often used to describe the transport of solutes in saturated porous media. Many investigators attribute this anomalous behavior to the fact that the spreading of solute is actually the result of the heterogeneity of subsurface materials and the wide distribution of flow paths and velocities available in such systems. An analysis using straightforward analytical equations confirms this hypothesis. An analytical equation based on a flow variance approach matches available field data when a variance descriptor (σ) of approximately 0.4 is employed. Also, current field data provide a basis for statistical selection of the variance parameter based on the level of concern related to the resulting calculated concentration. While the advection dispersion approach often yielded reasonable predictions, continued development of statistical and stochastic techniques will provide more defensible and mechanistically descriptive models.

INTRODUCTION AND SUMMARY

Development of mathematical models that accurately describe the transport of solutes in saturated and unsaturated porous media is an integral part in the assessment of the location and management of new and existing waste disposal facilities, as well as an important element in our fundamental understanding of geochemistry and diagenesis in the subsurface. A large number of computational algorithms are available to describe the behavior of solutes in subsurface flow systems. These range from direct solutions of governing equations based on assumed initial and boundary conditions to numerical finite element and finite difference methods. Despite the apparent diversity of methods, the various algorithms almost exclusively rely on the same governing equation--the advection and dispersion equation. Critical evaluation of this equation is important, affecting most of the available models. From a practical standpoint, any deficiencies identified should be put into perspective (i.e., how significant are differences between the model predictions and field measurements). Also, alternate methods, or the general path toward these methods, should be identified.

Recent data on laboratory- and field-scale dispersivity measurements [Silliman and Simpson, 1987; Gelhar et al., 1985; Moltz et al., 1986] document a phenomenon that is not consistent with the advection and dispersion approach; measured dispersivity values are a function of the measurement scale rather than a constant value. A variety of explanations have been developed for this phenomenon [Gelhar et al., 1985; Molz et al., 1986; Dagan, 1987; Greenkorn and Cala, 1986; Russo and Bresler, 1982; Konikow and Mercer, 1988; and Domenico and Robbins, 1984]. A consensus among the various investigators is emerging, however. Many geohydrologists and modelers attribute the effect to the heterogeneity found in the subsurface flow systems and the stochastic

nature of flow between various locations. As a result, a few alternate modeling approaches have been developed. These include several complex stochastic transport models [Gelhar, 1986; Mantoglou and Gelhar, 1987; Yeh et al., 1985; Duffy and Gelhar, 1986; Gutjahr and Gelhar, 1981; Simmons, 1982], and straightforward descriptions of statistically stratified flow fields [Matheron and de Marsily, 1980; Dagan, 1984; Jury, 1982; Jury et al., 1986.] To date, however, the relationship between the alternate methods and the classical advection dispersion approach has not been addressed in a definitive manner. The development of a straightforward alternate modeling approach, along with a comparison of the approach to classical methods, is presented below.

BACKGROUND

To meet the general objective described above, two analytical solutions describing solute transport were developed for the same boundary and initial conditions. One of these is based on the advection and dispersion equation, while the other is based on a heterogeneous media providing a distribution of flow paths between two locations--a flow variance approach. Note that the flow variance approach assumes that all of the spreading of a solute is due to the heterogeneity in available flow paths, and the solution is based on the transfer function similar to Jury and co-workers [Jury, 1982; Jury et al., 1986]; no diffusion or dispersion is assumed. The calculated concentrations at any time at various locations show that the two approaches are almost identical at a reference location; however, they diverge significantly at other locations.

Based on the two equations, a mathematical relationship between the assumed variance of the flow velocity distribution and dispersivity was derived. Variances were then calculated from the well documented data set

presented in Gelhar et al. [1985]. The resulting variance descriptors' values centered around a constant value of 0.4, and there was no effect of scale. This suggests that the flow variance approach may be a more mechanistically consistent description of subsurface transport. The probability distribution of the calculated variances provides modelers with a tool to calculate concentrations that have an explicit degree of conservatism built in. Although the analysis suggests that the classical advection dispersion approach may be sufficiently accurate for some transport calculations (e.g., calculating concentrations at a single receptor location or calculating relative concentrations to compare engineered options), additional development of alternate approaches is recommended. Specifically, stochastic and flow variance approaches appear to be attractive candidates for incorporation into both numerical and analytical models. The initial results with the flow variance approach provide a more satisfactory description of the field data than the advection dispersion approach.

