A Comment on Ahmadi and Ma (1990).

L.M. Liljegren
 Pacific Northwest National Laboratory
 Richland, Wa 99352

RECEIVED**FFR 2 4 1997****OSTI****Introduction**

A model to predict flow of a particulate mixture is described in Ahmadi and Ma (1990) and Ahmadi and Ma (1990); the full model, with closures, appears to result in predictions of unphysical behavior. Specifically, the model with its closure appears to result in spurious creation of energy. In this comment, the predictions of unphysical behavior will first be illustrated; the illustration will be followed by a some discussion of one closure that appears to cause the unphysical behavior in the case discussed.

Analysis of example problem

The unphysical behavior will be illustrated by examining the simplest possible behavior for of an isothermal, fully saturated two-phase mixture with incompressible fluid and particle constituents; that is, examining the special case described on page 342 by A&M (1990). The final, steady state, behavior of the case examined is neither turbulent, nor strictly speaking a flow. The case is selected because it illustrates the deficiency of the A&M model more clearly than an actual flow. Further discussion to justify use of this example to evaluate a proposed turbulence model is deferred.

Suppose we pour a mixture of particles and fluid into a very large vessel, containing a mixture of solid particles and a fluid phase with identical intrinsic densities (i.e. $\rho^f = \rho^i$; the superscript i indicates an intrinsic physical property, the addition of a superscript f indicates the fluid phase, the absence of any superscript other than i indicates the particle phase). We now induce turbulence by stirring the mixture vigorously. Subsequently, we allow the mixture to stand until it comes to steady state. One might expect that after a large period of time, the entire tank contents would be statistically homogeneous and motionless. In the absence of Brownian motion, the mean and fluctuating velocities of both phases would approach zero.

In the discussion that follows, it is shown that A&M (1990) model predicts that the kinetic energy of the particle and fluid phases in this case would be non-zero at steady state. The reader is referred to A&M for detailed discussion of their notations which are used here. Equation numbers from A&M are provided and denoted by their equation number followed by a I; those from M&A are followed by a II.

The following analysis is performed to illustrate that the A&M model predicts non-zero fluctuating kinetic energy in the example steady-state system described previously. In the selected coordinate system gravity acts downward in the 'z' direction. If the mixture is statistically homogeneous and at steady state, the "z" momentum equation for fluid and particle phases are respectively:

$$0 = -\rho^f v_z^f g_z - v_z^f \frac{\partial p^f}{\partial z} - D_o (\tilde{v}_z^f - \tilde{v}_z) \quad [4II] \quad [1]$$

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$$0 = \rho^i v g_z - v \frac{\partial p^f}{\partial z} + D_o(\tilde{v}_z^f - \tilde{v}_z) \quad [3II] \quad [2]$$

where p^f is described on page 332 of A&M as the mean pressure in the fluid phase. The superscript 'i' has been added to emphasize that the densities are intrinsic physical properties of the phases. As in Ma and Ahmadi, the intrinsic fluid density, ρ^f and intrinsic particle density ρ^i are constant.

Summing the two equations, it can be shown that A&M's model correctly predicts the mean pressure gradient varies hydrostatically in a motionless mixture containing neutrally buoyant particles.

$$\frac{\partial p^f}{\partial z} = (\rho^f v^f + \rho^i v) g_z = \rho^m g_z \quad [3]$$

The mixture density is defined $(\rho^f v^f + \rho^i v) = \rho^m$

M&A provide the transport equation for fluctuating kinetic energy for the special case of two-phase flow with incompressible fluid and particles on page 342. For the case under consideration, all spatial derivatives except pressure vanish and all temporal derivatives also vanish. So, the form of the equation applying to the fluid phase becomes

$$0 = \left[\frac{\mu^f}{\sigma^f \rho^f v^f k^f} \frac{\partial v^f p^f}{\partial z} \right] \frac{\partial v^f p^f}{\partial z} - \rho^f v^f \epsilon^f + 2D_o(k - ck^f) \quad [6II] \quad [4]$$

Substituting in the relation for the dissipation in the fluid phase

$$0 = \left[\frac{\mu^f}{\sigma^f \rho^f v^f k^f} \frac{\partial v^f p^f}{\partial z} \right] \frac{\partial v^f p^f}{\partial z} - \rho^f v^f \left[\frac{C^D}{\Lambda^f} (k^f)^{3/2} \right] + 2D_o(k - ck^f) \quad [10II, 11III] \quad [5]$$

where Λ^f represents a length scale of the fluid turbulence, and so is positive.