Classical Advection Dispersion Approach

The transport of solutes through a porous medium is typically represented in terms of advection modified by dispersion processes; both of these may be adjusted for retardation caused by interaction of the solute with alternate solution phases or the solid matrix. Dispersion is a mathematical construct that is used to represent the spreading of solute caused by molecular diffusion, velocity gradients across the pores, mixing of pore channels, and the effects of spatial heterogeneity in natural systems (e.g., layering or channeling). The one-dimensional form of the advection-dispersion equation for nonreactive solutes under uniform flow is:

$$\frac{\partial C}{\partial t} = -\bar{v} \frac{\partial C}{\partial x} + D \frac{\partial^2 C}{\partial x^2} \quad (1)$$

where: x = the coordinate along the flow line (length)

$\bar{v}$ = is the average linear flow velocity (length/time)

D = the longitudinal dispersion coefficient (length²/time)

C = the solute concentration (mass/length³)

The longitudinal dispersion coefficient is typically expressed as:

$$D = \alpha \bar{v} + D^* \quad (2)$$

where: α = the dispersivity (length)

D^* = the molecular diffusion coefficient in the medium
(length²/time).

Molecular diffusion is generally considered insignificant in practical field applications (based on peclet number and other considerations).

These simplified constructs form the basis of most analytical and numerical models currently used to describe solute transport. However, a growing body of data suggests that this approach is not entirely adequate to represent the complex flow and geochemical processes occurring in real subsurface systems. In particular, the dispersivity, α , that is theoretically a function of the porous medium has been shown to be dependent on the scale of measurement in both laboratory and field studies [Silliman and Simpson, 1987; Gelhar et al., 1985; Moltz et al., 1986]. Figure 1, adapted from Gelhar et al. [1985], demonstrates this relationship. In fact, many investigators have recommended approximating α using a direct relationship [Looney et al., 1987, Fjeld et al., 1987]:

$$\alpha = \beta L \quad (3)$$

where: L = the reference receptor location (length)

β = the dispersivity distance multiplier.

A β value of 0.1 is commonly recommended following inspection of figures similar to Figure 1 [Anderson, 1984; Looney et al., 1987; Fjeld et al., 1987].

To evaluate the relationship between the classical advection-dispersion approach and an alternate approach based on mathematical description of flow rate heterogeneity, analytical solutions for each will be developed. For a step function input, the following equation describes the concentration at a receptor location as a function of time [Freeze and Cherry, 1979]:

$$C(x,0) = 0 \quad x > 0$$

$$C(0,t) = C_{in} \quad t < 0$$

$$C(\infty,t) = 0 \quad t > 0$$

Boundary Conditions

relatively high D , x or ∇

$$C(x,t) = \frac{C_{in}}{2} \left[\operatorname{erfc} \left(\frac{x - \nabla t}{2 \sqrt{Dt}} \right) \right] \quad (4)$$

where $C(x,t)$ is the concentration at location x at time t based on advection and dispersion, erfc represents the complementary error function and x is any receptor location ($x > 0$). In this case, D is assumed to be defined by (2) and (3) for a reference receptor location.

Derivation of a Flow Variance-Based Equation

A straightforward method of calculating solute concentration in subsurface flow systems based on a transfer function has been proposed by Jury [1982] and Jury et al. [1986]. To date, this approach has been applied to partially saturated soil transport systems. However, the concept may be generalized to describe solute transport from a source zone to a receptor zone in a fully saturated subsurface flow system.

The transfer function approach assumes that a flow system, in this case an aquifer, may be entirely described in terms of a mathematical

transformation of the solute input function into an output function. The mathematical transformation used is selected to be consistent with the assumed governing physical and chemical processes. Thus, while the ultimate model is mechanistically consistent, it is not fully descriptive (e.g., specific variations in permeability caused by channels or barriers are not explicitly described). The transfer function model estimates the average and extreme behavior of solutes based on the field-measurable (or inferred) distribution of travel times in a porous medium. The transport of solutes in the sub-surface is assumed to be entirely defined by the mean flow velocity and a probabilistic description of the variability in flow velocity. The model assumes no dispersion, other than that which is implicit in the travel time variation.

Subject to these assumptions, the probability that a solute entering a flow system at a location ($x = 0$) will reach a reference receptor location ($x = L$) after a specified time at a mean flow velocity ($\bar{v}$) is:

$$P_L(t) = \int_0^{\bar{v}t} f_L(\bar{v}t') d(\bar{v}t') \quad (5)$$

where $f_L(\bar{v}t)$ is the probability density function (i.e., the probability that a solute will arrive between $\bar{v}t$ and $\bar{v}t+d(\bar{v}t)$ is $f_L(\bar{v}t) d(\bar{v}t)$) [Jury et al., 1982; Raats, 1978]. By representing the transfer as a probability density, all of the mechanisms that contribute to solute spreading are assumed to be functions of $\bar{v}t$. The system may be viewed as a bundle of tubes of different length within which fluid flows by piston flow.

By superposition, the concentration at the reference receptor location ($x = L$) for arbitrary variations in source concentrations is

$$\hat{C}_{(L,t)} = \int_0^{\infty} \hat{C}_{(0,(\bar{v}t - \bar{v}t'))} f_L(\bar{v}t') d(\bar{v}t') \quad (6)$$

where $\hat{C}_{(L,t)}$ = concentration at reference location (L) at time t

$\hat{C}_{(0,t)}$ = concentration at $x = 0$ at time t

The concentration at any receptor location is determined by assuming that the governing processes between $x = 0$ and $x = L$ are the same between all other receptor locations:

$$\hat{C}_{(x,t)} = \int_0^{\infty} \hat{C}_{(0,(\bar{v}t - \bar{v}t'))} \frac{L}{x} f_L(\bar{v}t'L/x) d(\bar{v}t') \quad (7)$$

where $C_{(x,t)}$ is the concentration at location x based on flow variance in the subsurface. Thus, a reference calibration function f_L allows statistical description of transport to all receptor locations.

Several measurements of flow velocity or hydraulic conductivity variations have been reported in the literature [Jury and Stolzy, 1982; White et al., 1986; Molz et al., 1986; Nielson et al., 1973; Van de Pol et al., 1977; Bigger and Neilson, 1976; Sharma et al., 1980; Smith et al., 1985; White et al., 1984]. These studies suggest that the calibration function, f_L is a lognormal distribution. Thus the travel time density function may be written as:

$$f_L(\bar{v}t) = \exp \left[\frac{-(\ln \bar{v}t - \mu)^2 / \sigma^2}{\sqrt{2\pi} \sigma \bar{v}t} \right] \quad (8)$$

where μ is the mean of the distribution of $\ln(\bar{v}t)$ and σ^2 is the corresponding variance. The standard deviation, σ , is a descriptor of the variance that is used in the model solutions. In a fully saturated flow system, μ will be approximately equal to $\ln(L)$. Several analytical solutions of (7) and (8) are available for various assumed input functions [Jury et al., 1982]. These include:

for a step function input -

$$\left. \begin{aligned} \hat{c}_{(x,0)} &= 0 & x > 0 \\ \hat{c}_{(0,t)} &= c_{in} & t > 0 \\ \hat{c}_{(0,t)} &= 0 & t < 0 \end{aligned} \right\} \text{Boundary Conditions}$$

$$\hat{c}_{(x,t)} = \frac{c_{in}}{2} \left\{ 1 + \operatorname{erf} \left[\frac{\ln(\bar{v}t L/x) - \ln(L)}{\sqrt{2} \sigma} \right] \right\} \quad (9)$$

$$= \frac{c_{in}}{2} \operatorname{erfc} \left[\frac{\ln(\bar{v}t /x)}{\sqrt{2} \sigma} \right]$$

Based on the previous studies, σ values should be in the range of 0.5 [Jury et al., 1986; Jury et al., 1982; Moltz et al., 1986]. A comparison of this approach, Equation (9), to the traditional advection dispersion approach, Equation (4), along with a straightforward transformation of the data set documented in Gelhar et al. [1985], provides an intuitive explanation of the observed dispersivity scale effect and provides a semiquantitative estimate of field values for σ .