A&M (1990) describe the physical significance of the first term in [5] on page 329 as follows: "There is also a secondary source term related to the product of the density-velocity correlation and mean pressure gradient field". In the more general transport equation for fluctuating kinetic energy provided in A&M (see [35I]), the term appeared as

$$\overline{v_i^{\alpha''} p^{\alpha}}_{,i} = \overline{v_i^{\alpha''}} \frac{\partial v p^f}{\partial z_i} = 0 \quad [6]$$

The closure for $\overline{v_i^{\alpha''}}$ appears inside brackets in [4] and [5]. Discussion regarding validity of the closure is deferred.

The second term in [5] represents dissipation of fluctuating kinetic energy in the fluid phase; the third term on the RHS of [5] represents transfer of fluctuating kinetic energy from the particle phase.

Likewise, the particle kinetic energy equation, with the appropriate substitution for the dissipation rate becomes

$$0 = \left[\frac{\mu^T}{\sigma^p \rho^i v k} \frac{\partial v}{\partial z} \frac{p^f}{k} \right] \frac{\partial v}{\partial z} \frac{p^f}{k} - \rho^i v a k^{3/2} - 2D_o(k - ck^f) \quad [5II \& 10II] \quad [7]$$

where a is an inverse length scale:

Summing the transport equations for fluctuating kinetic energy, [5] and [7], and substituting in the relation for the pressure gradient, [3], the transport equation for fluctuating kinetic energy becomes

$$\left[\frac{\mu^{fT} v^f}{\sigma^{fp} \rho^{fi} k^f} + \frac{\mu^T v}{\sigma^p \rho^i k} \right] (\rho_m^2 g_z^2) = \left[\rho^{fi} v^f \frac{C^{fD}}{\Lambda^f} (k^f)^{3/2} + \rho^i v a k^{3/2} \right] \quad [8]$$

dissipation

where

$$\mu^T = C^\mu \rho^i v d k^{1/2}; \quad \mu^{fT} = \frac{C^{f\mu} \rho^{fi} v^f k^{f2}}{\epsilon^f} = \frac{C^{f\mu} \rho^{fi} v^f k^{f2}}{a^f k^{f3/2}} = \frac{C^{f\mu} \rho^{fi} v^f k^{f1/2}}{a^f} \quad [8II \& 10II]$$

Substituting the definitions provided for μ^T , μ^{fT} results in

$$\left[\frac{C^{f\mu}}{C^{fD} \sigma^{fp}} \frac{\Lambda^f v^{f2}}{k^{f1/2}} + \frac{C^\mu}{\sigma^p} \frac{d v^2}{k^{1/2}} \right] (\rho_m^2 g_z^2) = \left[\rho^{fi} v^f \frac{C^{fD}}{\Lambda^f} (k^f)^{3/2} + \rho^i v a k^{3/2} \right] \quad [9]$$

dissipation

Equation [9] is the result sought for the purpose of discussing A&M's model and demonstrating a confusing feature of the model.

Details regarding coefficients $C^{f\mu}$ and C^μ , Prandtl numbers σ^{fp} , and σ^p are provided in A&M. Presumably the magnitudes of these adjustable parameters are specific to turbulent flows. However, the characteristics important to this discussion are that the parameters are strictly positive, and dimensionless.

As stated at the beginning of this note, one would expect the steady state behavior of a large vat containing neutrally buoyant particles and fluid to display zero mean velocity and zero fluctuating kinetic energy. In principle, the specific magnitude of k and k^f predicted by the model could be obtained by solving [7], and [9] and an additional equation that would specify either the length scale Λ^f or the turbulence dissipation rate ϵ^f in a particulate flow (n.b. equations [10II & 11 II] currently relate the three quantities Λ^f , ϵ^f and k^f . However, further restriction is required.)

Even without solving the equations it is possible to show that the predicted result is not $k = k_f = 0$ as might be expected for the problem posed. All coefficients in [9] are positive. Consequently, the LHS of [9], is a source of fluctuating kinetic energy; the right hand side is the dissipation rate. Because the source is non zero, either the fluid fluctuating kinetic energy or the particle fluctuating kinetic energy, or both, are not zero.

Root of the problem

Having identified that the model appears to predict unphysical behavior, the obvious question is why? One possible reason appears to be the choice of the closure for $\overline{v_i^{\alpha''}}$ which appears in [6]. The argument that follows suggests that the closure is incorrect when the mixture consists of phases that are individually incompressible.

Recall that the production term arose from [6] and the closure for the ensemble average of the velocity fluctuation $\overline{v_i^{\alpha''}}$. It is useful to discuss $\overline{v_i^{\alpha''}}$ and the proposed closure. The definition of $\overline{v_i^{\alpha''}}$ and the suggested closures provided by A&M for any particle phase α and a fluid phase f are respectively:

$$\overline{v_i^{\alpha''}} \equiv \frac{\overline{\rho^\alpha v_i^{\alpha'}}}{\rho^\alpha} = -\frac{\mu^{\alpha T}}{\sigma^{\alpha p} \rho^\alpha k^\alpha} \frac{\partial v^\alpha p^\alpha}{\partial x_i} [8I, 75I \& 65I] \quad [10]$$

$$\overline{v_i^{f''}} \equiv \frac{\overline{\rho^f v_i^{f'}}}{\rho^f} = -\frac{\mu^{f T}}{\sigma^{fp} \rho^f k^f} \frac{\partial p^f}{\partial x_i} = -\frac{\mu^{f T}}{\sigma^{fp} \rho^f k^f} \frac{\partial v^f p^f}{\partial x_i} [8I, 76I, 65I] \quad [11]$$

Where μ 's are turbulent viscosities and appear to have units of viscosity, the σ 's are parameters corresponding to Prandtl numbers and are presumably dimensionless and, according to [65I] on p 332, p^f is mean fluid pressure.

At least four comments can be made regarding the closure. First, it is dimensionally inconsistent. Second, the justification provided on page 334 appears to be inadequate. Third, the consequence of using the closure exactly as described in A&M is that the transport equation for fluctuating kinetic energy violates the first law of thermodynamics.

Fourth and finally, based on its definition, the correct closure for $\overline{v_i^{\alpha''}}$ appears to be zero for any phase with constant intrinsic density.

Interestingly, it appears the term $\overline{v_i^{\alpha''}}$ was have been set to zero in Cao and Ahmadi (1995); the discussion indicates that the model used there is taken from Ahmadi and Ma (1990). No discussion appears to indicate or justify the omission of the source term identified in A&M. This oversight is notable because the term omitted not only appeared in A&M; its physical significance was also discussed.

The discussion of each point follows.

Point 1: Dimensional inconsistency.

Assuming that the two turbulent viscosities have units of absolute viscosity, as they appear to in the definition provided in equation [74I] of A&M, then dimensionally the statement is

$$\frac{\text{length}}{\text{time}} = \frac{\text{mass}}{\text{length}^3} \frac{\text{length}}{\text{time}} \quad [12]$$

This may be a typo. If so it is persistent and appears both in [74I] of A&M and [5II] of M&A. It is for this reason that units for mass and length on the left hand and right hand side my [9] do not match.

Point 2. Little justification is provided for closure of $\overline{v_i^{\alpha''}}$.

The only justification for closures [75I] and [76I] are that they are "generalizations of that proposed by Ahmadi (1989) for single phase compressible turbulent flows".

This justification might be sufficient if the density-velocity covariance had identical meaning two-phase flows and in compressible single phase flows. During incompressible flow of a two-phase mixture, the superficial density of any phase, νp^i , varies only because its volume fraction ν varies. During compressible flow of a gas, the intrinsic density itself, ρ^i , varies; for gases, variation of the intrinsic density is governed by the ideal gas law. Volume fraction is not governed by the ideal gas law. Since the volume fraction and intrinsic density are distinct quantities governed by different physical laws, some modification is required to ensure that closures designed for compressible flows do not lead to serious anomalies when applied to flow of particulate mixtures with phases that are individually incompressible.

Point 3: Violation of the first law of thermodynamics.

The behavior predicted by the model violates the first law of thermodynamics, at least in the system described. In this particular system, fluctuating kinetic energy is created, and eventually dissipated to heat. The origin of this energy is entirely unclear; it seems to be self generating.

My choice of example is admittedly neither turbulent or flowing. However, as far as I can determine, the spurious energy production term would continue to exist even in a turbulent flow and is not peculiar to the case selected for analysis. Consequently, the model appears to predict that some energy is created; the only condition for this occurrence appears to be that body forces act on the mixture.

Point 4: Possible appropriate closure for $\overline{v_i^{\alpha''}}$ for mixtures with incompressible phases.

The correct closure for required for $\overline{v_i^{\alpha''}}$ in a mixture containing two phases with constant densities can be shown to be $\overline{v_i^{\alpha''}} = 0$. This differs from that suggested by A&M, which was reiterated as [10].

Unfortunately, the discussion is complicated due to an omission from A&M. Although the definition of the "mass-weighted average" is provided in terms of ensemble averages [8I], the exact definition of the "ensemble average" is not provided explicitly in their paper. Those familiar with multiphase flow literature are aware of two possible averages: that is the superficial and the in situ averages. Ensemble averages of both quantities exist. Unfortunately, the correct form of the closure for $\overline{v_i^{\alpha''}}$ differs depending on whether the ensemble average used by A&M is the ensemble average superficial velocity or the ensemble average in situ velocity.