Comparison of Alternate Models

Equation (9) defines the concentration at a receptor based on solute transport in a one-dimensional flow system in which the spreading of solute between the source and receptor is described by media/flow rate heterogeneity. Equation (4) defines the receptor concentration for this same system in which the spreading of solute is described by conventional advection and dispersion assumptions. By equating the underlying equations, the relationship between the descriptors (dispersion and variance) can be evaluated:

$$C_{(x,t)} = \hat{C}_{(x,t)}$$

$$\frac{C_{in}}{2} \operatorname{erfc} \left[\frac{x - \bar{v}t}{2 \sqrt{Dt}} \right] = \frac{C_{in}}{2} \operatorname{erfc} \left[\frac{\ln(\bar{v}t/x)}{\sqrt{2} \sigma} \right] \quad (10)$$

This is equivalent to assuming that the the various descriptors in each equation must correspond to an actual receptor concentration. If the resulting relationship between variance and dispersivity demonstrates the appropriate dispersivity scale effect (for constant values of σ), then one can conclude that the flow variance model is a more appropriate description of solute transport in subsurface systems. Equation 11 can be simplified to

$$\left[\frac{\bar{v}t - x}{\sqrt{2} \ln(\bar{v}t/x)} \right] \sigma^2 \frac{1}{t} = D \approx \alpha \bar{v} \approx \beta x \bar{v} \quad (11)$$

Table 1 presents the calculated D , α , and β values for various velocities, receptor distances, and times. Table 1 indicates that dispersivity is a function of receptor location for constant values of $\bar{v}$ and σ . Note also that the β value is constant for any given value of σ . Figure 2 shows the relationship between β and σ for locations in the vicinity of $\bar{v}t$. A dashed line is plotted on Figure 2 showing that $\sigma \approx 0.45$ for $\beta \approx 0.1$. Thus, at a σ value of 0.45, the often-used rule of thumb (dispersivity is 0.1 times the scale of the system) is met for all locations and all times.

Figures 2 and 3 show direct comparisons of the two modeling approaches for locations both closer than, and more distant than, the reference location. These figures show the front of the nonreactive solute passing the various locations as a function of time. The flow velocity ($\bar{v}$) in these figures is 10 (length/time), the σ is 0.45, and the reference receptor is located at 100 (length).

Since a constant dispersivity (or dispersion coefficient) is almost always used when modeling subsurface transport, Figures 3 and 4 indicate the type and magnitude of possible error when receptors are at a range of locations. When $x = \bar{v}t$ (i.e., the receptor is at the mean flow position), both models predict a relative concentration (C/C_0) of 0.5. Also, at $x = L$ (i.e., the receptor is at the reference location) the concentrations at all times are essentially identical. However, when $x \neq L$, the "tails" of the passing calculated plumes diverge significantly. If $x < L$, the advection dispersion approach predicts greater spreading of solute, while at $x > L$ the advection dispersion approach predicts less spreading of solute.

Using Equation 11, σ values based on the well documented data set in Gelhar et al. [1985] can be approximated. While the data demonstrate clearly that dispersivity is a function of receptor scale (Figures 1 and 5a), a plot of the approximate σ values (Figure 5b) has no scale dependence. Also, these σ values appear to be reasonably well described by a lognormal distribution (Figure 6). The approximated dispersivity scale effects based on the median ($\sigma = 0.4$), along with the 10% ($\sigma = 0.07$) and 90% ($\sigma = 1.05$) values, are plotted as lines on Figure 5a.

Finally, the data shown in Figure 6 may be used in a practical modeling approach to screen solute transport in simple systems (e.g., a small waste site). The resulting semiquantitative calculation may indicate if more robust modeling methods are warranted. Analytical solutions to the mathematical equations of contaminant transport have received considerable attention as tools for screening the hazard associated with active and inactive waste sites, or for assessing the suitability of a waste for disposal in a facility designed for nonhazardous materials. Most notably, the U.S. Environmental Protective Agency (EPA) has recommended a three-dimensional analytical model

for these purposes, the vertical and horizontal spread (VHS) model [EPA 1985a, EPA 1985b]. In general, analytical models are attractive screening tools because they are simple to apply (making the analysis of many constituents at many sites tractable) and they require a relatively small number of input parameters.