The probable form of the closure will be demonstrated as follows. First, the superficial and in situ velocities will be defined; the superscripts *i.s.* and *sup* will be used to indicate

"in situ" and "superficial" averages. Then it will be shown that when the intrinsic density of a phase is constant, its mass weighted average velocity $\bar{v}_i^\alpha$ is identical to its ensemble average in situ velocity, $\bar{v}_i^{\alpha, i.s.}$. It will be shown that the appropriate closure for $\bar{v}_i^{\alpha, i.s.}$ is zero for whenever the density of phase α is constant.

The superficial velocity of phase α , $\bar{V}_i^{\alpha, sup}$, is the ensemble average of its volumetric flux, as such, it is the product of its volume fraction v^α and the in situ ensemble average of its velocity $\bar{v}_i^{\alpha, i.s.}$. The two quantities are related as $\bar{V}_i^{\alpha, sup} = v^\alpha \bar{v}_i^{\alpha, i.s.} = \langle I^\alpha v_i^\alpha \rangle$ where the volume fraction of any phase α is the ensemble average of an indicator function I^α which takes on the value in the presence of phase α and 0 otherwise; so $\langle I^\alpha \rangle = v^\alpha$. The outer brackets to denote the ensemble average over the set of all realizations of a particular flow under consideration. For point particles, the ensemble average in situ velocity, $\bar{v}_i^{\alpha, i.s.}$, refers to the average of the velocities of the individual particles; the superficial velocity is the volumetric flux.

When the intrinsic density of the two phases are constant, the ensemble average in situ velocity $\bar{v}_i^{\alpha, i.s.}$ and the mass weighted velocity $\bar{v}_i^\alpha$ are identical.

$$\bar{v}_i^{\alpha, i.s.} \equiv \frac{\langle I^\alpha v_i^\alpha \rangle}{\langle I^\alpha \rangle} = \frac{\rho^{\alpha i} \langle I^\alpha v_i^\alpha \rangle}{\rho^{\alpha i} \langle I^\alpha \rangle} = \frac{\langle \rho^{\alpha i} I^\alpha v_i^\alpha \rangle}{\langle \rho^{\alpha i} I^\alpha \rangle} = \frac{\langle \rho^\alpha v_i^\alpha \rangle}{\langle \rho^\alpha \rangle} \equiv \bar{v}_i^\alpha \quad [8I]$$

It follows that the two velocity fluctuations defined in A&M are equal. This can be illustrated by applying the definition of each phase and using [13].

$$v_i^{\alpha'} \equiv \left(v_i^\alpha - \bar{v}_i^{\alpha, i.s.} \right) = \left(v_i^\alpha - \bar{v}_i^\alpha \right) \equiv v_i^{\alpha''} \quad [14]$$

The consequence is that if the relation [7I] (i.e. $\bar{v}_i^{\alpha'} = 0$) holds then $\bar{v}_i^{\alpha''} = 0$ must also be true.

It follows that when the intrinsic density of the two phases are each constant, the following closure applies to both the particle and fluid phases.

$$\bar{v}_i^{\alpha''} \bar{p}^{\alpha'} = \bar{v}_i^{\alpha''} \frac{\partial v^f}{\partial z_i} = 0 \quad [15]$$

Before closing, note that [13] and [14] do not hold when the density of phase α is variable; it is well known that the two averaged velocities are not equal in compressible single phase flows. The consequence is that if the particle phase has constant density while the fluid phase is compressible, then [13], [14] and [15] do hold for the particle phase, and do not hold for the fluid phase. Since A&M model appears to be motivated specifically to study rapid granular flows, it appear that [10] is inappropriate for the extremely rigid solid phase regardless of Mach number. Although [13]-[15] do not apply to a compressible fluid phase at high Mach numbers, it appears that some modification is required to allow permit application in rapid granular flows; this modification should separately consider variations in the volume fraction and the intrinsic density.

Conclusion

It would appear that unphysical behavior, which violates the first law of thermodynamics, is predicted by the model described in Ahmadi and Ma (1990); specifically according to the model, energy is created as a result of interactions between hydrostatic pressure gradients. This unphysical predictions are demonstrated in a situation with no flow.

It appears that unrealistic predictions result from applying a closure for the density-velocity covariance; this closure was taken directly from an analysis applying to compressible flow of a single phase gas. It appears that, for the special case of a mixture with incompressible phases (which is the case examined in A&M 1990), the correct closure for the term is zero. If so, any other choice of any non-zero closure would lead to creation or destruction of energy in some circumstance, including turbulent flow of a particulate mixture.

References:

- Ahmadi, G. and Ma, D. 1990 "A Thermodynamic Formulation for Dispersed Multiphase Turbulent Flows - I", *Int. J. Multiphase Flow* **16** pp 323-341
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- Cao, J. and Ahmadi, G. 1995 "Gas-Particle Two-Phase Turbulent Flow in a Vertical Duct", *Int. J. Multiphase Flow* **21** , 1203-1228.

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