Often, investigators assert that assuming minimal solute spreading (typically by using a low D) will yield a conservative (somewhat high) calculated estimate of concentration/risk. While this is sometimes the case, the opposite assumption (one of maximum solute spreading) will often yield a higher calculated risk due to significant levels reaching more distant receptors (this is especially true when considering radionuclides or degrading organics). Table 2 lists several cases in which assumption of relatively more (or less) solute spreading will yield results that tend to be conservative in terms of risk.

With these precepts in mind, one can select a σ value, or a bounding range of σ values, to screen the risk associated with a particular location. For cases where analysis of minimum spreading is desired, one might choose a σ of 0.07. This is equivalent to selecting a variance descriptor that will yield concentrations that are higher than 90% of the values reported in the literature. Similarly, if maximum spreading is desired, a σ of 1.05 might be used (yielding a spreading of solute that is greater than that predicted by 90% of the variance values in the literature). A more detailed listing of the range of reasonable σ values, derived from Figure 6, is provided in Table 3.

DISCUSSION

Field and laboratory measurements of dispersion in a wide variety of media suggest that dispersivity is not a function of the aquifer material, but instead it depends on the scale of the system. Several explanations of this

phenomenon have been developed. For example, Domenico and Robbins [1984] demonstrate that using an $(n-1)$ -dimensional model to model an n -dimensional physical system will introduce a dispersivity scale effect. Davis [1986] demonstrates that the scale effect is a mathematical result of representing aquifer properties as mean values. The emerging consensus carries the concept of aquifer properties further; for example Smith and Schwartz [1980] conclude that macroscopic dispersion results from large-scale spatial variations in hydraulic conductivity and that the use of large dispersivity values and uniform flow fields is an inappropriate description of transport of solutes in subsurface geological systems.

As discussed in Konikow and Mercer [1988], numerous investigators have provided insights to place on firmer ground the largely intuitive concept of spatial heterogeneity impacting apparent dispersivity. Gelhar and others have developed robust numerical methods that describe dispersion in a randomly varying porous medium using the Fourier transform [Gelhar et al., 1979; Matheron and de Marsily, 1980; Gelhar and Axness, 1983; Dieulin et al., 1981]. Statistical and Green's functions [Dagan, 1984; Jury, 1982; Jury et al., 1986], Monte Carlo methods [Schwartz, 1977; Smith and Schwartz, 1980], and random walk methods [Dagan, 1982] have also been applied to porous media. Results from describing fractured rock flow systems as networks suggest that fractured media do not behave as an equivalent continuum [Long et al., 1982; Robinson, 1983; Schwartz et al., 1983; Endo et al., 1984; Long and Witherspoon, 1985; Rasmussen et al., 1985; Andersson and Dverstorp, 1987]. Some investigators have viewed subsurface media as a system of separate channels, tubes, or strata in which flow proceeds independently at different speeds [Nereticks, 1984; Molz, 1986]. A large number of the techniques described above, along with the model described herein, are predicated

on a statistical description of aquifers and the resulting effects on solute spreading.

Two promising techniques that have been documented in recent publications and presentations are the application of fractal geometry mathematics to dispersion and the incorporation of geological models of depositional environment into transport models. Fractals [Mandelbrot, 1983] have unique mathematical properties that correspond to a scale-dependent description of heterogeneity or fractures [Ross, 1986; Wheatcraft and Tyler, 1988]. Understanding the depositional environment of a subsurface flow system is a key element in describing the site-specific statistical nature of the heterogeneities and potential for isolated flow paths in critical areas [Anderson, 1987; Fogg, 1986]. Significant developments in these emerging fields are expected.

The various investigations related to subsurface flow and transport systems have improved our understanding of these systems. None of the studies to date, however, has produced a definitive direct explanation of the scale dependence of apparent dispersivity. The direct comparison of an analytical solution based on classical advection dispersion assumptions to one based on a statistical flow variance assumption, combined with the active and preceding research, supports the concept that large-scale heterogeneities play a controlling role in solute spreading during subsurface transport. Further development of the various fractal geometry, geological, and statistical methods will ultimately provide usable numerical and analytical models that are descriptive and defensible.

CONCLUSION

A straightforward description of solute transport based on the heterogeneity of subsurface systems appears to eliminate the anomalous scale dependence seen in applications of the classical advection dispersion

approach. An alternate analytical solution, based on description of solute transport as nondispersive flow in a lognormal flow distribution field, yielded consistent results with variance descriptor values, σ , centering near 0.4. The concentrations calculated using the two models were identical at a reference receptor location, but diverged significantly at other receptor locations. The literature provides sufficient data to derive σ values to screen sites where detailed-measured transport parameter data are limited. The resulting screening may be sufficient to reduce the need for more robust field work and numerical models at every site. Based on the results, additional development of stochastic and flow variance approaches, to be incorporated into numerical and analytical models is recommended. Such development will improve the ability of modelers to adequately simulate the observable natural heterogeneity of subsurface geological materials, the framework of hydrologic transport systems.

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NOTATION

α	the longitudinal dispersivity, length
β	the dispersivity distance multiplier
$C(x,t)$	the concentration at location x at time t based on advection dispersion assumptions, mass/length ³
$C(x,t)$	the concentration at location x at time t based on flow variance assumptions, mass/length ³
D	the longitudinal dispersion coefficient, length ² /time
D^*	the molecular diffusion coefficient in the medium, length ² /time
L	reference receptor location, length
t	time
$\bar{v}$	average flow velocity, length/time
x	flow distance, length

TABLE 1

Dispersivity as a Function of $\bar{v}$, x and t

Input Data			Results								
v	x	t	$\sigma = 0.1$			$\sigma = 0.25$			$\sigma = 0.45$		
			D	α	β	D	α	β	D	A	β
50	10	0.19	2.5	0.05	0.005	15.6	0.3	0.03	50.7	1.01	0.101
50	100	1.9	25	0.5	0.005	156	3.0	0.03	507	10.1	0.101
50	1000	19	250	5.0	0.005	1560	30.0	0.03	5070	101	0.101
50	10000	190	2500	50	0.005	15600	300	0.03	50700	1010	0.101
5	100	19							50.7	10.1	0.101
10	100	9							101	10.1	0.101
50	100	1.9							507	10.1	0.101
100	100	0.9							1013	10.1	0.101

TABLE 2

Considerations Related to Selection of σ or α in Risk Screening Applications*

Conditions that tend to result in higher calculated risk for low assumed spreading (low σ or low α)

- Initial concentration $\leq$ health based standard
- Receptor location $\leq$ flow time $\times$ flow velocity adjusted for retardation
- Nondecaying chemical (or chemical that is growing in from parent or from reaction)
- Short "tail" of plume following source reduction is acceptable

Conditions that tend to result in higher calculated risk for high assumed spreading (high σ or α)

- Initial concentration $>$ or $\gg$ health based standard
- Receptor location $>$ flow time multiplied by the flow velocity adjusted for retardation
- Decaying constituent (radioactive, biological or chemical)
- Long "tail" of plume following source reduction may be important

* One or more conditions from a category may determine if a high or low assumption of solute spreading is conservative. Most real problems will have a combination of characteristics that should be used to select bounding coefficients.

TABLE 3

Range of σ Values Calculated from Existing Field-Measured Dispersivity

<u>Percentile</u>	<u>σ (Max. Spread)</u>	<u>σ (Min. Spread)</u>
2	0.03	2.40
5	0.04	1.60
10	0.07	1.05
15	0.10	0.85
20	0.15	0.70
25	0.17	0.64
30	0.20	0.55
40	0.30	0.50
50	0.40	0.40
60	0.50	0.30
70	0.55	0.20
75	0.64	0.17
80	0.70	0.15
85	0.85	0.10
90	1.05	0.07
95	1.60	0.04
98	2.40	0.03

Figure 1. Measured longitudinal dispersivity as a function of scale for saturated systems (from Gelhar et al., 1985).

Figure 2. Calculated dispersivity distance multiplier values (β) as a function of assumed variance descriptor (σ).

Figure 3. Comparison of alternate models for receptor locations closer than the reference location.

Figure 4. Comparison of alternate models for receptor locations more distant than the reference location.

Figure 5. Graphs of: a) longitudinal dispersivity (α) and b. variance descriptor (σ) as a function of scale. The lines on the dispersivity graph are the calculated dispersivity scale effect based on a range of σ values.

Figure 6. Probability plot showing the log-normal distribution of variance descriptor values (σ) calculated from the literature.